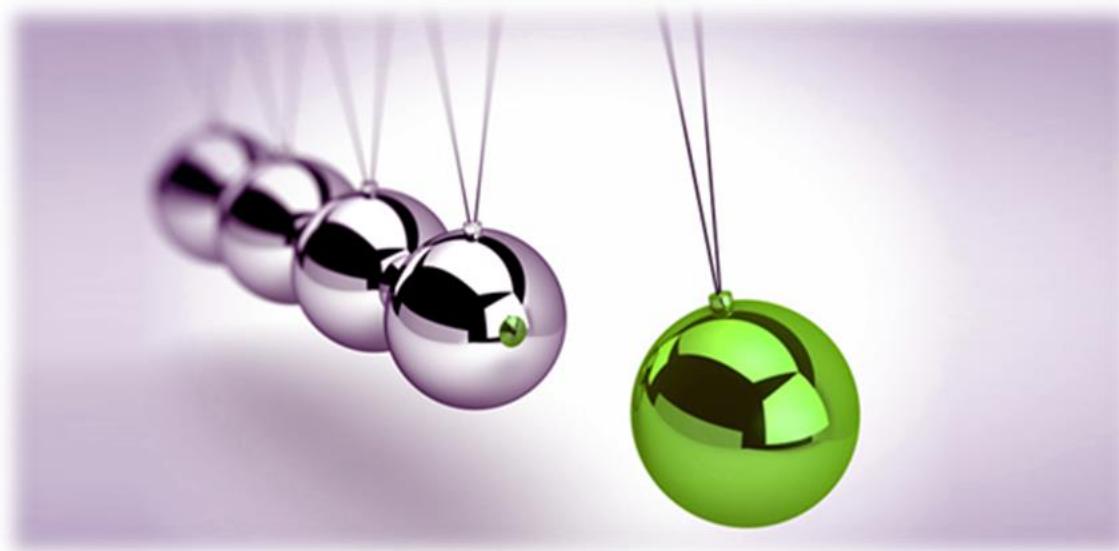




PHYSICS I - MECHANICS

Potential Energy and Energy Conservation



Doç. Dr. Ahmet Karatay
Ankara University
Engineering Faculty - Department of Engineering Physics

CHAPTER 9. Potential Energy and Energy Conservation

Learning Objectives

9.1 Potential Energy

9.2 Gravitational Potential Energy

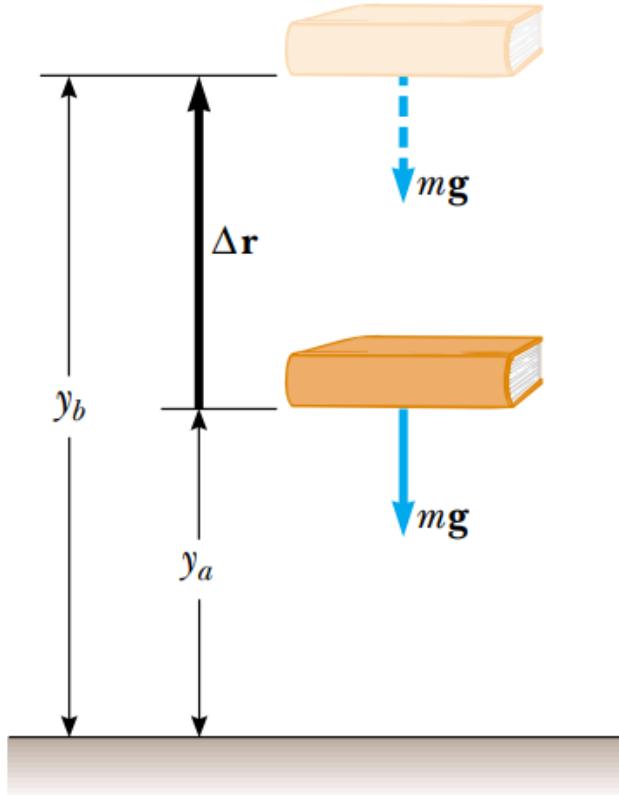
9.3 Elastic Potential Energy

9.4 Conservative and Nonconservative Forces

9.5 Force and Potential Energy

9.1 Potential Energy

We do some work on the system by lifting the book slowly through a height $\Delta y = y_b - y_a$ as seen in figure.



Because the energy change of the system is not in the form of kinetic energy or internal energy, it must appear as some other form of energy storage.

After lifting the book, we could release it and let it fall back to the position y_a and it gains kinetic energy.

While the book was at the y_b , it had potential to gain kinetic energy.

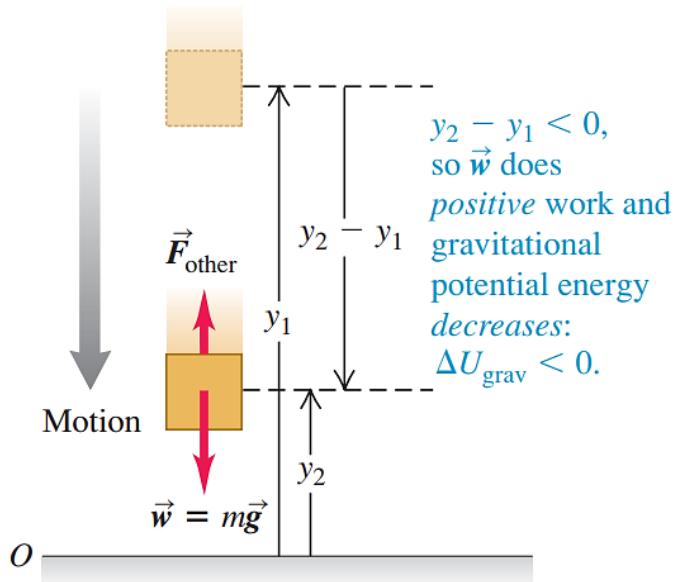
We call the energy storage mechanism before we release the book as *potential energy*.

The *potential energy* is energy associated with the *position* of a system rather than its motion. For this reason, energy associated with position is called **potential energy**.

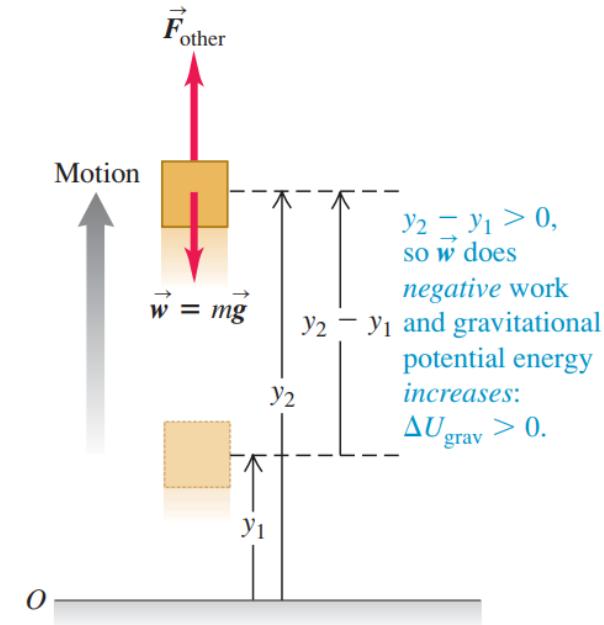
9.2 Gravitational Potential Energy

Gravitational potential energy is the energy associated with an object at a given location above the surface of the Earth.

