- METRICKÝ PROSTOR (M, ρ)

M množina, $\rho: M \times M \to \mathbb{R}$ metrika

// "vzdálenost"

$\cup$

- NORMOVANÝ PROSTOR $(V, \| \|)$

V lineární prostor, $\| \|: V \to \mathbb{R}$

// vektorový // norma

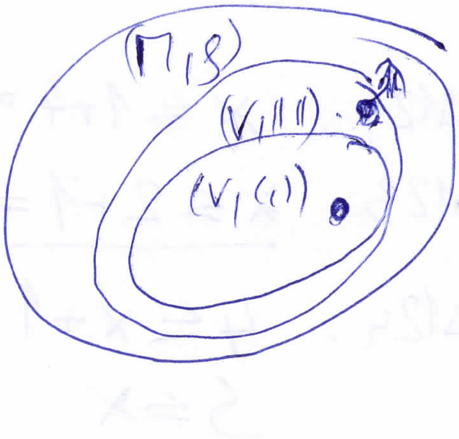
"velikost vektoru"

- UNITÁRNÍ PROSTOR $(V, (,))$

$\cup$

V lineární prostor + $(,): V \times V \to \mathbb{R}$

skalární součin



metrika indukovaná normou

$\rho(x,y) = \|x-y\|$

norma indukovaná s.s.

$\|x\| = \sqrt{(x,x)}$

Metrika ρ $\quad \forall x,y \in M$:

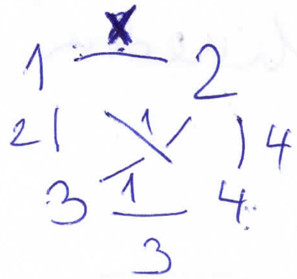
POSITIVITA 1) $\rho(x,y) \geq 0$, $\rho(x,y) = 0 \Leftrightarrow x = y$

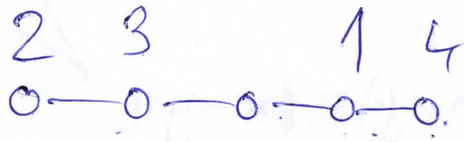
SYMETRIE 2) $\rho(x,y) = \rho(y,x)$

Δ nerovn. 3) $\rho(x,y) + \rho(y,z) \geq \rho(x,z)$

- Změňte 4 čísla tak, aby tabulka definovala metriku na $M = \{1,2,3,4\}$

	1	2	3	4
1	0	0	2	1
2	0	0	1	$-1 \to 4$
3	2	1	0	3
4	1	4	3	$1 \to 0$





($\Delta 124$: $x \leq 1+4 = 5$)

$\Delta 123$: $\underline{x \leq 2+1 = 3}$

$\Delta 124$: $4 \leq x+1$

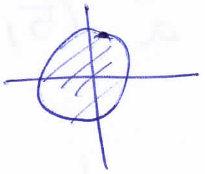
$\underline{3 \leq x}$

$\underline{\underline{x = 3}}$

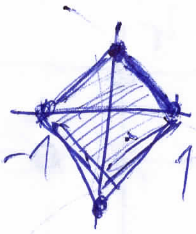
• $\mathbb{R}^n_2, \mathbb{R}^n_1, \mathbb{R}^n_\infty$. Načrtněte jednotkové koule se středem v 0 pro $n=2$.

$\mathbb{R}^2_2$: $\rho_2((0,0),(x,y)) \le 1$

$\sqrt{x^2+y^2} \le 1$

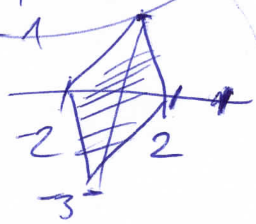


$\mathbb{R}^2_1$ $\rho_1((0,0),(x,y)) = |x|+|y| \le 1$



$\mathbb{R}^2_\infty$ $\rho_\infty((0,0),(x,y)) = \max\{|x|,|y|\} \le 1$

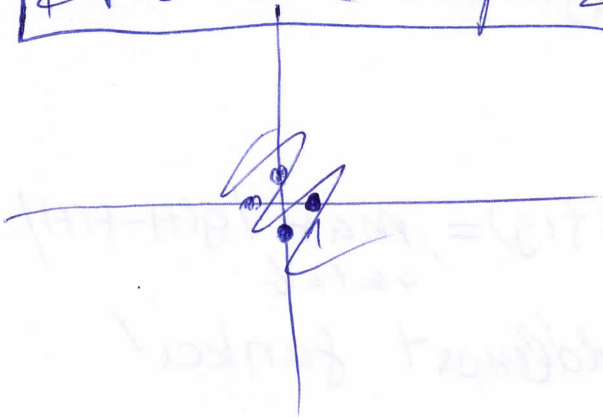
$\rho((x_1,y_1),(x_2,y_2)) = \frac{1}{2}|x_1-x_2| + \frac{1}{3}|y_1-y_2| \le 1$

