Propočítáné příklady CV.7,DÚ7

Monday, December 30, 2024 16:41

Cvičení 7 a 8 – Komplexní analýza 2024/2025
Týden 8 a 9

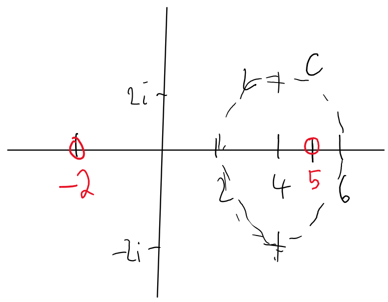
Úloha 2. *Spočtěte.*

(a)

$$\int_C \frac{z}{(z+2)(z-5)^2}\,dz, = 2\pi i \operatorname{res}_5\left(\frac{z}{(z+2)(z-5)^2}\right) = \frac{4}{49}\pi i$$

Úloha 2: (a) $\frac{4}{49}\pi i$

kde C je kladně orientovaná kružnice o rovnici $|z-4|=2$.



$\vee$ $z=5$ má $\frac{z}{(z+2)(z-5)^2}$ pól řádu 2

$\boxed{k=2}$

$$\lim_{z\to 2}\left(\cancel{(z-5)^2}\frac{z}{(z+2)\cancel{(z-5)^2}}\right)' = \lim_{z\to 5}\left(\frac{z}{z+2}\right)' =$$

$$= \lim_{z\to 5}\left(\frac{z+2-z}{(z+2)^2}\right) = \frac{2}{49}$$

(b)

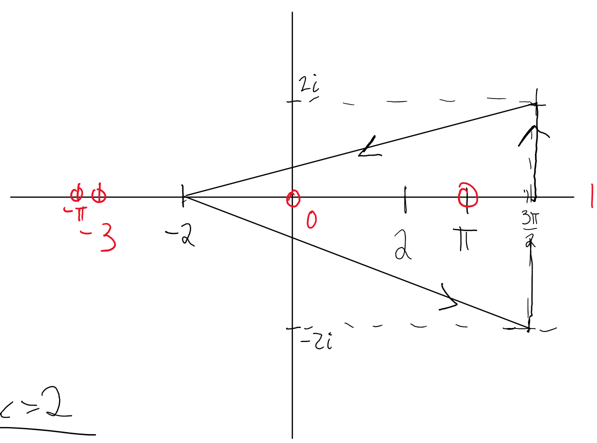
$$\int_C \frac{1}{z\sin z} + \frac{e^{\sin z}}{z+3}\,dz, = \int_C \frac{1}{z\sin z}dz + \underbrace{\int_C \frac{e^{\sin z}}{z+3}dz}_{0} = 2\pi i\left(\operatorname{res}_0\left(\frac{1}{z\sin z}\right) + \operatorname{res}_\pi\left(\frac{1}{z\sin z}\right)\right)$$

kde C je kladně orientovaná hranice trojúhelníka s vrcholy -2, $\frac{3\pi}{2}+2i$, $\frac{3\pi}{2}-2i$.

$$= 2\pi i \frac{1}{-\pi} = -2i$$ yes

Izol. singularity: 0, $k\pi$, $\cancel{-3}$

$\pi \to$ pól řádu 2



$k=2$

$f(z) = \frac{1}{z\sin z}$ v $z_0=0$ má izol. singul.

Pól řádu 2

Čit. nás. kor. $=0$

Jmen. nás. kor.:

$z\sin z\big|_{z=0}=0$

$(z\sin z)' = \sin z + z\cos z\big|_{z=0} = 0$

$(z\sin z)'' = \cos z + \cos z - z\sin z\big|_{z=0} = 2 \neq 0 \to$ jmen. nás. kor.: 2

$$\operatorname{res}_0 f(z) = \lim_{z\to 0}\left((z)^2\frac{1}{z\sin z}\right)' =$$

$$= \lim_{z\to 0}\left(\frac{z}{\sin z}\right)' = \lim_{z\to 0}\left(\frac{\sin z - z\cos z}{\sin^2 z}\right) \underset{\frac{0}{0}}{\overset{L'H}{=}} \lim_{z\to 0}\left(\frac{\cos z - \cos z + z\sin z}{2\cos^2 z}\right) = 0$$

$$\operatorname{res}_\pi f(z) = \lim_{z\to\pi}\left((z-\pi)\frac{1}{z\sin z}\right) = \lim_{z\to\pi}\left(\frac{z-\pi}{z\sin z}\right) \overset{L'H}{=} \lim_{z\to\pi}\frac{1}{\sin z + z\cos z} = \frac{1}{-\pi}$$

