Sec. 1: Regular diagrams, alternating knots

Regular projection

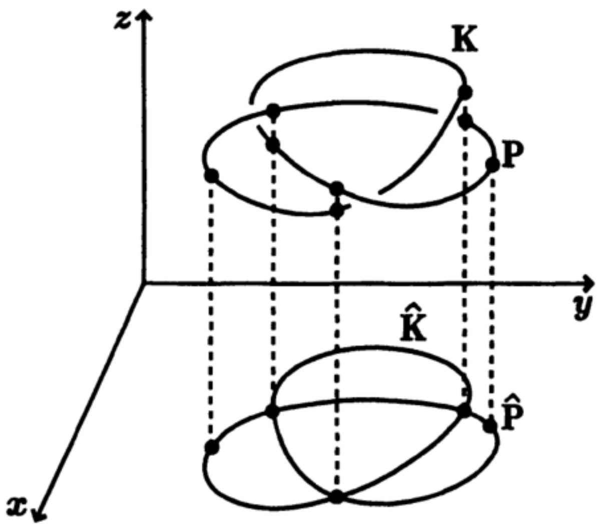
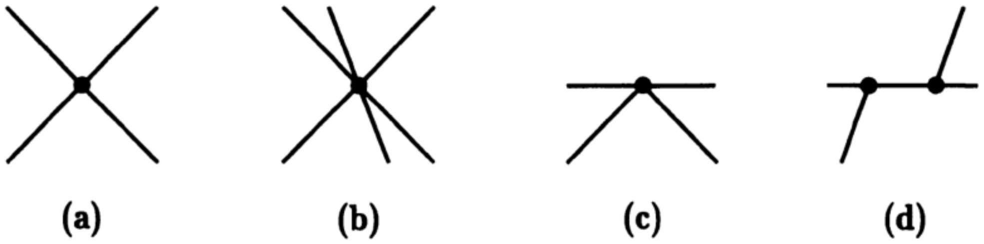
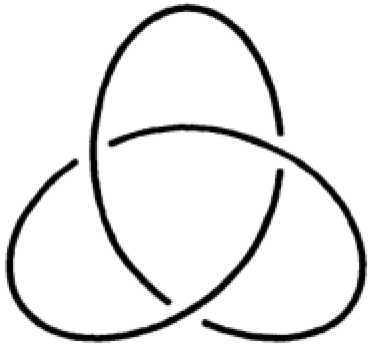


