Name&Surname: 9.01.23

No:

1 (30p.)	2 (30p.)	3(40p.)	Sum
			100

MTH0141&MTH105 CALCULUS I
2022-2023 FALL FINAL EXAM

Q.1. (a) Let f be a real function satisfying the following properties: $D_f = \mathbb{R}$, $f(0) = 0$, $f'(0) = 0$, $f'(x) > 0$ for all $x \in D_f - \{0\}$, f is left continuous at $x = 1$, f is not continuous at $x = 1$ and $\lim_{x \to 1^+} f(x) = -\infty$. Try to define such a function f and skecth a draft graph for f. (hint. you may use the well known real functions to write a piecewise function f satisfying these properties)

(b) If $f(0) = 0$, $\lim_{x \longrightarrow 0} \dfrac{f(x)}{x} = e$ and $g(x) = e^{f(x)} \cos^2(f(x))$, find $g'(0)$.

(c) Prove that $\dfrac{d}{dx}(\tan^{-1} x) = \dfrac{1}{1+x^2}$

Q. 2. (a) When an open top rectangular prism plate of metal, whose height is proportional with its side lengths (both the width and the length, respectively) , is heated in an oven, its height increases at the rate of $0.02cm/\min$. At what rate the plate's volume increasing when the height is $3cm$?

(b)

(c) Find the linerization of $f(x) = (1+x)^k$ at $x = 1$(k is a constant). Use this linerization to find an approximation for the function $g(x) = \sqrt[3]{\left(1 - \dfrac{x}{2+x}\right)^2}$ for values of x near 1.

Q. 3. Sketch the graph of the curve $y = \dfrac{x^3}{1-x}$. (Make all the needed calculations clearly and evaluate the needed limits carefully.

GOOD LUCK

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