$\llcorner$ pól řádu 1

(c)

$$\int_C \frac{1}{e^z-1} + \frac{z-\frac{\pi}{2}}{(\cos z)^2}\,dz, = 2\pi i \operatorname{res}_{\frac{\pi}{2}}\left(\frac{z-\frac{\pi}{2}}{(\cos z)^2}\right) = 2\pi i$$

(c) $2\pi i$

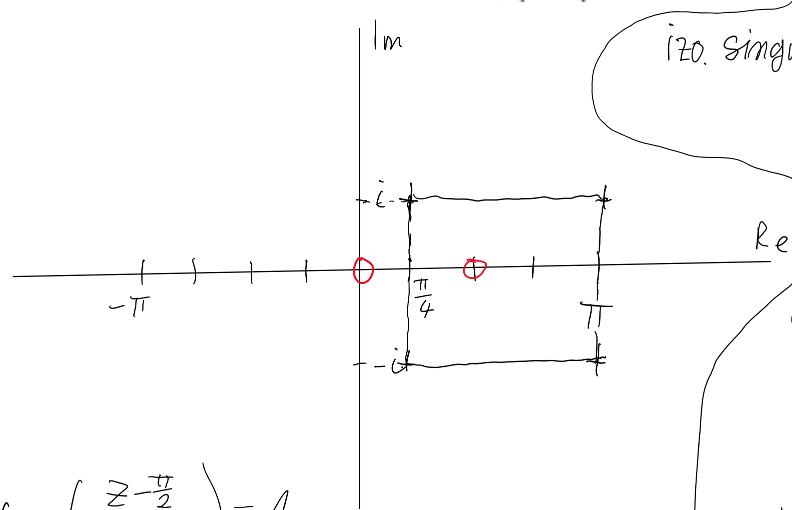
(c)

$$\int_C \frac{1}{e^z-1} + \frac{z-\frac{\pi}{2}}{(\cos z)^2}\,\mathrm{d}z, = 2\pi i \operatorname{res}_{\frac{\pi}{2}}\left(\frac{z-\frac{\pi}{2}}{(\cos z)^2}\right) = \underline{\underline{2\pi i}}$$

(c) $2\pi i$

kde C je kladně orientovaná hranice obdélníka s vrcholy $\frac{\pi}{4}+i$, $\frac{\pi}{4}-i$, $\pi+i$, $\pi-i$.

izo. singul.: $2k\pi,\ \frac{\pi}{2}+k\pi,\ k\in\mathbb{Z}$



Zajímá nás pouze izol. singul. v $\frac{\pi}{2}$

$$\text{Č}:\quad z-\frac{\pi}{2}\Big|_{z=\frac{\pi}{2}} = 0$$

$$\left(z-\frac{\pi}{2}\right)' = 1 \neq 0 \ \rightarrow \text{násobnost} = 1$$

$$\text{J}:\quad \cos z\Big|_{z=\frac{\pi}{2}} = 0$$

$$(\cos z)' = -\sin z\Big|_{z=\frac{\pi}{2}} = -1 \neq 0 \ \rightarrow \text{násobnost } 1\cdot \underline{2}$$