(a) A body moves downward



(b) A body moves upward



$$W_{\text{grav}} = Fs = -mg\hat{j}(y_2 - y_1)\hat{j} = mg y_1 - mg y_2$$

9.2 Gravitational Potential Energy

$$W_{grav} = Fs = w(y_1 - y_2) = \cancel{mgy_1} - \cancel{mgy_2} \quad U_{grav} = mgy$$

As the book falls from y_1 to y_2 , the work done by the gravitational force on the book is

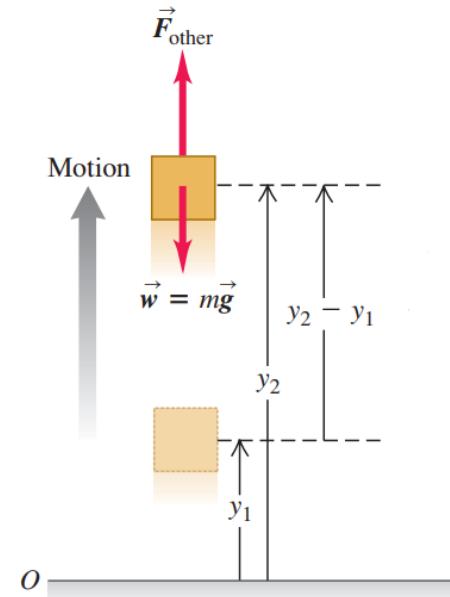
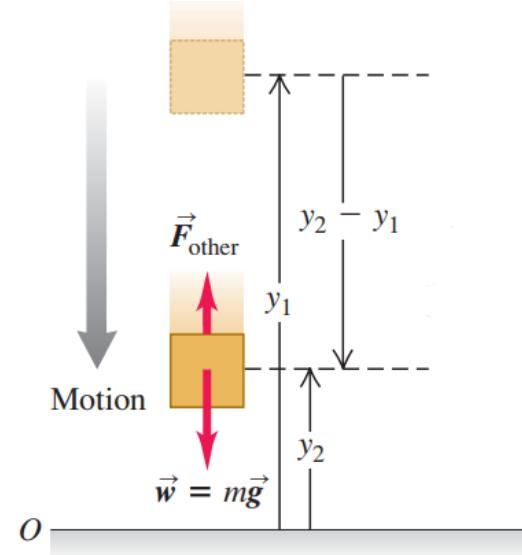
$$W_{grav} = Fs = -mg\hat{j}(y_2 - y_1)\hat{j} = mgy_1 - mgy_2$$

$$mgy_1 - mgy_2 = -(mgy_2 - mgy_1) = -(U_2 - U_1) = -\Delta U_{grav}$$

ΔU_{grav} is the gravitational potential energy of the system.

When the body moves up, y increases, the work done by the gravitational force is negative, and the gravitational potential energy increases $\Delta U_{grav} > 0$

When the body moves down, y decreases, the work done by the gravitational force is positive, and the gravitational potential energy decreases $\Delta U_{grav} < 0$



9.2 Gravitational Potential Energy

Conservation of Mechanical Energy (Gravitational Forces Only)

The work done on the book is equal to the change in the kinetic energy of the book:

$$W_{tot} = \Delta K = K_2 - K_1 = W_{grav} = mgy_1 - mgy_2$$

$$mgy_1 - mgy_2 = -(mgy_2 - mgy_1) = -(U_2 - U_1) = -\Delta U_{grav}$$

$$\Delta K = -\Delta U_{grav} \Rightarrow \Delta K + \Delta U_{grav} = 0$$

$$\Delta K = -\Delta U_{grav} = K_2 - K_1 = U_1 - U_2$$

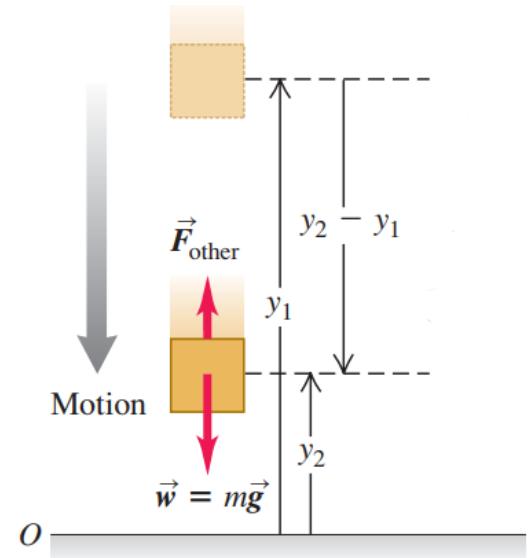
$$K_1 + U_1 = U_2 + K_2$$

The sum $K+U$ of kinetic and potential energy is called E , the **total mechanical energy of the system**.

$$K_1 + U_1 = U_2 + K_2 \quad E_1 = E_2$$

$$\frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_2^2 + mgy_2$$

conservation of mechanical energy



9.2 Gravitational Potential Energy

Example

You throw a 0.145-kg baseball straight up, giving it an initial velocity of magnitude 20.0 m/s. Find how high it goes, ignoring air resistance.

$$y_1 = 0$$

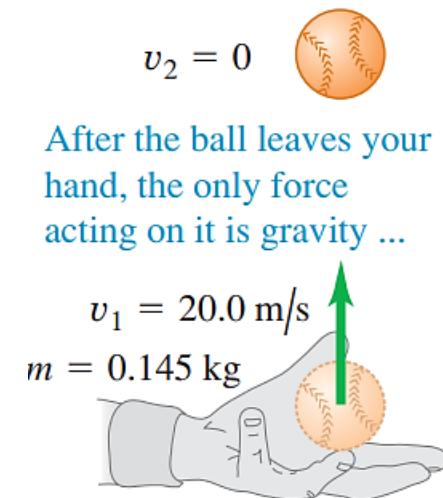
$$U_1 = mg y_1 = 0$$

$$K_2 = \frac{1}{2}mv_2^2 = 0$$

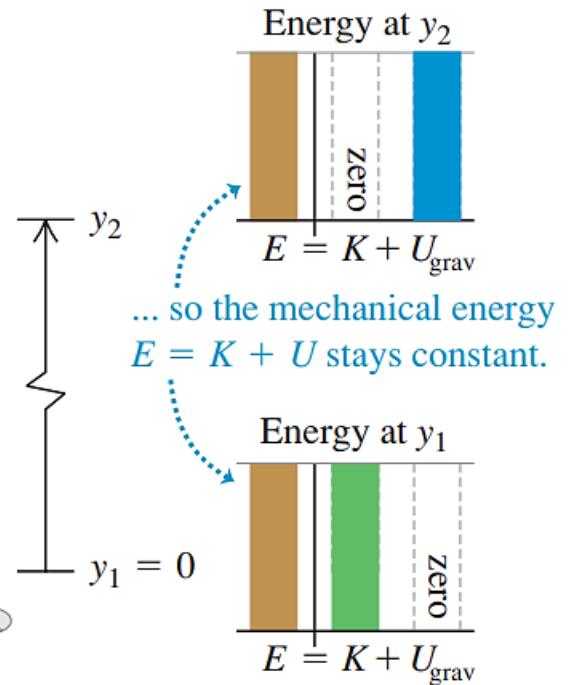
$$K_1 = U_2$$

$$K_1 = \frac{1}{2}mv_1^2 = mg y_2$$

$$y_2 = \frac{v_1^2}{2g} = \frac{(20.0 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = 20.4 \text{ m}$$



After the ball leaves your hand, the only force acting on it is gravity ...



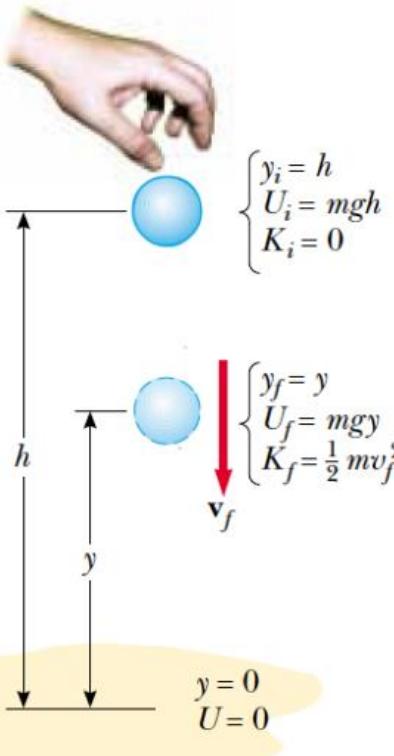
9.2 Gravitational Potential Energy

Example

A ball of mass m is dropped from a height h above the ground, as shown in figure.