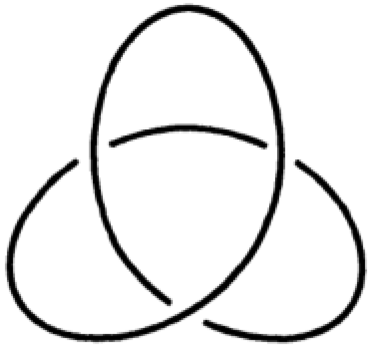
Figure 2.1.1

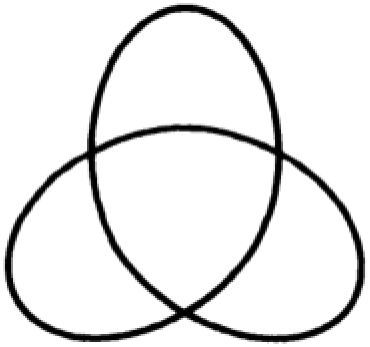


YES NO Figure 2.1.2 NO NO

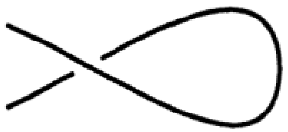
Regular diagram

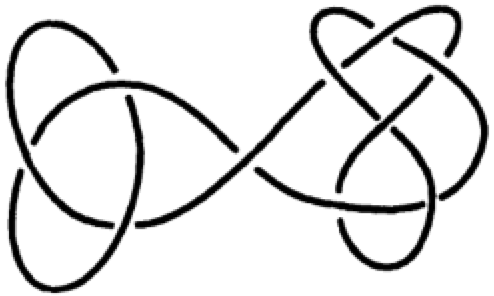




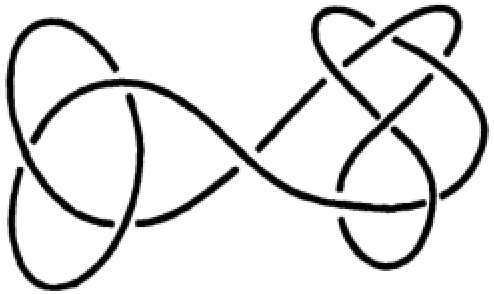


Reduced diagram

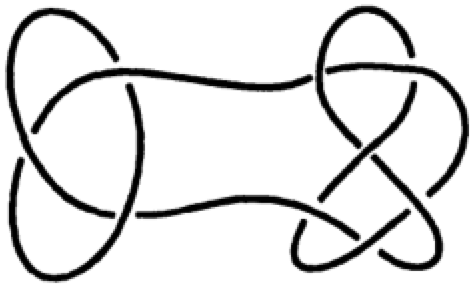




NO



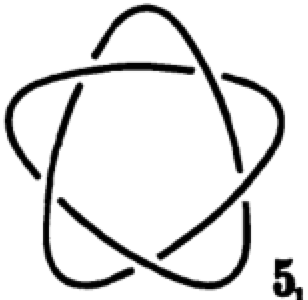
NO



YES

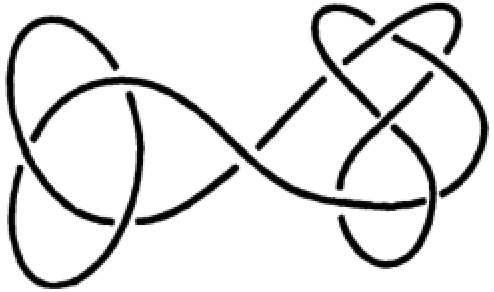
Alternating knots

YES

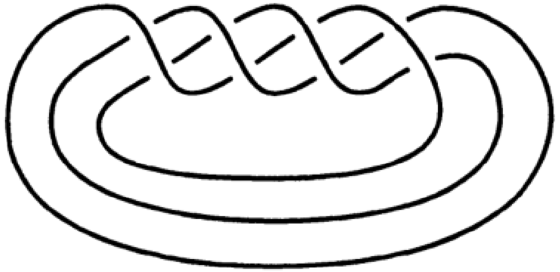


5_1

YES



NO



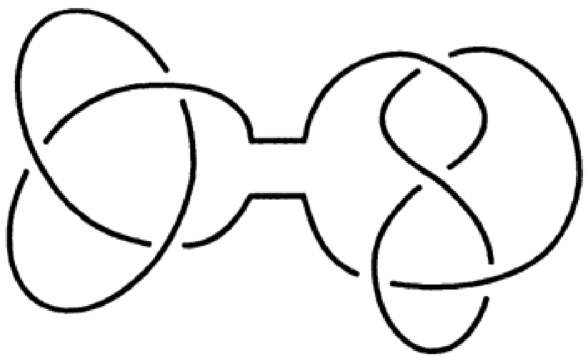
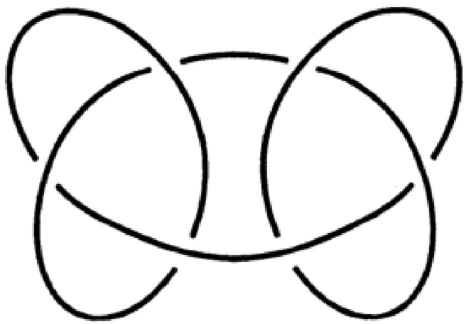


Figure 2.1.7



(a)

Exercise 2.1.3. Figures 1.5.6(a) and 2.1.7 are non-alternating diagrams for their respective knots. However, both of these knots are alternating knots. Show that they do possess alternating diagrams. (Hint: They have 6 and 7 crossings, respectively.)

