$\mathbb{F} = \{f_1, \dots f_n\} = \{f_1/2, \dots f_n/0\}$

$\mathbb{P} = \{p_1, \dots, p_m\} = \{p_1/3, \dots$

$\mathbb{F} = \{+, +1, 0\}$

$+/2,\ +1/1,\ 0/0$

$\mathbb{P} = \{\doteq\}$

$\doteq/2$

$T ::= x \quad (x \in \mathbb{X}) \mid f(\underbrace{T, \dots, T}_{\#_f}) \quad (\text{pro } f \in \mathbb{F})$

$F ::= p(\underbrace{T, \dots, T}_{\#_p}) \quad (\text{pro } p \in \mathbb{P}) \mid (F \wedge F) \mid (\neg F) \mid \dots \mid$

$\exists x. (F) \mid \forall x (F) \quad (\text{pro } x \in \mathbb{X})$

$\doteq(+(+1(x), 0()), 0()) \qquad (x+1)+0 \doteq 0$

$\mathbb{P} = \{=/2, \leq/2\}$

$\mathbb{F} = \emptyset$

$\forall y(\forall x((\leq(x,y) \vee \leq(y,x)) \vee =(x,y)))$

$\mathbb{F} = \{Socrates/0, Elvis/0\}$

$\mathbb{P} = \{človek/1, smrtelný/1\}$

$((\forall x.(človek(x) \to smrtelný(x)) \wedge človek(Sokrates))) \to$
$\to smrtelný(Sokrates))$

$((\forall x.(človek(x) \to smrtelný(x)) \wedge \neg smrtelný(Elvis))) \to$
$\to \neg človek(Elvis))$

volné/vázané proměnné $\quad p/2 \quad F/1$

$$\varphi \triangleq \forall x (p(F(x), y) \to \forall y . (p(F(x), y)))$$

vázaný $\quad$ volný $\quad$ vázané $\quad$ vázaný

$Q x . (\varphi)$

$Q \in \{\forall, \exists\}$

$$\exists y . \varphi(y)$$

realizace $\mathcal{M}$ jazyka PL1Ř L (FOL)

$\mathcal{M} = (M, \mathcal{L})$

$\mathcal{L}(f) : \#f = n$

$M^n \to M$

$M^{\#f} \to M$

$() \to M$

$\mathcal{L}(p) \subseteq M^{\#p}$

$M = \mathbb{N}$

$\mathcal{L}(\underline{+}) = +_{\mathbb{N}}$ $\{(0,0,0),(0,1,1),\dots\}$

$\mathcal{L}(\underline{+1}) = \lambda x . x +_{\mathbb{N}} 1$ $\quad +1(n) \mapsto n +_{\mathbb{N}} 1$

increment

$\mathcal{L}(\underline{0}) = 0_{\mathbb{N}}$

$\mathcal{L}(\underline{=}) = =_{\mathbb{N}}$

[illegible]

Π = List

$\mathcal{L}(0)$ = list* zero()

```
{
    return nullptr;
}
```

$\mathcal{L}(+1)$ = list* plus_one(list* val)

```
{
    list* of_the_jedi = new
                         list;
    of_the_jedi->next = val;
    return of_the_jedi;
}
```

```
struct list
{
    list* next;}
}
```

$\mathcal{L}(+)$ = list* plus(list* lhs, list* rhs)

```
{
    list* x = null;
    copy(x, lhs);      [lhs] -> [*]
    copy(x, rhs);      [rhs] -> [lhs] -> [*]
    return x;
}
```

$\mathcal{L}(\doteq)$ = ~~list*~~ bool equals(list* lhs, list* rhs) ohodnocení proměnných v realizaci