V $\frac{\pi}{2}$ pól řádu $2-1=1$

$$\operatorname{res}_{\frac{\pi}{2}}\left(\frac{z-\frac{\pi}{2}}{(\cos z)^2}\right) = 1$$

$$= \lim_{z\to\frac{\pi}{2}}\left(\left(z-\frac{\pi}{2}\right)\frac{z-\frac{\pi}{2}}{(\cos z)^2}\right) = \lim_{z\to\frac{\pi}{2}} \frac{\left(z-\frac{\pi}{2}\right)^2}{\cos^2 z}$$

$$= \left(\frac{0}{0}\right) \overset{L'H}{=} \lim_{z\to\frac{\pi}{2}} \frac{2\left(z-\frac{\pi}{2}\right)}{-2(\sin z)(\cos z)} = \left(\frac{0}{0}\right) \overset{L'H}{=} \lim_{z\to\frac{\pi}{2}} \frac{2}{-(\sin 2z)'} = \lim_{z\to\frac{\pi}{2}} \frac{2}{-2\cos 2z} = \underline{1}$$

(a) $\int_{-\infty}^{\infty} \frac{1}{x^2-6x+25}\,\mathrm{d}x = \int_{-\infty}^{\infty} \frac{e^{i0x}}{x^2-6x+25} = \int_{-\infty}^{\infty} \frac{e^{i0x}}{(x-3)^2+4^2} = \int_{-\infty}^{\infty} \frac{e^{i0x}}{((x-3)-4i)((x-3)+4i)}$

$$\underline{\alpha = 0} \rightarrow \int_{-\infty}^{\infty} \boxed{\ } = 2\pi i \sum_{w\in S^+} \operatorname{res}_w \boxed{\ }$$

Izol. singularity v

$\rightarrow x = 3+4i$

$x = 3-4i$

$$\operatorname{res}_{3+4i} \frac{1 \ \rightarrow \text{násobnost}=0}{\underbrace{(x-3-4i)}_{\text{násobnost}=1}(x-3+4i)} \underset{\text{dosaz}}{=} \frac{1}{\underset{\text{derivace}}{1}\cdot(8i)} = \frac{1}{8i}$$

$$\int_{-\infty}^{\infty} \frac{1}{x^2-6x+25}\,dx = 2\pi i \cdot \frac{1}{4i} = \underline{\underline{\frac{1}{4}\pi}}$$

(a) $\frac{\pi}{4}$

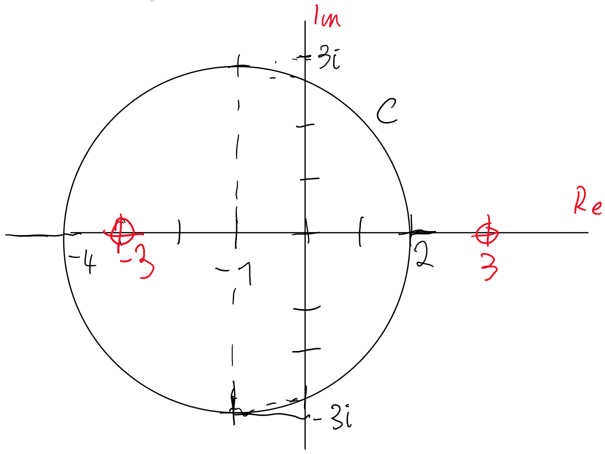
Úloha 3 *Určete, čemu se rovná* $f(z)$

$\int_{-\infty}$

Úloha 3. *Určete, čemu se rovná*

$$\int_C \underbrace{\left(\frac{2}{(z+3)^5}+\cancel{\frac{3}{z-3}}+\frac{4}{z+3}\right)}_{f(z)}\mathrm{d}z, = 2\pi i\left(\mathrm{res}_{-3}\left(\frac{2}{(z+3)^5}\right) + \mathrm{res}_{-3}\left(\frac{4}{z+3}\right)\right)$$

kde C je kladně orientovaná kružnice o rovnici $|z+1|=3$.



singularity v

$z=-3$, $z=3$

$$\mathrm{res}_{-3} f(z) = 4$$

$$\int_C \frac{2}{(z+3)^5}+\frac{4}{z+3}+\frac{3}{z-3}\,dz = \underbrace{\int_C \frac{2}{(z+3)^2}+\boxed{\frac{4}{z+3}}\,dz}_{\text{rozvoj}} + \underbrace{\int_C \frac{3}{z-3}\,dz}_{0} = 2\pi i\cdot 4 = 8\pi i$$