Exercise 2.1.4. *(Taniyama)* Let K_1 and K_2 be alternating knots. Suppose that they have alternating diagrams with n_1 and n_2 crossing points, respectively. Show that the connected sum of K_1 and K_2 has an alternating diagram with exactly $n_1 + n_2$ crossing points.

Sec. 2: Gauss codes (DT notation)

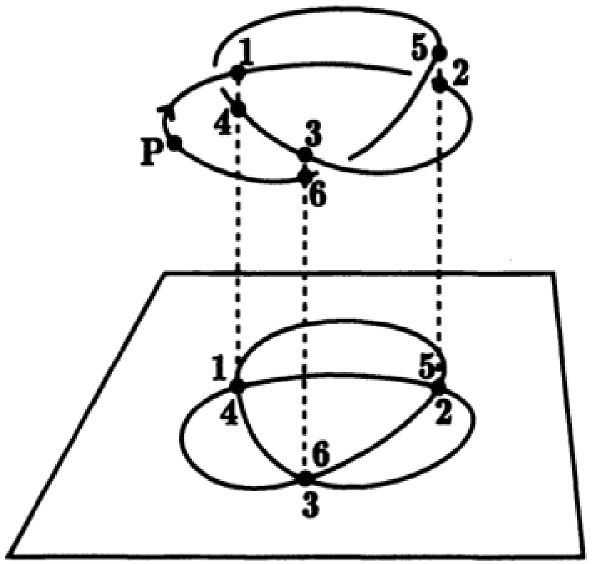
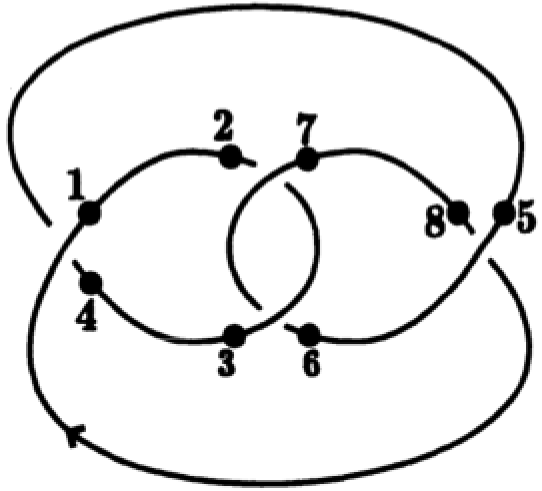
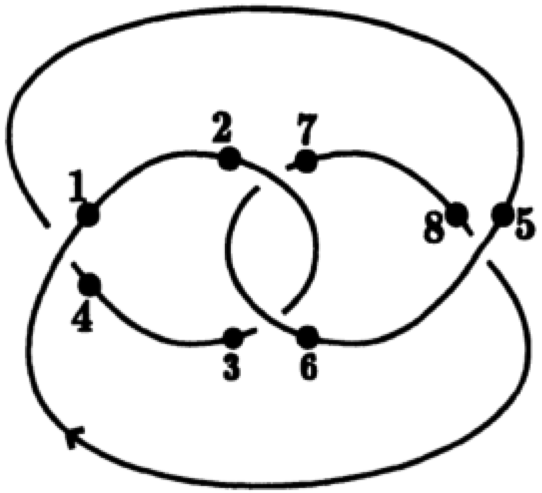


Figure 2.2.2

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & -5 & 6 & -1 & 2 & -3 \end{pmatrix}$$





$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & -7 & 6 & -1 & 8 & -3 & 2 & -5 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 7 & -6 & -1 & 8 & 3 & -2 & -5 \end{pmatrix}.$$

Sec. 2: Gauss codes (DT notation)

