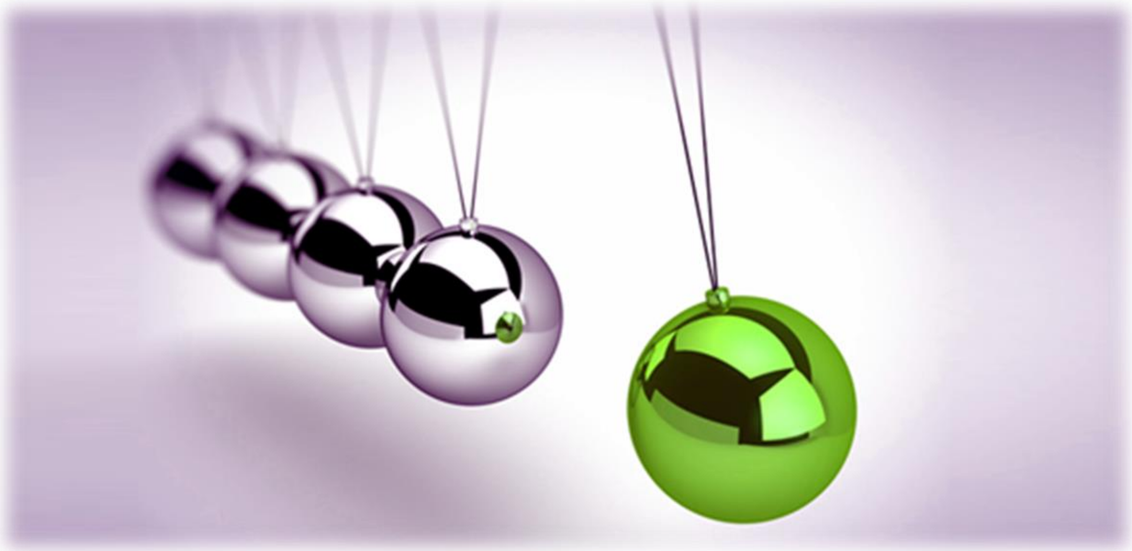
PHYSICS I - MECHANICS
MOTION in ONE DIMENSION



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CHAPTER 3. MOTION in ONE DIMENSION

Learning Objectives

3.1 Position, Displacement, Velocity and Speed

3.2 Instantaneous Velocity and Speed

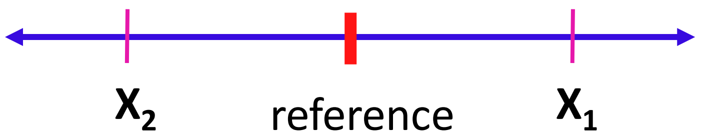
3.3 Acceleration

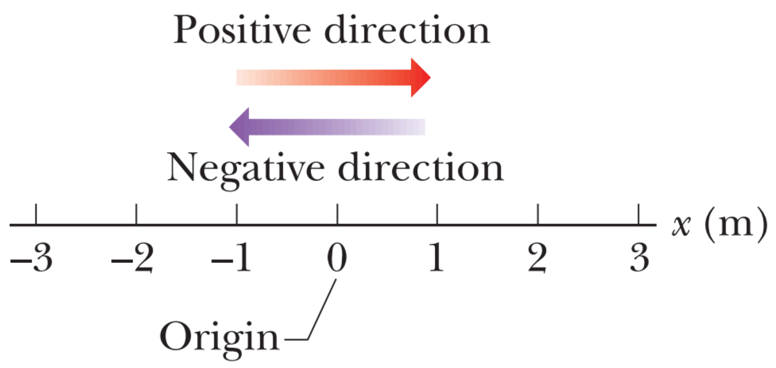
3.4 One-Dimensional Motion with Constant Acceleration

