This is a fix of the algorithm for computing the maximum direct simulation on a nondeterministic finite automaton (NFA) from [1]. An NFA is a quintuple $\mathcal{A} = (Q, \Sigma, \delta, I, F)$ where Q is a finite set of *states*, Σ is a finite nonempty *alphabet*, $\delta \colon Q \times \Sigma \to 2^Q$ is the *transition function*, $I \subseteq Q$ is the set of *initial states*, and $F \subseteq Q$ is the set of *final states*. We define $\delta^r \colon Q \times \Sigma \to 2^Q$ to be the reverse of δ, i.e., $\delta^r(q', a) = \{q \in Q \mid q' \in \delta(q, a)\}$.

A *(direct) simulation* is a relation $\preceq \,\subseteq Q \times Q$ such that if $p \preceq q$, then

1. if $p \in F$ then $q \in F$ and

2. for all $a \in \Sigma$ it holds that if $p' \in \delta(p, a)$, then $\exists q' \in \delta(q, a)$ such that $p' \preceq q'$.

Algorithm 1: Computation of the maximum direct simulation $\preceq$

Input: NFA $\mathcal{A} = (Q, \Sigma, \delta, I, F)$
Output: $R = \,\preceq$

```
 1 foreach q ∈ Q, a ∈ Σ do                          // preprocessing
 2  |   compute δ^r(q, a) as a linked list;
 3 worklist ← empty;
 4 R ← ∅;
 5 foreach p ∈ Q, q ∈ Q, a ∈ Σ do                    // initial refinement
 6  |   cnt_a(p, q) ← |δ(q, a)|;
 7  |   if (p ∈ F ∧ q ∉ F) ∨ (δ(p, a) ≠ ∅ ∧ δ(q, a) = ∅) then
 8  |    |   R ← R ∪ {(p, q)};
 9  |    |   worklist.enqueue((p, q));
10 while worklist ≠ empty do                         // propagate until fixpoint
11  |   (p', q') ← worklist.dequeue();
12  |   foreach a ∈ Σ do
13  |    |   foreach q ∈ δ^r(q', a) do
14  |    |    |   cnt_a(p', q) ← cnt_a(p', q) − 1;
15  |    |    |   if cnt_a(p', q) = 0 then    // q can't go over a above p'
16  |    |    |    |   foreach p ∈ δ^r(p', a) do
17  |    |    |    |    |   if (p, q) ∉ R then
18  |    |    |    |    |    |   R ← R ∪ {(p, q)};
19  |    |    |    |    |    |   worklist.enqueue((p, q));
20 return Q² \ R;
```

$$q \xrightarrow{\;\;a\;\;} q'$$
$$\Big\downarrow \preceq \qquad\qquad \Big\downarrow \not\preceq$$
$$p \xrightarrow{\;\;a\;\;} p'$$
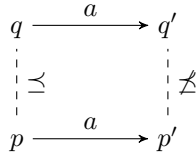
References

[1] Lucian Ilie, Gonzalo Navarro, and Sheng Yu. On NFA reductions. In Juhani Karhumäki, Hermann A. Maurer, Gheorghe Paun, and Grzegorz Rozenberg, editors, *Theory Is Forever, Essays Dedicated to Arto Salomaa on the Occasion of His 70th Birthday*, volume 3113 of *Lecture Notes in Computer Science*, pages 112–124. Springer, 2004.