String Disequalities

Let $\mathbb{X}$ be a set of *(string) variables* and Σ be a (finite) alphabet. A *string assignment* is a mapping $\sigma\colon \mathbb{X} \to \Sigma^*$, giving each variable a string value. An *automata assignment* is a mapping α assigning every variable $x \in \mathbb{X}$ a *deterministic finite automaton* (DFA). A *string disequality* is a formula of the form $x_1 \dots x_n \neq y_1 \dots y_m$ where $x_1, \dots, x_n, y_1, \dots, y_m \in \mathbb{X}$ (there can be multiple occurrences of the same variable in a disequality). A *system of string disequalities* is a conjunction of string disequalities. A string assignment σ is a *model* of a string disequality $x_1 \dots x_n \neq y_1 \dots y_m$, written as $\sigma \models x_1 \dots x_n \neq y_1 \dots y_m$, iff $\sigma(x_1) \dots \sigma(x_n) \neq \sigma(y_1) \dots \sigma(y_m)$. For instance, for $\sigma = \{x \mapsto ab, y \mapsto bab\}$, it holds that $\sigma \models xy \neq yxx$ because $\sigma(x)\sigma(y) = abbab$ and $\sigma(y)\sigma(x)\sigma(x) = bababab$. For a system of string disequalities S, we write $\sigma \models S$ (σ is a model of S) iff $\sigma \models D$ for every disequality D in S.

Problem statement. STRINGDISEQUALITIES

Input: A system of string disequalities S and an automata assignment α.

Output: *true* iff there exists a string assignment σ such that $\sigma \models S$ and for all $x \in \mathbb{X}$, it holds that $\sigma(x) \in \mathcal{L}(\alpha(x))$, *false* otherwise.

Task 1 *Characterize as precisely as possible the complexity of* STRINGDISEQUALITIES.

Hint: start with the lower bound.