- Neglecting air resistance, determine the speed of the ball when it is at a height y above the ground.
- Determine the speed of the ball at y if at the instant of release it already has an initial upward speed v_i at the initial altitude h .

a) From the conservation of mechanical energy:



$$K_f + U_f = K_i + U_i$$

$$\frac{1}{2}mv_f^2 + mg(y - h) = 0 + mgh$$

$$v_f^2 = 2g(h - y)$$

$$v_f = \sqrt{2g(h - y)}$$

b) $\frac{1}{2}mv_f^2 + mg(y - h) = \frac{1}{2}mv_i^2 + mgh$

$$v_f^2 = v_i^2 + 2g(h - y)$$

$$v_f = \sqrt{v_i^2 + 2g(h - y)}$$

9.2 Gravitational Potential Energy

Example (When Forces Other Than Gravity Do Work)

Suppose your hand moves upward by 0.50 m while you are throwing the 0.145-kg baseball straight up. The ball leaves your hand with an upward velocity of 20.0 m/s.

- Find the magnitude of the force (assumed constant) that your hand exerts on the ball.
- Find the speed of the ball at a point 15.0 m above the point where it leaves your hand. Ignore air resistance.

$$W_{tot} = W_{other} + W_{grav}$$

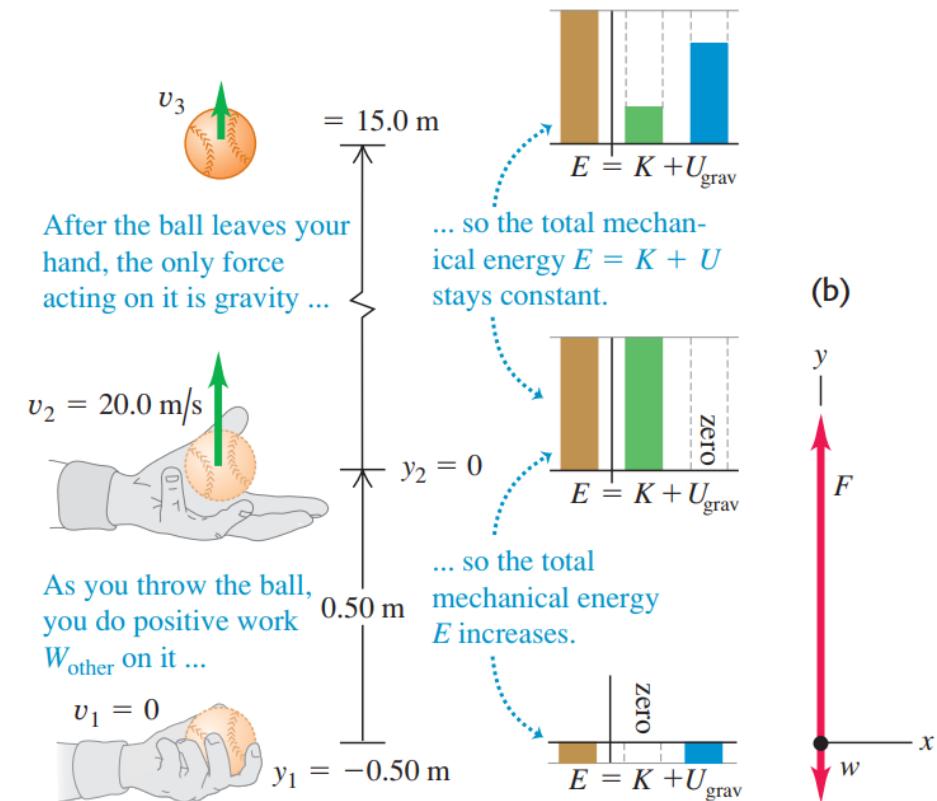
$$W_{other} + W_{grav} = K_2 - K_1$$

$$W_{grav} = (U_2 - U_1)$$

$$W_{other} + (U_2 - U_1) = K_2 - K_1$$

$$K_1 + U_1 + W_{other} = K_2 + U_2$$

$$\frac{1}{2}mv_1^2 + mgy_1 + W_{other} = \frac{1}{2}mv_2^2 + mgy_2$$



9.2 Gravitational Potential Energy

Example

a)

$$y_1 = -0.50 \text{ m}$$

$$v_1 = 0$$

$$U_1 = mgy_1 = (0.145 \text{ kg}) \left(9.80 \frac{\text{m}}{\text{s}^2} \right) (-0.50 \text{ m}) = -0.71 \text{ J}$$

$$K_1 = 0$$

$$y_2 = 0$$

$$K_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}(0.145 \text{ kg})(20.0 \text{ m/s})^2 = 29.0 \text{ J}$$

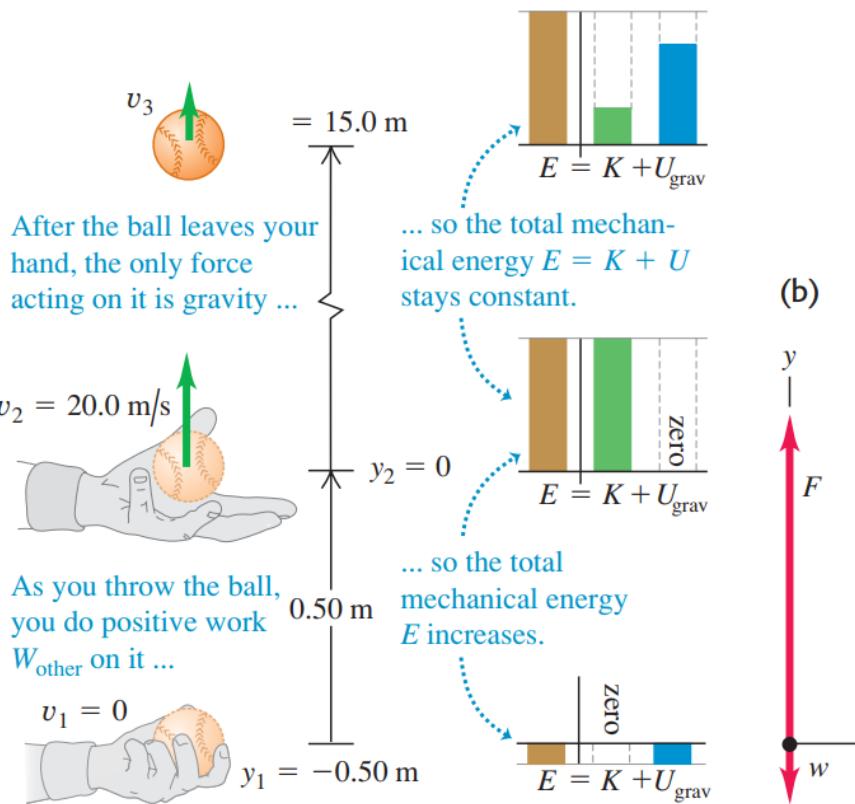
$$v_2 = 20 \text{ m/s}$$

$$U_{\text{grav},2} = mgy_2 = (0.145 \text{ kg})(9.80 \text{ m/s}^2)(0) = 0$$

$$K_1 + U_{\text{grav},1} + W_{\text{other}} = K_2 + U_{\text{grav},2}$$

$$W_{\text{other}} = (K_2 - K_1) + (U_{\text{grav},2} - U_{\text{grav},1})$$

$$= (29.0 \text{ J} - 0) + [0 - (-0.71 \text{ J})] = 29.7 \text{ J}$$



$$F = \frac{W_{\text{other}}}{y_2 - y_1} = \frac{29.7 \text{ J}}{0.50 \text{ m}} = 59 \text{ N}$$

