VL10 Randomized Algorithms

7. February

Agenda

- Las Vegas and Monte Carlo
- Genetic Algorithms (GA)

Las Vegas and Monte Carlo

- Las Vegas are randomized algorithms that always give the correct result but can be very resource-intensive.
- **Monte Carlo** is a class of randomized algorithms that use repeated sampling to approximate the exact solution.

```
/* Las Vegas algorithm */
while(true) {
    k = RandomInteger(a.length);
    if (a[k] == 1)
        return k;
}

/* Monte Carlo algorithm */
nr_samples = 500;
while(nr_samples > 0) {
```

```
k = RandomInteger(a.length);

if (a[k] == 1)
    return k;

nr_samples--;
}

throw new Exception("Algorithm failed!");
```

Estimate the Value of Pi

```
n = 1000000;
count = 0;

for ( i = 0 ; i < n ; i++ ) {
    x = RandomInteger(2) - 1;
    y = RandomInteger(2) - 1;

    if ( x * x + y * y < 1 ) {
        count++;
    }
}</pre>
```

```
print( "pi = ", 4 * count / n );
```

Genetic Algorithms

- Class of evolutionary algorithms that uses techniques inspired by evolutionary biology such as inheritance, recombination, mutation, and selection
 - 1. The evolution starts with a population of randomly generated individuals. The result of this step is called **generation**.
 - 2. **Fitness** of each individual is computed and evaluated. If desired fitness is reached, the algorithm terminates.
 - 3. Regarding to their fitness, multiple individuals are selected to form a new generation. New individuals are created using recombination and mutation.
 - 4. Go to 2

Chromosome

■ Chromosome is the basic descriptive

data element of an individual, usually stored as vector data

- binary number
- array of integers, letters, etc.
- Every chromosome represents a single solution candidate for the problem.

Fitness

■ The fitness function quantifies the similarity of an individual with the optimal solution.

Selection

- Arbitrary
- Ranking
- Tournament
- Roulette wheel

Recombination, Mutation

- Recombination takes typically two chromosomes and produces a new chromosome.
 - Systematically

- Or in any different manner
- Mutation is operation that randomly changes information of a chromosome.
 - Useful to escape from local maximum

Example: Bit Count

Maximize the number of non-zero bits in a word (16 bit integer value).

- 1. Generate 4 random integers (4 is the population size)
- 2. Compute fitness for each, terminate if fitness reaches 16
- 3. Keep the first two with best fitness for the next generation
- 4. Drop the other two
- 5. Create third via recombination of 1st and 2nd
- 6. Create fourth via mutation of 1st (change one bit)
- 7. Go to 2

```
{ 'code' => 47427 }
iteration = 1
           { 'fitness' => 9,
"code" => 47459 \},
           \{ \text{'code'} => 46946, 
'fitness' => 9 },
           \{ \text{ 'code'} => 46946 \},
           { 'code' => 39267 }
iteration = 2
           { 'fitness' => 9,
code' => 47459 \},
           { 'code' => 46946,
'fitness' => 9 },
           { 'code' => 46946 },
           { 'code' => 39267 }
iteration = 3
           { 'fitness' => 9,
code' => 47459 \},
           { 'code' => 46946,
'fitness' => 9 },
           \{ \text{ 'code'} => 46946 \},
           { 'code' => 63843 }
iteration = 4
           { 'fitness' => 10,
'code' => 63843 },
           { 'fitness' => 9,
'code' => 47459 },
```

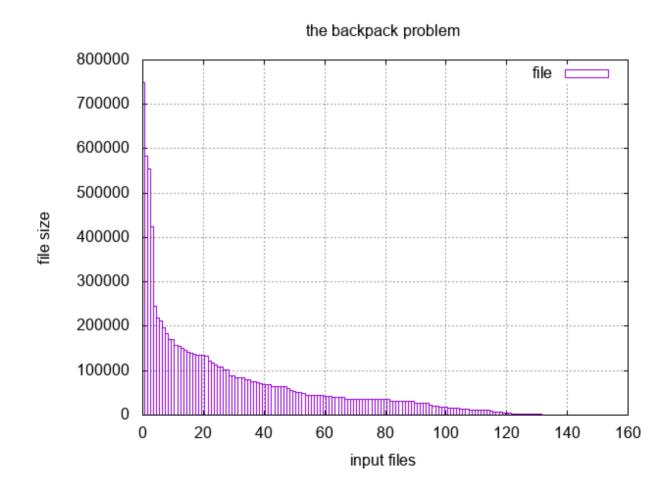
```
{ \text{'code'} => 63843 },
           { 'code' => 61795 }
iteration = 5
           { 'fitness' => 10,
'code' => 63843 },
           { 'fitness' => 10,
'code' => 63843 },
           { \text{'code'} => 63843 },
           { 'code' => 63779 }
iteration = 6
           { 'fitness' => 10,
'code' => 63843 },
           { 'fitness' => 10,
'code' => 63843 },
           { \text{'code'} => 63843 },
           { 'code' => 63811 }
iteration = 42
           { 'fitness' => 16,
'code' => 65535 },
           \{ \text{'code'} => 65519, 
'fitness' => 15 },
           { \text{'code'} => 65535 },
           { 'code' => 64511 }
```

all done, maximum found

Example: The Backpack Problem

Find a subset of objects that fit into a backpack with given capacity.

Example: target capacity 1.44 MB, files in the /bin directory



Chromosome

Binary string, 1 for file that's included, o for file

that's excluded.

Fitness Function

Sum of files that are included. Individuals with sum higher than target capacity are not fit.

Convergence

```
Fit: 1 not-fit: 99 best:
0.737444774624838
1/100 Mutating 99 candidates.
Fit: 43 not-fit: 57 best:
0.994665382738092
2/100 Mutating 57 candidates.
Fit: 45 not-fit: 55 best:
0.994665382738092
3/100 Mutating 55 candidates.
Fit: 58 not-fit: 42 best:
0.994665382738092
Dropping 8 candidates.
4/100 Mutating 50 candidates.
Fit: 61 not-fit: 39 best:
0.994665382738092
Dropping 11 candidates.
5/100 Mutating 50 candidates.
Fit: 57 not-fit: 43 best:
0.994665382738092
```

```
Dropping 7 candidates.
6/100 Mutating 50 candidates.
Fit: 54 not-fit: 46 best:
0.994665382738092
96/100 Mutating 50 candidates.
Fit: 55 not-fit: 45 best:
0.999981456327333
Dropping 5 candidates.
97/100 Mutating 50 candidates.
Fit: 53 not-fit: 47 best:
0.999981456327333
Dropping 3 candidates.
98/100 Mutating 50 candidates.
Fit: 51 not-fit: 49 best:
0.999981456327333
Dropping 1 candidates.
99/100 Mutating 50 candidates.
Fit: 52 not-fit: 48 best:
0.999981456327333
```