Úloha 3: $8\pi i$

residuum=4

singularita v $z=3$ není uvnitř C

(b) $\int_{-\infty}^{\infty} \frac{x^2 e^{i0\pi}}{(x^2+4)^2}\mathrm{d}x = \int_{-\infty}^{\infty}\frac{x^2}{(x^2+4)^2}dx = \mathrm{res}_{+2i}\Big(\underbrace{\frac{x^2}{(x^2+4)^2}}_{f(x)}\Big)$

izol. singularita v $x=\pm 2i$

$\alpha = 0$ beru izol. singul. s kl. imaginar. hodnotami

Ve $+2i$ je pól řádu 2 $k=2$

$$\mathrm{res}_{2i} f(x) = \lim_{x\to 2i}\left(\cancel{(x-2i)^2}\frac{x^2}{(x+2i)^2\cancel{(x-2i)^2}}\right)' = \lim_{x\to 2i}\frac{2x(x+2i)^2 - x^2\cdot 2(x+2i)}{(x+2i)^4}$$

$$= \frac{4i\cdot 16i^2 - 4i^2\cdot 8i}{4^4} = \frac{-i(64-32)}{256} = -i\,\frac{32}{256} = -\frac{i}{8}$$

$$\int_{-\infty}^{\infty}\frac{x^2}{(x^2+4)^2}dx = 2\pi i\cdot\frac{-i}{8} = \frac{\pi}{4}$$

(b) $\frac{\pi}{4}$

$\int^{\infty} \quad -2ix \quad \left(\frac{e^{-2ix}}{}\right) + \mathrm{res}\left(\frac{e^{-2ix}}{}\right)$

$$(c)\ \int_{-\infty}^{\infty} \frac{e^{-2ix}}{\underbrace{(x^2+1)}_{\pm i}\underbrace{(x^2+9)}_{\pm 3i}}\,dx = -2\pi i\left(\mathrm{res}_{-i}\left(\frac{e^{-2ix}}{(x^2+1)(x^2+9)}\right)+\mathrm{res}_{-3i}\left(\frac{e^{-2ix}}{(x^2+1)(x^2+9)}\right)\right)$$

$\alpha = -2$

$$= -2\pi i\left(\frac{e^{-2i(-i)}}{\cancel{-16i}\ 8} + \frac{e^{-2i(-3i)}}{\cancel{48i}\ 24}\right) = \frac{\pi e^{-2}}{8} - \frac{\pi e^{-6}}{24}$$

$$\mathrm{res}_{-i}\left(\frac{e^{-2ix}}{(x^2+1)(x^2+9)}\right) = \frac{e^{-2ix}}{2x\ (x^2+9)}\Bigg|_{x=-i} =$$

(c) $\left(\frac{e^{-2}}{8} - \frac{e^{-6}}{24}\right)\pi$ ✓

V $-i$ pól řádu 1

$x^2+1\,\big|_{-i} = 0$

$2x\,\big|_{-i} \neq 0$

$$\mathrm{res}_{-3i}\left(\frac{e^{-2ix}}{(x^2+1)(x^2+9)}\right) = \frac{e^{-2ix}}{(x^2+1)\ 2x}\Bigg|_{x=-3i} = \frac{e^{-2(-3i)}}{-8\cdot(-6i)}$$

d) $$(d)\ \int_{-\infty}^{\infty} \frac{e^{1ix}}{(x^2-2x+2)^2}\,dx = 2\pi i\left(\mathrm{res}_{1+i}\frac{e^{ix}}{(x^2-2x+2)^2}\right) = 2\pi i\cdot\underbrace{\frac{e^{-1-i}}{2i}} = \pi e^{i-1}$$