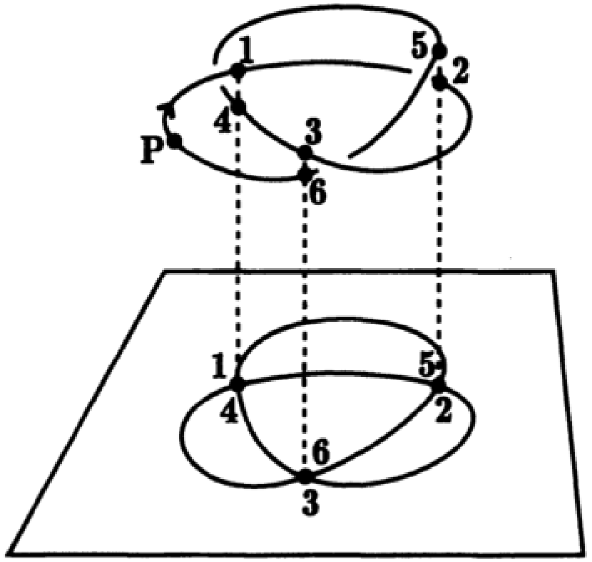
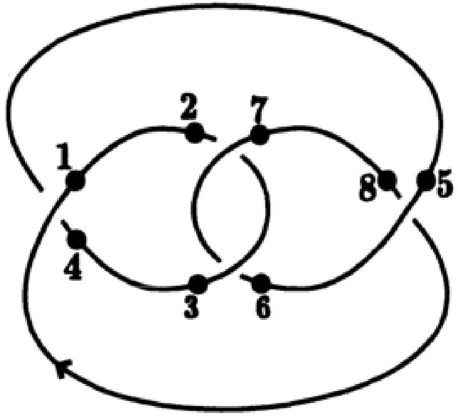


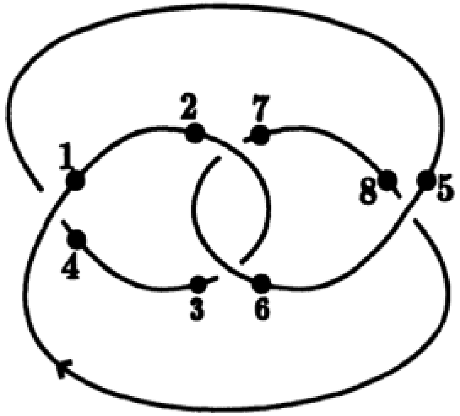
Figure 2.2.2

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & -5 & 6 & -1 & 2 & -3 \end{pmatrix}$$

Record only even numbers: (4, 6, 2)

Why sufficient??? Observe: odd <-> even. Why???





$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & -7 & 6 & -1 & 8 & -3 & 2 & -5 \end{pmatrix}$$

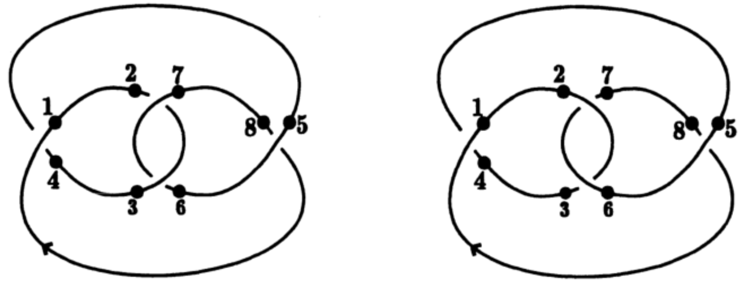
$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 7 & -6 & -1 & 8 & 3 & -2 & -5 \end{pmatrix}.$$

(4, 6, 8, 2) (4, -6, 8, -2)

(4,6,2)