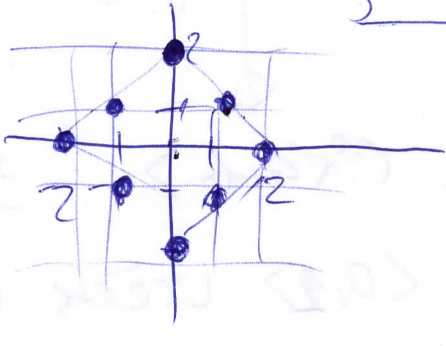


• ~~To samé pro~~ $\mathbb{Z}^2$ ~~poloměr~~

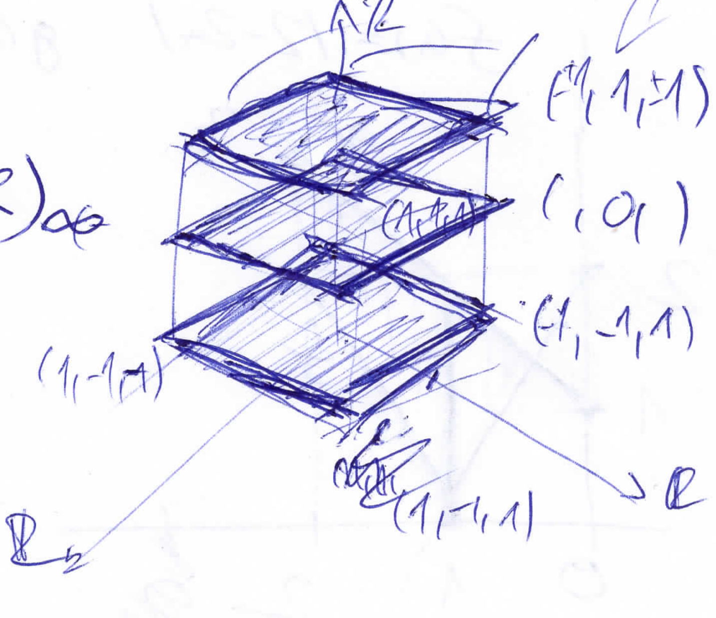
$\boxed{\text{Kružnice o pol.} = 2}$ ~~koule = 2~~

$\boxed{\rho((x,y),(0,0)) = 2}$

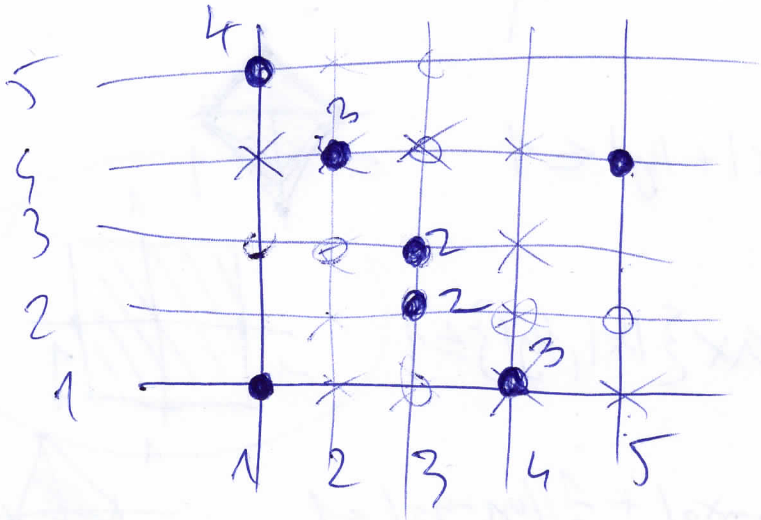




• a pro $(\mathbb{R}\times\mathbb{Z}\times\mathbb{R})_\infty$



- Na $\{1,2,3,4,5\}^2_\infty$ znázorněte body, pro které jsou stejně vzdáleny od $(1,1)$ a $(5,4)$.