$\alpha = 1$ → kořeny s klad. imaginární složkou

Kořeny:

$$(x^2-2x+2)^2 = ((x-1)^2+1^2)^2 = \big(((x-1)-i)(x-1+i)\big)^2$$

* $1+i$, $1-i$ } 2 násobné kořeny jmenovatele

→ mě zajímají kořeny s $\underset{\geq 0}{\text{kladnou}}$ imaginární složkou *$(1+i)$ → v tomto bodě pól řádu 2

Použiju vzoreček $\mathrm{res}_{z_0} f(z) = \frac{1}{(k-1)!}\lim_{z\to z_0}\left((z-z_0)^k f(z)\right)^{(k-1)}$ derivace, k je řád pólu

$$\mathrm{res}_{1+i}\frac{1}{((x-1-i)(x-1+i))^2} = \frac{1}{(2-1)!}\lim_{z\to 1+i}\left(\cancel{(z-1-i)^2}\frac{e^{ix}}{\cancel{(z-1-i)^2}(z-1+i)^2}\right)' = \lim_{z\to 1+i}\left(\frac{e^{ix}}{(z-1+i)^2}\right)' =$$

$$= \lim_{z\to 1+i}\left(\frac{ie^{ix}(z-1+i)^2 - e^{ix}2(z-1+i)}{(z-1+i)^4}\right) = \frac{e^{i-1}(-4i-4i)}{16} = \frac{e^{i-1}\cdot(-8i)}{16} = \frac{e^{i-1}}{2i}$$

$$\frac{1}{x^2} = x^{-2} = -2x^{-3}$$

DÚ:

DÚ:

Cvičení 7 – Komplexní analýza 2024/2025
Dobrovolná domácí cvičení

Úloha 1. *Spočtěte*

$$\operatorname{res}_{\frac{\pi}{2}} \frac{e^{iz} - i + \sin(z - \frac{\pi}{2})}{(z - \frac{\pi}{2})^2 \cos z}.$$

Úloha 2. *Spočtěte*

$$\int_C \frac{z - \frac{\pi}{2}i}{e^z - i} + \frac{z^2}{(z^2+1)^2} + \frac{\sin z}{(z+1)^4}\,\mathrm{d}z,$$

kde C je kladně orientovaná hranice trojúhelníka s vrcholy $0,\ 2+2\pi i,\ -2+2\pi i$.

Úloha 3. *Určete, čemu se rovná*

$$\int_C \frac{3}{z-i} + \frac{1}{(z-i)^2} + \frac{2}{(z+i)^2} + 4(z+i)\,\mathrm{d}z,$$

kde C je kladně orientovaná kružnice o rovnici $|z+i| = 1$.

Cvičení 8 – Komplexní analýza 2024/2025
Dobrovolná domácí cvičení

Úloha 1. *Spočtěte*

$$\int_{-\infty}^{\infty} \frac{x^2}{(x^2+4)(x^2+9)}\,\mathrm{d}x.$$

Úloha 2. *Spočtěte*

$$\int_{-\infty}^{\infty} \frac{x}{(x^2-4x+13)^2} e^{-2ix}\,\mathrm{d}x.$$

Úloha 3. *Spočtěte*

$$\int_{-\infty}^{\infty} \frac{e^{ix}}{(x^2-4ix-3)(x^2+1)}\,\mathrm{d}x.$$

Cvičení 7

Úloha 1. *Spočtěte*

$$\operatorname{res}_{\frac{\pi}{2}} \frac{e^{iz} - i + \sin(z - \frac{\pi}{2})}{(z - \frac{\pi}{2})^2 \cos z}. = \frac{i}{2}$$

$$= \frac{-i}{-2} = \frac{i}{2}.$$

V $\frac{\pi}{2}$ Pól řádu $3-2=1$ → vzoreček $\lim_{z\to\frac{\pi}{2}} (z-\frac{\pi}{2}) \frac{e^{iz}-i+\sin(z-\frac{\pi}{2})}{(z-\frac{\pi}{2})^2\cos z}$

