This is a fix of the algorithm for computing the maximum direct simulation on a nondeterministic finite automaton (NFA) from [1]. An NFA is a quintuple $\mathcal{A} = (Q, \Sigma, \delta, I, F)$ where $Q$ is a finite set of *states*, $\Sigma$ is a finite nonempty *alphabet*, $\delta\colon Q \times \Sigma \to 2^Q$ is the *transition function*, $I \subseteq Q$ is the set of *initial states*, and $F \subseteq Q$ is the set of *final states*. We define $\delta^r\colon Q \times \Sigma \to 2^Q$ to be the reverse of $\delta$, i.e., $\delta^r(q', a) = \{q \in Q \mid q' \in \delta(q, a)\}$.

A *(direct) simulation* is a relation $\preceq\, \subseteq Q \times Q$ such that if $p \preceq q$, then

1. if $p \in F$ then $q \in F$ and
2. for all $a \in \Sigma$ it holds that if $p' \in \delta(p, a)$, then $\exists q' \in \delta(q, a)$ such that $p' \preceq q'$.

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**Algorithm 1:** Computation of the maximum direct simulation $\preceq$

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**Input:** NFA $\mathcal{A} = (Q, \Sigma, \delta, I, F)$
**Output:** Maximum direct simulation $\preceq$ on $\mathcal{A}$

**foreach** $q \in Q, a \in \Sigma$ **do** `// preprocessing`
  compute $\delta^r(q, a)$ as a linked list;
*worklist* $\leftarrow$ empty;
$R \leftarrow \emptyset$;
**foreach** $p \in Q, q \in Q, a \in \Sigma$ **do** `// initial refinement`
  $\texttt{cnt}_a(p, q) \leftarrow |\delta(q, a)|$;
  **if** $(p \in F \wedge q \notin F) \vee (\delta(p, a) \neq \emptyset \wedge \delta(q, a) = \emptyset)$ **then**
    $R \leftarrow R \cup \{(p, q)\}$;
    *worklist*.enqueue($(p, q)$);
**while** *worklist* $\neq$ empty **do** `// propagate until fixpoint`
  $(p', q') \leftarrow$ *worklist*.dequeue();
  **foreach** $a \in \Sigma$ **do**
    **foreach** $q \in \delta^r(q', a)$ **do**
      $\texttt{cnt}_a(p', q) \leftarrow \texttt{cnt}_a(p', q) - 1$;
      **if** $\texttt{cnt}_a(p', q) = 0$ **then** // $q$ can't go over $a$ above $p'$
        **foreach** $p \in \delta^r(p', a)$ **do**
          **if** $(p, q) \notin R$ **then**
            $R \leftarrow R \cup \{(p, q)\}$;
            *worklist*.enqueue($(p, q)$);
**return** $Q^2 \setminus R$;

---

$$\begin{tikzcd}
q \arrow[r, "a"] \arrow[d, dash, dashed, "\preceq"'] & q' \arrow[d, dash, dashed, "\not\preceq"] \\
p \arrow[r, "a"] & p'
\end{tikzcd}$$

## References


[1] Lucian Ilie, Gonzalo Navarro, and Sheng Yu. On NFA reductions. In Juhani Karhumäki, Hermann A. Maurer, Gheorghe Paun, and Grzegorz Rozenberg, editors, *Theory Is Forever, Essays Dedicated to Arto Salomaa on the Occasion of His 70th Birthday*, volume 3113 of *Lecture Notes in Computer Science*, pages 112–124. Springer, 2004.