3.5 Freely Falling Objects

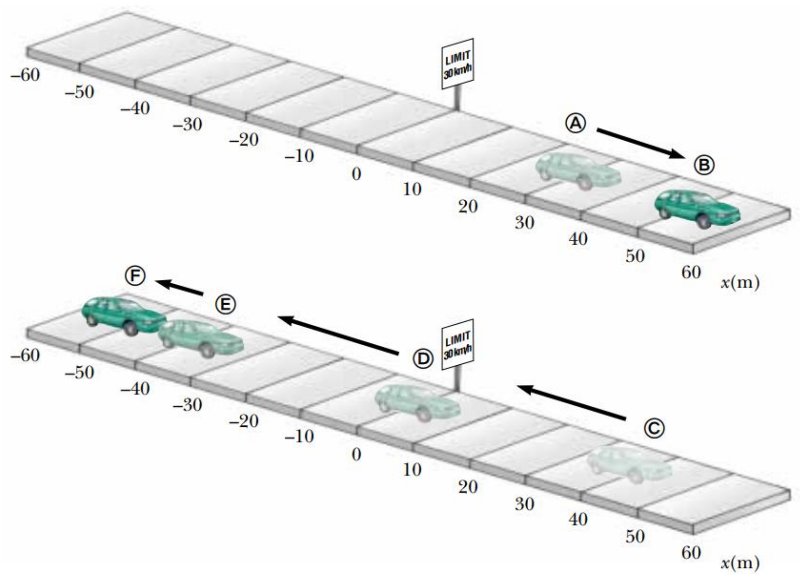
3.6 Graphical Integration in Motion Analysis

3.1 Position, Displacement, Velocity and Speed





Position: A particle's **position** is the location of the particle with respect to a chosen reference point that we can consider to be the origin of a coordinate system.



A car moves back and forth along a straight line taken to be the *x* axis.

Position of the Car at Various Times

Position	t(s)	x(m)
Ⓐ	0	30
Ⓑ	10	52
Ⓒ	20	38
Ⓓ	30	0
Ⓔ	40	−37
Ⓕ	50	−53

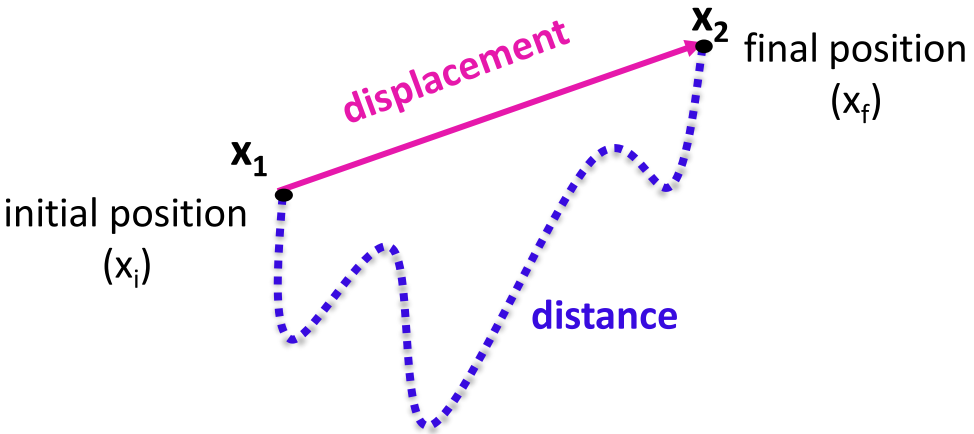
3.1 Position, Displacement, Velocity and Speed

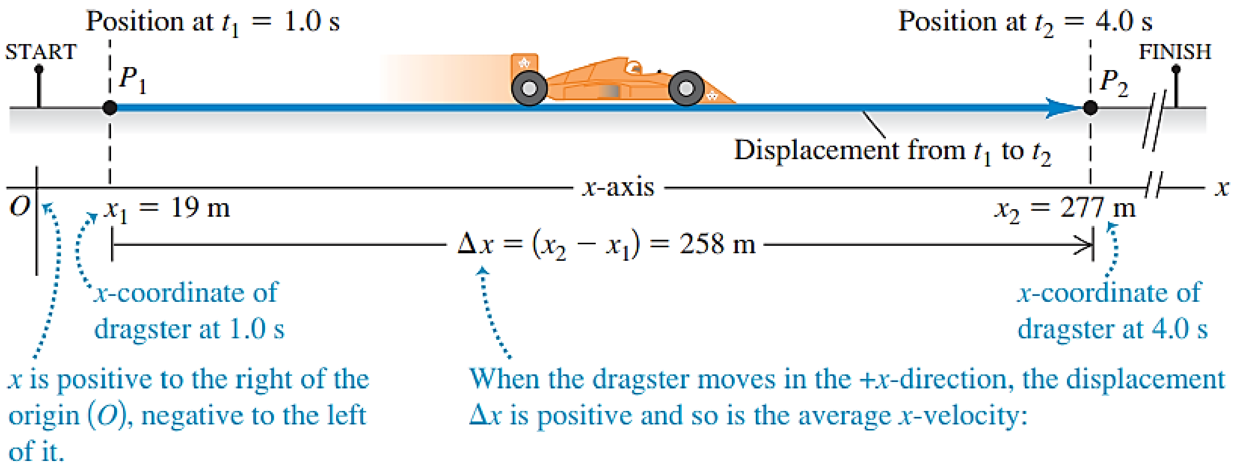
Displacement: A change from position x_1 to position x_2 is called a displacement **Δx.** It is a vector. $\Delta x = x_2 - x_1$

