$\varphi \equiv \forall x \exists y.\ (x<y) \wedge \exists z.\ (y<z)$

$\boxed{\varphi \equiv \forall x \exists y \forall z.\ (x<y) \vee (y<z)}$ PRENEX. FORMA

$\cancel{\forall x}\ \exists u (p(u,y) \to \exists y \exists z (\cancel{\forall x}.\ g(y,z) \to \exists v.\ p(v,z)))$

1. $\exists u.(p(u,y) \to \exists y \exists z.(g(y,z) \to \exists v.\ p(v,z)))$

VOLNÁ

VÁZANÁ

2. $\exists u.(p(u,y) \to \exists \bar{y} \exists z.(g(\bar{y},z) \to \exists v.\ p(v,z)))$

3. ✓ 4. ✓

5. $\exists u \exists \bar{y}.(p(u,y) \to \exists z.(g(\bar{y},z) \to \exists v.\ p(v,z)))$

$\exists u \exists \bar{y} \exists z.\ p(u,y) \to (g(\bar{y},z) \to \exists v.\ p(v,z))$

$\exists u \exists \bar{y} \exists z \exists v.\ p(u,y) \to (g(\bar{y},z) \to p(v,z))$

$$\left(\underline{\exists u}.\, p(u,y)\right) \to \exists \bar{y}\, \exists z \left(g(\bar{y}, z) \to \exists v.\, p(v,z)\right)$$

$$\forall u.\quad p(u,y) \to \exists \bar{y}\, \exists z \left(g(\bar{y}, z) \to \exists v.\, p(v,z)\right)$$

$$\forall u\, \exists \bar{y}\, \exists z\, \exists v.\ p(u,y) \to \left(g(\bar{y}, z) \to p(v,z)\right)$$

TEORIE: JAZYK L, TEORIE JE MNOŽINA FORMULÍ NAD L

KAŽDÉ $\varphi \in T$ NAZÝVÁME AXIOMEM

JAZYK S BIN SYMBOLEM „<"

T_{ω}

(A1) $\forall x, y. \quad (x<y) \to \neg(y<x)$

(A2) $\forall x, y, z. \quad (x<y) \to ((y<z) \to (x<z))$

$$\equiv$$

$$((x<y) \land (y<z)) \to (x<z)$$

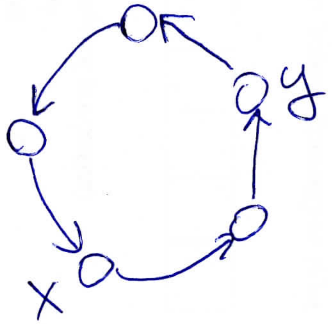
MODEL TEORIE T

- REALIZACE JAZYKA $\mathcal{M}$, T. Ž. $\forall \varphi \in T$. $\mathcal{M} \models \varphi$

OZNAČUJEME $\mathcal{M} \models T$

$(\mathbb{Z}, <) \models T_w$

$(\mathbb{Z}, >) \models T_w$



$(x<y) \wedge (y<x)$

NENÍ MODELEM T_w

$\models T_u$

AXIOMY

- 3 AX. VÝROKOVÉ LOGIKY
- Ax. substituce
  $\vdash \forall x\, \varphi \rightarrow \varphi[t]_x$
- Ax. kvantifikátoru
  $[\forall x(\boxed{\varphi} \rightarrow \psi)] \rightarrow [\varphi \rightarrow (\forall x \psi)]$
  x není volná ve φ
- Ax. rovnosti

ODVOZ. PRAVIDLA

- MODUS PONENS
- PRAVIDLO GENERALIZACE

$$\frac{\varphi}{\forall x\, \varphi}$$

KDE x je volná proměnná ve φ

$\vdash s_1 = t_1 \to (s_2 = t_2 \to (\dots (s_n = t_n \to (f(s_1, \dots, s_n) = f(t_1, \dots, t_n))) \dots)$

$\vdash s_1 = t_1 \to (s_2 = t_2 \to (\dots (s_n = t_n \to (p(s_1, \dots, s_n) \to p(t_1, \dots, t_n)) \dots)$

Lemma I.12.

