Linear Rational Automata

Consider the following formal model of *linear rational automata*, which are automata with one register (denoted as r) that can contain a rational number (we denote rationals by $\mathbb{Q}$) with an unlimited precision and manipulate this register using affine transformations. Formally, a *linear rational automaton* (LRA) is a quintuple $\mathcal{A} = (Q, q_0, r_0, F, \delta)$ where

- Q is a finite non-empty set of *states*,
- $q_0 \in Q$ is the *initial state*,
- $r_0 \in \mathbb{Q}$ is the *initial value* of register r,
- $F \subseteq Q$ is the set of *final states*, and
- δ is a set of *transitions*, each of them of one of the following forms:
  1. $p \xrightarrow{r:=ar+c} q$ denoting a transition from state p to state q that updates the value of register r from r_{old} to $a \cdot r_{old} + c$ for $a, c \in \mathbb{Q}$ and
  2. $p \xrightarrow{\texttt{odd}(\lfloor r \rfloor)} q$ denoting a transition that is executable only if $\lfloor r \rfloor$ is an odd integer (the transition does not modify r).

A *run* of $\mathcal{A}$ is a sequence of consecutive transitions that respects the semantics of the transitions, it is *accepting* if it ends in a state from F, and a LRA is *fertile* if it contains at least one accepting run.

Task 1 *Characterize as precisely as possible the complexity of deciding whether a given LRA is fertile.*