9.2 Gravitational Potential Energy

Example

b)

$$y_3 = 15 \text{ m}$$

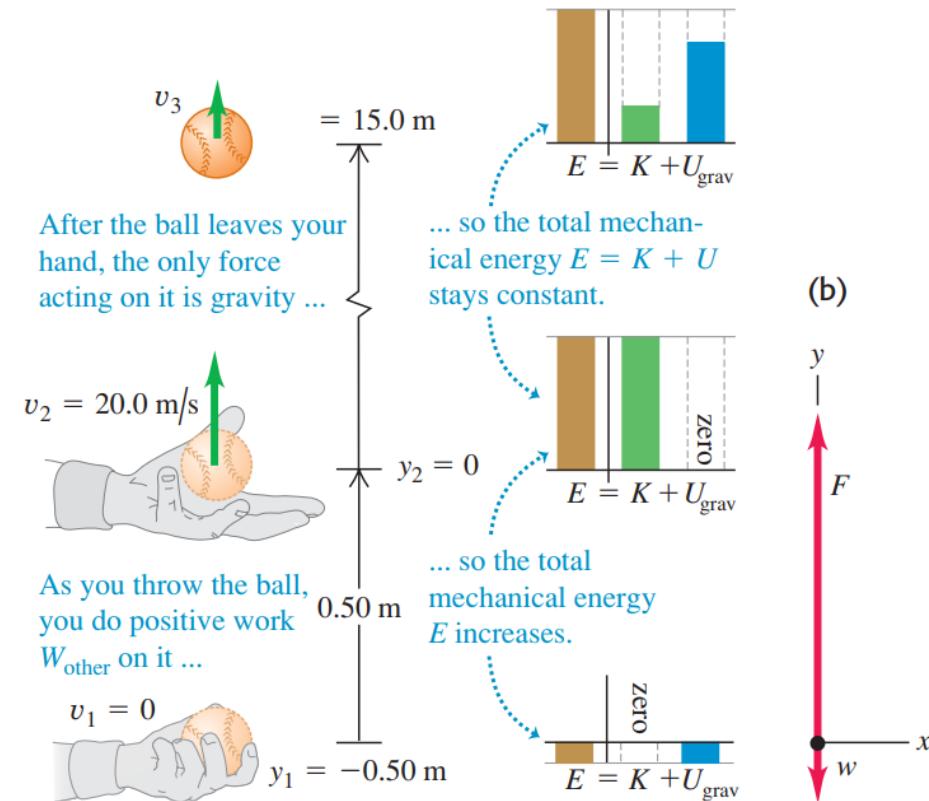
$$K_2 + U_{\text{grav},2} = K_3 + U_{\text{grav},3}$$

$$U_{\text{grav},3} = mgy_3 = (0.145 \text{ kg})(9.80 \text{ m/s}^2)(15.0 \text{ m}) = 21.3 \text{ J}$$

$$\begin{aligned} K_3 &= (K_2 + U_{\text{grav},2}) - U_{\text{grav},3} \\ &= (29.0 \text{ J} + 0 \text{ J}) - 21.3 \text{ J} = 7.7 \text{ J} \end{aligned}$$

Since $K_3 = \frac{1}{2}mv_{3y}^2$, we find

$$v_{3y} = \pm \sqrt{\frac{2K_3}{m}} = \pm \sqrt{\frac{2(7.7 \text{ J})}{0.145 \text{ kg}}} = \pm 10 \text{ m/s}$$



9.2 Gravitational Potential Energy

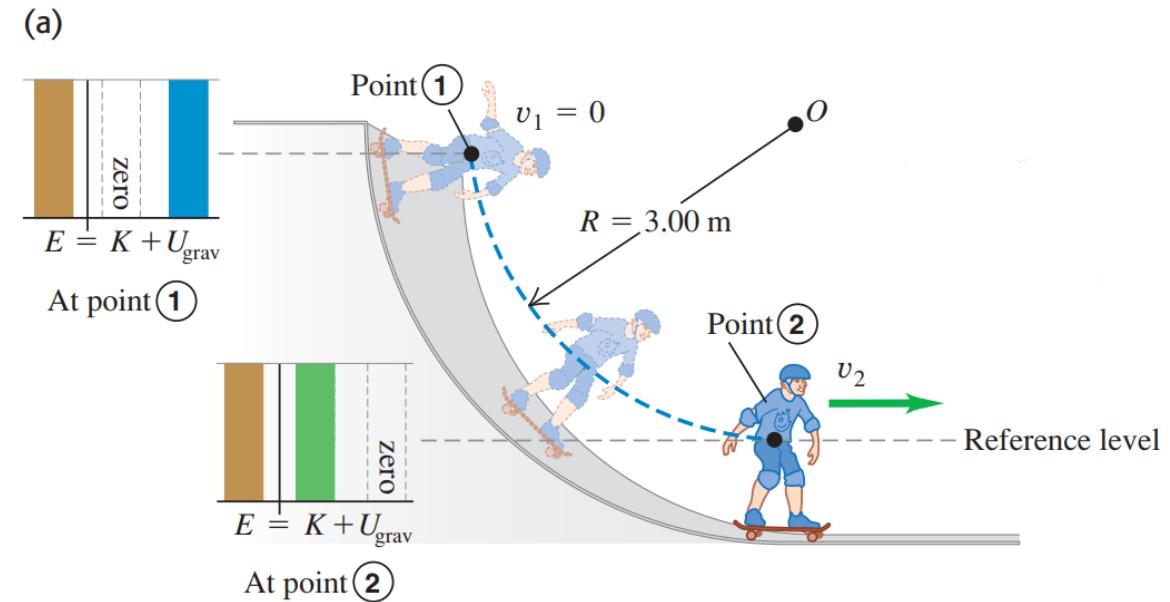
Example

A person skateboards from rest down a curved, frictionless ramp. He moves through a quarter-circle with Radius $R=3.0\text{ m}$. He and his skateboard have a total mass of 25.0 kg .

- Find his speed at the bottom of the ramp.
- Find the normal force that acts on him at the bottom of the curve.

a) $K_1 = 0 \quad U_{\text{grav},1} = mgR$
 $K_2 = \frac{1}{2}mv_2^2 \quad U_{\text{grav},2} = 0$

$$K_1 + U_{\text{grav},1} = K_2 + U_{\text{grav},2}$$
$$0 + mgR = \frac{1}{2}mv_2^2 + 0$$
$$v_2 = \sqrt{2gR}$$
$$= \sqrt{2(9.80\text{ m/s}^2)(3.00\text{ m})} = 7.67\text{ m/s}$$



