MP

VD: Γ L

$\Gamma \cup \{A\} \vdash B \iff \Gamma \vdash A \to B$

a) $\vdash \neg A \to (A \to B)$

A1 $\neg A \to (\neg B \to \neg A)$

VD $\neg A \vdash \neg B \to \neg A$

A3 $(\neg B \to \neg A) \to (A \to B)$

MP $\neg A \vdash A \to B$

VD $\vdash \neg A \to (A \to B)$

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b) $\vdash \neg\neg A \to A$

1 (a) $\vdash \neg\neg A \to (\neg A \to \neg\neg\neg A)$

2 VD $\neg\neg A \vdash \neg A \to \neg\neg\neg A$

3 A3 $(\neg A \to \neg\neg\neg A) \to (\neg\neg A \to A)$

4 MP $\neg\neg A \vdash \neg\neg A \to A$

5 VD $\neg\neg A \vdash A$

6 VD $\vdash \neg\neg A \to A$

c) $\vdash A \to \neg\neg A$

(b) $\neg\neg\neg A \to \neg A$

(A3) $(\neg\neg\neg A \to \neg A) \to A \to \neg\neg A$

MP $\vdash A \to \neg\neg A$

d) $(A \to B) \to (\neg B \to \neg A)$

1 predp. $A \to B \vdash A \to B$

2 predp. $\neg\neg A \vdash \neg\neg A$

3 (b) $\vdash \neg\neg A \to A$

4 ~~VD~~ MP $\neg\neg A$ ~~$\to B$~~ $\vdash A$

5 MP 4,4. $A \to B, \neg\neg A \vdash B$

6 (c) $\vdash B \to \neg\neg B$

7 MP $A \to B, \neg\neg A \vdash \neg\neg B$

8 VD $A \to B \vdash \neg\neg A \to \neg\neg B$

9 A3 $(\neg\neg A \to \neg\neg B) \to (\neg B \to \neg A)$

10 MP $A \to B \vdash \neg B \to \neg A$

11 VD $\vdash (A \to B) \to (\neg B \to \neg A)$ ↓triv.

$\frac{A, A \to B}{B}$

pr. $A \to B \vdash A \to B$

(b) $\neg\neg A \to A$

VD $\neg\neg A \vdash A$

$\exists x\, p(x,y)$ < $t = {}^{y}_{x}$ nová sub$ za y — $\exists x\, p(x,x)$

X

$\vdash \varphi \overset{MG}{\Longrightarrow} \vdash \forall x \varphi$ $\quad\not\Longrightarrow\quad \vdash \varphi \to \forall x \varphi$

ax. subs. $\vdash \forall x \varphi \to \varphi \iff \forall x \varphi \vdash \varphi$

VD $T \vdash \varphi \to \psi \iff T \cup \{\varphi\} \vdash \psi$

pokud φ je uzavřená (všechny proměnné vázané)

$\varphi \overset{MG}{\vdash} \forall x \varphi$ $\quad\not\Longrightarrow\quad \varphi \to \forall x \varphi$

$\forall x \varphi \vdash \varphi \overset{VD}{\iff} \vdash \forall x \varphi \to \varphi$ (AS)

2. pred. sym φ

$\varphi(x,y) \vdash \varphi(y,x)$

1	Pred	$\varphi(x,y)$	5 MP	$\forall y\, \varphi(y,\check{x})$	9 AS $\vdash \forall \check{x}(\varphi(\check{y},\check{x}) \to \varphi(y,x))$
2	MG	$\forall y \varphi(x,y)$	6 AS	$\forall y\, \varphi(y,\check{x}) \to \varphi(y,\check{x})$	10 MP $\varphi(x,y) \vdash \varphi(y,x)$
3	MG	$\forall \check{x} \forall y\, \varphi(x,y)$	7 MP	$\varphi(y,\check{x})$	
4	AS	$\forall \check{x} \forall y\, \varphi(x,y) \to \forall y\, \varphi(y,\check{x})$	8 MG	$\forall \check{x}\, \varphi(y,\check{x})$	

$y \to \check{x}$

$\varphi(x,y) \rightsquigarrow \forall x \varphi(x,y) \rightsquigarrow \forall y \forall x \varphi(x,y)$
$\rightsquigarrow \forall x \varphi(x,z) \rightsquigarrow \varphi(y,z) \rightsquigarrow \forall z\, \varphi(y,z)$
$\rightsquigarrow \varphi(y,x)$

př. $\varphi(x,y)$

MG $\forall x \varphi(x,y)$

MG $\forall y \forall x \varphi(x,y)$

AS $\forall y \forall x \varphi(x,y) \to \forall x \varphi(x,z)$

MP $\forall x \varphi(x,z)$

AS $\forall x \varphi(x,z) \to \varphi(y,z)$

MP $\varphi(y,z)$

MG $\forall z\, \varphi(y,z)$

AS $\forall z\, \varphi(y,z) \to \varphi(y,x)$

MP $\varphi(y,x)$

$\varphi(x,y) \vdash \varphi(y,x)$

$\forall x(\varphi \to \psi) \to (\exists x\varphi \to \exists x\psi)$

$T = (A \to B) \to (\neg B \to \neg A)$

$\eta = \forall x(\varphi \to \psi) \to (\forall x\varphi \to \forall x\psi)$

0) předp. $\forall x(\varphi \to \psi)$

1) AS $\vdash \forall x(\varphi \to \psi) \to (\varphi \to \psi)$

2) MP $\varphi \to \psi$

3) T $(\varphi \to \psi) \to (\neg\psi \to \neg\varphi)$

4) MP ~~$\varphi \to \psi$~~ $\neg\psi \to \neg\varphi$

5) MG $\forall x(\neg\psi \to \neg\varphi)$

6) η ~~$\forall x(\varphi \to \psi) \to (\forall x\varphi \to \forall x\psi)$~~ $\forall x(\neg\psi \to \neg\psi) \to (\forall x\neg\psi \to \forall x\neg\varphi)$

7) MP $\forall x\neg\psi \to \forall x\neg\varphi$

8) T $\vdash (\forall x\neg\psi \to \forall x\neg\varphi) \to (\neg\forall x\neg\varphi \to \neg\forall x\neg\psi)$

9) MP $\forall x(\varphi \to \psi) \vdash \neg\forall x\neg\varphi \to \neg\forall x\neg\psi$

10) zjednodušení $\neg\forall x\neg \equiv \exists x$ $\forall x(\varphi \to \psi) \vdash \exists x\varphi \to \exists x\psi$

11) VD $\vdash \forall x(\varphi \to \psi) \to (\exists x\varphi \to \exists x\psi)$

uzavř.

$\alpha \equiv$

$M \models \alpha$	$M \models \neg\alpha$
X	X
✓	X
X	✓

2|4

2 4

2-4 ≠ 4-2

1) $\alpha \equiv p(x,y) \to f(x,y) = f(y,x)$

2 2 2,2 2,2

2) $p(e,x) \to p(y, f(x,y))$

3) $\exists x \forall y (p(x,y) \to \neg p(y,x))$

$\forall x \exists y (p(x,y) \wedge p(y,x))$

Realizace M jazyka L: = bin. fun. f, nul. fun. e, pred p (2), univerzum $\mathbb{Z}$

$f_M(a,b) = a-b$

$e_M = 0$ $\quad p_M(a,b) \Leftrightarrow \exists k \in \mathbb{Z}\ ka = b$