On Checking the Monotonicity of Parametric Interval Solution of Linear Structural Systems

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Abstract. If the solution of a parametric linear system is monotone as a function of interval parameters, then an interval hull of parametric solution can be computed easily. Some attempts to solve the problem of checking the monotonicity of parametric solution have been made in the literature. However, no complete algorithm has been given; only some directions for further research were presented. In this paper some investigations on checking monotonicity of parametric interval solution have been made. A method based on author's earlier research is presented. Some illustrative examples of structural mechanical systems are included to check the performance of the method.

Keywords: parametric linear systems, monotonicity, hull solution.

1 Introduction

In structural analysis many factors rise uncertainty or imprecision. They are connected either to external factors, such as boundary conditions or applied loads, or to internal factors, such as mechanical or geometric characteristics [1,6,7,8,9,21]. Imprecise values can be modelled using probability distributions, compact intervals or fuzzy numbers – these are the most popular approaches. Problems involving uncertain parameters should be solved properly to bound all possible responses of mechanical system.

This paper focuses on linear systems of structure mechanics with interval parameters. Methods for solving parametric linear systems have been developed in recent years [4,3,12,11,19,18]. To solve a system means to enclose parametric solution by an interval vector. The quality of the enclosure depends on the number and the width of interval parameters. The narrowest possible enclosure is called the interval hull solution.

The combinatorial approach and the monotonicity approach have been favoured by many authors in solving linear problems [14,15]. The combinatorial solution is computed as a convex hull of the solutions to 2^k (k is a number of interval parameters) real linear systems corresponding to all combinations of the endpoints of the parameter intervals. Many numerical examples show that this method yields very good results when the parameter intervals are relatively narrow.

R. Wyrzykowski et al. (Eds.): PPAM 2007, LNCS 4967, pp. 1400–1409, 2008.

When the parametric solution is monotone with respect to all the parameters, the solution set hull can be computed by solving at most $2n$ real systems. Generally it is very difficult to check the monotonicity of the solution. Some attempts to solve this problem – check the sign of derivatives of the parametric solution as a function of parameters – have been described in [5,13,16]. However, no complete algorithm has been given, and only some directions for further research were presented instead.

In this paper a new method for calculating enclosures for the derivatives of the parametric solution is presented. The new method, called here *Method for Checking the Monotonicity* (MCM for short), is based on the parametric *Direct Method* developed by I. Skalna [18].

The paper is organized as follows. The second section contains preliminaries on solving parametric interval linear systems with two disjoint sets of parameters. In the third section, the monotonicity approach for computing interval hull solution is outlined. This is followed by a description of the MCM method. Next, some illustrative examples of truss structures and the results of computational experiments are presented. The paper ends with summary conclusions.

2 Preliminaries

Italic faces will be used for real quantities, while bold italic faces will denote their interval counterparts. Let $\mathbb{IR}$ denote a set of real compact intervals $\boldsymbol{x} = [\underline{x}, \overline{x}] = \{x \in \mathbb{R} \mid \underline{x} \leqslant x \leqslant \overline{x}\}$. For two intervals $a, b \in \mathbb{IR}$, $a \geqslant b$, $a \leqslant b$ and $a = b$ will mean that, resp., $\underline{a} \geqslant \overline{b}$, $\overline{a} \geqslant \underline{b}$, and $\underline{a} = \underline{b}$ z $\overline{a} = \overline{b}$. $\mathbb{IR}^n$ will denote interval vectors, $\mathbb{IR}^{n\times n}$ square interval matrices [10]. The midpoint $\check{x} = (\underline{x} + \overline{x})/2$ and the radius $\mathrm{r}(\boldsymbol{x}) = (\overline{x} - \underline{x})/2$ are applied to interval vectors and matrices componentwise.

Consider linear algebraic system

$$A(p)x(p) = b(p) \tag{1}$$

with coefficients being affine–linear functions of a vector $p \in \mathbb{R}^k$

$$a_{ij}(p) = \alpha_{ij0} + \alpha_{ij}^{\mathrm{T}} \cdot p\,, \qquad b_j(p) = \beta_{j0} + \beta_j^{\mathrm{T}} \cdot p\ \ , \tag{2}$$

where $\alpha_{ij0}, \beta_{j0} \in \mathbb{R}$, $\alpha_{ij} = \{\alpha_{ij\nu}\} \in \mathbb{R}^k$, $\beta_j = \{\beta_{j\nu}\} \in \mathbb{R}^k$, $i, j = 1, \ldots, n$.