$$e^{iz} - i + \sin(z-\tfrac{\pi}{2})\Big|_{z=\frac{\pi}{2}} = 0$$

$$= \lim_{z\to\frac{\pi}{2}} \frac{e^{iz}-i+\sin(z-\frac{\pi}{2})}{(z-\frac{\pi}{2})\cos z} \overset{L'H}{=} \lim_{z\to\frac{\pi}{2}} \frac{ie^{iz}+\cos(z-\frac{\pi}{2})}{\cos z - (z-\frac{\pi}{2})\sin z}$$

$$\left(e^{iz} - i + \sin(z-\tfrac{\pi}{2})\right)' = i\cdot e^{iz} + \cos(z-\tfrac{\pi}{2})\Big|_{z=\frac{\pi}{2}} = -1+1 = 0 \overset{L'H}{=} \lim_{z\to\frac{\pi}{2}} \frac{-e^{iz}-\sin(z-\frac{\pi}{2})}{-\sin z - \sin z - (z-\frac{\pi}{2})\cos z}$$

$$\left(e^{iz} - i + \sin(z-\tfrac{\pi}{2})\right)'' = -e^{iz} - \sin(z-\tfrac{\pi}{2})\Big|_{z=\frac{\pi}{2}} = -i - 0 \neq 0 \quad = \frac{-i}{-2} = \frac{i}{2}$$

Úloha 2. *Spočtěte*

$$\int_C \frac{z - \frac{\pi}{2}i}{e^z - i} + \frac{z^2}{(z^2+1)^2} + \frac{\sin z}{(z+1)^4}\,\mathrm{d}z, = \left(\operatorname{res}_{\frac{\pi}{2}} \frac{z-\frac{\pi}{2}i}{e^z-i} + \operatorname{res}_i \frac{z^2}{(z^2+1)^2}\right)\cdot 2\pi i = \frac{-i}{4}\cdot 2\pi i = \frac{\pi}{2}$$

kde C je kladně orientovaná hranice trojúhelníka s vrcholy $0,\ 2+2\pi i,\ -2+2\pi i$.

$$I = 2\pi i \cdot \left(-\frac{i}{4}\right) = \frac{\pi}{2}.$$

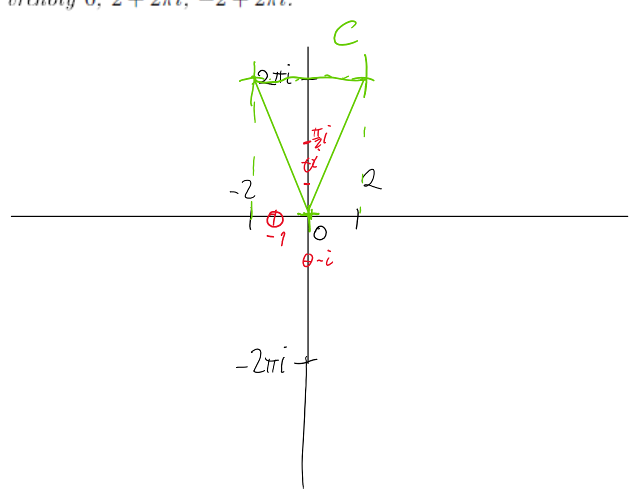
Izolované singularity:

$(e^z - i) = 0 \to z = \frac{\pi}{2}i + 2k\pi i$

$(z^2+1)^2 = 0 \to z = \pm i$

$(z+1)^4 = 0 \to z = -1$

$$\operatorname{res}_{\frac{\pi}{2}i} \frac{\overbrace{z - \frac{\pi}{2}i}^{\text{násobnost}=1}}{\underbrace{e^z - i}_{\text{násobnost}=1}} = 0$$



$$\operatorname{res}_i \frac{z^2}{\underbrace{(z^2+1)^2}_{\text{v } z=i \text{ pól řádu 2}}} = \frac{1}{(2-1)!}\lim_{z\to i}\left(\cancel{(z-i)^2}\frac{z^2}{(z+i)^2\cancel{(z-i)^2}}\right)' = \lim_{z\to i}\left(\frac{2z(z+i)^2 - z^2\cdot 2(z+i)}{(z+i)^4}\right)$$

