

# Fully Automated Shape Analysis Based on Forest Automata<sup>†</sup>

Parosh A. Abdulla   **Lukáš Holík**   Bengt Jonsson  
**Ondřej Lengál**   Cong Quy Trinh   Adam Rogalewicz  
Jiří Šimáček   **Tomáš Vojnar**

Brno University of Technology, Czech Republic  
Uppsala University, Sweden

5<sup>th</sup> WAVAS

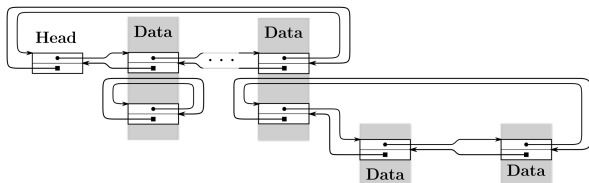
---

<sup>†</sup>Published in *Proc. of CAV'11, CAV'13, ATVA'13*

# Shape Analysis

## ■ Shape analysis:

- reasoning about programs with dynamic linked data structures
- notoriously **difficult**: infinite sets of complex graphs

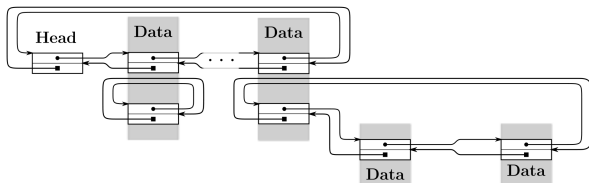


- **memory safety**: invalid dereferences, double free, memory leakage
- **error line reachability** (assertions), **shape invariance** (testers), ...

# Shape Analysis

## ■ Shape analysis:

- ▶ reasoning about programs with dynamic linked data structures
- ▶ notoriously **difficult**: infinite sets of complex graphs



- ▶ **memory safety**: invalid dereferences, double free, memory leakage
- ▶ **error line reachability** (assertions), **shape invariance** (testers), ...

## ■ Existing **solutions**:

- ▶ often specialized (lists)
- ▶ require human help (loop invariants, inductive predicates)
- ▶ low scalability

## ■ Separation Logic

😊 local reasoning: well scalable

☹️ fixed abstraction

## ■ Separation Logic

- 😊 local reasoning: **well scalable**
- 😞 **fixed abstraction**

## ■ Abstract Regular Tree Model Checking (ARTMC)

- 😊 uses tree automata (TA): **flexible** and **refinable abstraction**
- 😞 monolithic encoding of the heap: **limited scalability**

# The Forest Automata-based Approach

- Introduced at CAV'11.

# The Forest Automata-based Approach

- Introduced at CAV'11.
- Combines
  - 😊 flexibility of ARTMC

# The Forest Automata-based Approach

- Introduced at CAV'11.
- Combines
  - 😊 flexibility of ARTMCwith
  - 😊 scalability of SL



# The Forest Automata-based Approach

- Introduced at CAV'11.
- Combines
  - ☺ flexibility of ARTMCwith
  - ☺ scalability of SLby
  - splitting heaps into tree components

# The Forest Automata-based Approach

- Introduced at CAV'11.

- Combines

☺ flexibility of ARTMC

with

☺ scalability of SL

by

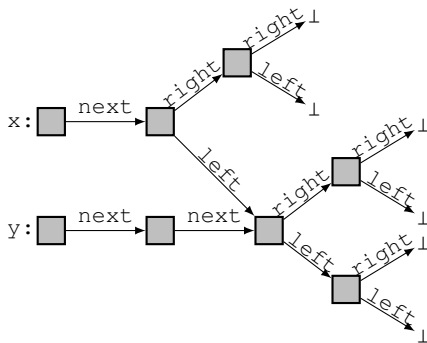
- splitting heaps into tree components

and

- using tree automata to represent sets of tree components of heaps

# Heap Representation

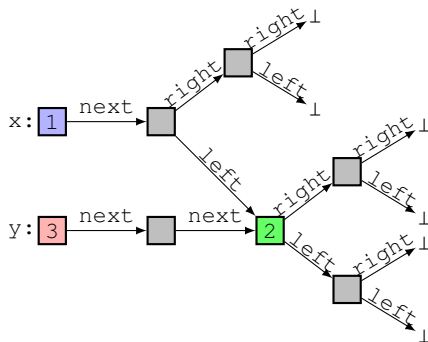
## ■ Forest decomposition of a heap



# Heap Representation

- Forest decomposition of a heap

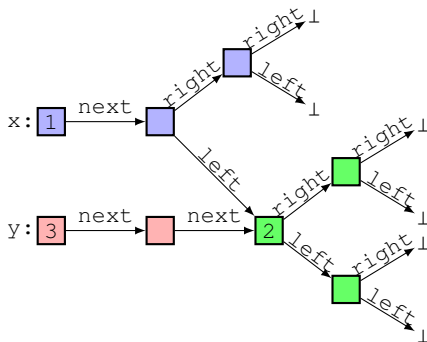
- **Forest decomposition** of a heap
- ▶ Identify **cut-points** ← nodes referenced:
    - by variables, or
    - multiple times



# Heap Representation

## ■ Forest decomposition of a heap

- ▶ Identify **cut-points** ← nodes referenced:
  - ▶ Split the heap into **tree components**
- by variables, or
  - multiple times

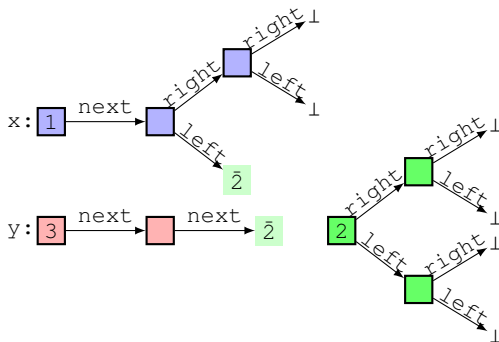


# Heap Representation

## ■ Forest decomposition of a heap

- ▶ Identify **cut-points**
- ▶ Split the heap into **tree components**
- ▶ **References** are explicit

- nodes referenced:
- by variables, or
- multiple times



# Heap Representation

■ a heap  $h \mapsto$  a forest  $(\uparrow_1, \uparrow_2, \dots, \uparrow_n)$

# Heap Representation

- a **heap**  $h \mapsto$  a **forest**  $(\blacktriangleright_1, \blacktriangleright_2, \dots, \blacktriangleright_n)$
- a **set of heaps**  $\mathcal{H} \mapsto \{(\blacktriangleright_1, \blacktriangleright_2, \dots, \blacktriangleright_n), (\blacktriangleright'_1, \blacktriangleright'_2, \dots, \blacktriangleright'_m), \dots\}$



# Heap Representation

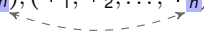
■ a **heap**  $h \mapsto$  a **forest**  $(\blacktriangleright_1, \blacktriangleright_2, \dots, \blacktriangleright_n)$

■ a **set of heaps**  $\mathcal{H} \mapsto \{(\blacktriangleright_1, \blacktriangleright_2, \dots, \blacktriangleright_n), (\blacktriangleright'_1, \blacktriangleright'_2, \dots, \blacktriangleright'_m), \dots\}$

▸ split  $\mathcal{H}$  into classes of forests with:

1 the same number of trees

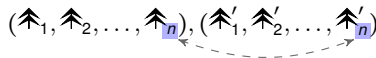
$(\blacktriangleright_1, \blacktriangleright_2, \dots, \blacktriangleright_n), (\blacktriangleright'_1, \blacktriangleright'_2, \dots, \blacktriangleright'_n)$



# Heap Representation

- a **heap**  $h \mapsto$  a **forest**  $(\blacktriangleright_1, \blacktriangleright_2, \dots, \blacktriangleright_n)$
- a **set of heaps**  $\mathcal{H} \mapsto \{(\blacktriangleright_1, \blacktriangleright_2, \dots, \blacktriangleright_n), (\blacktriangleright'_1, \blacktriangleright'_2, \dots, \blacktriangleright'_m), \dots\}$ 
  - split  $\mathcal{H}$  into classes of forests with:

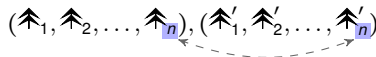
- 1 the same number of trees
- 2 having the same references



# Heap Representation

- a **heap**  $h \mapsto$  a **forest**  $(\blacktriangleright_1, \blacktriangleright_2, \dots, \blacktriangleright_n)$
- a **set of heaps**  $\mathcal{H} \mapsto \{(\blacktriangleright_1, \blacktriangleright_2, \dots, \blacktriangleright_n), (\blacktriangleright'_1, \blacktriangleright'_2, \dots, \blacktriangleright'_m), \dots\}$ 
  - split  $\mathcal{H}$  into classes of forests with:

- 1 the same number of trees
- 2 having the same references
- 3 in the same order

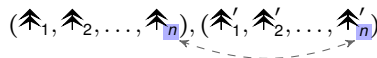


# Heap Representation

- a **heap**  $h \mapsto$  a **forest**  $(\blacktriangleright_1, \blacktriangleright_2, \dots, \blacktriangleright_n)$
- a **set of heaps**  $\mathcal{H} \mapsto \{(\blacktriangleright_1, \blacktriangleright_2, \dots, \blacktriangleright_n), (\blacktriangleright'_1, \blacktriangleright'_2, \dots, \blacktriangleright'_m), \dots\}$

- ▶ split  $\mathcal{H}$  into classes of forests with:

- 1 the same number of trees
- 2 having the same references
- 3 in the same order



- ▶ i.e., with the same **interconnection** of **tree components**

# Heap Representation

■ a **heap**  $h \mapsto$  a **forest**  $(\blacktriangleright_1, \blacktriangleright_2, \dots, \blacktriangleright_n)$