In practice, right-hand vector and matrix elements depend on two disjoint sets of parameters $p, q \in \mathbb{R}^k$. Hence, the system (1) can be replaced by the system

$$A(p)x(p, q) = b(q)\ \ , \tag{3}$$

and linear dependencies (2) by

$$a_{ij}(p) = \alpha_{ij0} + \alpha_{ij}^{\mathrm{T}} \cdot p\,, \qquad b_j(q) = \beta_{j0} + \beta_j^{\mathrm{T}} \cdot q\ \ , \tag{4}$$

where $\alpha_{ij0}, \beta_{j0} \in \mathbb{R}$, $\alpha_{ij} = \{\alpha_{ij\nu}\} \in \mathbb{R}^k$, $\beta_j = \{\beta_{j\nu}\} \in \mathbb{R}^l$, $i, j = 1, \ldots, n$.

Now assume that some model parameters are unknown. The real vectors p and q are replaced by interval vectors $\boldsymbol{p}$ and $\boldsymbol{q}$ (the real elements are represented by point intervals). This gives a family of the systems

$$A(p)x(p,\, q) = b(q), \quad p \in \boldsymbol{p},\, q \in \boldsymbol{q} \ , \tag{5}$$

which is usually written in a symbolic compact form

$$A(\boldsymbol{p})x(\boldsymbol{p},\, \boldsymbol{q}) = b(\boldsymbol{q}) \ , \tag{6}$$

and is called the *parametric interval linear system.* Parametric (*united*) solution set of the system (6) is defined [2,3,12,17] as

$$S(\boldsymbol{p},\, \boldsymbol{q}) = \{\, x \mid \exists p \in \boldsymbol{p},\ \exists q \in \boldsymbol{q},\ \ A(p)x(p,\, q) = b(q)\} \ . \tag{7}$$

If the solution set $\boldsymbol{S} = S(\boldsymbol{p},\, \boldsymbol{q})$ is bounded, then its interval hull exists and is defined as $\square \boldsymbol{S} = [\inf \boldsymbol{S},\, \sup \boldsymbol{S}] = \bigcap \{\boldsymbol{y} \in \mathbb{IR}^n \mid \boldsymbol{S} \subseteq \boldsymbol{y}\}$. $\square \boldsymbol{S}$ is called an *interval hull solution.* In order to guarantee that the solution set is bounded, the matrix $A(\boldsymbol{p})$ must be regular, i.e. $A(p)$ must be non-singular for all parameters $p \in \boldsymbol{p}$.

Two other solutions are defined for parametric linear systems [5]. Any vector $\boldsymbol{x} = [\underline{x},\, \overline{x}] \in \mathbb{IR}^n$ such that

$$\inf_{s \in S} s_i \leqslant \underline{x}_i \quad \text{and} \quad \sup_{s \in S} s_i \geqslant \overline{x}_i, \quad i = 1, \ldots, n \ ,$$

is referred to as the *inner solution* (*approximation* [12]) for (6). Respectively, any vector $\boldsymbol{x} = [\underline{x},\, \overline{x}] \in \mathbb{IR}^n$ such that

$$\inf_{s \in S} s_i \geqslant \underline{x}_i \quad \text{and} \quad \sup_{s \in S} s_i \leqslant \overline{x}_i, \quad i = 1, \ldots, n \ .$$

is referred to as the *outer solution* (*approximation*) for (6). The quality of outer approximation can be estimated by means of inner approximation.

3 Monotonicity Approach

When the parametric solution is monotone with respect to all parameters p_i ($i = 1, \ldots, k,$) and q_j ($j = 1, \ldots, l,$) then the interval hull solution can be computed by solving at most $2n$ real systems.

