CFD-based Aerodynamic Design Optimisation

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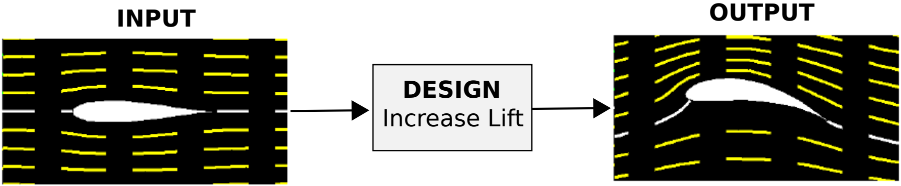
Overview

- Brief History of Aerodynamic Design
  - What is aerodynamic design?
  - Pre-1930's
  - NACA series airfoils
  - CFD-based design
- CFD-based design optimisation
- State-of-the-art: Adjoint-based optimisation
- Example: hypersonic inlet design
- Questions

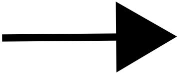
What is aerodynamic design?

- Series of steps that engineers follow to find a solution to a problem.
- The process always begins with an explicit goal (e.g. increase lift, decrease drag).
- Often involves design under constraint: cost, structural limits, etc.



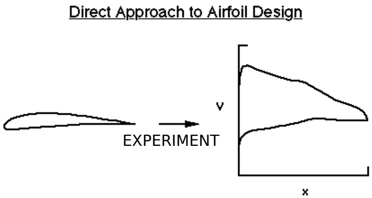
Pre-1930s

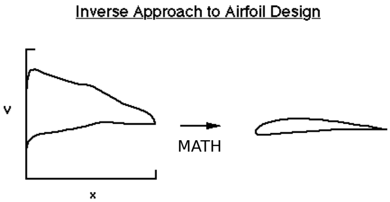
- Engineers relied on experiments.
- Wind tunnel testing $\longrightarrow$ measure forces/moments/flow visualisation.
- Intuition based design.
- Expensive and slow design process.



NACA Series

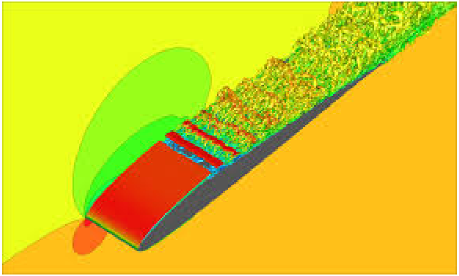
- Airfoil shapes were mathematically derived.
- Two approaches:
  1. Geometric mathematical formulation.
  2. Inverse design: specify desired pressure distribution and employ advanced mathematics (e.g. potential flow) to derive the required geometrical shape.
- Good for basic wing geometries
- What about multi-element wings?
  or complex flow? (e.g. turbulent flow, large separation)





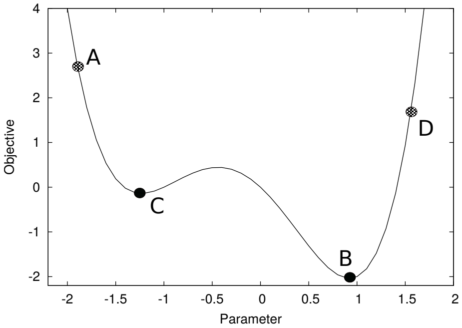
CFD-based design

- "Virtual" wind tunnel.
- Fast and inexpensive to run several hundred simulations.
- Can model turbulence, large separation, etc.
- Easy to visualise entire flowfield in detail.
- Back to intuition based design.



Aerodynamic Design Optimisation

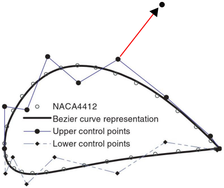
- Is the pursuit of a design which is optimal in that it **minimises** a certain **objective functional** while satisfying given **constraints**.
- Typical objective functions for aerodynamic design: lift and drag.
- Employ numerical algorithm to find optimum:
  1. Local methods (e.g. gradient-based search).
  2. Global methods (e.g. bio-inspired, evolutionary algorithms).



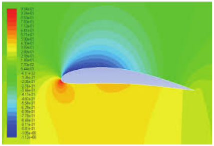
CFD-based Design Optimisation

- In gradient-based optimisation we need:
  1. Objective function evaluation $\longrightarrow$ CFD solution.
  2. Objective function gradients $\longrightarrow$ Finite-differences.

$$\frac{\partial J}{\partial D} \approx \frac{J(D+h) - J(D)}{h}$$



Flow Solver $\longrightarrow$



Post-process $\longrightarrow$ Drag

- 15 Bézier points $=$ 16 flow solutions for one design iteration.
- Computationally expensive!