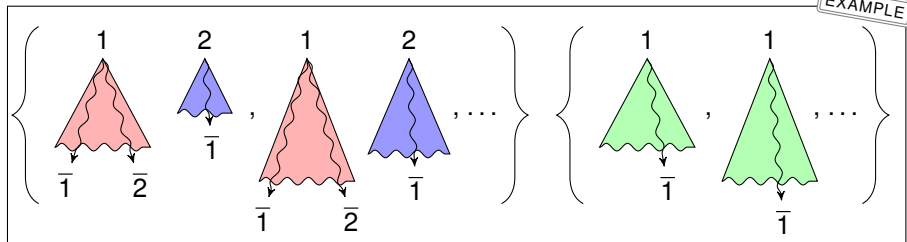
■ a **set of heaps**  $\mathcal{H} \mapsto \{(\blacktriangleright_1, \blacktriangleright_2, \dots, \blacktriangleright_n), (\blacktriangleright'_1, \blacktriangleright'_2, \dots, \blacktriangleright'_m), \dots\}$

▸ split  $\mathcal{H}$  into classes of forests with:

- 1 the same number of trees
- 2 having the same references
- 3 in the same order

$(\blacktriangleright_1, \blacktriangleright_2, \dots, \blacktriangleright_n), (\blacktriangleright'_1, \blacktriangleright'_2, \dots, \blacktriangleright'_n)$

▸ i.e., with the same **interconnection** of **tree components**



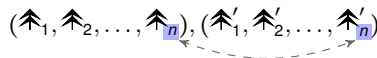
# Heap Representation

■ a **heap**  $h \mapsto$  a **forest**  $(\blacktriangleright_1, \blacktriangleright_2, \dots, \blacktriangleright_n)$

■ a **set of heaps**  $\mathcal{H} \mapsto \{(\blacktriangleright_1, \blacktriangleright_2, \dots, \blacktriangleright_n), (\blacktriangleright'_1, \blacktriangleright'_2, \dots, \blacktriangleright'_m), \dots\}$

▸ split  $\mathcal{H}$  into classes of forests with:

- 1 the same number of trees
- 2 having the same references
- 3 in the same order



▸ i.e., with the same **interconnection** of **tree components**

■ **Cartesian representation** of classes of  $\mathcal{H}$ :

$$\{(\blacktriangleright_1, \blacktriangleright_2, \dots, \blacktriangleright_n), (\blacktriangleright'_1, \blacktriangleright'_2, \dots, \blacktriangleright'_n), \dots\}$$

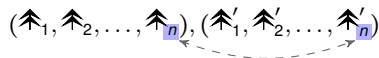
# Heap Representation

■ a **heap**  $h \mapsto$  a **forest**  $(\uparrow_1, \uparrow_2, \dots, \uparrow_n)$

■ a **set of heaps**  $\mathcal{H} \mapsto \{(\uparrow_1, \uparrow_2, \dots, \uparrow_n), (\uparrow'_1, \uparrow'_2, \dots, \uparrow'_m), \dots\}$

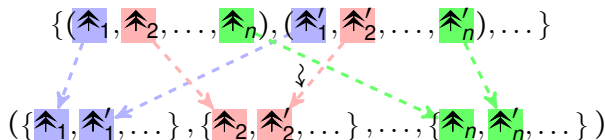
▸ split  $\mathcal{H}$  into classes of forests with:

- 1 the same number of trees
- 2 having the same references
- 3 in the same order



▸ i.e., with the same **interconnection** of **tree components**

■ **Cartesian representation** of classes of  $\mathcal{H}$ :



▸ We assume working with **rectangular classes**, i.e., for a class  $C$ ,  
 $(\uparrow, \uparrow), (\uparrow, \uparrow) \in C \Rightarrow (\uparrow, \uparrow), (\uparrow, \uparrow) \in C$ , otherwise  $C$  is

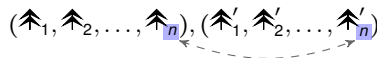
# Heap Representation

■ a **heap**  $h \mapsto$  a **forest**  $(\uparrow_1, \uparrow_2, \dots, \uparrow_n)$

■ a **set of heaps**  $\mathcal{H} \mapsto \{(\uparrow_1, \uparrow_2, \dots, \uparrow_n), (\uparrow'_1, \uparrow'_2, \dots, \uparrow'_m), \dots\}$

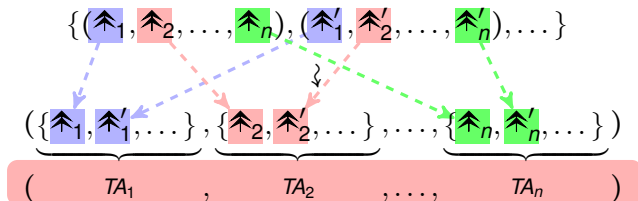
▸ split  $\mathcal{H}$  into classes of forests with:

- 1 the same number of trees
- 2 having the same references
- 3 in the same order



▸ i.e., with the same **interconnection** of **tree components**

■ **Cartesian representation** of classes of  $\mathcal{H}$ :



▸ We assume working with **rectangular classes**, i.e., for a class  $C$ ,  
 $(\uparrow, \uparrow), (\uparrow, \uparrow) \in C \Rightarrow (\uparrow, \uparrow), (\uparrow, \uparrow) \in C$ , otherwise  $C$  is



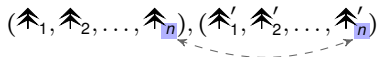
# Heap Representation

■ a **heap**  $h \mapsto$  a **forest**  $(\uparrow_1, \uparrow_2, \dots, \uparrow_n)$

■ a **set of heaps**  $\mathcal{H} \mapsto \{(\uparrow_1, \uparrow_2, \dots, \uparrow_n), (\uparrow'_1, \uparrow'_2, \dots, \uparrow'_m), \dots\}$

▸ split  $\mathcal{H}$  into classes of forests with:

- 1 the same number of trees
- 2 having the same references
- 3 in the same order

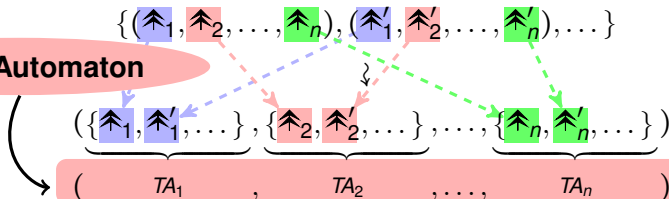


▸ i.e., with the same **interconnection** of **tree components**

■ **Cartesian representation** of classes of  $\mathcal{H}$ :

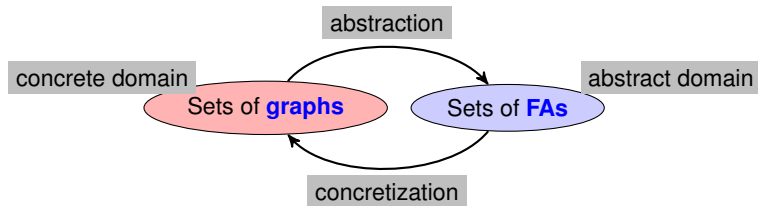
$$\{(\uparrow_1, \uparrow_2, \dots, \uparrow_n), (\uparrow'_1, \uparrow'_2, \dots, \uparrow'_n), \dots\}$$

**Forest Automaton**

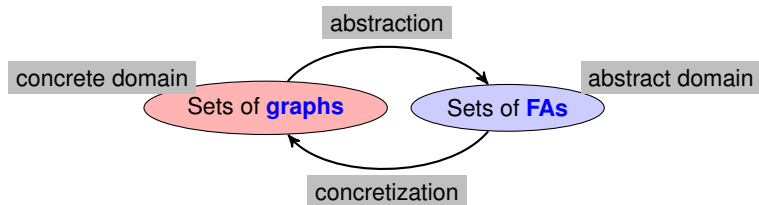


▸ We assume working with **rectangular classes**, i.e., for a class  $C$ ,  
 $(\uparrow, \uparrow), (\uparrow, \uparrow) \in C \Rightarrow (\uparrow, \uparrow), (\uparrow, \uparrow) \in C$ , otherwise  $C$  is

# Abstract Interpretation



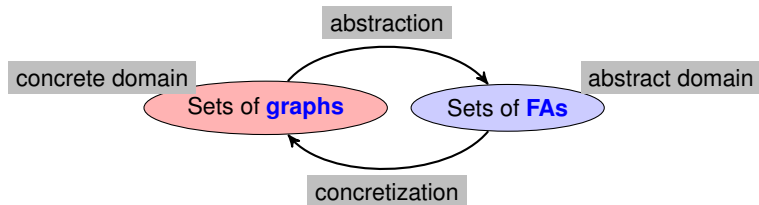
# Abstract Interpretation



## Statements

- `x := new T()`
- `delete(x)`
- `x := null`
- `x := y`
- `x := y.next`
- `x.next := y`
- `if/while (x == y)`

# Abstract Interpretation

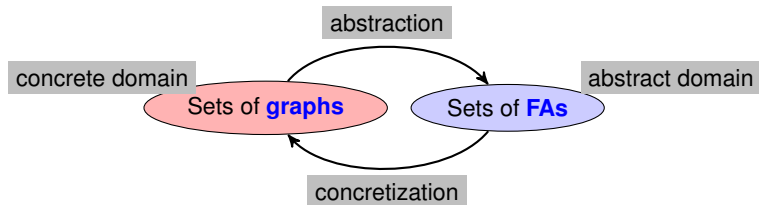


## Statements

- `x := new T()`
- `delete(x)`
- `x := null`
- `x := y`
- `x := y.next`
- `x.next := y`
- `if/while (x == y)`

## Abstract Transformers

# Abstract Interpretation



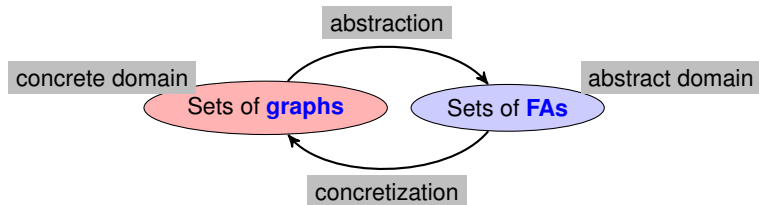
## Statements

- `x := new T()`
- `delete(x)`
- `x := null`
- `x := y`
- `x := y.next`
- `x.next := y`
- `if/while (x == y)`