Let $E^s = \{e \in \mathbb{R}^s \mid e_i \in \{-1, 0, 1\}, i = 1, \ldots, s\}$. For $\boldsymbol{a} \in \mathbb{IR}^n$, $e \in E^n$, $a_i^e = \underline{a}$ if $e_i = -1$, $a_i^e = \check{a}$ if $e_i = 0$, and $a_i^e = \overline{a}$ if $e_i = 1$.

Theorem 1. *Let $A(\mathbf{p})$ be regular and let functions $x_i(p,\, q) = \left\{A^{-1}(p) \cdot b(q)\right\}_i$ be monotone on interval boxes $\boldsymbol{p} \in \mathbb{IR}^k$, $\boldsymbol{q} \in \mathbb{IR}^l$ with respect to each parameter p_m ($m = 1, \ldots, k$), q_r ($r = 1, \ldots, l$). Then*

$$\square S(\boldsymbol{p},\, \boldsymbol{q})_i = \left[\left\{A(p^{-e})^{-1} b(q^{-f})\right\}_i,\, \left\{A(p^{e})^{-1} b(q^{f})\right\}_i\right] \ , \tag{8}$$

where $e_\nu = \mathrm{sign}\frac{\partial x_i}{\partial p_\nu}(p,\, q)$, $f_\mu = \mathrm{sign}\frac{\partial x_i}{\partial q_\mu}(p,\, q)$.

Now consider the family of parametric linear equations (5) and assume $a_{ij}(p)$ and $b_i(q)$ $(i, j = 1, \ldots, n)$ are continuously differentiable functions in, resp., $\boldsymbol{p}$ and $\boldsymbol{q}$. Global monotonicity properties of the solution with respect to each parameter p_m, q_r can be verified by checking the sign of derivatives $\frac{\partial x}{\partial p_m}(p, q)$, $\frac{\partial x}{\partial q_r}(p, q)$ on the domains $\boldsymbol{p}$ and $\boldsymbol{q}$.

Differentiation of (5) with respect to p_m $(m = 1, \ldots, k)$ and q_r $(r = 1, \ldots, l)$ results in

$$A(p)\frac{\partial x}{\partial p_m}(p, q) = -\frac{\partial A(p)}{\partial p_m}x(p, q) \ , \tag{9}$$

$$A(p)\frac{\partial x}{\partial q_r}(p, q) = \frac{\partial b}{\partial q_r}(q) \ . \tag{10}$$

The following estimation of the derivatives $\frac{\partial x}{\partial p_m}(p, q)$ $(m = 1, \ldots, k)$ and $\frac{\partial x}{\partial q_r}(p, q)$ $(r = 1, \ldots, l)$ has been proposed in [16]:

$$\frac{\partial x}{\partial p_m}(p, q) \subseteq \boldsymbol{B}\Delta^m \boldsymbol{x}^*, \quad p \in \boldsymbol{p}, q \in \boldsymbol{q} \ , \tag{11}$$

$$\frac{\partial x}{\partial q_r}(p, q) \subseteq \boldsymbol{B}\delta^m, \quad p \in \boldsymbol{p}, q \in \boldsymbol{q} \ , \tag{12}$$

where $\boldsymbol{B}$ approximates $\square \left\{A^{-1}(p) \mid p \in \boldsymbol{p}\right\}$, $\Delta^m = -\frac{\partial A}{\partial p_m}(p)$, $\delta^m = \frac{\partial b}{\partial q_r}(q)$ and $\boldsymbol{x}^*$ is the initial enclosure for the parametric solution set (7). For the family of systems originating from equation (1) the derivatives can be estimated, as suggested by Kolev [5], by means of a formula similar to (12):

$$\frac{\partial x}{\partial p_m}(p) \subseteq \boldsymbol{B}(\delta^m - \Delta^m \boldsymbol{x}^*), \quad p \in \boldsymbol{p} \ . \tag{13}$$

The main drawback of the approach involving approximation of the inverse matrix is that when transforming equations (9) and (10) into (11) and (12), information about the system dependencies is lost.