Adjoint-based Design Optimisation

- Type of CFD-based optimisation.
- Most efficient means of calculating objective function gradients.
- All gradients evaluated via 1 **flow** solution & 1 **adjoint** solution, e.g. computational expense $<<$ 16 flows solutions required for finite-differences.
- Derived via a mathematical trick using Lagrange multipliers.
- Adjoint system of equations:

$$\left(\frac{\partial \mathbf{R}}{\partial \mathbf{Q}}\right)^T \lambda = -\left(\frac{\partial J}{\partial \mathbf{Q}}\right)^T$$

- Objective function gradient:

$$\frac{dL}{d\mathbf{D}} = \frac{\partial J}{\partial \mathbf{D}} + \lambda^T \frac{\partial \mathbf{R}}{\partial \mathbf{D}}$$

Adjoint-based Design Optimisation

- objective function:

$$J(\mathbf{Q}(\mathbf{D}), \mathbf{D})$$

- derivative of objective function with respect to design parameters:

$$\frac{dJ}{d\mathbf{D}} = \frac{\partial J}{\partial \mathbf{D}} + \frac{\partial \mathbf{J}}{\partial \mathbf{Q}} \frac{\partial \mathbf{Q}}{\partial \mathbf{D}}.$$

- The $\partial \mathbf{Q} / \partial \mathbf{D}$ term requires finite-difference approximations

$$\frac{\partial \mathbf{Q}}{\partial \mathbf{D}} \approx \frac{Q(D+h) - Q(D)}{h}$$

- how can we remove it? $\longrightarrow$ adjoint method!

Adjoint-based Design Optimisation

- reformulate objective function as a Lagrange function:

$$L(\mathbf{Q}(\mathbf{D}), \mathbf{D}, \lambda) = J(\mathbf{Q}(\mathbf{D}), \mathbf{D}) + \lambda^T \mathbf{R}(\mathbf{Q}(\mathbf{D}), \mathbf{D}),$$

- where the Navier-Stokes (or Euler) equations are (e.g. conservative!):

$$\mathbf{R}(\mathbf{Q}(\mathbf{D}), \mathbf{D}) = 0,$$

- derivative of Lagrange function with respect to design parameters:

$$\frac{dL}{d\mathbf{D}} = \frac{\partial J}{\partial \mathbf{D}} + \left(\frac{\partial J}{\partial \mathbf{Q}} + \lambda^T \frac{\partial \mathbf{R}}{\partial \mathbf{Q}}\right)\frac{\partial \mathbf{Q}}{\partial \mathbf{D}} + \lambda^T \frac{\partial \mathbf{R}}{\partial \mathbf{D}}.$$

- set bracketed term to 0 and rearrange:

$$\left(\frac{\partial J}{\partial \mathbf{Q}} + \lambda^T \frac{\partial \mathbf{R}}{\partial \mathbf{Q}}\right) = 0 \quad \longrightarrow \quad \left(\frac{\partial \mathbf{R}}{\partial \mathbf{Q}}\right)^T \lambda = -\left(\frac{\partial J}{\partial \mathbf{Q}}\right)^T.$$

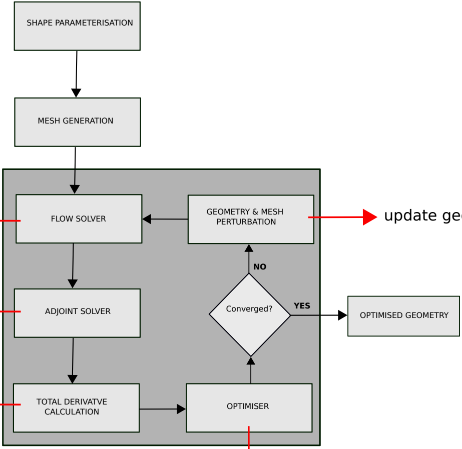
Adjoint-based Design Optimisation

- How do we compute:

$$\frac{\partial J}{\partial \mathbf{D}}, \; \frac{\partial J}{\partial \mathbf{Q}}, \; \frac{\partial \mathbf{R}}{\partial \mathbf{Q}}, \; \frac{\partial \mathbf{R}}{\partial \mathbf{D}}?$$

- We need to:
  1. run flow solver to reach steady-state.
  2. use flow solution + linearisation method to calculate.
- Popular linearisation methods:
  - Hand differentiation of flow solver code.
  - Finite differences.
  - Complex-step differentiation.
  - Algorithmic differentiation.
- Finite differences are computationally cheaper now since **R** is the residual **NOT** a converged flow solution (e.g. **Q**).