Distance: It is the length of actual path traveled. It is a scaler.

displacement of a particle

$$\Delta x \equiv x_f - x_i$$



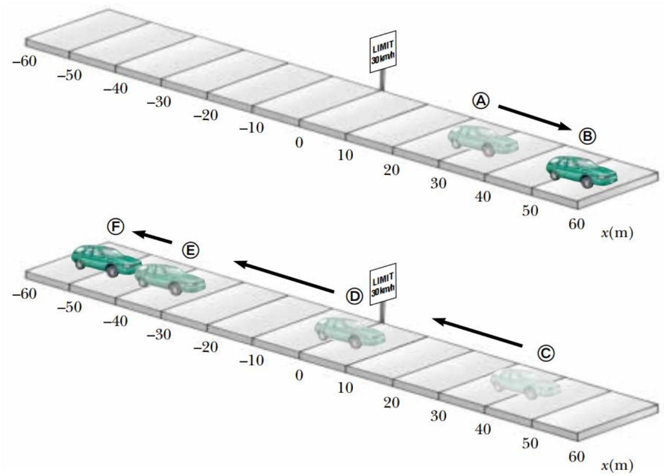


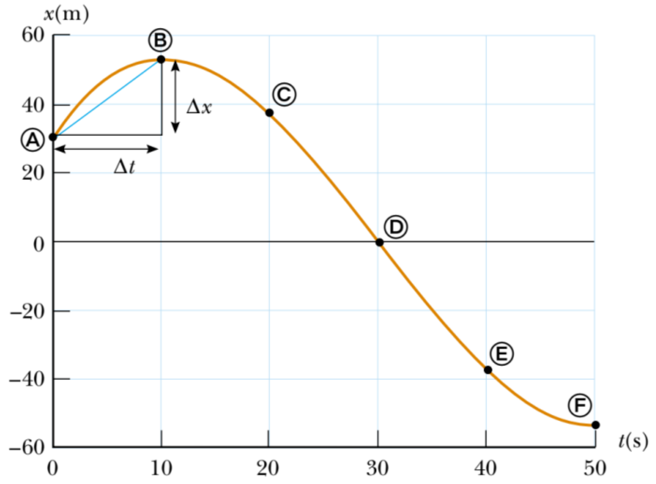
3.1 Position, Displacement, Velocity and Speed

Velocity:

The **average velocity** $\bar{v}_x$ of a particle is defined as the particle's displacement Δx divided by the time interval Δt during which that displacement occurs:

$$\boxed{\bar{v}_x \equiv \frac{\Delta x}{\Delta t}}$$





$\bar{v}_x$= (52 m - 30 m)/(10 s - 0) = 2.2 m/s

Speed:

The **average speed** of a particle, a scalar quantity, is defined as **the total distance traveled divided by the total time interval required to travel that distance:**

$$\boxed{\text{Average speed} = \frac{\text{total distance}}{\text{total time}}}$$

Does NOT include DIRECTION!!