9.2 Gravitational Potential Energy

Example

b)

$$v_2 = \sqrt{2gR}$$

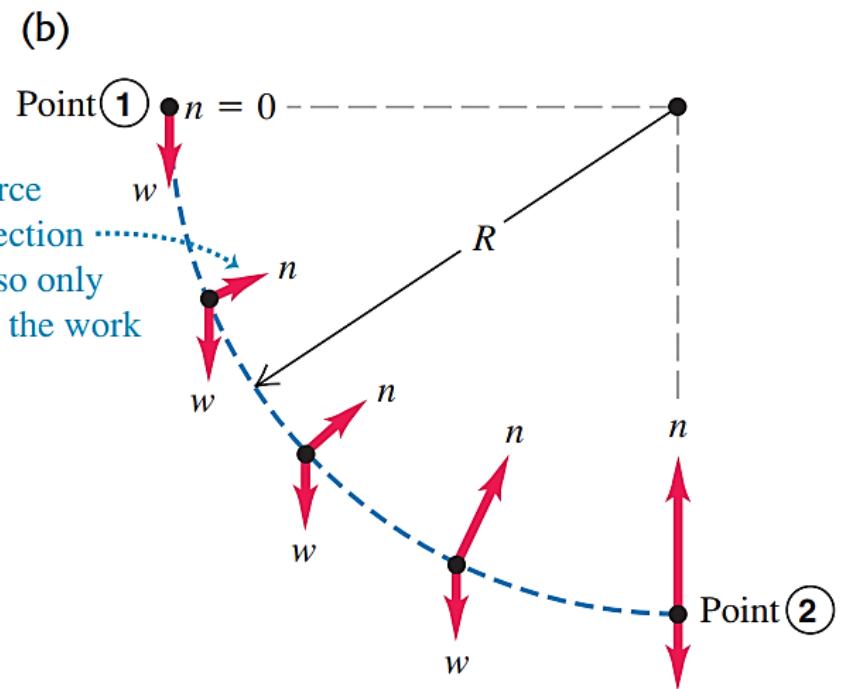
$$a_{\text{rad}} = \frac{v_2^2}{R} = \frac{2gR}{R} = 2g$$

$$\sum F_y = n + (-w) = ma_{\text{rad}} = 2mg$$

$$n = w + 2mg = 3mg$$

$$= 3(25.0 \text{ kg})(9.80 \text{ m/s}^2) = 735 \text{ N}$$

At each point, the normal force acts perpendicular to the direction of Throcky's displacement, so only the force of gravity (w) does the work on him.



9.2 Gravitational Potential Energy

Example

We want to slide a 12-kg crate up a 2.5-m-long ramp inclined at 30° . A worker, ignoring friction, calculates that he can do this by giving it an initial speed of 5.0 m/s at the bottom and letting it go. But friction is not negligible; the crate slides only 1.6 m up the ramp, stops, and slides back down.

- Find the magnitude of the friction force acting on the crate, assuming that it is constant.
- How fast is the crate moving when it reaches the bottom of the ramp?

$$K_1 = \frac{1}{2}(12 \text{ kg})(5.0 \text{ m/s})^2 = 150 \text{ J}$$

$$U_{\text{grav},1} = 0$$

$$K_2 = 0$$

$$U_{\text{grav},2} = (12 \text{ kg})(9.8 \text{ m/s}^2)(0.80 \text{ m}) = 94 \text{ J}$$

$$W_{\text{other}} = -fs$$

$$K_1 + U_{\text{grav},1} + W_{\text{other}} = K_2 + U_{\text{grav},2}$$

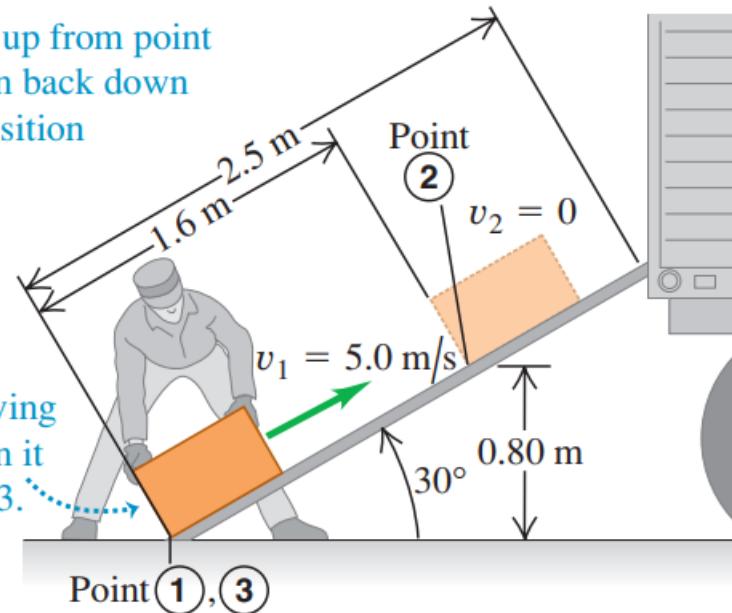
$$\begin{aligned} W_{\text{other}} &= -fs = (K_2 + U_{\text{grav},2}) - (K_1 + U_{\text{grav},1}) \\ &= (0 + 94 \text{ J}) - (150 \text{ J} + 0) = -56 \text{ J} = -fs \end{aligned}$$

$$f = \frac{W_{\text{other}}}{s} = \frac{56 \text{ J}}{1.6 \text{ m}} = 35 \text{ N}$$

(a)

The crate slides up from point 1 to point 2, then back down to its starting position (point 3).

The crate is moving at speed v_3 when it returns to point 3.



9.2 Gravitational Potential Energy

Example

$$W_{\text{other}} = W_{\text{fric}} = -2fs = -2(56 \text{ J}) = -112 \text{ J}$$

$$K_1 = 150 \text{ J} \text{ and } U_{\text{grav},1} = 0$$

$$K_1 + U_{\text{grav},1} + W_{\text{other}} = K_3 + U_{\text{grav},3}$$

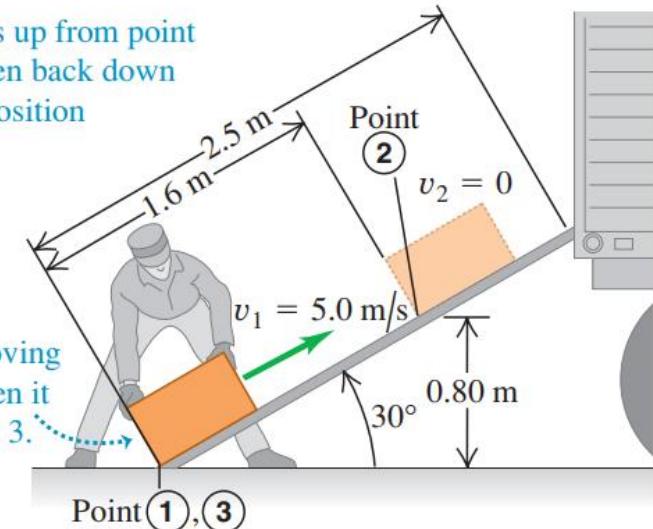
$$\begin{aligned} K_3 &= K_1 + U_{\text{grav},1} - U_{\text{grav},3} + W_{\text{other}} \\ &= 150 \text{ J} + 0 - 0 + (-112 \text{ J}) = 38 \text{ J} \end{aligned}$$

$$K_3 = \frac{1}{2}mv_3^2$$

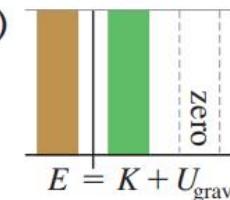
$$v_3 = \sqrt{\frac{2K_3}{m}} = \sqrt{\frac{2(38 \text{ J})}{12 \text{ kg}}} = 2.5 \text{ m/s}$$

(a)

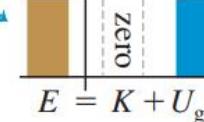
The crate slides up from point 1 to point 2, then back down to its starting position (point 3).



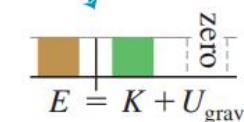
(b)



At point ①



At point ②



At point ③

The force of friction does negative work on the crate as it moves, so the total mechanical energy $E = K + U_{\text{grav}}$ decreases.