$e : X \longrightarrow M$

$t[e]$

$+1(0)$

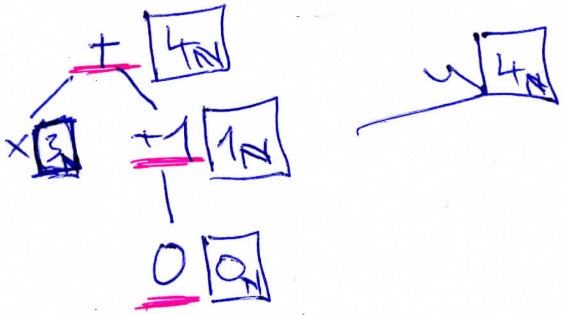
$\exists y\,(x \underline{+} 1(0) \underline{\doteq} y)$

$e(x) = 3$

$e(y/4)\ \|\exists y\,(x \underline{+} (\underline{+1}(\underline{0})) \underline{\doteq} y)\|$

$e' = \{x \mapsto 3_N, y \mapsto 4_N\}$

$\boxed{4_N}\ x \underline{+} (\underline{+1}(\underline{0})) \underline{\doteq} y\ \boxed{4_N}$



φ je splněna v M

$\forall e \in X \to M : M \models \varphi$

φ je pravdivá v M s ohodnocením e

φ je splnitelná v M

$\exists e \in X \to M :$ — " —

φ je logicky platná

$\forall$ realizace $M : M \models \varphi$

$\models \varphi$

$\forall x(\forall y(p(x,y) \to \exists z(p(z,y))))$

$\varphi \hat{=} \forall x (\exists y (y + 1 \hat{=} x))$

$e(x) = 0$

$M_{\mathbb{N}} \not\models \varphi$

$M = \mathbb{Z}$

$M_{\mathbb{Z}} \models \varphi$

je formule

$\exists y (y + 1 = 0)$

pravdivá

$\exists y (y = -1)$

$L: \mathbb{P} = \{p/2\}$

$\varphi \triangleq \exists x(p(x,x))$

$\mathcal{M}_{\mathbb{Z},=} = (\mathbb{Z}, \mathcal{L}(p) = =_{\mathbb{Z}})$

$\mathcal{M}_{\mathbb{Z},=} \models \varphi$

$\mathcal{M}_{\mathbb{Z},>} = (\mathbb{Z}, \mathcal{L}(p) = >_{\mathbb{Z}})$

$\mathcal{M}_{\mathbb{Z},>} \not\models \varphi$

$\mathcal{M}_{NSD} = (\mathbb{Z}, \mathcal{L}(p)(x,y) \iff NSD(x,y) = 1)$

$\mathcal{M}_{NSD} \overset{?}{\models} \exists x . p(x,x)$

$e = \{x \mapsto 1\}$ $p(\cancel{x,x})$

$$\mathcal{M}_{NSD} \overset{?}{\models} \psi \triangleq \exists x(\underline{\exists y}(p(x,y)))$$

$$\mathcal{M}_{NSD} \models \psi$$

$$\mathcal{M}_{NSD} \overset{?}{\models} \neg\psi =$$

$$\Leftrightarrow$$

$$\mathcal{M}_{NSD} \overset{?}{\not\models} \psi$$

$$\mathcal{M}_{NSD} \not\models \neg\psi$$

$$M_{NSD} \overset{?}{\models} p(x,y) \to \exists z (p(z,x))$$

teorie $T = \{\varphi_1, \dots, \varphi_n\}$

$L \quad \mathbb{F} = \{0/0, 1/0, +/2\}$

$\mathbb{P} = \{\doteq/2\}$

$T_L = \{$ 1) $\forall x(\neg(x + 1 \doteq 0))$,

2) $\forall x(\forall y((x + 1 \doteq y + 1) \to x \doteq y))$,

3) $\forall x(x + 0 \doteq x)$,

4) $\forall x(\forall y(x + (y + 1) \doteq (x + y) + 1))$

$\} \cup \{(\varphi(0) \wedge (\forall x(\varphi(x) \to \varphi(x + 1)))) \to \forall x(\varphi(x)) \mid$ pro formuli φ jazyka L s 1 volnou proměnnou$\}$

$M = (\mathbb{N}, \dots)$

$\mathbb{R}_0^+ \qquad Int(x)$