In [13] it is stated that global monotonicity of the parametric solution of the system $A(\boldsymbol{p})x(\boldsymbol{p}) = b(\boldsymbol{p})$ can be verified by solving k parametric linear systems in a global domain $\boldsymbol{p} \in \mathbb{IR}^k$

$$A(p)\frac{\partial x(p)}{\partial p_m} = \frac{\partial b(p)}{\partial p_m} - \frac{\partial A(p)}{\partial p_m}\boldsymbol{x}^* \ .$$

However it is not clearly explained how to handle $\boldsymbol{x}^*$ in the context of solving parametric linear systems restricted to the domain $\boldsymbol{p}$.

4 Description of the MCM Method

In this paper an approach based on a Direct Method [18] for solving parametric linear systems is proposed. Bear in mind that $a_{ij}(p)$, $b_j(q)$ are affine linear functions. This implies that $\frac{\partial a_{ij}}{\partial p_m}$, $\frac{\partial b_i}{\partial q_r}$ are constant on, resp., $\boldsymbol{p}$ and $\boldsymbol{q}$. Hence, the approximations of $\frac{\partial x}{\partial p_m}(p, q)$, $\frac{\partial x}{\partial q_r}(p, q)$ can be obtained by solving the following k parametric linear systems

$$A(\boldsymbol{p})\frac{\partial x}{\partial p_m} = b'^m(\boldsymbol{x}^*) \ , \tag{14}$$

and l parametric linear systems

$$A(\boldsymbol{p})\frac{\partial x}{\partial q_r} = b''^r \ , \tag{15}$$

where $b_j'^m(x^*) = -\alpha_{ijm}x_j^*$, $b_j''^r = \beta_{jr}$, $j = 1, \ldots, n$, $x^* \in \boldsymbol{x}^*$.

For a fixed i, $1 \leqslant i \leqslant n$, let $\boldsymbol{D}_{im}^p$ denotes the estimate for $\frac{\partial x_i}{\partial p_m}(p, q)$, and $\boldsymbol{D}_{ir}^q$ the estimate for $\frac{\partial x_i}{\partial q_r}(p, q)$ obtained by solving equations (14) and (15) using the MCM method. Assume that each estimate $\boldsymbol{D}_{im}^p$, $\boldsymbol{D}_{ir}^q$ ($m = 1, \ldots, k$; $r = 1, \ldots, l$; $i = 1, \ldots, n$) meets one of the following conditions:

$$\boldsymbol{D}_{i(\cdot)}^{(\cdot)} \geqslant 0 \tag{16a}$$

$$\boldsymbol{D}_{i(\cdot)}^{(\cdot)} \leqslant 0 \tag{16b}$$

$$\boldsymbol{D}_{i(\cdot)}^{(\cdot)} = 0 \tag{16c}$$

Based on equations (16a), (16b) and (16c), vectors $\underline{p}^i$, $\overline{p}^i$, $\underline{q}^i$, $\overline{q}^i$ are defined:

$$\underline{p}_m^i = \begin{cases} \underline{p}_m \text{ if } \boldsymbol{D}_{im}^p \geqslant 0 \\ \overline{p}_m \text{ if } \boldsymbol{D}_{im}^p \leqslant 0 \\ \check{p}_m \text{ if } \boldsymbol{D}_{im}^p = 0 \end{cases}, \quad \overline{p}_m^i = \begin{cases} \overline{p}_m \text{ if } \boldsymbol{D}_{im}^p \geqslant 0 \\ \underline{p}_m \text{ if } \boldsymbol{D}_{im}^p \leqslant 0 \\ \check{p}_m \text{ if } \boldsymbol{D}_{im}^p = 0 \end{cases}, \quad m = 1, \ldots, k \ , \tag{17}$$

$$\underline{q}_r^i = \begin{cases} \underline{q}_r \text{ if } \boldsymbol{D}_{ir}^q \geqslant 0 \\ \overline{q}_r \text{ if } \boldsymbol{D}_{ir}^q \leqslant 0 \\ \check{q}_r \text{ if } \boldsymbol{D}_{ir}^q = 0 \end{cases}, \quad \overline{q}_r^i = \begin{cases} \overline{q}_r \text{ if } \boldsymbol{D}_{ir}^q \geqslant 0 \\ \underline{q}_r \text{ if } \boldsymbol{D}_{ir}^q \leqslant 0 \\ \check{q}_r \text{ if } \boldsymbol{D}_{ir}^q = 0 \end{cases}, \quad r = 1, \ldots, l \tag{18}$$