9.3 Elastic Potential Energy

There are many situations in which we encounter potential energy that is not gravitational in nature. To be specific, we'll consider storing energy in an ideal spring. To keep such an ideal spring stretched by a distance x , we must exert a force

$$F = -kx$$

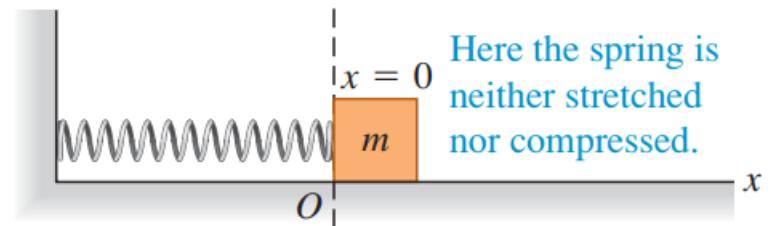
As the block moves from one position to another position how much work does the elastic (spring) force do on the block?

We found that the work we must do on the spring to move one end from x_1 to x_2 is

$$W = \frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2 \quad (\text{work done on a spring})$$

$$W_{\text{el}} = \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2 \quad (\text{work done by a spring})$$

(a)



Here the spring is
neither stretched
nor compressed.

9.3 Elastic Potential Energy

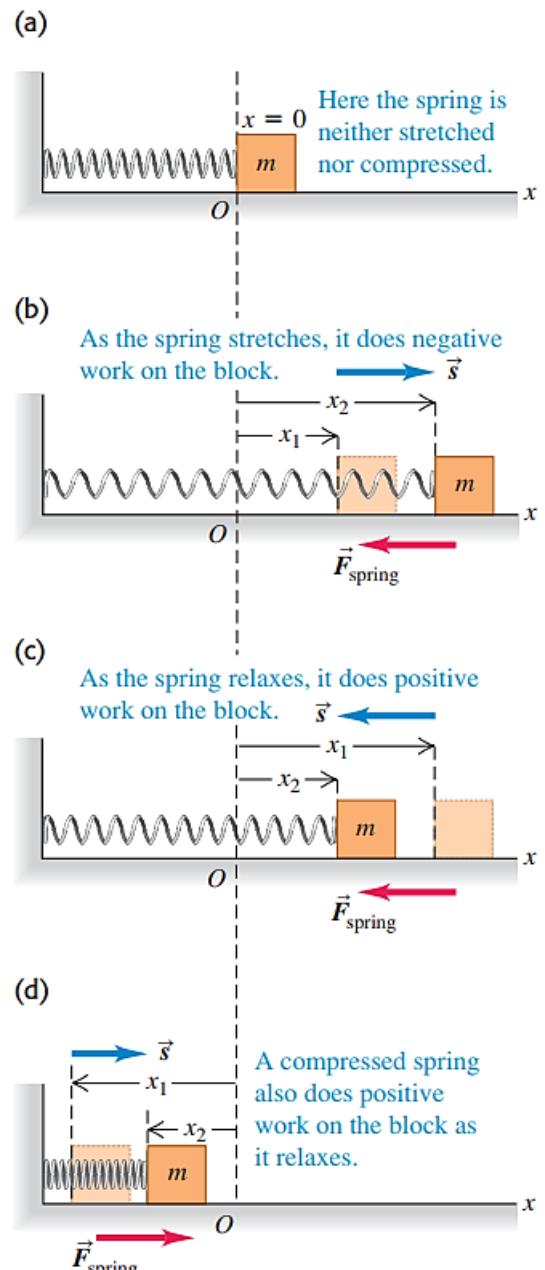
The body is at $x=0$ when the spring is neither stretched nor compressed.

When x_1 and x_2 are both positive and $x_2 > x_1$, the spring does negative work on the block, which moves in the $+x$ direction while the spring pulls on it in the $-x$ direction. The spring stretches farther, and the block slows down.

When x_1 and x_2 are both positive and $x_1 > x_2$, the spring does positive work as it relaxes and the block speeds up.

If the spring can be compressed as well as stretched, x_1 or x_2 or both may be negative, but the expression for W_{el} is still valid.

x_1 and x_2 are negative, but x_2 is less negative than x_1 ; the compressed spring does positive work as it relaxes, speeding the block up.



9.3 Elastic Potential Energy

We can express the work done by the spring in terms of a given quantity at the beginning and end of the displacement. This quantity is $\frac{1}{2}kx^2$ and we define it to be the elastic potential energy:

$$U_{el} = \frac{1}{2}kx^2$$

The work done on the block by the elastic force in terms of the change in elastic potential energy:

$$W_{el} = \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2 \quad W_{el} = U_{el;1} - U_{el;2} = -\Delta U_{el}$$

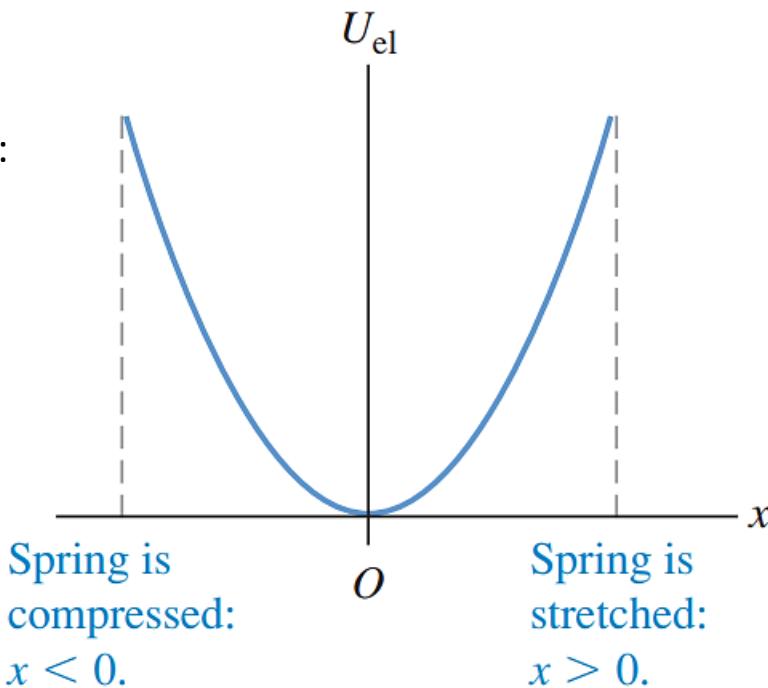
The work–energy theorem says that $W_{tot} = K_2 - K_1$ no matter what kind of forces are acting on a body. If the elastic force is the only force that does work on the body, then

$$W_{tot} = W_{el} = U_{el,1} - U_{el,2}$$

The work–energy theorem, $W_{tot} = K_2 - K_1$, then gives us

$$K_1 + U_{el,1} = K_2 + U_{el,2} \quad (\text{if only the elastic force does work})$$

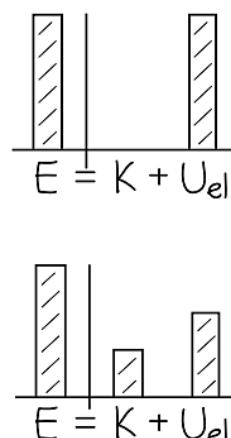
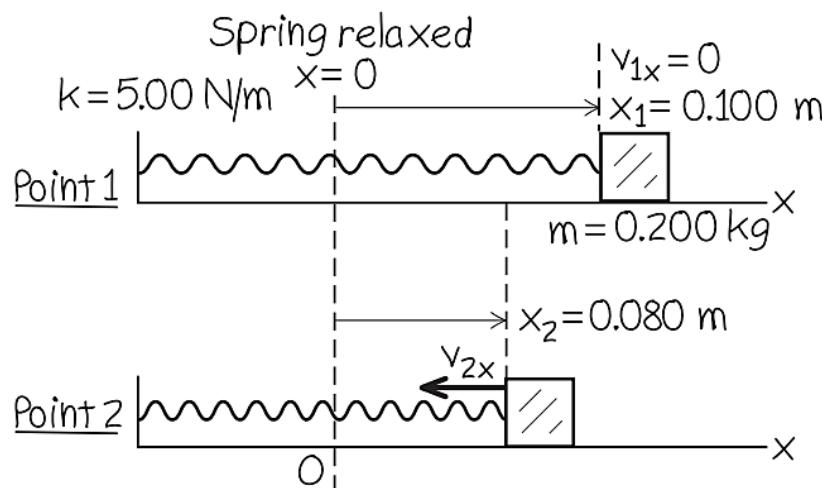
$$\frac{1}{2}mv_1^2 + \frac{1}{2}kx_1^2 = \frac{1}{2}mv_2^2 + \frac{1}{2}kx_2^2 \quad (\text{if only the elastic force does work})$$