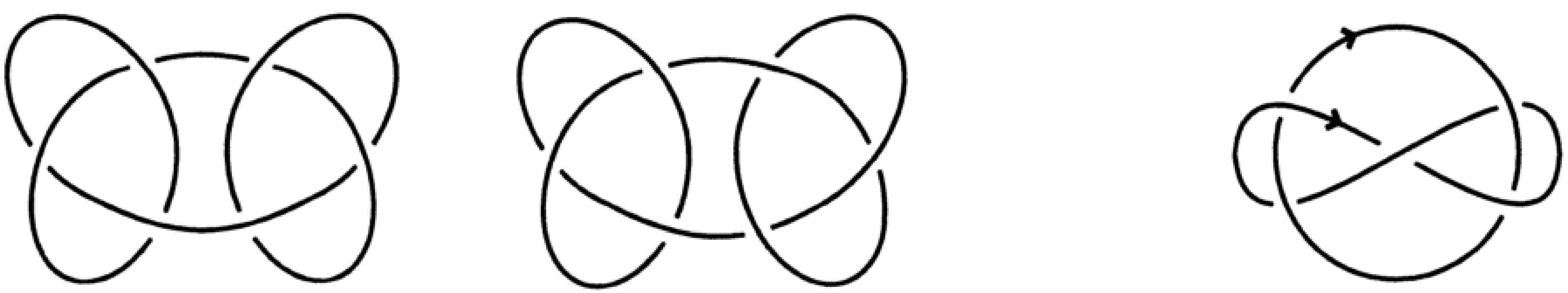
(4,6,8,2)

(4, -6, 8, -2)

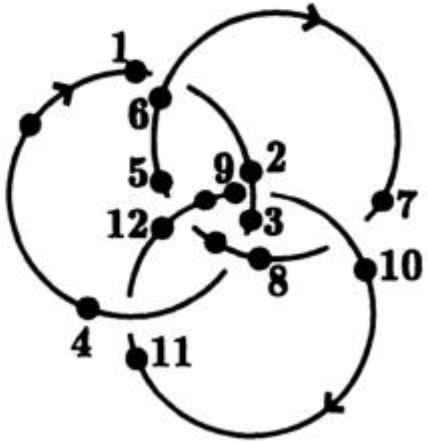


Exercise 2.2.3. Suppose a sequence $(a_1, a_2, \ldots, a_n)$ is a code of a knot K. Show that the same sequence can be a code for the mirror image of K.

Exercise 2.2.2. Determine the codes for



Example 2.2.2. The code for the (regular diagram of) Borromean rings in Figure 2.2.4 is $(-6, -8 \mid -12, -10 \mid -2, -4)$.



Exercise 2.2.1. Show that if all the signs in a given code agree, then it is a code of an alternating diagram; show that the converse also holds.

Exercise 2.2.4. *Find all* knots or links that have the following codes:

(a) $(4, 8, -12, 2, 14, 16, -6, 10)$

(b) $(6, 8, 22, 20, 4, -16, -26, -10, -24, -12, 2, -14, -18)$

(c) $(6, 10, 2, -12 \mid 4, -8)$

Exercise 2.2.5. Show that there cannot exist a knot with the code $(8, 10, 2, 4, 6)$.

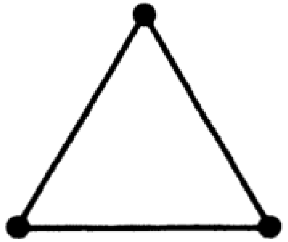
Exercise 2.2.6. Use the code of a knot to show that the number of knots and links that have regular diagrams with n crossing points is at most $2^n n!$

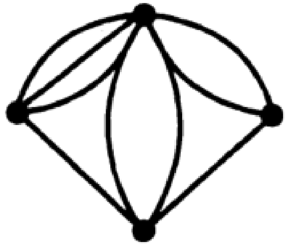
Lambda(n) = the number of prime knots that have a regular diagram with with n crossings

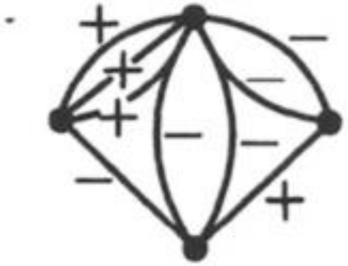
n	0	1	2	3	4	5	6	7	8	9	10	11	12	13
$\lambda(n)$	1	0	0	1	1	2	3	7	21	49	165	552	2176	9988

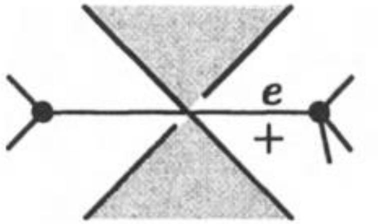
It is natural, of course, that as n increases, $\lambda(n)$ begins to increase rather rapidly. Actually, it was only a few years ago that it was *proven* that if n is large, then at the very least $\lambda(n)$ is bigger than n^2 [ES1]. Before this result was announced, basically all that could be said was that $\lambda(n) \geq 1$ for large n!