$$= \frac{2i\cdot 4i^2 - i^2\cdot 2\cdot 2i}{(2i)^4} = \frac{-8i+4i}{2^4} = \frac{-4i}{16} = \frac{-i}{4}$$

$$(z^2+1)\Big|_{z=i} = 0$$

$$(z^2+1)' = 2z\Big|_{z=i} \neq 0$$

Úloha 3. *Určete, čemu se rovná*

$$\int_C \frac{3}{z-i} + \frac{1}{(z-i)^2} + \frac{2}{(z+i)^2} + 4(z+i)\,\mathrm{d}z, = 2\pi i\left(\underbrace{\operatorname{res}_{-i}\frac{2}{(z+i)^2}}_{0}\right) = 0$$

kde C je kladně orientovaná kružnice o rovnici $|z+i| = 1$.

$I = 2\pi i \cdot 0 = 0.$

1 izol. singul.

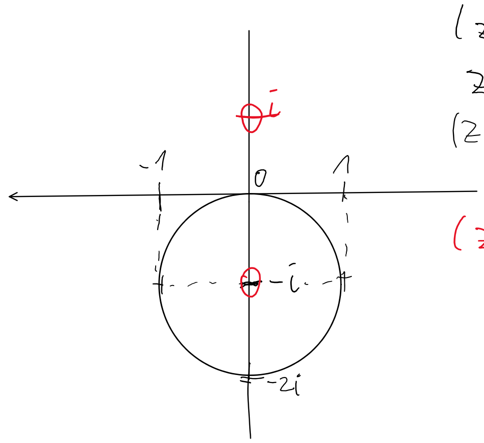
$z-i \to i$

$(z-i)^2 \to i$

$(z+i)^2 = -i$

$$\operatorname{res}_{-i}\frac{2}{(z+i)^2} = \cancel{\lim_{z\to -i}\frac{d}{dz}\left(\frac{2}{(z+i)^2}\cdot(z+i)^2\right)} = 0$$

pól řádu 2



Úloha 1. *Spočtěte*

$$\int_{-\infty}^{\infty}\frac{x^2 e^{\alpha z}}{(x^2+4)(x^2+9)}\,\mathrm{d}x. = 2\pi i\left(\operatorname{res}_{2i}\frac{x^2}{(x^2+4)(x^2+9)} + \operatorname{res}_{3i}\frac{x^2}{(x^2+4)(x^2+9)}\right) = 2\pi i\cdot\frac{1}{10i} = \frac{\pi}{5}$$

$\alpha = 0 \to$ kořeny s kladnou imag.

$(x^2+4) \to \pm 2i$

$(x^2+9) \to \pm 3i$

$\frac{\pi}{5}.$

$$\operatorname{res}_{2i}\frac{x^2}{\underbrace{(x^2+4)(x^2+9)}_{\text{pól ř. 1}}} = \frac{x^2}{2x(x^2+9)}\Big|_{x=2i} = \frac{-4}{4i(-4+9)} = -\frac{1}{5i} = -\frac{2}{10i}$$

nástroj Jm. 1 $\operatorname{res}_{z_0}\frac{P(z)}{Q(z)} = \frac{P(z_0)}{Q'(z_0)}$

$+$

$$\operatorname{res}_{3i}\frac{x^2}{(x^2+4)(x^2+9)} = \frac{x^2}{(x^2+4)\,2x}\Big|_{x=3i} = \frac{-9}{(-9+4)\cdot 6i} = \frac{-9}{-30i} = \frac{3}{10i}$$

Úloha 2. *Spočtěte*

$$\int_{-\infty}^{\infty} \frac{x}{(x^2-4x+13)^2} e^{-2ix}\,dx. = 2\pi i \, res_{2-3i} \left(\underbrace{\frac{x\, e^{-2ix}}{((x-2+3i)(x-2-3i))^2}}_{\xi} \right)$$

