

This is a fix of the algorithm for computing the maximum direct simulation on a nondeterministic finite automaton (NFA) from [1]. An NFA is a quintuple $\mathcal{A} = (Q, \Sigma, \delta, I, F)$ where Q is a finite set of *states*, Σ is a finite nonempty *alphabet*, $\delta: Q \times \Sigma \rightarrow 2^Q$ is the *transition function*, $I \subseteq Q$ is the set of *initial states*, and $F \subseteq Q$ is the set of *final states*. We define $\delta^r: Q \times \Sigma \rightarrow 2^Q$ to be the reverse of δ , i.e., $\delta^r(q', a) = \{q \in Q \mid q' \in \delta(q, a)\}$.

A (*direct*) *simulation* is a relation $\preceq \subseteq Q \times Q$ such that if $p \preceq q$, then

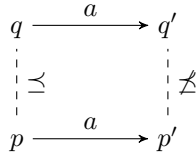
1. if $p \in F$ then $q \in F$ and
2. for all $a \in \Sigma$ it holds that if $p' \in \delta(p, a)$, then $\exists q' \in \delta(q, a)$ such that $p' \preceq q'$.

Algorithm 1: Computation of the maximum direct simulation \preceq

Input: NFA $\mathcal{A} = (Q, \Sigma, \delta, I, F)$
Output: Maximum direct simulation \preceq on \mathcal{A}

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1 foreach  $q \in Q, a \in \Sigma$  do                                     // preprocessing
2   | compute  $\delta^r(q, a)$  as a linked list;
3  $worklist \leftarrow$  empty;
4  $R \leftarrow \emptyset$ ;
5 foreach  $p \in Q, q \in Q, a \in \Sigma$  do                             // initial refinement
6   |  $cnt_a(p, q) \leftarrow |\delta(q, a)|$ ;
7   | if  $(p \in F \wedge q \notin F) \vee (\delta(p, a) \neq \emptyset \wedge \delta(q, a) = \emptyset)$  then
8   |   |  $R \leftarrow R \cup \{(p, q)\}$ ;
9   |   |  $worklist.enqueue((p, q))$ ;
10 while  $worklist \neq$  empty do                                     // propagate until fixpoint
11   |  $(p', q') \leftarrow worklist.dequeue()$ ;
12   | foreach  $a \in \Sigma$  do
13   |   | foreach  $q \in \delta^r(q', a)$  do
14   |   |   |  $cnt_a(p', q) \leftarrow cnt_a(p', q) - 1$ ;
15   |   |   | if  $cnt_a(p', q) = 0$  then //  $q$  can't go over  $a$  above  $p'$ 
16   |   |   |   | foreach  $p \in \delta^r(p', a)$  do
17   |   |   |   |   | if  $(p, q) \notin R$  then
18   |   |   |   |   |   |  $R \leftarrow R \cup \{(p, q)\}$ ;
19   |   |   |   |   |   |  $worklist.enqueue((p, q))$ ;
20 return  $Q^2 \setminus R$ ;
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References

- [1] Lucian Ilie, Gonzalo Navarro, and Sheng Yu. On NFA reductions. In Juhani Karhumäki, Hermann A. Maurer, Gheorghe Paun, and Grzegorz Rozenberg, editors, *Theory Is Forever, Essays Dedicated to Arto Salomaa on the Occasion of His 70th Birthday*, volume 3113 of *Lecture Notes in Computer Science*, pages 112–124. Springer, 2004.