3.1 Position, Displacement, Velocity and Speed

Example:

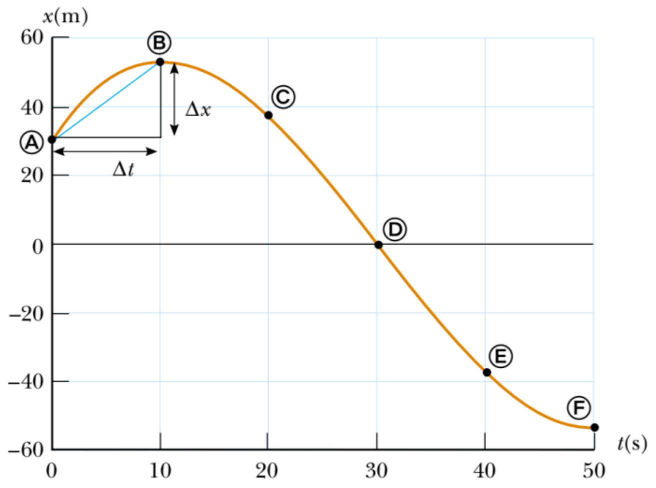
Find the displacement, average velocity, and average speed of the car between positions A and F.

Solution From the position–time graph, note that x_A =30 m at t_A = 0 s and that x_F = -53 m at t_F = 50 s.

$$\Delta x = X_F - X_A = -53\ m - 30\ m = -83\ m$$

This result means that the car ends up 83 m in the negative direction from where it started.

Position of the Car at Various Times		
Position	***t*(s)**	***x*(m)**
Ⓐ	0	30
Ⓑ	10	52
Ⓒ	20	38
Ⓓ	30	0
Ⓔ	40	−37
Ⓕ	50	−53



$$\bar{v}_x = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} = \frac{x_\mathrm{F} - x_\mathrm{A}}{t_\mathrm{F} - t_\mathrm{A}}$$

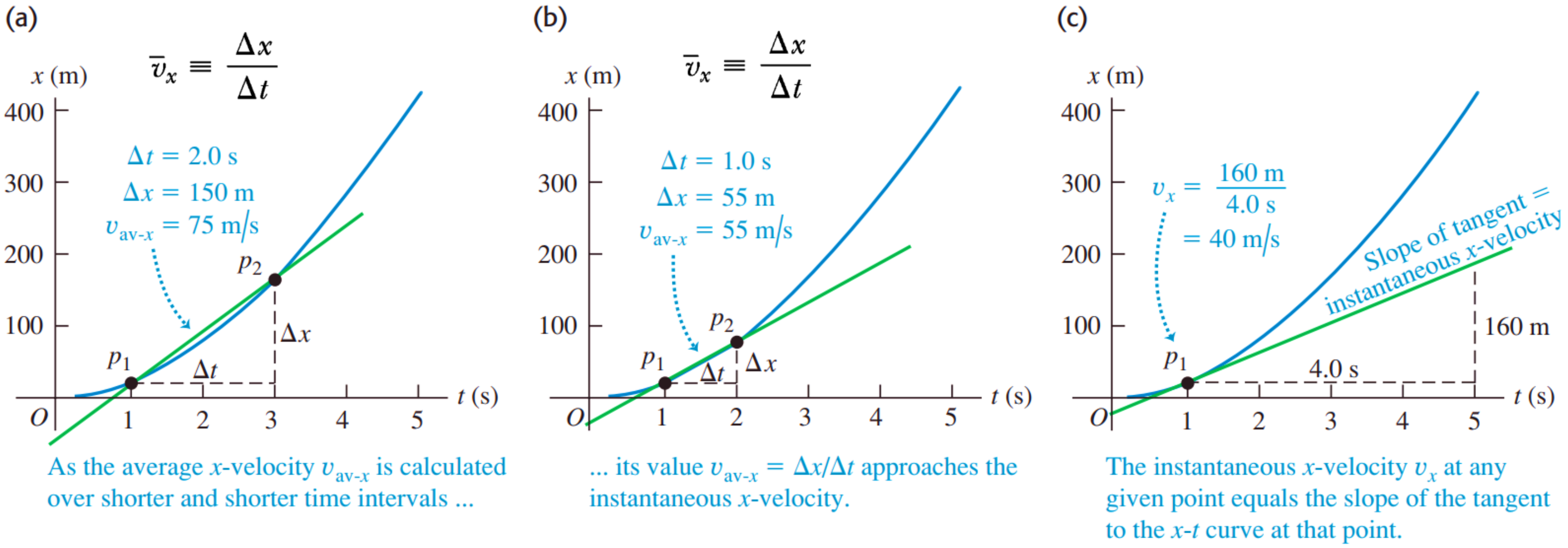
$$= \frac{-53\text{ m} - 30\text{ m}}{50\text{ s} - 0\text{ s}} = \frac{-83\text{ m}}{50\text{ s}}$$

$$= \boxed{-1.7\text{ m/s}}$$

$$\text{Average speed} = \frac{127\text{ m}}{50\text{ s}} = \boxed{2.5\text{ m/s}}$$