## Abstract Transformers

append a TA

# Abstract Interpretation



## Statements

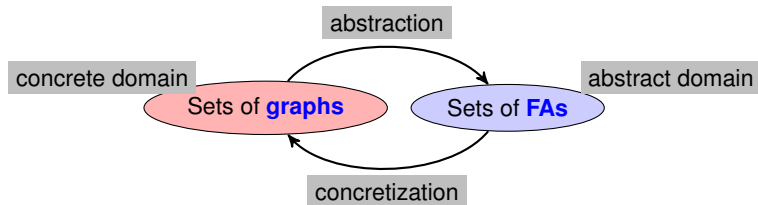
- `x := new T()`
- `delete(x)`
- `x := null`
- `x := y`
- `x := y.next`
- `x.next := y`
- `if/while (x == y)`

## Abstract Transformers

append a TA

remove a TA

# Abstract Interpretation



## Statements

- `x := new T()`
- `delete(x)`
- `x := null`
- `x := y`
- `x := y.next`
- `x.next := y`
- `if/while (x == y)`

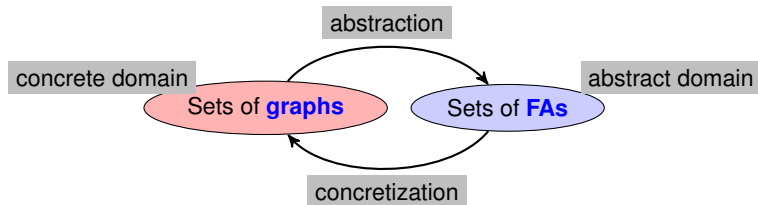
## Abstract Transformers

append a TA

remove a TA

modify transitions

# Abstract Interpretation



## Statements

- `x := new T()`
- `delete(x)`
- `x := null`
- `x := y`
- `x := y.next`
- `x.next := y`
- `if/while (x == y)`

## Abstract Transformers

append a TA

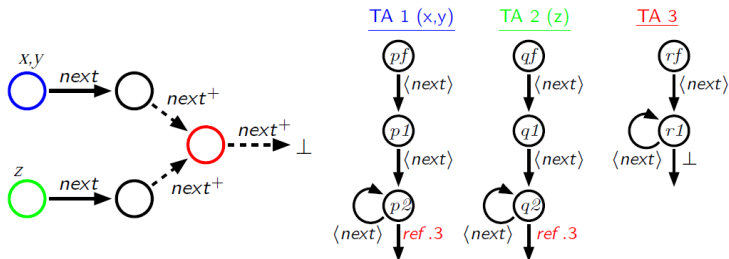
remove a TA

modify transitions

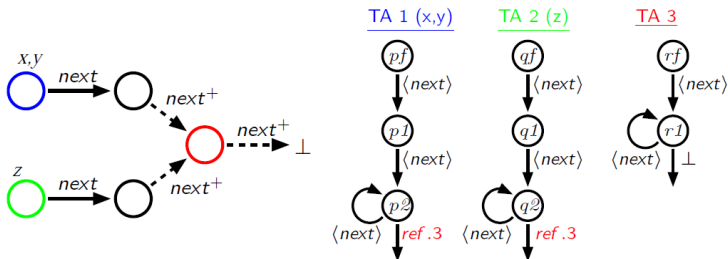
check symbols on transitions



# Abstract Transformers for Pointer Updates

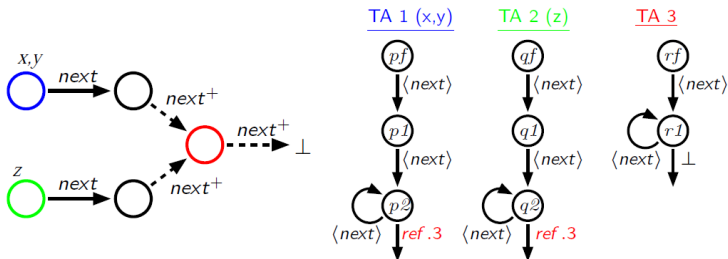


# Abstract Transformers for Pointer Updates

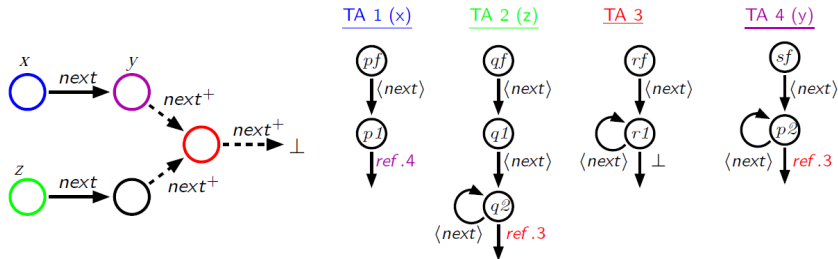


■  $y := x.next$

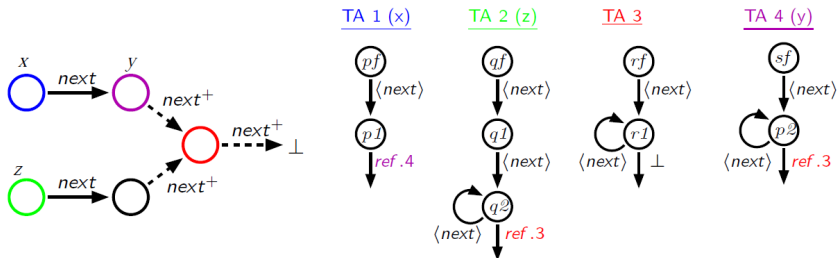
# Abstract Transformers for Pointer Updates



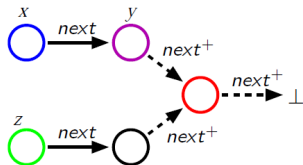
## ■ $y := x.next$



# Abstract Transformers for Pointer Updates

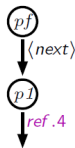


# Abstract Transformers for Pointer Updates

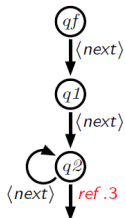


■  $x.next := z;$

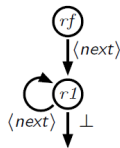
TA 1 ( $x$ )



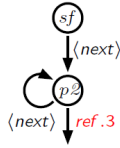
TA 2 ( $z$ )



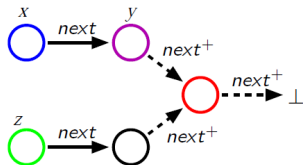
TA 3



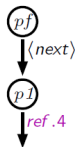
TA 4 ( $y$ )



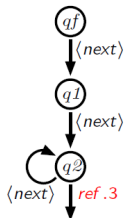
# Abstract Transformers for Pointer Updates



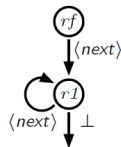
TA 1 (x)



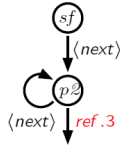
TA 2 (z)



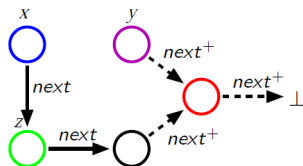
TA 3



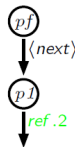
TA 4 (y)



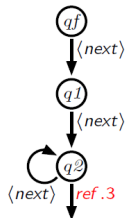
■ `x.next:=z;`



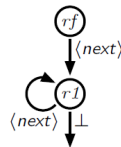
TA 1 (x)



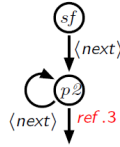
TA 2 (z)



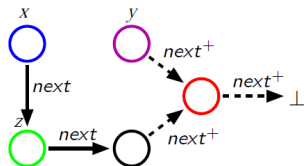
TA 3



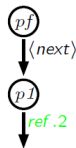
TA 4 (y)



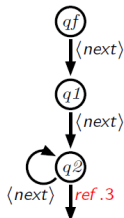
# Abstract Transformers for Pointer Updates



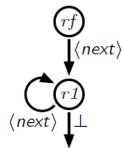
TA 1 ( $x$ )



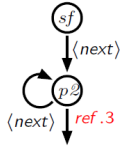
TA 2 ( $z$ )



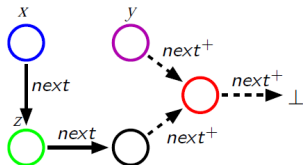
TA 3



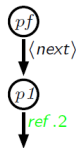
TA 4 ( $y$ )



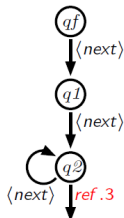
# Abstract Transformers for Pointer Updates



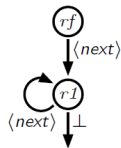
TA 1 ( $x$ )



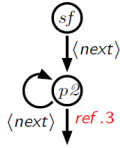
TA 2 ( $z$ )



TA 3



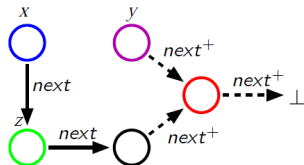
TA 4 ( $y$ )



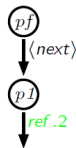
■  $Z := X;$



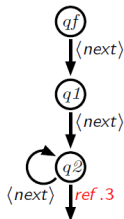
# Abstract Transformers for Pointer Updates



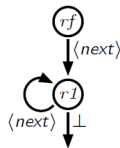
TA 1 (x)



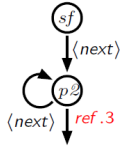
TA 2 (z)



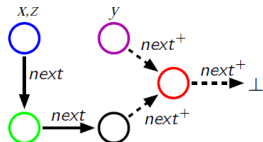
TA 3



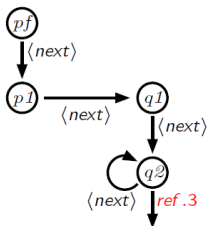
TA 4 (y)



