Affine Integer Automata

Consider the following formal model of *affine integer automata*, which are automata with *two* registers (denoted as r_1, r_2) that can contain integer numbers (each register one number) and manipulate this register using affine transformations. Formally, an *affine integer automaton* (AIA) is a sextuple $\mathcal{A} = (Q, q_0, r_1^I, r_2^I, F, \delta)$ where

- Q is a finite non-empty set of *states*,
- $q_0 \in Q$ is the *initial state*,
- $r_1^I, r_2^I \in \mathbb{Z}$ are the *initial values* of register r_1 and r_2,
- $F \subseteq Q$ is the set of *final states*, and
- δ is a set of *transitions*, each of them of the form $p \xrightarrow[r_2:=a_2r_2+c_2]{r_1:=a_1r_1+c_1} q$, denoting a transition from state p to state q that updates the values of register r_1 and r_2 from $r_{1,old}$ to $a_1 \cdot r_{1,old} + c_1$ and from $r_{2,old}$ to $a_2 \cdot r_{2,old} + c_2$ for $a_1, a_2, c_1, c_2 \in \mathbb{Z}$.

A *run* of $\mathcal{A}$ is a sequence of consecutive transitions that respects the semantics of the transitions, it is *accepting* if it ends in a state from F, and an AIA is *converging* if for at least one accepting run, it holds that at the end of the run, the values of r_1 and r_2 are the same.

Task 1 *Characterize as precisely as possible the complexity of deciding whether a given AIA is converging.*