$$\rho((x,y),(1,1)) = \rho((x,y),(5,4))$$

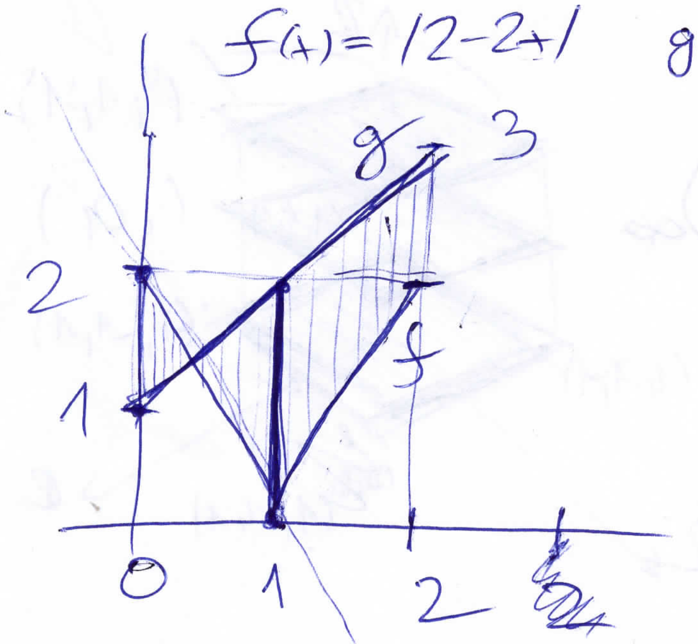
$$\max\{|x-1|,|y-1|\} = \max\{|x-5|,|y-4|\}$$

spojité funkce

- Na $C^0\langle a,b\rangle$ $\rho(f,g) = \max_{a\le t\le b} |g(t)-f(t)|$

Na $\langle 0,2\rangle$ určete vzdálenost funkcí

$f(t) = |2-2t|$ $g(t) = t+1$



$f(1) = 0$
$g(1) = 2$

$\underline{\underline{\rho(f,g) = 2}}$

Norma $\| \ \| : V \to R$

1) $\|x\| \geq 0$, $\|x\| = 0 \iff x = \vec{0}$ POSIT.

2) $\|\alpha x\| = \alpha \cdot \|x\|$ HOMOG.

3) $\|x\| + \|y\| \geq \|x+y\|$

Metriky indukované normou $\rho(x,y) = \|x-y\|$

$\mathbb{R}^2_2$ $\|(x,y)\|_2 = \sqrt{x^2+y^2}$

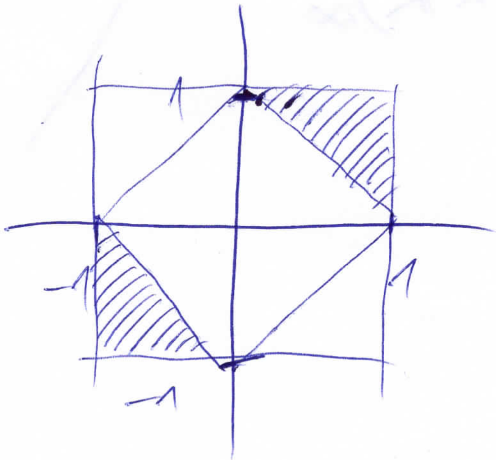
$\mathbb{R}^2_1$ $\|(x,y)\|_1 = |x| + |y|$

$\mathbb{R}^2_\infty$ $\|(x,y)\|_\infty = \max\{|x|, |y|\}$

C^0 $\|f\| = \max_{t} |f(t)|$

- Na $\mathbb{R}^2$ ($\mathbb{R}^2_1$ a $\mathbb{R}^2_\infty$) znázorněte graficky množinu $\{(x,y) \in \mathbb{R}^2 \mid x \cdot y \geq 0, \|(x,y)\|_\infty \leq 1 \leq \|(x,y)\|_1\}$

$\max\{|x|, |y|\} \leq 1 \leq |x| + |y|$



Skalární součin $(\,,\,) : V \times V \to \mathbb{R}$

1) $(x,x) \geq 0$, $(x,x)=0 \Leftrightarrow x=\vec{0}$ POSIT.