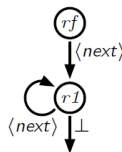
■  $Z := X;$



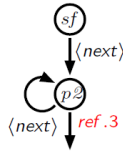
TA 1 (x,z)



TA 3



TA 4 (y)

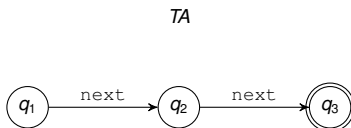


- **Abstraction** on forest automata ( $TA_1, \dots, TA_n$ )

- **Abstraction** on forest automata  $(TA_1, \dots, TA_n)$ 
  - **collapse** states of component TAs  $\leadsto (TA_1^\alpha, \dots, TA_n^\alpha)$

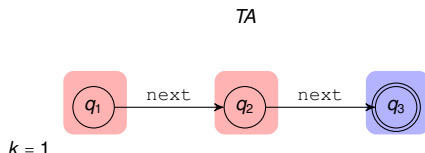
- **Abstraction** on forest automata  $(TA_1, \dots, TA_n)$ 
  - **collapse** states of component TAs  $\rightsquigarrow (TA_1^\alpha, \dots, TA_n^\alpha)$
  - **finite-height** abstraction (from ARTMC)
    - collapse states with languages whose prefixes match **up to height  $k$**

- **Abstraction** on forest automata ( $TA_1, \dots, TA_n$ )
  - **collapse** states of component TAs  $\leadsto (TA_1^\alpha, \dots, TA_n^\alpha)$
  - **finite-height** abstraction (from ARTMC)
    - collapse states with languages whose prefixes match **up to height  $k$**



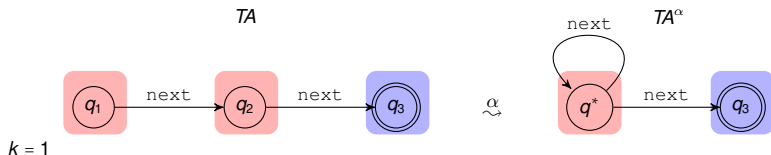
# Widening

- **Abstraction** on forest automata ( $TA_1, \dots, TA_n$ )
  - **collapse** states of component TAs  $\leadsto (TA_1^\alpha, \dots, TA_n^\alpha)$
  - **finite-height** abstraction (from ARTMC)
    - collapse states with languages whose prefixes match **up to height  $k$**



# Widening

- **Abstraction** on forest automata ( $TA_1, \dots, TA_n$ )
  - **collapse** states of component TAs  $\rightsquigarrow (TA_1^\alpha, \dots, TA_n^\alpha)$
  - **finite-height** abstraction (from ARTMC)
    - collapse states with languages whose prefixes match **up to height  $k$**



# Nondeterministic Tree Automata

- For efficiency reasons, we **never determinize** TAs.
- All operations done on NTAs, including:
  - **inclusion checking**: based on **antichains** and **simulations**,
    - discarding macro-states during an implicit subset construction,
    - inclusion on (normalized) FA can be checked **component-wise**—used for detecting the **fixpoint**
  - **size reduction**: based on **simulation equivalences**.
    - collapsing simulation-equivalent states.



# Summary

The so-far-presented:

# Summary

The so-far-presented:

- 😊 works well for **singly linked lists** (SLLs), **trees**,  
SLLs with **head/tail pointers**, trees with **root pointers**, ...

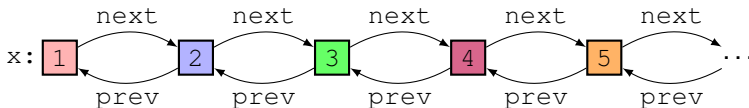
# Summary

$$(\uparrow_1, \uparrow_2, \dots, \uparrow_n) \approx (\uparrow'_1, \uparrow'_2, \dots, \uparrow'_n)$$

...

The so-far-presented:

- 😊 works well for **singly linked lists (SLLs)**, **trees**,  
SLLs with **head/tail pointers**, trees with **root pointers**, ...
- 😞 fails for more complex data structures
  - ▶ **unbounded** number of **cut-points**  $\leadsto \infty$  **classes** of  $\mathcal{H}$



- doubly linked lists (DLLs), circular lists, nested lists,
- trees with parent pointers,
- skip lists

## ■ Hierarchical Forest Automata

- FAs are **symbols** (**boxes**) of FAs of a **higher level**
- a **hierarchy** of FAs


## ■ Hierarchical Forest Automata

- FAs are **symbols** (**boxes**) of FAs of a **higher level**
- a **hierarchy** of FAs
- intuition: replace **repeated subgraphs** by a **single symbol**, **hiding** some cut-points

# Hierarchical Forest Automata

## ■ Hierarchical Forest Automata

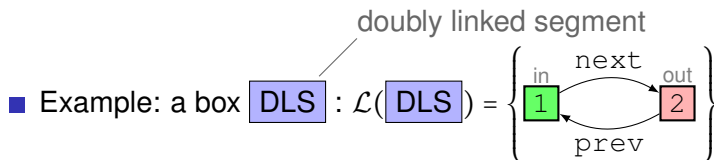
- FAs are **symbols** (**boxes**) of FAs of a **higher level**
- a **hierarchy** of FAs
- intuition: replace **repeated subgraphs** by a **single symbol**, **hiding** some cut-points

- Example: a box  doubly linked segment

# Hierarchical Forest Automata

## ■ Hierarchical Forest Automata

- FAs are **symbols** (**boxes**) of FAs of a **higher level**
- a **hierarchy** of FAs
- intuition: replace **repeated subgraphs** by a **single symbol**, **hiding** some cut-points



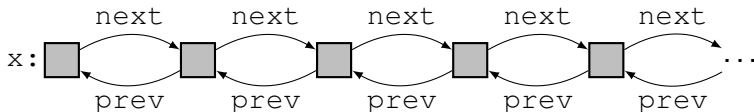
# Hierarchical Forest Automata

## ■ Hierarchical Forest Automata

- FAs are **symbols** (**boxes**) of FAs of a **higher level**
- a **hierarchy** of FAs
- intuition: replace **repeated subgraphs** by a **single symbol**, **hiding** some cut-points

■ Example: a box **DLS** :  $\mathcal{L}(\text{DLS}) = \left\{ \begin{array}{c} \text{in} \quad \text{next} \quad \text{out} \\ \text{1} \quad \text{2} \\ \text{prev} \end{array} \right\}$

doubly linked segment





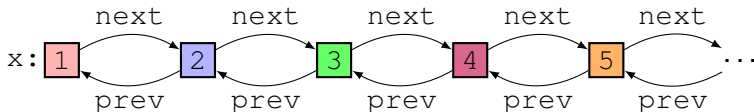
# Hierarchical Forest Automata

## ■ Hierarchical Forest Automata

- FAs are **symbols** (**boxes**) of FAs of a **higher level**
- a **hierarchy** of FAs
- intuition: replace **repeated subgraphs** by a **single symbol**, **hiding** some cut-points

■ Example: a box **DLS** :  $\mathcal{L}(\text{DLS}) = \left\{ \begin{array}{c} \text{in} \quad \text{next} \quad \text{out} \\ \boxed{1} \quad \quad \quad \boxed{2} \\ \text{prev} \end{array} \right\}$

doubly linked segment



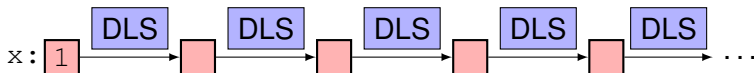
# Hierarchical Forest Automata

## ■ Hierarchical Forest Automata

- FAs are **symbols** (**boxes**) of FAs of a **higher level**
- a **hierarchy** of FAs
- intuition: replace **repeated subgraphs** by a **single symbol**, **hiding** some cut-points

■ Example: a box **DLS** :  $\mathcal{L}(\text{DLS}) = \left\{ \begin{array}{c} \text{in} \xrightarrow{\text{next}} \text{out} \\ \text{out} \xrightarrow{\text{prev}} \text{in} \end{array} \right\}$

doubly linked segment



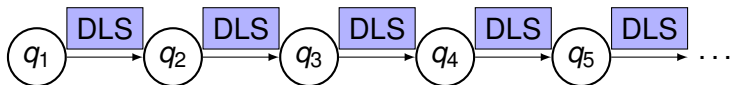
# Hierarchical Forest Automata

## ■ Hierarchical Forest Automata

- FAs are **symbols** (**boxes**) of FAs of a **higher level**
- a **hierarchy** of FAs
- intuition: replace **repeated subgraphs** by a **single symbol**, **hiding** some cut-points

■ Example: a box **DLS** :  $\mathcal{L}(\text{DLS}) = \left\{ \begin{array}{c} \text{in} \xrightarrow{\text{next}} \text{out} \\ \text{prev} \xleftarrow{\quad} \end{array} \right\}$

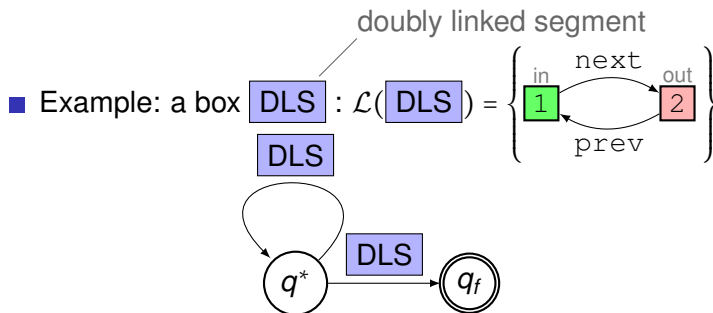
doubly linked segment



# Hierarchical Forest Automata

## ■ Hierarchical Forest Automata

- FAs are **symbols** (**boxes**) of FAs of a **higher level**
- a **hierarchy** of FAs
- intuition: replace **repeated subgraphs** by a **single symbol**, **hiding** some cut-points



## The Challenge

How to find the “right” boxes?

## The Challenge

How to find the “right” boxes?

