

16. Elektromagnetická vlna ve vodiči, skin efekt

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Maxwellky:

$$\textcircled{1} \operatorname{div} \vec{B} = 0 \quad \textcircled{2} \operatorname{div} \vec{D} = S_a \quad \textcircled{3} \operatorname{rot} \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \textcircled{4} \operatorname{rot} \vec{H} = \vec{J}_a + \frac{\partial \vec{D}}{\partial t}$$

- Ve vodivém prostředí platí: $\vec{D} = \epsilon \vec{E}$, $\vec{B} = \mu \vec{H}$, $\vec{J}_a = \sigma \vec{E}$
- Zrotujeme $\textcircled{3}$: $\operatorname{rot} \operatorname{rot} \vec{E} = -\frac{\partial}{\partial t} \operatorname{rot}(\mu \vec{H})$ / $\operatorname{rot} \vec{H} = \sigma \vec{E} + \epsilon \frac{\partial \vec{E}}{\partial t}$

$$\bullet \operatorname{grad} \operatorname{div} \vec{E} - \Delta \vec{E} = -\sigma \mu \frac{\partial \vec{E}}{\partial t} - \epsilon \mu \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\bullet \underbrace{\Delta \vec{E} - \epsilon \mu \frac{\partial^2 \vec{E}}{\partial t^2}}_{\text{vlnová rovnice}} - \underbrace{\sigma \mu \frac{\partial \vec{E}}{\partial t}}_{\text{útlum vlny}} = 0$$

$$\bullet \text{Zfurijeme: } (-k^2 + \epsilon \mu \omega^2 + i \omega \sigma \mu) \vec{E} = 0$$

$$\bullet \text{Disperzní rel.: } -k^2 + \epsilon \mu \omega^2 + i \omega \sigma \mu = 0$$

$$k = \sqrt{\epsilon \mu \omega^2 + i \omega \sigma \mu}$$

vysoká vodivost
tento člen vyhraje

$$k \approx \sqrt{i \omega \sigma \mu} = \sqrt{\frac{\omega \sigma \mu}{2}} + i \sqrt{\frac{\omega \sigma \mu}{2}}$$

$$\bullet \text{Zkoumáme } \left. \begin{array}{l} \text{prostorové} \\ \text{chování} \\ \text{vlny} \end{array} \right\} \psi(x) = A e^{i(kx)} = A \exp\left(i\left(\sqrt{\frac{\omega \sigma \mu}{2}} + i\sqrt{\frac{\omega \sigma \mu}{2}}\right)x\right)$$

$$= A \underbrace{e^{-\sqrt{\frac{\omega \sigma \mu}{2}}x}}_{\text{útlum}} \underbrace{e^{i\sqrt{\frac{\omega \sigma \mu}{2}}x}}_{\text{sin a cos}}$$

$$\bullet \text{Charakteristická hloubka } \delta \dots A = \frac{A_0}{e}$$

$$\sqrt{\frac{\omega \sigma \mu}{2}} x = 1$$

$$\delta = \sqrt{\frac{2}{\omega \sigma \mu}} \quad ; \text{ skinová hloubka } \rightarrow \text{útlum amplitudy e-krát (skin efekt)}$$