9.3 Elastic Potential Energy

Example

A glider with mass $m=0.20 \text{ kg}$ sits on a frictionless horizontal air track, connected to a spring with force constant $k=5.0 \text{ N/m}$. You pull on the glider, stretching the spring 0.100 m , and release it from rest. The glider moves back toward its equilibrium position ($x=0$). What is its velocity when $x=0.080 \text{ m}$?



$$K_1 = \frac{1}{2}mv_{1x}^2 = \frac{1}{2}(0.200 \text{ kg})(0)^2 = 0$$

$$U_1 = \frac{1}{2}kx_1^2 = \frac{1}{2}(5.00 \text{ N/m})(0.100 \text{ m})^2 = 0.0250 \text{ J}$$

$$K_2 = \frac{1}{2}mv_{2x}^2$$

$$U_2 = \frac{1}{2}kx_2^2 = \frac{1}{2}(5.00 \text{ N/m})(0.080 \text{ m})^2 = 0.0160 \text{ J}$$

$$K_2 = K_1 + U_1 - U_2 = 0 + 0.0250 \text{ J} - 0.0160 \text{ J} = 0.0090 \text{ J}$$

$$v_{2x} = \pm \sqrt{\frac{2K_2}{m}} = \pm \sqrt{\frac{2(0.0090 \text{ J})}{0.200 \text{ kg}}} = \pm 0.30 \text{ m/s}$$

We choose the negative root because the glider is moving in the $-x$ direction. Our answer is $v_{2x} = -0.30 \text{ m/s}$

9.3 Elastic Potential Energy

Example

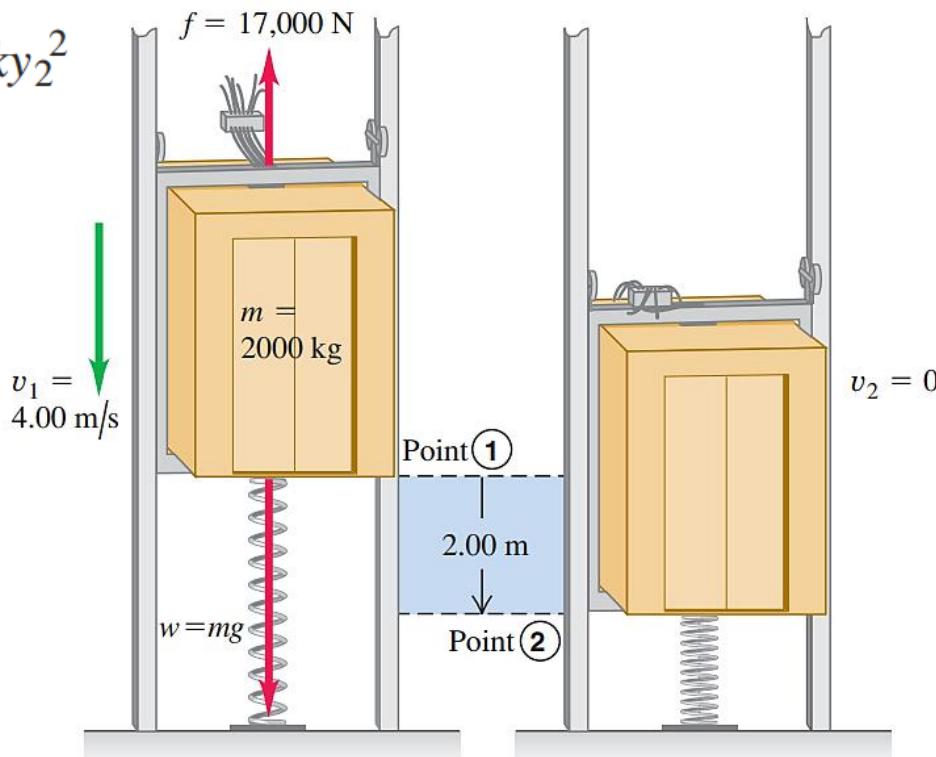
A 2000-kg elevator with broken cables in a test rig is falling at 4.0 m/s when it contacts a cushioning spring at the bottom of the shaft. The spring is intended to stop the elevator, compressing 2.00 m. During the motion a safety clamp applies a constant 17,000-N frictional force to the elevator. What is the necessary force constant k for the spring?

$$K_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}(2000 \text{ kg})(4.00 \text{ m/s})^2 = 16,000 \text{ J} \quad U_2 = mgy_2 + \frac{1}{2}ky_2^2$$

$$mgy_2 = (2000 \text{ kg})(9.80 \text{ m/s}^2)(-2.00 \text{ m}) = -39,200 \text{ J}$$

$$W_{\text{other}} = -(17,000 \text{ N})(2.00 \text{ m}) = -34,000 \text{ J}$$

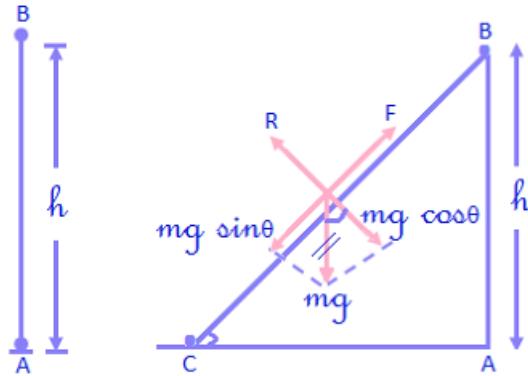
$$\begin{aligned} K_1 + 0 + W_{\text{other}} &= 0 + (mgy_2 + \frac{1}{2}ky_2^2) \\ k &= \frac{2(K_1 + W_{\text{other}} - mgy_2)}{y_2^2} \\ &= \frac{2[16,000 \text{ J} + (-34,000 \text{ J}) - (-39,200 \text{ J})]}{(-2.00 \text{ m})^2} \\ &= 1.06 \times 10^4 \text{ N/m} \end{aligned}$$



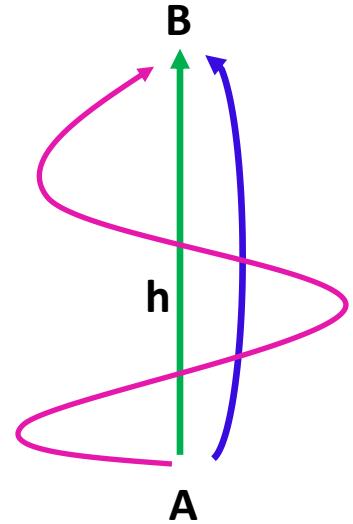
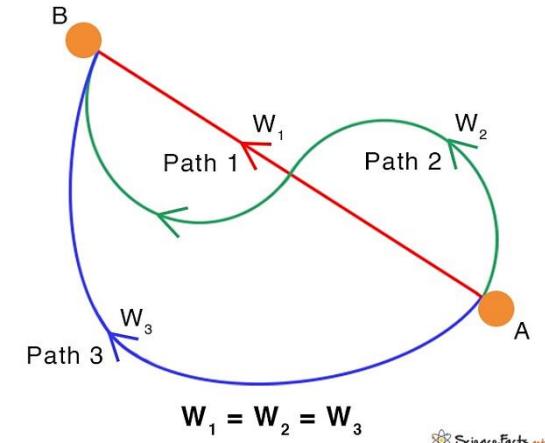
9.4 Conservative and Nonconservative Forces

Conservative Forces

If the work done by the force on a body depends only upon the initial and final positions of the body and is independent of the path taken, the force is the **conservative force**.