- CAV'11 — database of boxes
- CAV'13 — automatic discovery

# Learning of Boxes

- compromise between

# Learning of Boxes

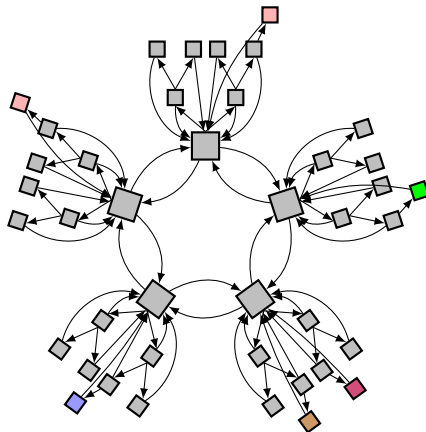
- compromise between
  - **reusability**: use on different heaps of the same kind  
     $\leadsto$  use **small** boxes



# Learning of Boxes

## ■ compromise between

- **reusability**: use on different heaps of the same kind  
     $\leadsto$  use **small** boxes



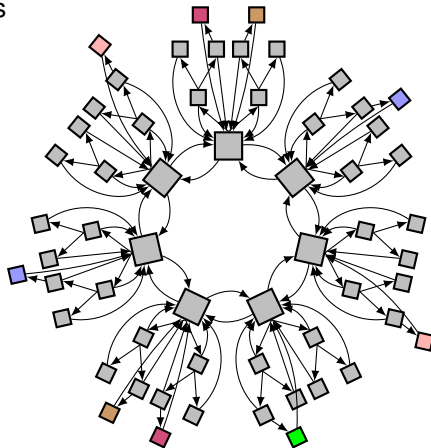


# Learning of Boxes

## ■ compromise between

- **reusability**: use on different heaps of the same kind

~> use **small** boxes

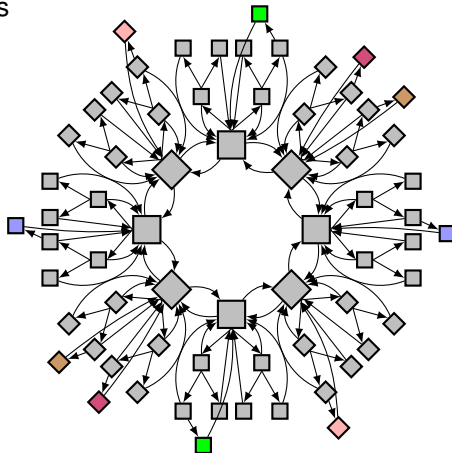


# Learning of Boxes

## ■ compromise between

- **reusability**: use on different heaps of the same kind

↪ use **small** boxes

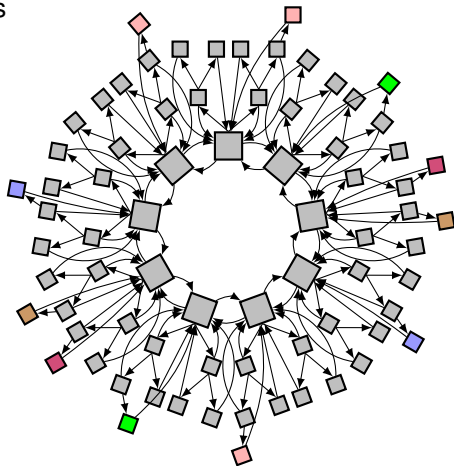


# Learning of Boxes

## ■ compromise between

- **reusability**: use on different heaps of the same kind

~> use **small** boxes



# Learning of Boxes

- compromise between
  - **reusability**: use on different heaps of the same kind  
     $\leadsto$  use **small** boxes

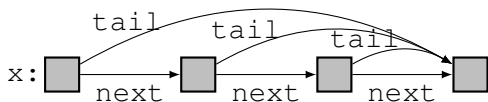
# Learning of Boxes

- compromise between
  - **reusability**: use on different heaps of the same kind
    - ↪ use **small** boxes
  - ability to **hide cut-points**
    - ↪ do not use **too small** boxes

# Learning of Boxes

## ■ compromise between

- **reusability**: use on different heaps of the same kind
  - ↪ use **small** boxes
- ability to **hide cut-points**
  - ↪ do not use **too small** boxes

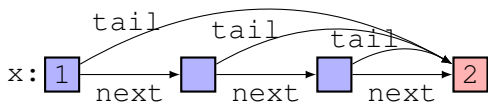




# Learning of Boxes

## ■ compromise between

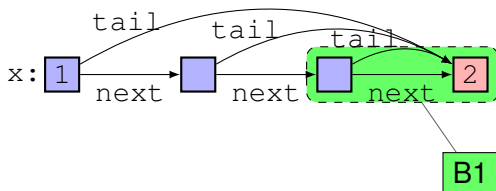
- **reusability**: use on different heaps of the same kind
  - ↪ use **small** boxes
- ability to **hide cut-points**
  - ↪ do not use **too small** boxes



# Learning of Boxes

## ■ compromise between

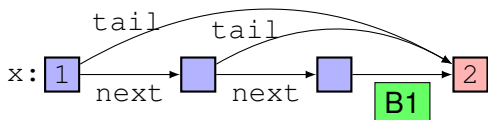
- ▶ **reusability**: use on different heaps of the same kind
  - ↪ use **small** boxes
- ▶ ability to **hide cut-points**
  - ↪ do not use **too small** boxes



# Learning of Boxes

## ■ compromise between

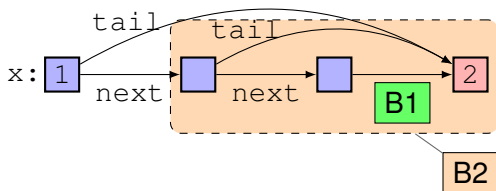
- ▶ **reusability**: use on different heaps of the same kind
  - ↪ use **small** boxes
- ▶ ability to **hide cut-points**
  - ↪ do not use **too small** boxes



# Learning of Boxes

## ■ compromise between

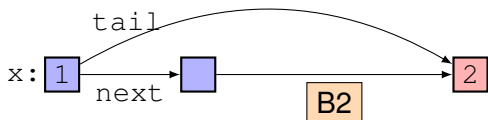
- ▶ **reusability**: use on different heaps of the same kind
  - ↪ use **small** boxes
- ▶ ability to **hide cut-points**
  - ↪ do not use **too small** boxes



# Learning of Boxes

## ■ compromise between

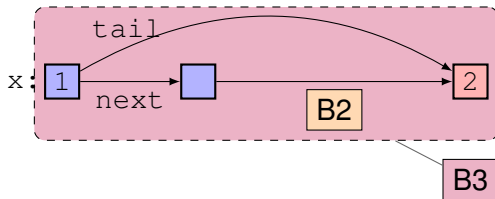
- **reusability**: use on different heaps of the same kind
  - ↪ use **small** boxes
- ability to **hide cut-points**
  - ↪ do not use **too small** boxes



# Learning of Boxes

## ■ compromise between

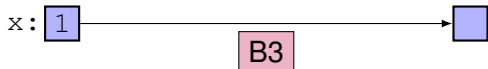
- ▶ **reusability**: use on different heaps of the same kind
  - ↪ use **small** boxes
- ▶ ability to **hide cut-points**
  - ↪ do not use **too small** boxes



# Learning of Boxes

## ■ compromise between

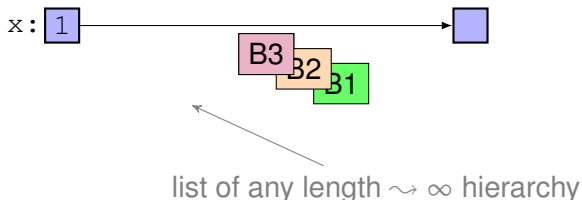
- **reusability**: use on different heaps of the same kind  
     $\leadsto$  use **small** boxes
- ability to **hide cut-points**  
     $\leadsto$  do not use **too small** boxes



# Learning of Boxes

## ■ compromise between

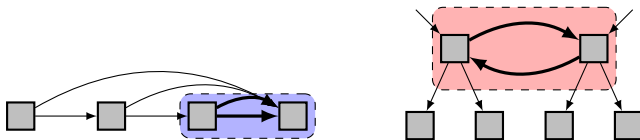
- **reusability**: use on different heaps of the same kind  
     $\leadsto$  use **small** boxes
- ability to **hide cut-points**  
     $\leadsto$  do not use **too small** boxes





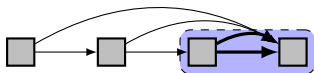
# Learning of Boxes: Knots

- 1 Smallest subgraphs meaningful to be folded:

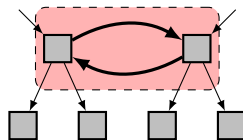


# Learning of Boxes: Knots

- 1 Smallest subgraphs meaningful to be folded:



- 2 Handle interface



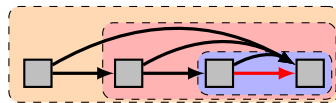
# Learning of Boxes: Knots

- 1 Smallest subgraphs meaningful to be folded:



- 2 Handle interface

- **compose** intersecting knots



prevent  $\infty$  nesting

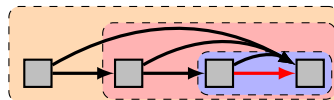
# Learning of Boxes: Knots

- 1 Smallest subgraphs meaningful to be folded:



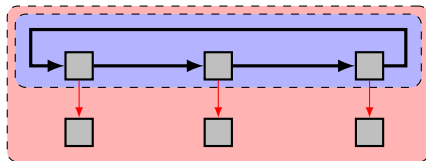
- 2 Handle interface

- **compose** intersecting knots



prevent  $\infty$  nesting

- **enclose** paths from inner nodes to leaves

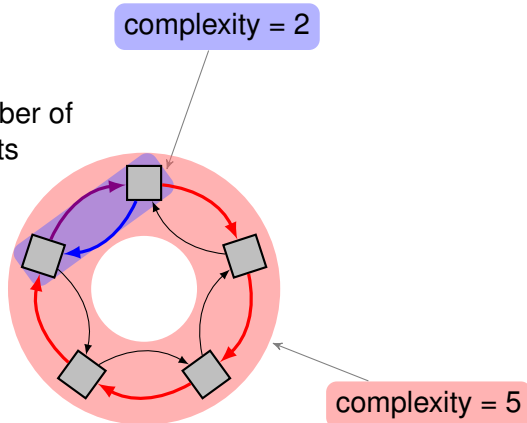


prevent  $\infty$   
interface nodes

- 3 Complexity: max number of cutpoints in basic knots

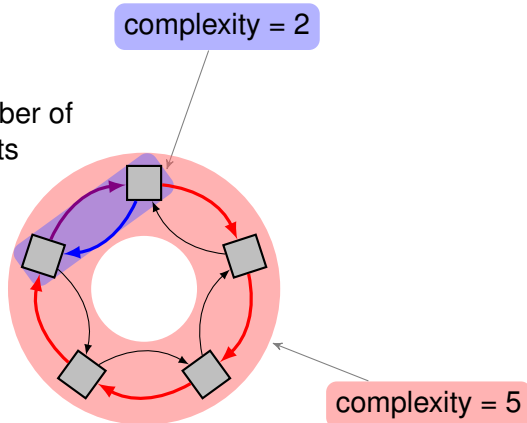
# Learning of Boxes: Knots

- 3 Complexity: max number of cutpoints in basic knots



# Learning of Boxes: Knots

- 3 Complexity: max number of cutpoints in basic knots



- find basic knots with  $1, 2, \dots$  cut-points

# Widening Revisited

- **learning** and **folding** of boxes in the abstraction loop



# Widening Revisited

- learning and folding of boxes in the abstraction loop

## The Goal

Fold boxes that will, after abstraction, appear on cycles of automata.

⇒ hide unboundedly many cut-points

# Widening Revisited

- learning and folding of boxes in the abstraction loop

## The Goal

Fold boxes that will, after abstraction, appear on cycles of automata.

⇒ hide unboundedly many cut-points

1 **Algorithm:** Abstraction Loop

2 *Unfold solo boxes*

3 **repeat**

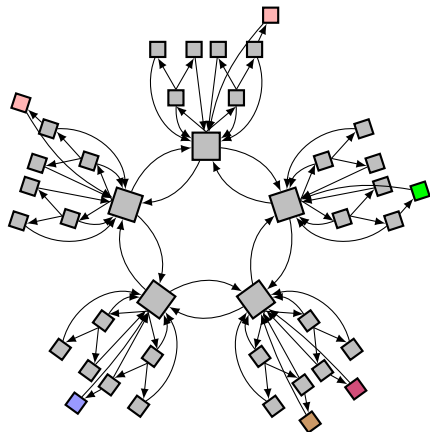
4     *Abstract*

5     *Fold*

6 **until** fixpoint

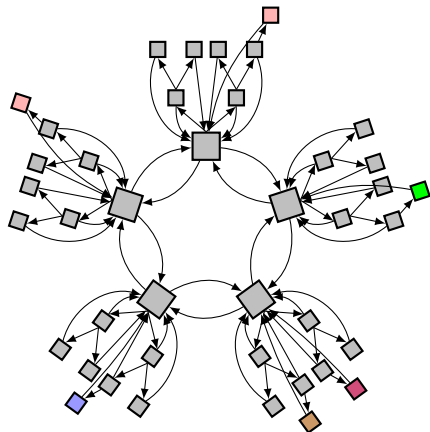
not on a cycle

# Learning of Boxes: Example



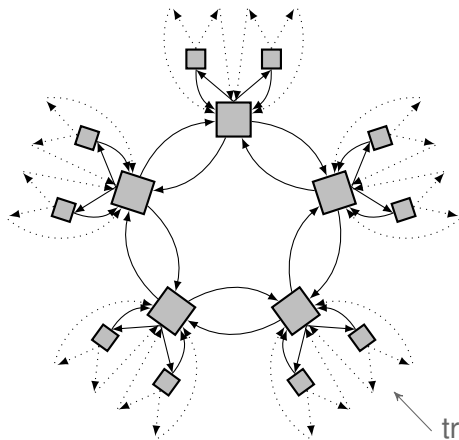
- 1 *Unfold solo boxes*
- 2 **repeat**
- 3     *Abstract*
- 4     *Fold*
- 5 **until** *fixpoint*

# Learning of Boxes: Example



- 1 **Unfold solo boxes**
- 2 **repeat**
- 3     *Abstract*
- 4     *Fold*
- 5 **until** *fixpoint*

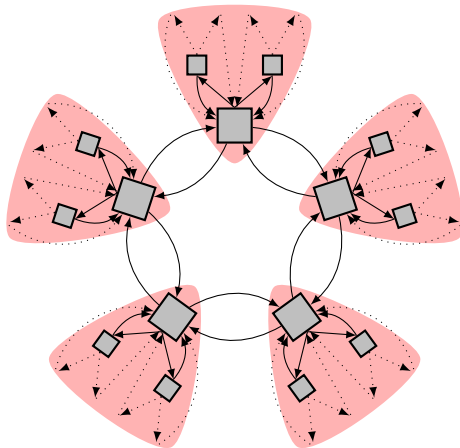
# Learning of Boxes: Example



- 1 *Unfold solo boxes*
- 2 **repeat**
- 3 *Abstract*
- 4 *Fold*
- 5 **until** *fixpoint*

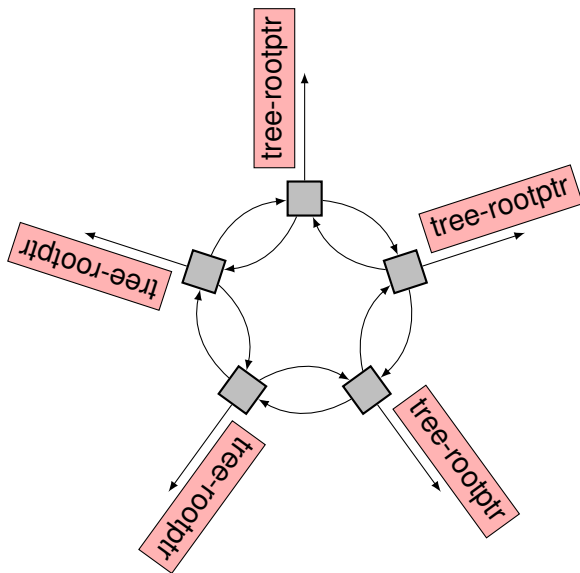
tree with root ptrs of any height

# Learning of Boxes: Example



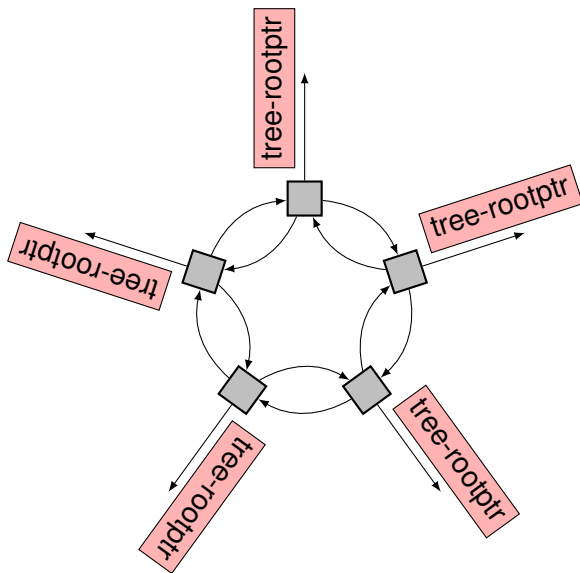
- 1 *Unfold solo boxes*
- 2 **repeat**
- 3     *Abstract*
- 4     *Fold*
- 5 **until** *fixpoint*

# Learning of Boxes: Example



- 1 *Unfold solo boxes*
- 2 **repeat**
- 3     *Abstract*
- 4     *Fold*
- 5 **until** *fixpoint*

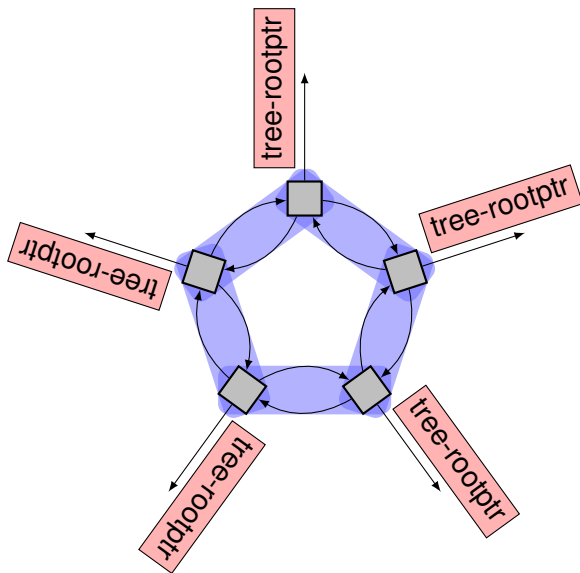
# Learning of Boxes: Example



- 1 *Unfold solo boxes*
- 2 **repeat**
- 3     *Abstract*
- 4     *Fold*
- 5 **until** *fixpoint*