Then by Theorem 1 the i-th component of the interval hull solution

$$\square S(\mathbf{p}, \mathbf{q})_i = \left[\left\{A(\underline{p}^i)^{-1}b(\underline{q}^i)\right\}_i, \left\{A(\overline{p}^i)^{-1}b(\overline{q}^i)\right\}_i\right] \ .$$

Now suppose that the sign of some derivatives $\frac{\partial x_i}{\partial p_m}(p, q)$, $\frac{\partial x_i}{\partial q_r}(p, q)$ hasn't been determined and let $I = \{1, \ldots, k\}$, $J = \{1, \ldots, l\}$,

$$\begin{array}{l} I_+^i = \{m \in I \mid \boldsymbol{D}_{im}^p \geqslant 0\}, \ J_+^i = \{r \in J \mid \boldsymbol{D}_{ir}^q \geqslant 0\}, \\ I_-^i = \{m \in I \mid \boldsymbol{D}_{im}^p \leqslant 0\}, \ J_-^i = \{r \in I \mid \boldsymbol{D}_{ir}^q \leqslant 0\}, \\ I_0^i = \{m \in I \mid \boldsymbol{D}_{im}^p = 0\}, \ J_0^i = \{r \in I \mid \boldsymbol{D}_{ir}^q = 0\} \ . \end{array}$$

Then for each i, $1 \leqslant i \leqslant n$, new vectors of parameters

$$\boldsymbol{p}^i = \left\{\boldsymbol{p}_j\right\}_{j \in \{I \setminus (I_+^i \cup I_-^i \cup I_0^i)\}}, \quad \boldsymbol{q}^i = \left\{\boldsymbol{q}_j\right\}_{j \in \{J \setminus (J_+^i \cup J_-^i \cup J_0^i)\}} \ ,$$

are composed and a new linear systems

$$A(\boldsymbol{p}^i)x(\boldsymbol{p}^i,\ \boldsymbol{q}^i) = b(\boldsymbol{q}^i)$$

are considered. The process of determining the sign of derivatives restarts and continues separately for each new system, until no further improvement is obtained. The number of repetitions depends on the number and width of the interval parameters.

5 Numerical Examples

To check the performance of the Method for Checking the Monotonicity (MCM) some illustrative examples of structural mechanical systems have been provided. The results of the MCM method are compared with the results of the *Evolutionary Optimization Method* [20] (EOM for short) – which calculates inner approximation of the hull solution, and the results of the *Based on Inverse Matrix* method (BIM for short) – which uses the inverse of the parametric interval matrix to enclose derivatives.

Example 1. (*21-bar plane truss structure*) For the plane truss structure shown in Fig. 1 the displacements of the nodes are computed. The truss is subjected to downward forces $P_1 = P_2 = P_3 = 30$[kN] as depicted in the figure; Young's modulus $Y = 7.0 \times 10^{10}$[Pa], cross-section area $C = 0.003$[m^2], and length $L = 2$[m]. Assume the stiffness of all bars is uncertain by $\pm 5\%$. This gives 21 interval parameters.

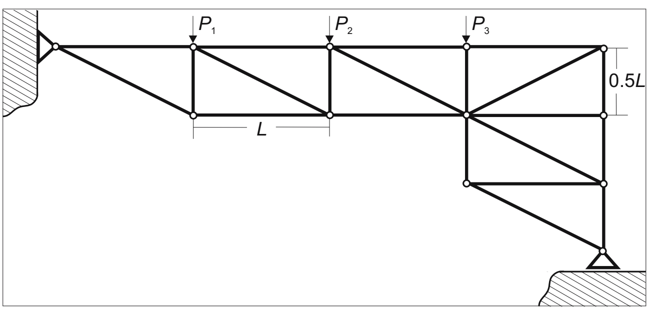

Fig. 1. Example 1: 21-bar plane truss structure

The results obtained are presented in Table 1. The number of derivatives with a definite sign is denoted by *ndsm* – for the MCM method and, respectively, *ndsb* – for the BIM method. Columns 3, 4, 5 contain the displacements of the truss nodes. Both the MCM method and the EOM method yield the same result – the interval hull solution. The results of the MCM method and the BIM method

