Lab 2

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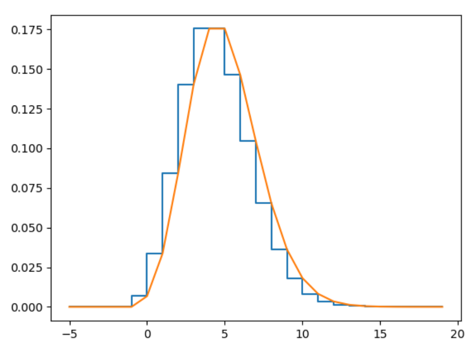
1 Problem 1

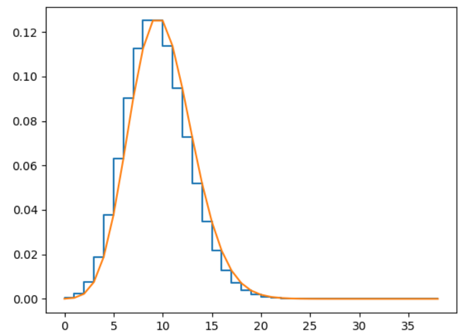
In this section, we look at detecting gamma rays. We want to detect gamma rays against a background of cosmic rays that follow a Poisson distribution. We assume that in one day there's an average of 5 cosmic rays and 8 gamma rays detected by the detector.

1.1

For one day our Poisson background distribution looks like:

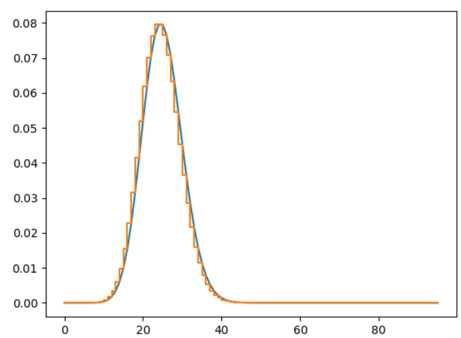


For two days, we can look at the probability as a sum by taking the convolution of our two daily distributions. We then see the probability of events for two days summing to an average number that's twice the average for one day. In our example below, the average for two days is 10 events which is twice the average for one day (5 events).



1.2

If we look at five days we can see a Poisson distribution that averages at 25 events. In the image below one can clearly see this is still a Poisson distribution even after summing over 5 days (step function with the shape of a normal distribution).



Conceptually, it makes sense that over the course of a larger interval, the background would remain Poisson distributed because we're still counting the number of events in a time interval. We haven't changed the measurement method just increased the time interval. Mathematically, we're convolving two of the same function and our result will still be of the same function type.

1.3

As you average days, the probability distribution becomes narrower around the mean. This makes sense since the most common number of events will have an extremely high probability as you add up more trials while less probable values will become even less likely as you average. This narrows the function so that it's closer to the mean with decreasing heights as you move outwards. Here's a distribution averaged over 10 days and one averaged over 100 days:

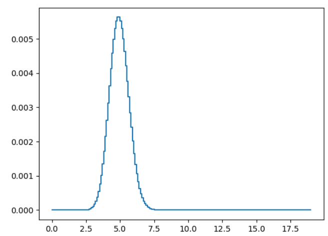


Figure 1: 10 day average

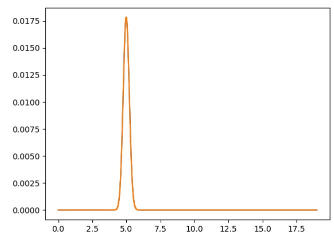


Figure 2: 100 day average

1.4

Let N be a number of days of observation and Y be the number of gamma rays detected per day. We let $N = 15$ and $Y = 8$. Assuming we saw $M = N*Y = 120$ gamma rays in an N day period, what is the associated sigma value of this measurement? To calculate this we need to compare our measurement value to the appropriate distribution. The correct distribution will also be for an N day period. Since we have the distribution for one day we can take the convolution of this distribution 14 times ($\#ofconvolutions = \#ofdays - 1$). This will give us the distribution of the sum of 15 days with the appropriate average mean of 5 events per day.

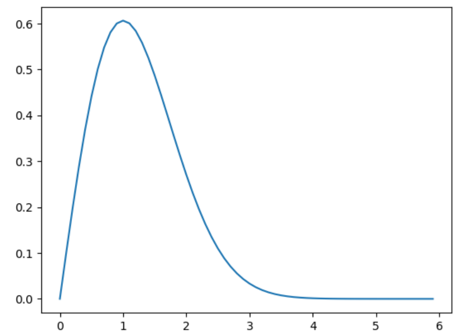
To find the probability of getting our measurement value we integrate our distribution from 120 to ∞. Once we have our probability we can plug this into a ppf function to return our associated sigma value on a normal distribution. See the following code:

```
N=15
M=N*Y
probability = 1 - stats.poisson.cdf(M,X*(15))
sigma = stats.norm.ppf(probability)
print(sigma)
OUT: -4.84
```

So in this case we get a value of 4.84σ.

2 Problem 2

In this section we examined a skewed continuous distribution, the Rayleigh distribution. Here's a standard Rayleigh distribution:



2.1

Now we look at how this distribution changes as we average over more observing intervals. We need to take the convolution with each trial distribution then divide the x-axis by the number of intervals. we use the code:

. We can see the distribution becomes more Gaussian as we average more. Here's a comparison of two distributions with increasing numbers of trials:

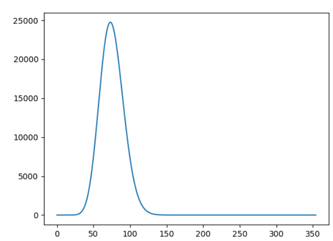


Figure 3: 6 trials averaged

Figure 4

As the number of trials go up the distribution looks more and more like a Gaussian distribution with an equal slope on either side of the peaked average.

3 Problem 3

3.1 Version 1

3.2 Version 2