- Gravitational Force
- Spring Force
- Electric Force



The work done by a conservative force always has four properties:

1. It can be expressed as the difference between the initial and final values of a potential-energy function.
2. It is reversible.
3. It is independent of the path of the body and depends only on the starting and ending points.
4. When the starting and ending points are the same, the total work is zero.

When the only forces that do work are conservative forces, the total mechanical energy is constant.

$$E = K + U$$

9.4 Conservative and Nonconservative Forces

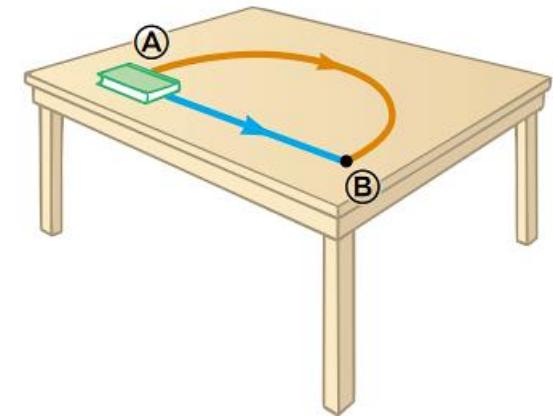
Nonconservative Forces

Nonconservative forces acting within a system cause a change in the mechanical energy E_{mech} of the system. The work done depends on the path, so the friction force cannot be conservative.

The work done against friction is greater along the brown path than along the blue path

Because the work done depends on the path, friction is a nonconservative force

Nonconservative forces acting in a system cause a *change* in the mechanical energy of the system



9.5 Force and Potential Energy

The conservative forces can be derived from an expression of the potential energy of the system. Suppose that there is a force exerting on this system and does work:

?

$$W = \int \vec{F}_x \cdot d\vec{x} = \Delta K = -U = -\Delta U(x)$$

If the point of application of the force undergoes an infinitesimal displacement dx , we can express the infinitesimal change in the potential energy of the system dU as

$$\vec{F}_x \cdot d\vec{x} = -dU(x)$$

$$(F_x \hat{\mathbf{i}}) \cdot (d_x \hat{\mathbf{i}}) = -dU(x)$$

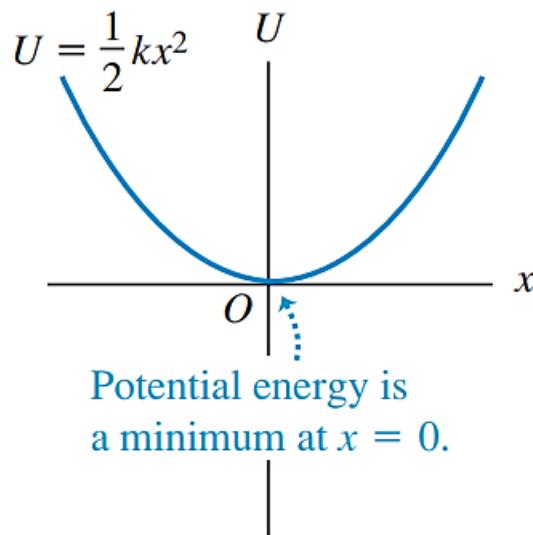
$$F_x d_x = -dU(x)$$

$$F_x = -\frac{dU}{dx}$$

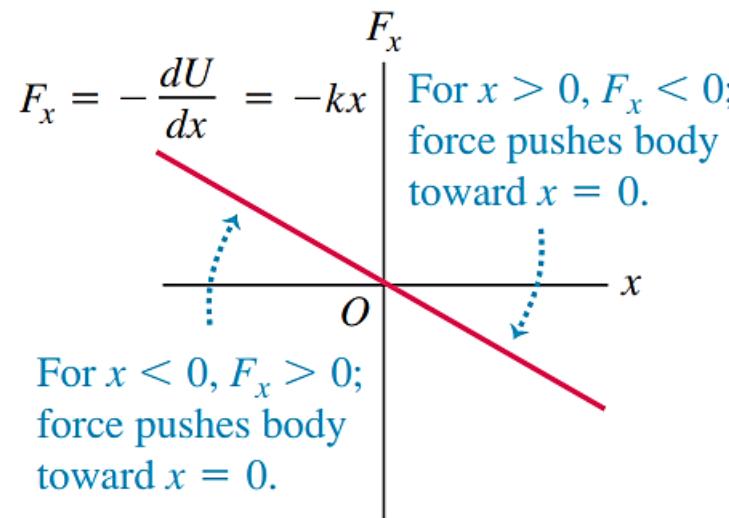
That is, **the x component of a conservative force acting on an object within a system equals the negative derivative of the potential energy of the system with respect to x .**

$$\mathbf{F} = -\frac{\partial U}{\partial x} \hat{\mathbf{i}} - \frac{\partial U}{\partial y} \hat{\mathbf{j}} - \frac{\partial U}{\partial z} \hat{\mathbf{k}}$$

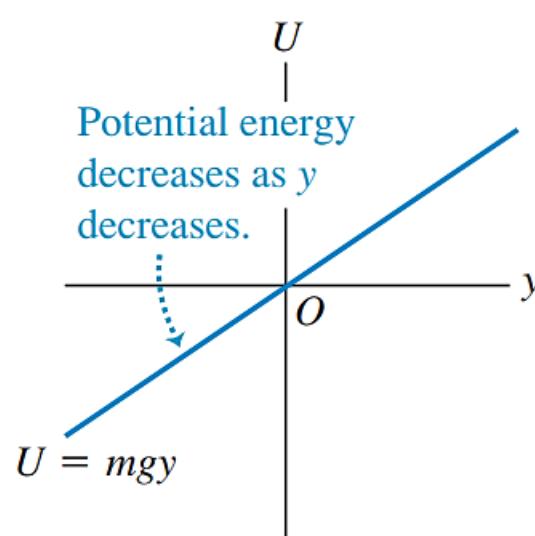
9.5 Force and Potential Energy



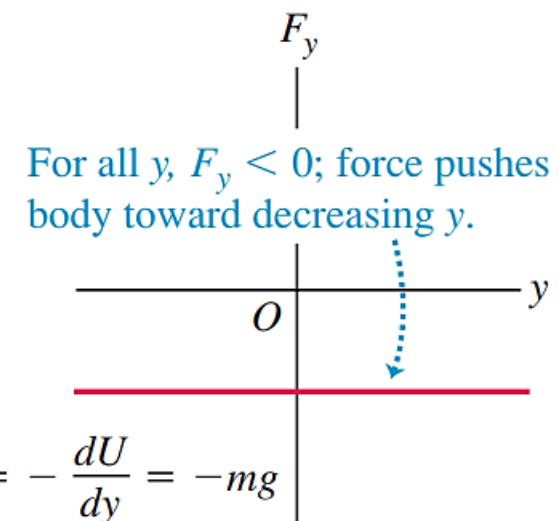
Potential energy is a minimum at $x = 0$.



For $x < 0$, $F_x > 0$; force pushes body toward $x = 0$.



Spring potential energy and force as functions of x



$$F_y = -\frac{dU}{dy} = -mg$$

Gravitational potential energy and force as functions of y