Eilmer CFD-based Optimisation



Eilmer, OpenFOAM, etc.

update geometry & mesh

$$\left(\frac{\partial \mathbf{R}}{\partial \mathbf{Q}}\right)^T \lambda = -\left(\frac{\partial J}{\partial \mathbf{Q}}\right)^T$$

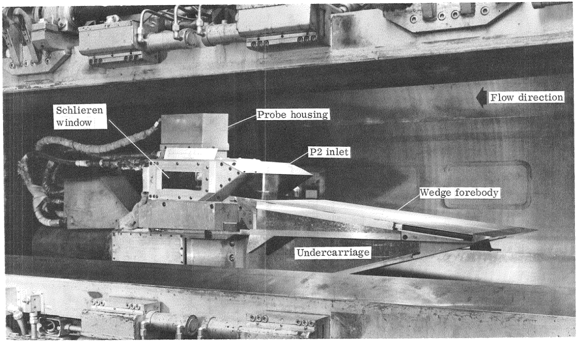
$$\frac{dL}{d\mathbf{D}} = \frac{\partial J}{\partial \mathbf{D}} + \lambda^T \frac{\partial \mathbf{R}}{\partial \mathbf{D}}$$

e.g. gradient descent

$$D_j^{n+1} = D_j^n - \alpha \frac{dL}{dD_j}$$

Hypersonic inlet optimisation - history

- P2 hypersonic inlet.
  - Designed for generic hypersonic vehicle at NASA in 70's.
  - Used low-fidelity inviscid & viscous methods in design procedure.
  - Design objective: **compression ratio of 2** across the cowl shock & approximately **uniform static pressure distribution at the throat**.



P2 hypersonic inlet installed in NASA wind tunnel (flow right to left)

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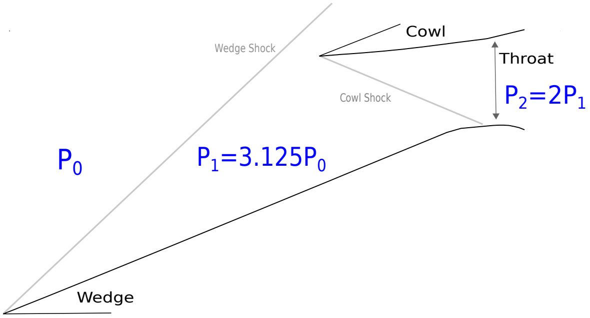
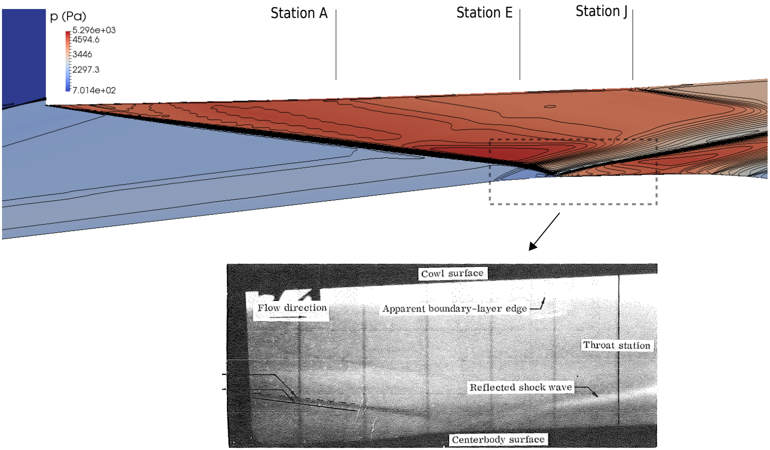


P2 hypersonic inlet schematic (flow right to left).

Hypersonic inlet optimisation - baseline flowfield

- Reflected cowl shock $\longrightarrow$ non-uniform pressure across throat.

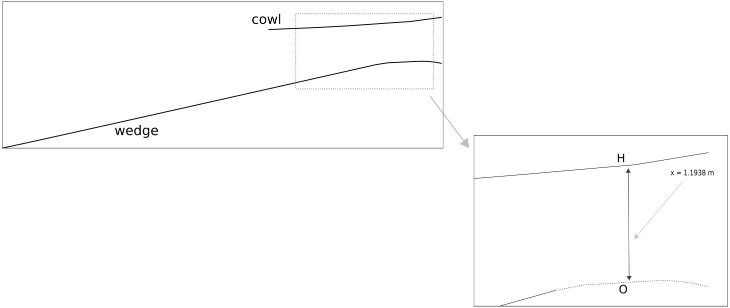


Baseline P2 hypersonic inlet RANS flow solution with experimental insert.