$\alpha = -2$

kořeny se zápor. imag. složkou. $2-3i$: pól řádu 2

$(x-2)^2 + 3^2 = (x-2+3i)(x-2-3i)$

$x = 2 \pm 3i$

$$res_{2-3i}\,\xi = \lim_{z\to 2-3i} \frac{d}{dz}\left(\cancel{(x-2+3i)^2} \frac{x e^{-2ix}}{\cancel{(x-2+3i)^2}(x-2-3i)^2}\right)$$

$$= \lim_{z\to 2-3i}\left(\frac{(e^{-2ix} - 2ix e^{-2ix})(x-2-3i)^2 - xe^{-2ix}(2(x-2-3i))}{(x-2-3i)^4}\right)$$

$$= \frac{(e^{-4i-6} - 2i(2-3i)e^{-4i-6})\underbrace{(-6i)^2}_{6^2} - ((2-3i)e^{-4i-6}(2\cdot(-6i)))}{(-6i)^4}$$

$$= \frac{6^2 - 2^2\cdot 6^2\cdot i + 6^3 - (-24i-18)}{6^4} e^{-4i-6}$$

$$= \frac{6^2 - 2^2\cdot 6^2 i + 6^3 + 6\cdot 2^2 i + 6\cdot 3}{6^4} e^{-4i-6}$$

$$= e^{0}\, \frac{2\cdot 3 - 2^3\cdot 3 i + 2^2\cdot 3^2 + 2^2 i + 3}{2^3 3^3} = \frac{63 - 20i}{8\cdot 27} e^{-4i-6}$$

$$= \left(\frac{7}{24} - \frac{5}{54} i\right) e^{-4i-6}$$

$$I = 2\pi i \qquad = \left(\frac{7}{72} i + \frac{5}{27}\right)\pi e^{-4i-6}$$

Skoro stejné

$$\left(\frac{7}{27} - \frac{i}{3}\right)\pi e^{-6-4i}.$$

Úloha 3. *Spočtěte*

$$\int_{-\infty}^{\infty} \frac{e^{ix}}{(x^2-4ix-3)(x^2+1)}\,dx. = 2\pi i\left(res_i f(x) + res_{3i} f(x)\right) = I$$

$\alpha = 1$

kořeny jmenovatele s kladnou imag.

$V\, x = i$ pól řádu 2

$V\, x = 3i$ pól řádu 1

$(x^2+1) = 0 \to \pm i \to i$

$(x^2-4ix-3) = 0$

$(x-2i)^2 + 1 = (x-2i+i)(x-2i-i) = \underbrace{(x-i)}_{i}\underbrace{(x-3i)}_{3i}$

$$(x^2-4ix-3)=0$$

$$(x-2i)^2+1=(x-2i+i)(x-2i-i)=(x-i)(x-3i)$$

$$\operatorname{res}_i \frac{e^{ix}}{(x-i)^2(x-3i)(x+i)} = \lim_{z\to i}\left(\cancel{(z-i)^2}\frac{e^{ix}}{\cancel{(z-i)^2}(z-3i)(z+i)}\right)' =$$

$$= \lim_{z\to i}\frac{ie^{ix}(z-3i)(z+i)-e^{ix}\overbrace{(2z-2i)}^{i}}{\underbrace{(z-3i)^2(z+i)^2}_{4\cdot(-1)\cdot(-1)}} = \frac{\cancel{i}e^{-1}(-2\cancel{i})\cancel{(-1)}}{4} = -\frac{1}{2}e^{-1}$$

$$I = 2\pi i\left(-\frac{e^{-3}}{16i}-\frac{e^{-1}}{2}\right) = -\pi\left(\frac{e^{-3}}{8}+e^{-1}i\right) \qquad -\left(\frac{e^{-3}}{8}+\frac{e^{-1}}{2}\right)\pi.$$

$$\operatorname{res}_{3i}\frac{e^{ix}}{(x-i)^2(x-3i)(x+i)} = \left.\frac{e^{ix}}{(x-i)^2(x+i)}\right|_{x=3i} = \frac{e^{-3}}{(-4)\cdot(4i)} = \frac{e^{-3}}{-16i}$$