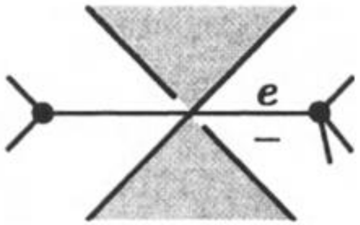
Sec. 2: Knot graphs

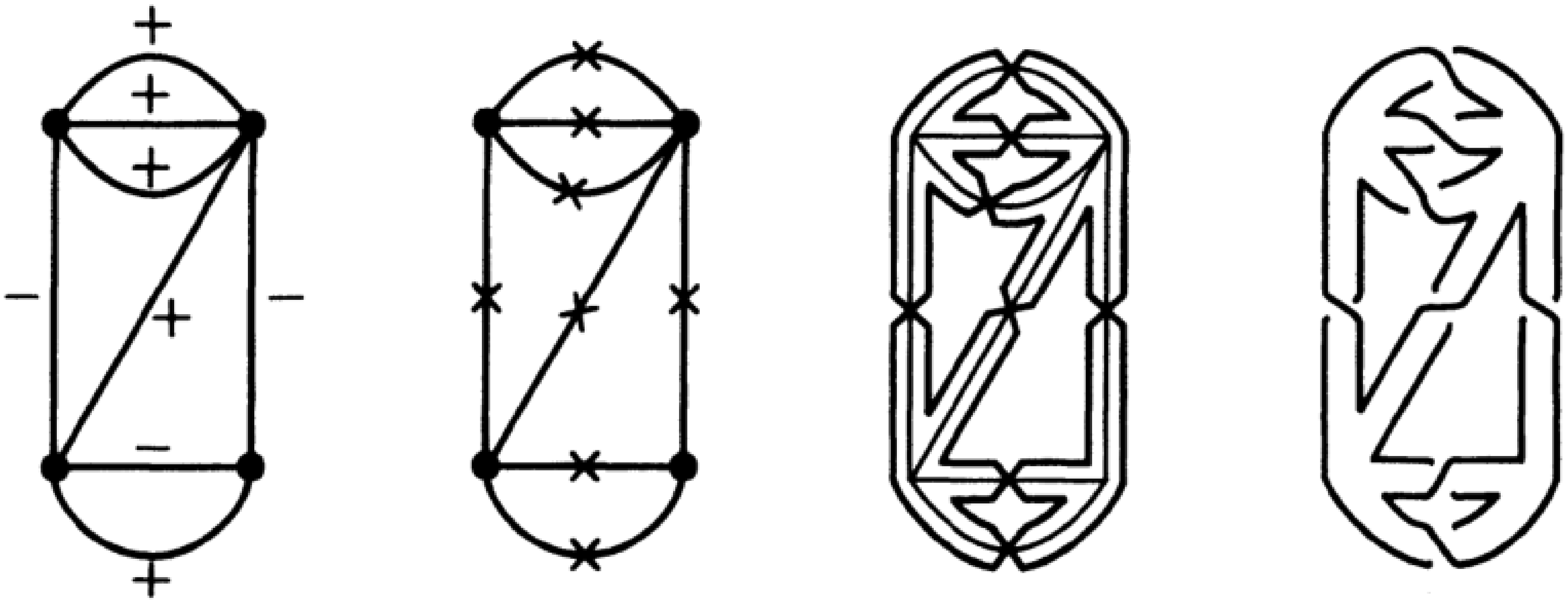


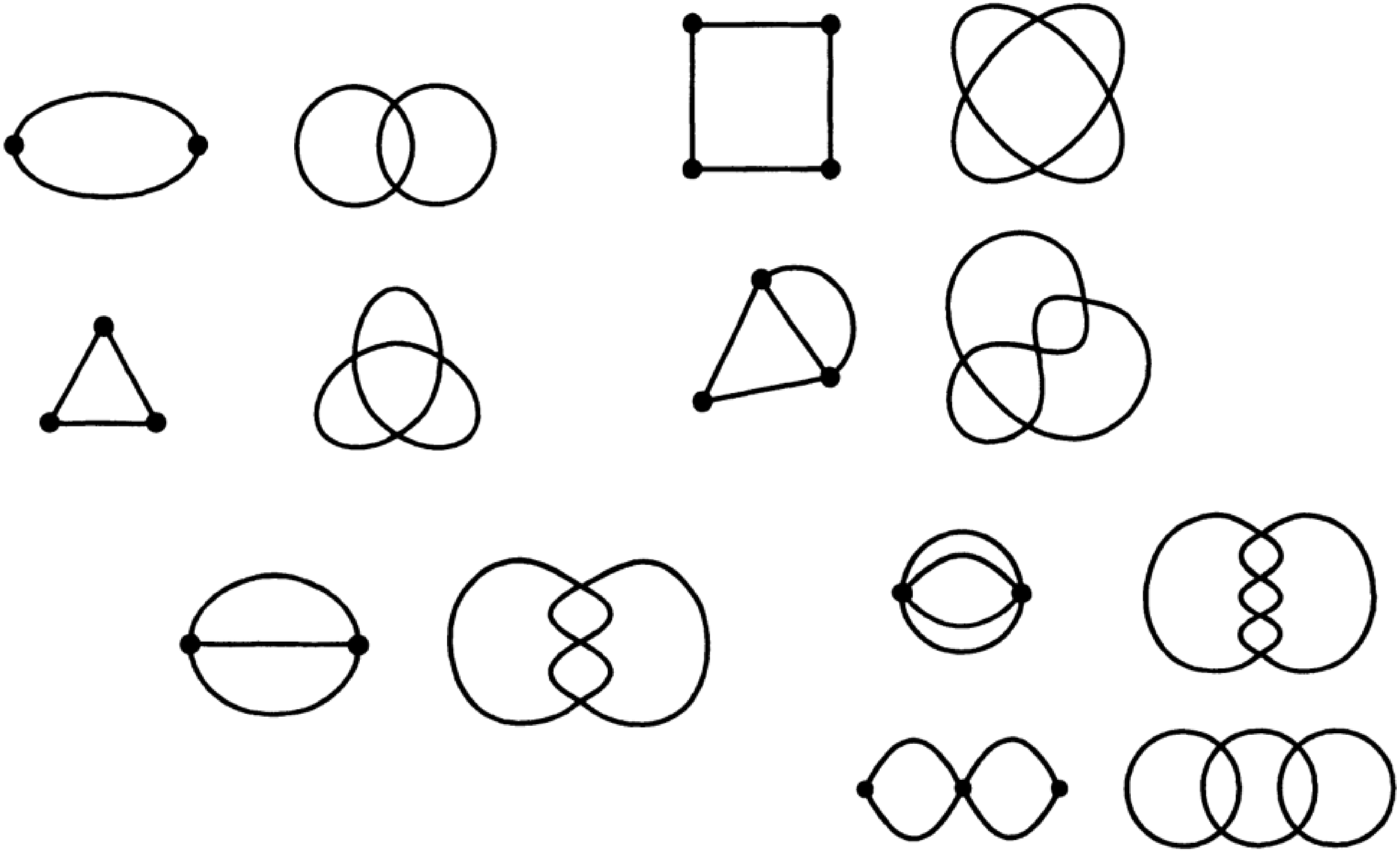












Exercise 2.3.3. List all the knots (and links) that correspond to connected plane graphs that have 5 and 6 positive edges. Moreover, determine which of these knots are equivalent.

Exercise 2.3.1. Show that a regular diagram that is also an alternating diagram corresponds to a graph G with the same sign on all the edges. Moreover, show that these are the only possible kind of graphs.

Knotinfo.org

- Other notations
- Pictures
- Invariants