$\vdash \varphi$

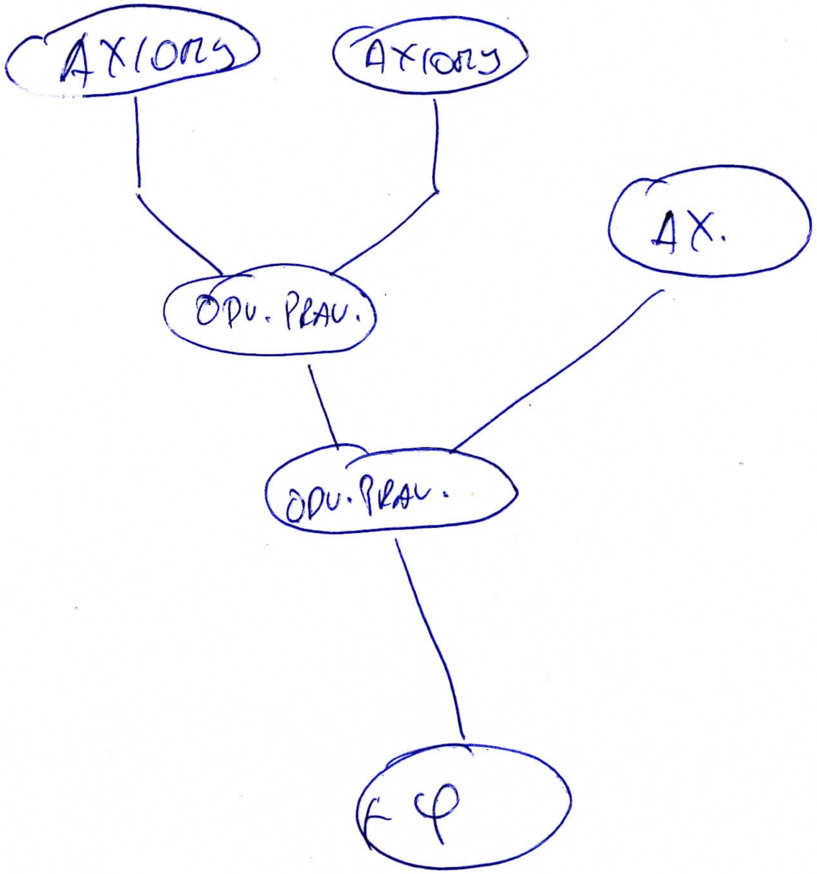
$\vdash x = y \to (\varphi \to \varphi_x[y])$

V.D

$x = y \vdash \varphi \to \varphi_x[y]$

M.P.

$x = y \vdash \varphi_x[y]$



AX.

V TEORII USPOŘÁDÁNÍ T_m DOKAŽTE, ŽE

$$\forall x, y, z, g.\ (x<y) \to ((y<z) \to ((z<g) \to (x<g)))$$

---

① Ax trans. $\vdash \forall x, y, z.\ (x<y) \to ((y<z) \to (x<z))$

Ax. substituce

② $\vdash [\forall x, y, z.\ [(x<y) \to ((y<z) \to (x<z))]] \to [(y<z) \to ((z<g) \to (y<g))]$

MODUS PONENS

③ $\vdash (y<z) \to ((z<g) \to (y<g))$

④ V.D

$(y<z) \vdash (z<g) \to (y<g)$

V.D

$(y<z), (z<g) \vdash (y<g)$

5 substituci $x \leftarrow x,\ y \leftarrow y,\ z \leftarrow g$

$$\vdash [\forall x,y,z.\ (x<y) \to ((y<z) \to (x<z))] \to [(x<y) \to ((y<g) \to (x<g))]$$

~~6 V.D~~ M.P

6 $\vdash (x<y) \to [(y<g) \to (x<g)]$

7 V.D $(x<y) \vdash (y<g) \to (x<g)$

8 M.P $(y<z), (z<g), (x<y) \vdash (X<g)$

V.D $(y<z), (x<y) \vdash (z<g) \to (x<g)$

V.D $(x<y) \vdash (y<z) \to ((z<g) \to (x<g))$

V.D $\vdash (x<y) \to ((y<z) \to ((z<g) \to (x<g)))$

4x ax. generalizace

JAZYK	P.S.	next(x,y)
		list(x)
	F.S.	nil
JAZYK S ROVNOSTÍ		

TEORIE

(A1) $\vdash \forall x.\ x = nil \rightarrow list(x)$

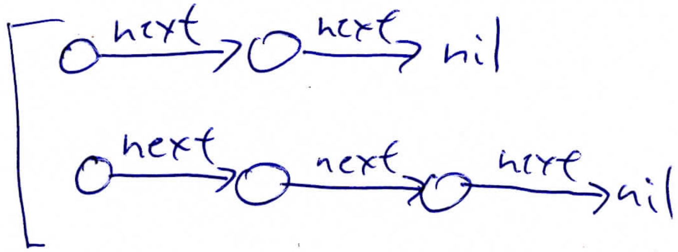
(A2) $\vdash \forall x,y.\ list(y) \wedge next(x,y) \wedge unique\_next(x) \rightarrow list(x)$

$unique\_next(x) \equiv \forall y.z.\ next(x,y) \rightarrow (next(x,z) \rightarrow x = z)$