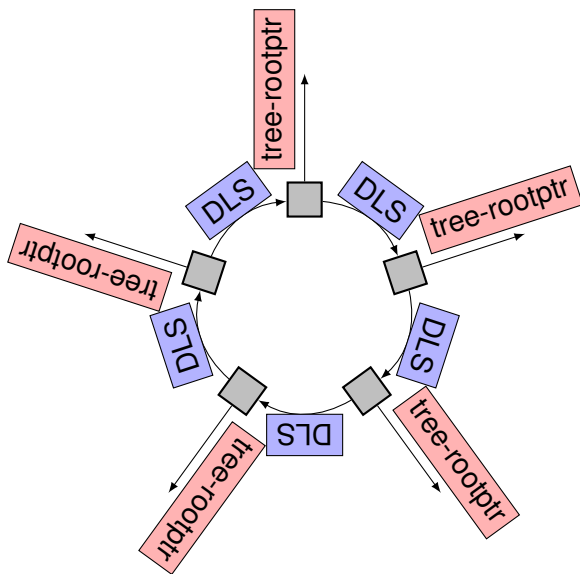


# Learning of Boxes: Example



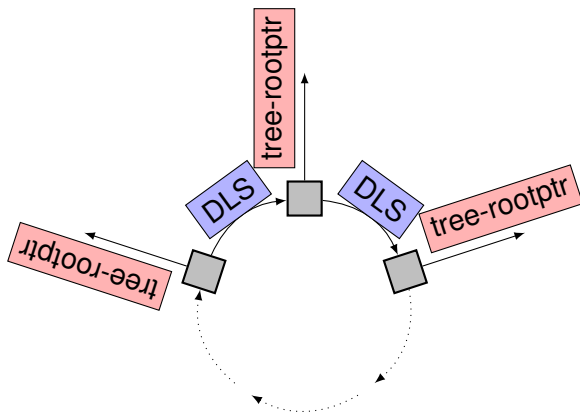
- 1 *Unfold solo boxes*
- 2 **repeat**
- 3     *Abstract*
- 4     *Fold*
- 5 **until** *fixpoint*

# Learning of Boxes: Example



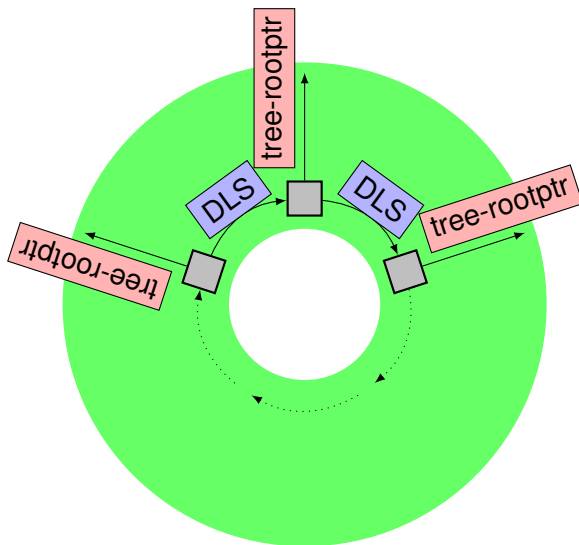
- 1 *Unfold solo boxes*
- 2 **repeat**
- 3     *Abstract*
- 4     *Fold*
- 5 **until** *fixpoint*

# Learning of Boxes: Example



- 1 *Unfold solo boxes*
- 2 **repeat**
- 3     *Abstract*
- 4     *Fold*
- 5 **until** *fixpoint*

# Learning of Boxes: Example



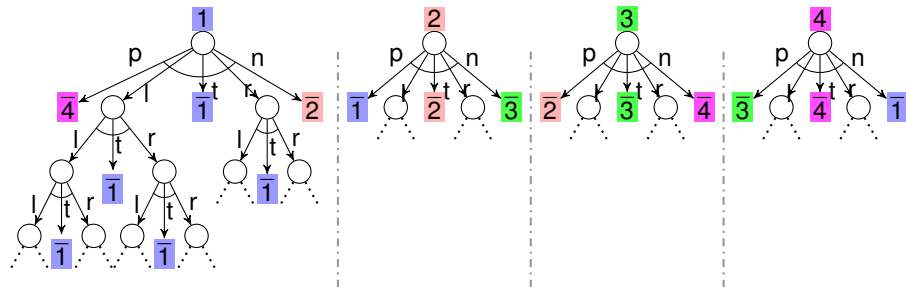
- 1 *Unfold solo boxes*
- 2 **repeat**
- 3     *Abstract*
- 4     *Fold*
- 5 **until** *fixpoint*

# Learning of Boxes: Example

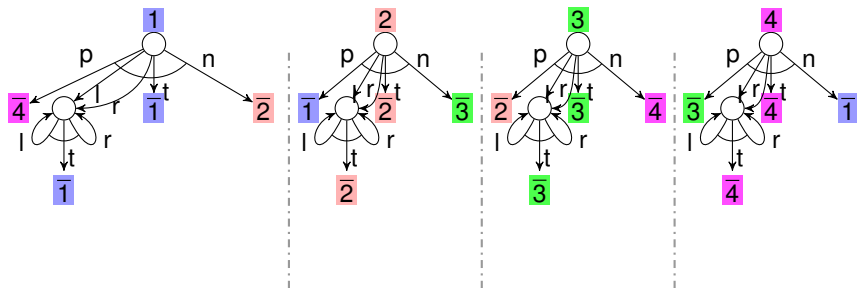
circular-DLL-of  
-trees-rootptr

- 1 *Unfold solo boxes*
- 2 **repeat**
- 3     *Abstract*
- 4     *Fold*
- 5 **until** *fixpoint*

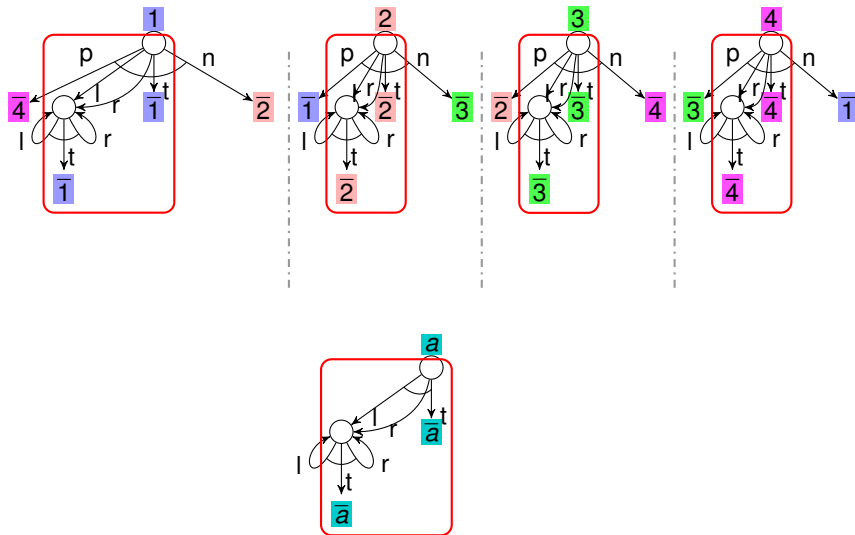
# Learning, Folding, and Abstraction on FA



# Learning, Folding, and Abstraction on FA

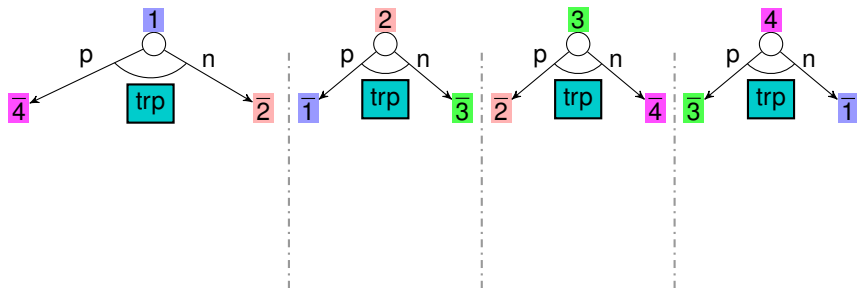


# Learning, Folding, and Abstraction on FA

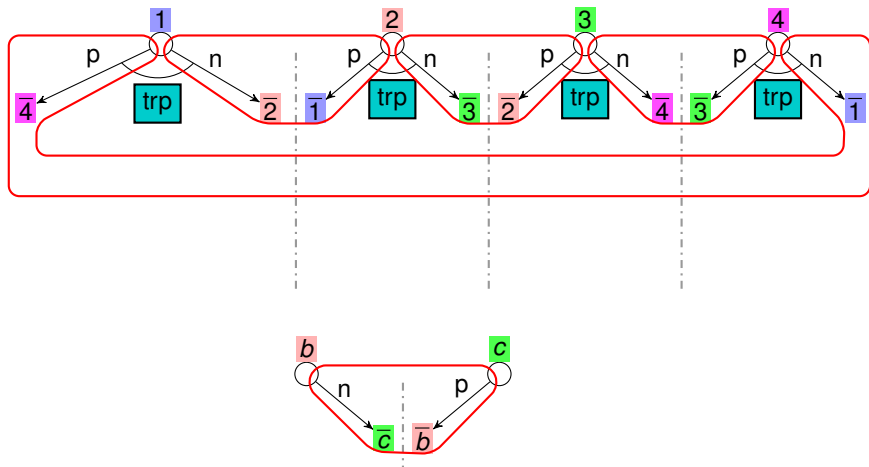




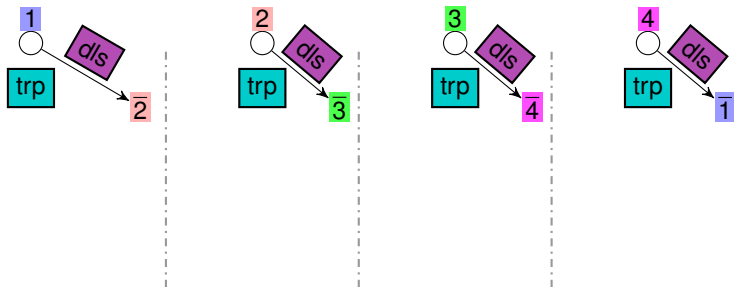
# Learning, Folding, and Abstraction on FA



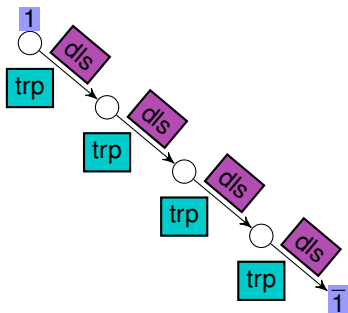
# Learning, Folding, and Abstraction on FA