Table 1. Comparison of the solutions for Example 1

ndsm	*ndsb*	MCM [$\times 10^{-4}$]	BIM [$\times 10^{-4}$]	EOM [$\times 10^{-4}$]
18	8	[−275.20, −247.45]	[−280.67, −241.10]	[−275.20, −247.45]
21	4	[−12.03, −10.88]	[−12.03, −10.88]	[−12.03, −10.88]
21	9	[−226.23, −204.68]	[−227.01, −203.59]	[−226.23, −204.68]
21	5	[−18.05, −16.33]	[−18.05, −16.33]	[−18.05, −16.33]
21	12	[−135.69, −122.77]	[−135.69, −122.77]	[−135.69, −122.77]
21	6	[−12.60, −10.31]	[−12.60, −10.31]	[−12.60, −10.31]
21	11	[−32.74, −26.97]	[−32.75, −26.95]	[−32.74, −26.97]
21	7	[−7.16, −4.30]	[−7.16, −4.30]	[−7.16, −4.30]
21	7	[1.22, 1.65]	[1.22, 1.65]	[1.22, 1.65]
20	7	[20.62, 25.25]	[19.93, 25.81]	[20.62, 25.25]
20	9	[−226.23, −204.68]	[−227.44, −203.15]	[−226.23, −204.68]
21	7	[20.62, 25.25]	[20.12, 25.62]	[20.62, 25.25]
21	12	[−137.20, −124.13]	[−137.21, −124.10]	[−137.20, −124.13]
21	7	[15.18, 19.24]	[14.84, 19.47]	[15.18, 19.24]
21	11	[−31.24, −25.25]	[−31.24, −25.60]	[−31.24, −25.25]
20	7	[15.18, 19.24]	[14.70, 19.61]	[15.18, 19.24]
21	6	[−0.14, 0.14]	[−0.14, 0.14]	[−0.14, 0.14]
21	9	[12.80, 16.58]	[12.73, 16.66]	[12.80, 16.58]
21	5	[−1.50, −1.36]	[−1.50, −1.36]	[−1.50, −1.36]
21	8	[1.53, 4.93]	[1.40, 5.09]	[1.53, 4.93]
21	11	[−28.42, −22.69]	[−28.43, −22.7]	69 [−2.84, −2.27]

differ for 9 elements. The number of derivatives with the definite sign *ndsm* is 2–5 times greater than *ndsb*. The MCM method generated the result four time faster than the BIM method.

Example 2. (Baltimore bridge built in 1870)
Consider the plane truss structure shown in Figure 2 subjected to downward forces of $P_1 = 80[kN]$ at node 11, $P_2 = 120[kN]$ at node 12 and P_1 at node 15; Young's modulus $Y = 2.1 \times 10^{11}$ [Pa], cross-section area $C = 0.004[m^2]$, and length $L = 1[m]$. Assume that the stiffness of 23 bars is uncertain by ±5%. This gives 23 interval parameters.

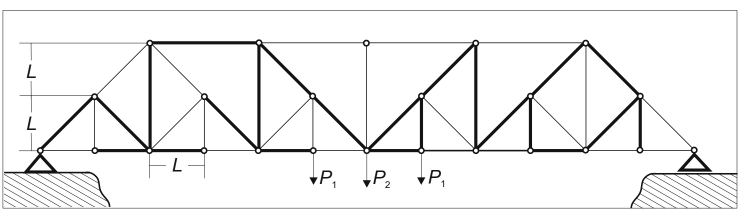

Fig. 2. Example 2: Baltimore bridge (built in 1870)

Table 2. Comparison of the number of derivatives of definite sign for Example 2

MCM	MCM	BIM	BIM
Num. of elems	*ndsm*	Num. of elems	*ndsb*
33	23	6	5
3	22	17	4
5	11	14	3
3	10	5	2
1	8	1	1
		1	0