Hypersonic inlet optimisation - objective function

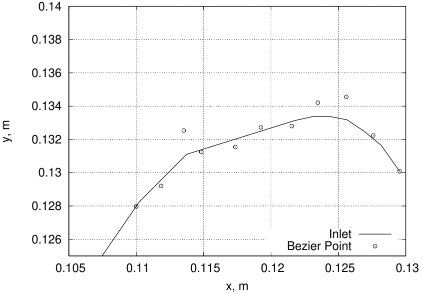
$$J = \frac{\int_0^H (p(x,y) - p^*)^2 dA}{A_{\text{tot}}}$$

$$p^* = r_p p_\infty \longrightarrow \quad r_p = 6.25 \ \& \ p_\infty = 701.4 \ Pa$$

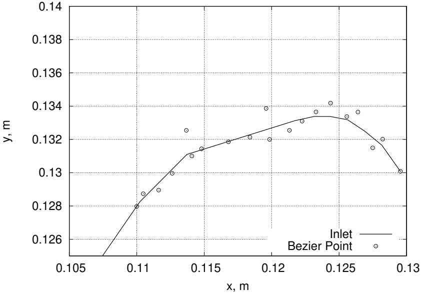


Objective function integral path.

Hypersonic inlet optimisation - parameterisation

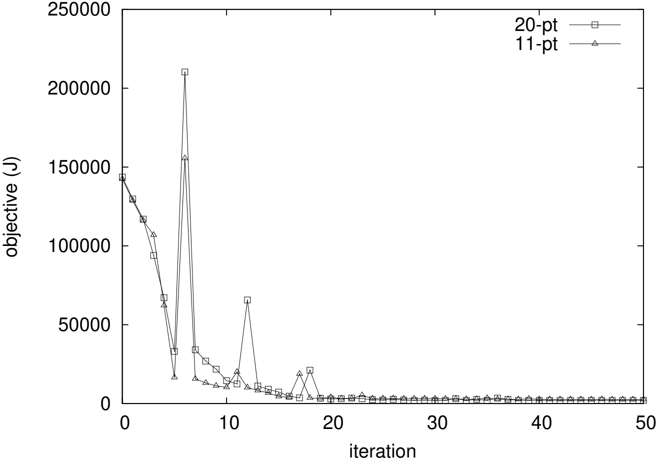


11-points Bézier curve representation of baseline P2 inlet.



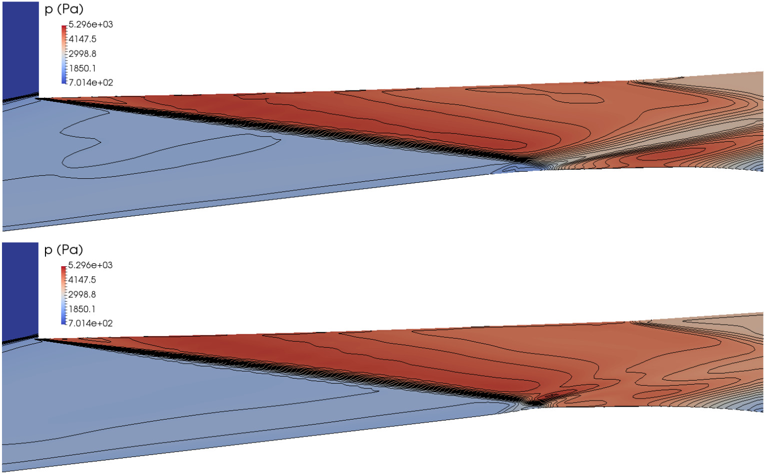
20-points Bézier curve representation of baseline P2 inlet.

Hypersonic inlet optimisation - objective function history



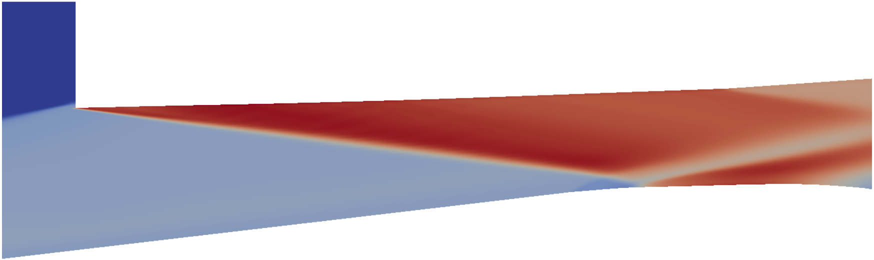
Objective function history plot.

Hypersonic inlet optimisation - result



P2 inlet; baseline (above), 20-point optimised (below).

Hypersonic Inlet Optimisation - video



Questions?