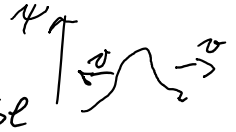
14. Vlnová rovnice

Wednesday, January 15, 2025 19:28



- Máme nějakou vlnu co nemění svůj tvar a pohybuje se
- Vlnová funkce: $\Psi(t,\vec{x}) = f(x \pm vt)$; $\eta = x \pm vt$
- 2× derivujeme podle prostoru

$$\frac{\partial \Psi}{\partial x} = \frac{\partial f}{\partial \eta} = \frac{\partial f}{\partial \eta} \cdot \frac{\partial \eta}{\partial x} = \frac{\partial f}{\partial \eta} \cdot 1$$

$$\frac{\partial^2 \Psi}{\partial x} = \frac{\partial^2 f}{\partial \eta^2}$$

- 2× podle času

$$\frac{\partial \Psi}{\partial t} = \frac{\partial f}{\partial \eta} \cdot \frac{\partial \eta}{\partial t} = \frac{\partial f}{\partial t} \cdot (\pm v)$$

$$\frac{\partial^2 \Psi}{\partial t^2} = v^2 \frac{\partial^2 f}{\partial \eta^2}$$

- Spojíme:

$\vec{x}$: $\frac{\partial^2 f}{\partial \eta^2} = \frac{\partial^2 \Psi}{\partial x^2}$

t: $\frac{\partial^2 f}{\partial \eta^2} = \frac{1}{v^2} \frac{\partial^2 \Psi}{\partial t^2}$

$$\Rightarrow \frac{\partial^2 \Psi}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 \Psi}{\partial t^2}$$

3D

$$\Delta \Psi = \frac{1}{v^2} \frac{\partial^2 \Psi}{\partial t^2}$$

Vlnová rovnice: $\Delta \Psi - \frac{1}{v^2} \frac{\partial^2 \Psi}{\partial t^2} = 0$

$$\Box \Psi = 0$$