Table 3. Comparison of the solutions for Example 2

MCM	MCM	BIM	BIM	EOM
ndsm	$[\times 10^{-4}]$	*ndsb*	$[\times 10^{-4}]$	$[\times 10^{-4}]$
10	$[-24.63, -22.90]$	2	$[-25.10, -22.38]$	$[-24.45, -23.08]$
10	$[18.37, 19.71]$	2	$[17.96, 20.10]$	$[18.55, 19.53]$
10	$[-24.63, -22.90]$	2	$[-25.10, -22.38]$	$[-24.45, -23.08]$
11	$[-40.81, -38.12]$	3	$[-41.33, -37.55]$	$[-40.52, -38.42]$
11	$[18.40, 19.68]$	3	$[18.15, 19.91]$	$[18.55, 19.53]$
11	$[-56.97, -53.36]$	3	$[-57.61, -52.66]$	$[-56.59, -53.76]$
11	$[-56.97, -53.36]$	3	$[-57.61, -52.66]$	$[-56.59, -53.76]$
8	$[16.23, 18.20]$	2	$[16.01, 18.41]$	$[16.65, 17.79]$
11	$[10.40, 12.38]$	4	$[10.20, 12.58]$	$[10.62, 12.15]$
22	$[-56.37, -53.96]$	5	$[-57.28, -53.00]$	$[-56.37, -53.96]$
22	$[-24.19, -23.31]$	5	$[-24.53, -22.96]$	$[-24.19, -23.31]$
22	$[-24.19, -23.31]$	5	$[-24.60, -22.89]$	$[-24.19, -23.31]$

The results of the MCM and the BIM methods are summarized in Table 2 and Table 3. Table 2 contains the number of elements (even columns) for which the corresponding number of derivatives have definite sign (odd columns).

Table 3 contains those elements of the solution for which the number of derivatives with definite sign *ndsm* is less than 23 (remaining elements are equal to the elements of the interval hull solution). The results of the MCM method, the BIM method, and the EOM method are presented in subsequent columns. It can be seen from the table that the result of the MCM method is far better than the result of the BIM method. The number of derivatives with the definite sign *ndsm* is 3–5 times greater than *ndsb*. The MCM method generated the result four time faster than the BIM method.

6 Conclusions

Checking the sign of the derivatives is a clue to test the global monotonicity of the solution of parametric linear systems. The global monotonicity enables calculating the interval hull solution easily by solving at most $2n$ real systems.

A new method for estimating the sign of the derivatives of the parametric interval solution is presented. The quality of the results depends on the size of the problem, and the number and width of interval parameters. In general case the MCM method produces very tight enclosure for the interval hull solution. It is quite obvious that the bigger is the number of derivatives with definite sign, the narrower are bounds of the approximate solution. If the number of derivatives with definite sign differs significantly from the number of interval parameters, then the EOM method can be used to check the overestimation.

To show the superiority of the MCM method over the methods based on calculating the inverse of the interval matrix some illustrative examples of structural mechanical systems are provided. It can be seen from the results that the MCM method produces narrower bounds on the hull solution and is much faster.

The presented methodology can be applied to any problem which requires solving linear systems with input data dependent on uncertain parameters. It can and used by, or combined with other methods for solving parametric linear systems (e.g. Evolutionary Optimization Method, Global Optimization).