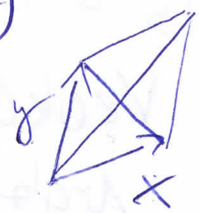
2) $(x,y) = (y,x)$ SYMETRIE

3) $(\lambda x_1 + x_2, y) = \lambda (x_1,y) + (x_2,y)$ LINEARITA

- Norma indukovaná s.s : $\|x\| = \sqrt{(x,x)}$

- normovaný = unitární $\Longleftrightarrow$

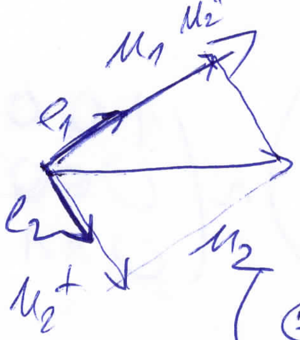
$$\|x+y\|^2 + \|x-y\|^2 = 2\left(\|x\|^2 + \|y\|^2\right)$$



- Gramm – Schmidt ortogonalizace

stand. s.s $\downarrow$ $\| \, \|_2$

- Najděte ortogonální bázi prostoru W generovaného vektory $u_1 = (2,1,2)$, $u_2 = (1,1,0)$, $u_3 = (0,-1,2)$



$$e_1 = \frac{u_1}{\|u_1\|} = \frac{(2,1,2)}{\sqrt{2^2+1^2+2^2}} = \frac{1}{3}(2,1,2) = \left(\frac{2}{3}, \frac{1}{3}, \frac{2}{3}\right)$$

$$u_2^{\perp} = u_2 - (u_2, e_1) \cdot e_1$$

$$\left(\text{ZK: } (u_2^{\perp}, e_1) = (u_2, e_1) - (u_2, e_1)\overbrace{(e_1, e_1)}^{1} = 0 \right)$$

$$u_2^{\perp} = (1,1,0) - 1 \cdot \left(\frac{2}{3}, \frac{1}{3}, \frac{2}{3}\right) = \left(\frac{1}{3}, \frac{2}{3}, -\frac{2}{3}\right)$$

$$e_2 = \frac{u_2^{\perp}}{\|u_2^{\perp}\|} = \left(\frac{1}{3}, \frac{2}{3}, -\frac{2}{3}\right)$$

$$u_3^\perp = u_3 - (u_3, e_1)e_1 - (u_3, e_2)e_2$$

$$= (0,-1,2) - 1\cdot\left(\tfrac{2}{3},\tfrac{1}{3},\tfrac{2}{3}\right) - (-2)\cdot\left(\tfrac{1}{3},\tfrac{2}{3},-\tfrac{2}{3}\right)$$

$$= (0,0,0) \Longrightarrow u_3 \text{ lin. závislé na } u_1 u_2$$

$$\langle u_1, u_2, u_3\rangle = \left\langle \left(\tfrac{2}{3},\tfrac{1}{3},\tfrac{2}{3}\right), \left(\tfrac{1}{3},\tfrac{2}{3},-\tfrac{2}{3}\right)\right\rangle = \left\langle \left(\tfrac{1}{\sqrt2},\tfrac{1}{\sqrt2},0\right), \left(\tfrac{1}{2},-\tfrac{1}{2},2\right)\right\rangle$$

$$\begin{array}{c} u_2 \\ u_1 \\ u_3 \end{array}\begin{pmatrix} 1 & 1 & 0 \\ 2 & 1 & 2 \\ 0 & -1 & 2 \end{pmatrix} \sim \boxed{\begin{pmatrix} 1 & 1 & 0 \\ 0 & -1 & 2 \\ \cancel{0} & \cancel{-1} & \cancel{2} \end{pmatrix}}$$

$$(1,1,0) \rightsquigarrow \underline{\tfrac{1}{\sqrt2}(1,1,0)}$$

$$(0,-1,2) - \left(-\tfrac{1}{\sqrt2}\right)\tfrac{1}{\sqrt2}(1,1,0) = \left(\tfrac{1}{2},-\tfrac{1}{2},2\right)$$

- V euklidovském prostoru $\mathbb{R}^4$ nalezněte ortogon. bázi W generovaného $u_1 = (1,1,1,1)$, $u_2 = (1,1,1,-1)$, $u_3 = (1,-1,-1,1)$, $u_4 = (-1,1,1,1)$

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & -1 \\ 1 & -1 & -1 & 1 \\ -1 & 1 & 1 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & -2 \\ 0 & -2 & -2 & 0 \\ 0 & 2 & 2 & 2 \end{pmatrix} \sim$$

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & -2 \\ 0 & -2 & -2 & 0 \\ \cancel{0} & \cancel{0} & \cancel{0} & \cancel{2} \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

$$W = \langle (1,0,0,0), (0,0,0,1), (0,1,1,0)\rangle$$