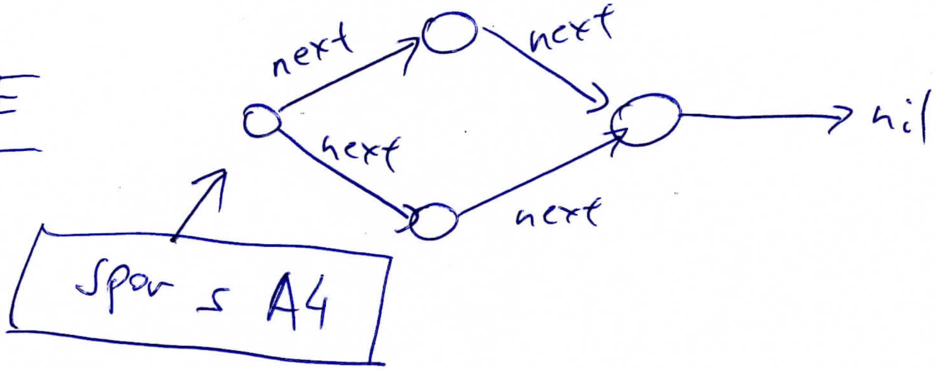
(A3) $\vdash \forall x. list(x) \wedge x \neq nil \rightarrow \exists y. (next(x,y) \wedge list(y))$

(A4) $\vdash \forall x. list(x) \wedge x \neq nil \rightarrow unique\_next(x)$

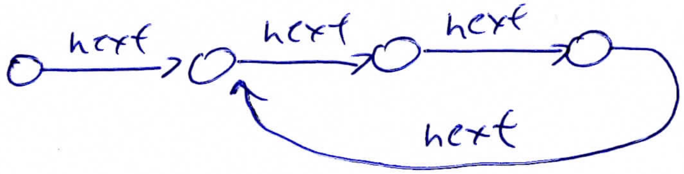
ANO


next
next
nil
next
next
next
nil

NE


next
next
next
next
nil
Spor s A4

ANO


next
next
next
next

UVAŽUJME TEORII LISTŮ T_{SLL}.

- DOPLŇTE TEORII T_{SLL} O NOVÝ AXIOM A DOKAŽTE, ŽE $\varphi \equiv \forall x, y.\ next(x, y) \land y = nil \rightarrow list(x)$

BUDEME POTŘEBOVAT DOPLNIT (A5) $\forall x, y.\ next(x, y) \rightarrow UNIQUE\_NEXT(x)$

φ: $(A \land B) \rightarrow C \equiv A \rightarrow (B \rightarrow C)$

$\varphi \equiv \forall x, y.\ next(x, y) \rightarrow (y = nil \rightarrow list(x))$

---

1 Ax. (A1) $\vdash \forall x.\ x = nil \rightarrow list(x)$

2 Ax substituce

$\vdash [\forall x.\ x = nil \rightarrow list(x)] \rightarrow y = nil \rightarrow list(y)$

$[\ \vdash \forall x.\ \varphi \rightarrow \varphi_x[y]\ ]$

3. M.P 1 + 2

$\vdash y = nil \rightarrow list(y)$

4 V.D $y = nil \vdash list(y)$

(5) Ax. (A2)

$\vdash \forall x,y.\ list(y) \wedge next(x,y) \wedge unique\_next(x) \rightarrow list(x)$

$\equiv$

$\vdash \forall x,y.\ list(y) \rightarrow (next(x,y) \rightarrow (unique\_next(x) \rightarrow list(x)))$

(6) Ax. substituce

$\vdash [\forall x,y.\ list(y) \rightarrow (next(x,y) \rightarrow (unique\_next(x) \rightarrow list(x)))]$
$\rightarrow list(y) \rightarrow (next(x,y) \rightarrow (unique\_next(x) \rightarrow list(x)))$

(7) M.P. (5) + (6)

$\vdash list(y) \rightarrow (next(x,y) \rightarrow (unique\_next(x) \rightarrow list(x)))$

(8) M.P (4) + (7)

$y = nil \vdash next(x,y) \rightarrow (unique\_next(x) \rightarrow list(x))$

(9.) V.D

$y = nil, next(x,y) \vdash unique\_next(x) \rightarrow list(x)$

10 Ax. (A5) $\vdash \forall x.y.\ next(x,y) \rightarrow unique\_next(x)$

11 Ax. substituce

$$\vdash [\forall x,y.\ next(x,y) \rightarrow unique\_next(x)] \rightarrow [next(x,y) \rightarrow unique\_next(x)]$$

12 M.P 10 + 11

$$\vdash next(x,y) \rightarrow unique\_next(x)$$

13 V.D

$$next(x,y) \vdash unique\_next(x)$$

14 M.P 9 + 13

$$y=nil, next(x,y), \cancel{next(x,y)} \vdash list(x)$$

15 2x V.D

$$next(x,y) \vdash y=nil \rightarrow list(x)$$

$$\vdash next(x,y) \rightarrow (y=nil \rightarrow list(x))$$

16 PRAVIDLO GENERALIZACE

$$\vdash \forall y.\ next(x,y) \rightarrow (y=nil \rightarrow list(x))$$

$$\vdash \forall x,y.\ next(x,y) \rightarrow (y=nil \rightarrow list(x))$$