References

1. Aughenbaugh, J., Paredis, C.: Why are intervals and imprecisions important in engineering design? In: Muhannah, R.L.R. (ed.) Proceedings of the NSF Workshop on Reliable Engineering Computing (REC), Svannah, Georgia USA, February 22–24, 2006, pp. 319–340 (2006)
2. Jansson, C.: Interval linear systems with symmetric matrices, skew-symmetric matrices and dependencies in the right hand side. Computing 46(3), 265–274 (1991)
3. Kolev, L.: A method for outer interval solution of linear parametric systems. Reliable Computing 10, 227–239 (2004)
4. Kolev, L.: Outer solution of linear systems whose elements are ane functions of interval parameters. Reliable Computing 6, 493–501 (2002)
5. Kolev, L.: Solving linear systems whose elements are non-linear functions of intervals. Numerical Algorithms 37, 213–224 (2004)
6. Lallemand, B., Plessis, G., Tison, T., Level, P.: Modal Behaviour of Structures Defined by Imprecise Geometric Parameters #125. In: Proc. SPIE, Proceedings of IMAC-XVIII: A Conference on Structural Dynamics. Society of Photo-Optical Instrumentation Engineers (SPIE) Conference, January 2000, vol. 4062, p. 1422 (2000)
7. Muhanna, R.L., Erdolen, A.: Geometric uncertainty in truss systems: an interval approach. In: Muhanna, R.L. (ed.) Proceedings of the NSF Workshop on Reliable Engineering Computing: Modeling Errors and Uncertainty in Engineering Computations, Savannah, Georgia USA, February 22-24, 2006, pp. 239–247 (2006)
8. Muhanna, R., Kreinovich, V., Solin, P., Cheesa, J., Araiza, R., Xiang, G.: Interval finite element method: New directions. In: Muhannah, R.L. (ed.) Proceedings of the NSF Workshop on Reliable Engineering Computing (REC), Svannah, Georgia USA, February 22–24, 2006 (2006)
9. Neumaier, A.: Worst case bounds in the presence of correlated uncertainty. In: Muhannah, R.L. (ed.) Proceedings of the NSFWorkshop on Reliable Engineering Computing (REC), February 22–24, 2006, pp. 113–114 (2006)

10. Neumaier, A.: Interval Methods for Systems of Equations. Encyclopedia of Mathematics and its Applications. Cambridge University Press, Cambridge (1990)
11. Popova, E.: Quality of the solution of parameter-dependent interval linear systems. ZAMM 82(10), 723–727 (2002)
12. Popova, E.D.: On the solution of parametrised linear systems. In: Kraemer, W., J.W.v.G. (eds.) Scientific Computing, Validated Numerics, Interval Methods, pp. 127–138. Kluwer Acad. Publishers, Dordrecht (2001)
13. Popova, E., Iankov, R., Bonev, Z.: Bounding the response of mechanical structures with uncertainties in all the parameters. In: Muhannah, R.L. (ed.) Proceedings of the NSF Workshop on Reliable Engineering Computing (REC), Svannah, Georgia USA, February 22–24, 2006, pp. 245–265 (2006)
14. Pownuk, A.: Calculations of displacement in elastic and elastic-plastic structures with interval parameters. In: 33rd Solid Mechanics Conference, Zakopane, Poland, September 2000, pp. 160–161 (2000)
15. Rao, S., Berke, L.: Analysis of uncertain structural systems using interval analysis. AIAA J. 35(4), 727–735 (1997)
16. Rohn, J.: A method for handling dependent data in interval linear systems. Technical report 911, Academy of Sciences of the Czech Republic, Czech Republic (2004)
17. Rump, S.: Verification methods for dense and sparse systems of equations. In: Herzberger, J. (ed.) Topics in validated computations: proceedings of IMACS-GAMM International Workshop on Validated Computation, Oldenburg, Germany, August 30–September 3, 1993. Studies in Computational Mathematics, vol. 5, pp. 63–135. Elsevier, Amsterdam, The Netherlands (1994)
18. Skalna, I.: A method for outer interval solution of parametrized systems of linear interval equations. Reliable Computing 12(2), 107–120 (2006)
19. Skalna, I.: Methods for solving systems of linear equations of structure mechanics with interval parameters. Computer Assisted Mechanics and Engineering Sciences 10(3), 281–293 (2003)
20. Skalna, I.: Evolutionary optimization method for approximating the solution set hull of parametric linear systems. In: Boyanov, T., Dimova, S., Georgiev, K., Nikolov, G. (eds.) NMA 2006. LNCS, vol. 4310, pp. 361–368. Springer, Heidelberg (2007)
21. Zalewski, B., Mullen, R., Muhanna, R.: Bounding the response of mechanical structures with uncertainties in all the parameters. In: Muhannah, R.L. (ed.) Proceedings of the NSF Workshop on Reliable Engineering Computing (REC), Svannah, Georgia USA, February 22–24, 2006, pp. 439–456 (2006)