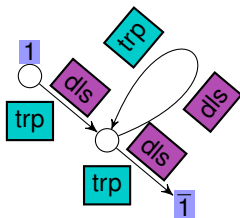
# Learning, Folding, and Abstraction on FA



## Learning, Folding, and Abstraction on FA



# Learning, Folding, and Abstraction on FA



# Experimental Results

- implemented in the **Forester** tool

# Experimental Results

- implemented in the **Forester** tool
- comparison with Predator (a state-of-the-art tool for lists)
  - winner of [HeapManipulation](#) and [MemorySafety](#) of SV-COMP'13

# Experimental Results

- implemented in the **Forester** tool
- comparison with Predator (a state-of-the-art tool for lists)
  - ▶ winner of [HeapManipulation](#) and [MemorySafety](#) of SV-COMP'13

Table : Results of the experiments [s]

Example	FA	Predator	Example	FA	Predator
SLL (delete)	0.04	0.04	DLL (reverse)	0.06	0.03
SLL (bubblesort)	0.04	0.03	DLL (insert)	0.07	0.05
SLL (mergesort)	0.15	0.10	DLL (insertsort <sub>1</sub> )	0.40	0.11
SLL (insertsort)	0.05	0.04	DLL (insertsort <sub>2</sub> )	0.12	0.05
SLL (reverse)	0.03	0.03	DLL of CDLLs	1.25	0.22
SLL+head	0.05	0.03	DLL+subdata	0.09	T
SLL of 0/1 SLLs	0.03	0.11	CDLL	0.03	0.03
SLL <sub>Linux</sub>	0.03	0.03	tree	0.14	Err
SLL of CSLLs	0.73	0.12	tree+parents	0.21	T
SLL of 2CDLLs <sub>Linux</sub>	0.17	0.25	tree+stack	0.08	Err
skip list <sub>2</sub>	0.42	T	tree (DSW) <sup>Deutsch-Schorr-Waite</sup>	0.40	Err
skip list <sub>3</sub>	9.14	T	tree of CSLLs	0.42	Err

timeout

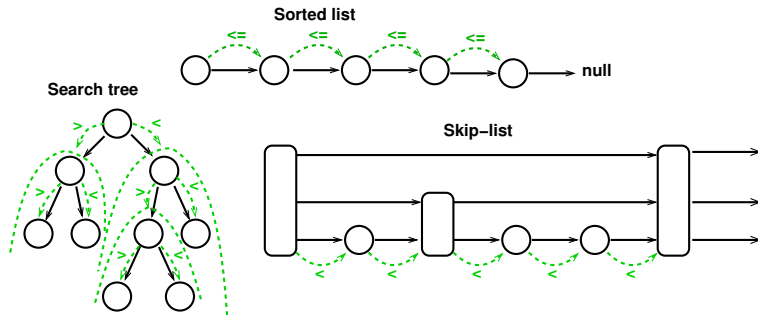
false positive



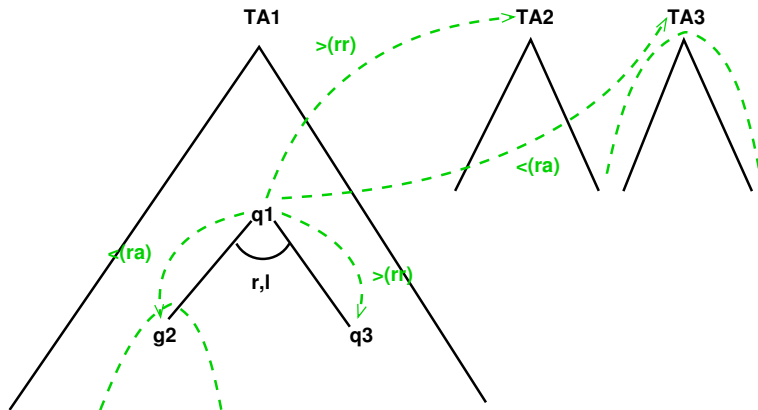
# Tracking Relations Over Data Values

- Verify data-related properties such as sortedness.
- Verify memory safety even if it depends on relations over data.

# Motivation



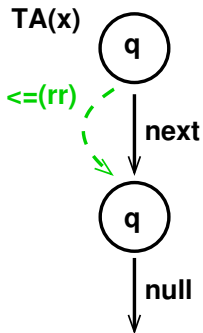
# Forest Automata with Data Constraints



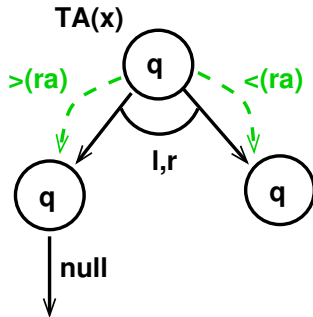
$$q1 \xrightarrow{r,l} (q2, q3) : \{0 <_{ra} 1, 0 <_{rr} 2, 0 <_{ra} TA2, 0 >_{rr} TA3\}$$

# Examples of Encoded Structures

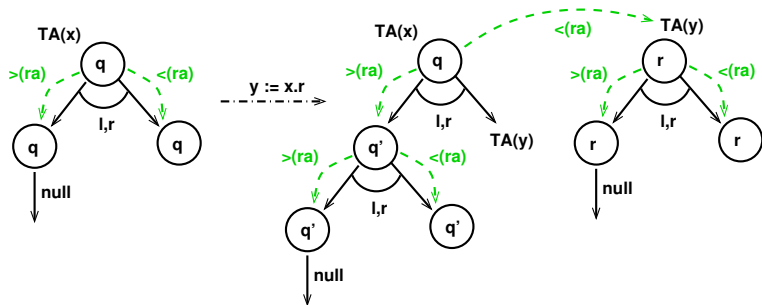
Sorted list



Search tree

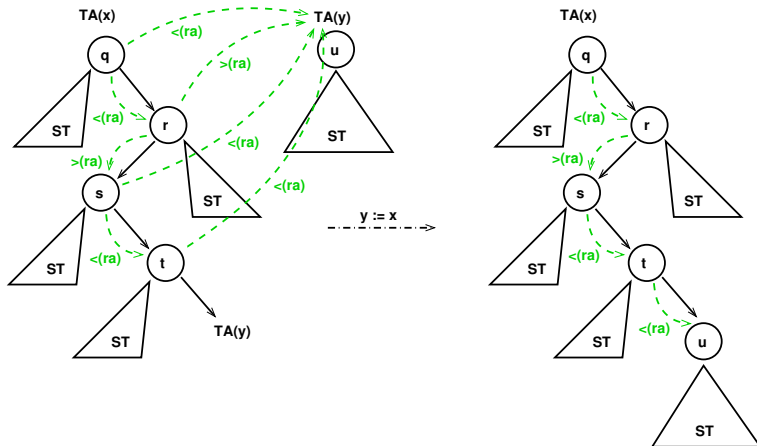


# Update



# More Complex Update

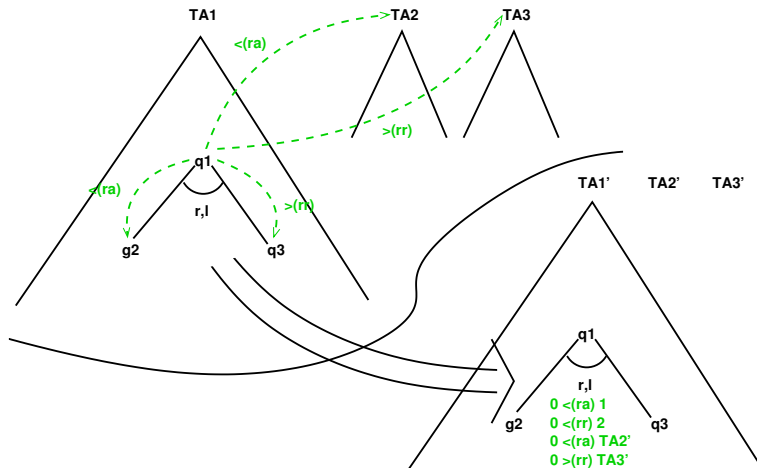
An intermediate state of a traversal of a search tree



# Needed Machinery

- FA machinery must be extended with handling data constraints.
- Particularly, we need to be able to do:
  - Language Inclusion Check
  - Simulation Reduction
  - Abstraction
- This is done with a help of
  - Saturation which infers valid data constraints from existing ones.
  - Translation to ordinary FA and subsequent use of ordinary FA algorithms.

# Translation to Plain FA





# Experimental Results

Example	time	Example	time
SLL insert	0.06	DLL insert	0.14
SLL delete	0.08	DLL delete	0.38
SLL reverse	0.07	DLL reverse	0.16
SLL bubblesort	0.13	DLL bubblesort	0.39
SLL insertsort	0.10	DLL insertsort	0.43

Example	time	Example	time
BST insert	6.87	SL <sub>2</sub> insert	9.65
BST delete	114.00	SL <sub>2</sub> delete	10.14
BST left rotate	7.35	SL <sub>3</sub> insert	56.99
BST right rotate	6.25	SL <sub>3</sub> delete	57.35

# Conclusion

Shape analysis with **forest automata**:

- fully **automated**, very **flexible**

# Conclusion

Shape analysis with **forest automata**:

- fully **automated**, very **flexible**
- the **Forester** tool

- ▶ <http://www.fit.vutbr.cz/research/groups/verifit/tools/forester>

# Conclusion

Shape analysis with **forest automata**:

- fully **automated**, very **flexible**

- the **Forester** tool

  - <http://www.fit.vutbr.cz/research/groups/verifit/tools/forester>

- successfully verified:

  - (singly/doubly linked (circular)) **lists** (of ( . . . ) lists)
  - **trees** (with additional pointers)
  - **skip lists**
  - tracking **ordering** relations

- not covered here:

  - support for **pointer arithmetic**

# Future Work

- CEGAR loop
  - **red-black** trees, ...
- concurrent data structures
  - lockless skip lists, ...
- recursive boxes
  - B+ trees, ...