Lorentzova transformace

Wednesday 15 January 2025 14:00

Galileova transformace – klasická mechanika –> předpokládá absolutní čas a prostor

$$\boxed{\begin{aligned} \tilde{x} &= x - vt & \qquad \tilde{x} &= x + vt \\ \tilde{y} &= y & &-\|- \\ \tilde{z} &= z & &-\|- \end{aligned}}$$

je to v rozporu s chováním el. mag. pole

nemůžeme skládat rychlosti jako v klasické mechanice

Lorentzova transformace

$$(1) \quad \tilde{x} = \gamma \cdot (x - vt)$$

$$(2) \quad x = \gamma \cdot (\tilde{x} + v\tilde{t})$$ inverzní transformace

Světlo se šíří v obou soustavách stejně, dle Maxwell. rovnic

$P: \; x = c \cdot t$

$\tilde{P}: \; \tilde{x} = c \cdot \tilde{t}$

$$(1) \quad c\tilde{t} = \gamma \cdot (ct - vt)$$

$$(2) \quad ct = \gamma \cdot (c\tilde{t} + v\tilde{t})$$

$(1) \cdot (2)$

$$c^2 t\tilde{t} = \gamma^2 \cdot c^2 t\tilde{t} - v^2 t\tilde{t})$$

$$c^2 \cancel{t\tilde{t}} = \gamma^2 \cdot (c - v) \cdot (c + v) \cdot \cancel{t\tilde{t}}$$

$$c^2 = \gamma^2 \cdot (c - v) \cdot (c + v)$$

$$\gamma^2 = \frac{c^2}{c^2 - v^2} = \frac{c^2}{c^2 - v^2}$$

$$= \frac{1}{1 - \frac{v^2}{c^2}}$$

$$\gamma = \sqrt{\frac{1}{1 - \frac{v^2}{c^2}}}$$

lorentzův faktor

$\tilde{t} = ?$ z (1) dosadit $\tilde{x}$ do (2)

$$x = \gamma \cdot [\gamma \cdot (x - vt) + v\tilde{t}]$$

$$x = \gamma^2 x - \gamma^2 vt + \gamma v \tilde{t}$$

$$\gamma v \tilde{t} = x - \gamma^2 x + \gamma^2 vt$$

$$\tilde{t} = \frac{x \cdot (1 - \gamma^2)}{\gamma v} + \gamma t$$

$$\tilde{t} = \gamma \cdot \left(\frac{1 - \gamma^2 x}{\gamma^2 v} - t\right)$$

$$\tilde{t} = \gamma \left[t + \left(\frac{1}{\gamma^2} - 1\right) \cdot \frac{x}{v}\right]$$

$$\frac{1}{\gamma^2} = 1 - \frac{v^2}{c^2}$$

$$\tilde{t} = \gamma \cdot \left[t + \left(1 - \frac{v^2}{c^2} - 1\right) \frac{x}{v}\right]$$

$$\tilde{t} = \gamma \cdot \left(t - \frac{vx}{c^2}\right)$$

LT:

$$\tilde{x} = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} \; ; \quad \tilde{t} = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} \; ; \quad \tilde{y} = y, \quad \tilde{z} = z$$

$$f(t; \vec{x}) = x_0, x_1, x_2, x_3$$

$$\begin{pmatrix} \tilde{x}_0 \\ \tilde{x}_1 \\ \tilde{x}_2 \\ \tilde{x}_3 \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$\det = 1 \cdot 1 \cdot (\gamma^2 - \gamma^2 \beta^2)$$

$$= \gamma^2 \cdot (1 - \beta^2)$$

$$\beta = \frac{v}{c}$$