3.2 Instantaneous Velocity and Speed

We need to know the velocity of a particle at a particular instant in time, rather than the average velocity over a finite time interval.



The ***instantaneous velocity*** $\boldsymbol{v_x}$ equals the limiting value of the ratio Δ_x/Δ_t as Δ_t approaches zero

$$v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

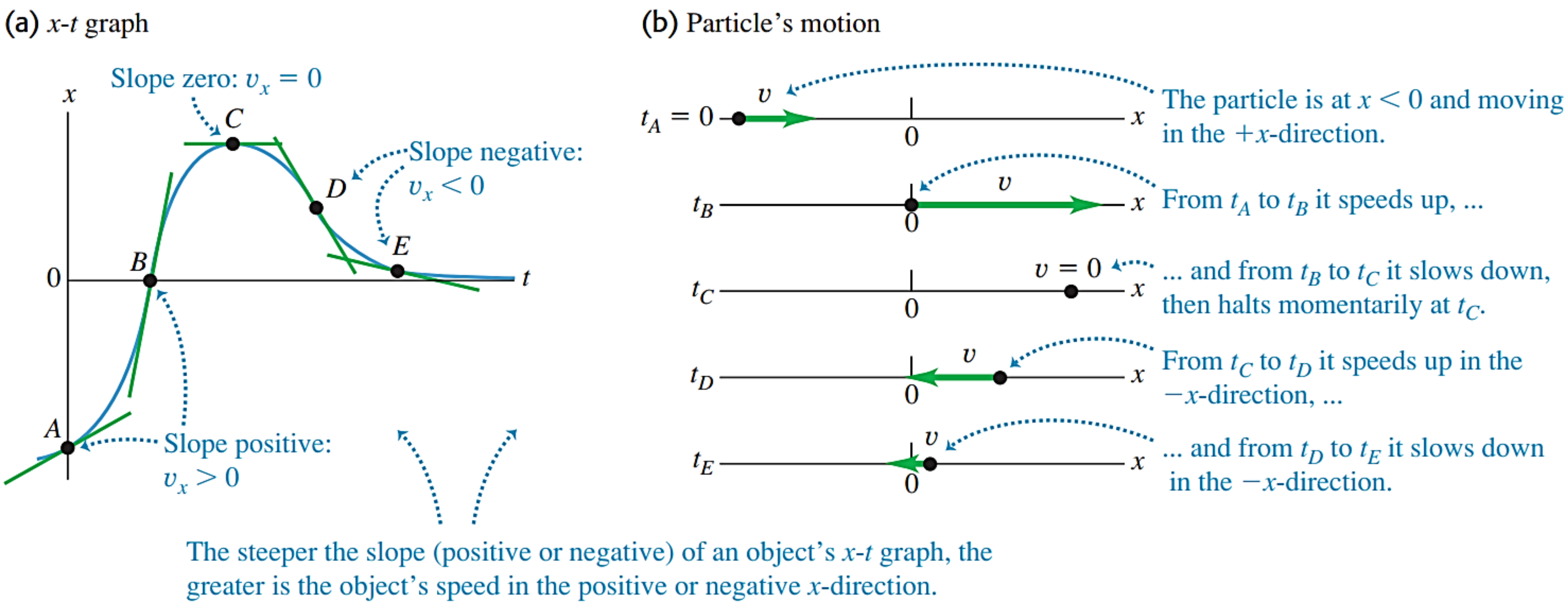
$$v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

The ***instantaneous speed*** of a particle is defined as the magnitude of its instantaneous velocity.

3.2 Instantaneous Velocity and Speed

The x-t graph of the motion of a particular particle

A motion diagram



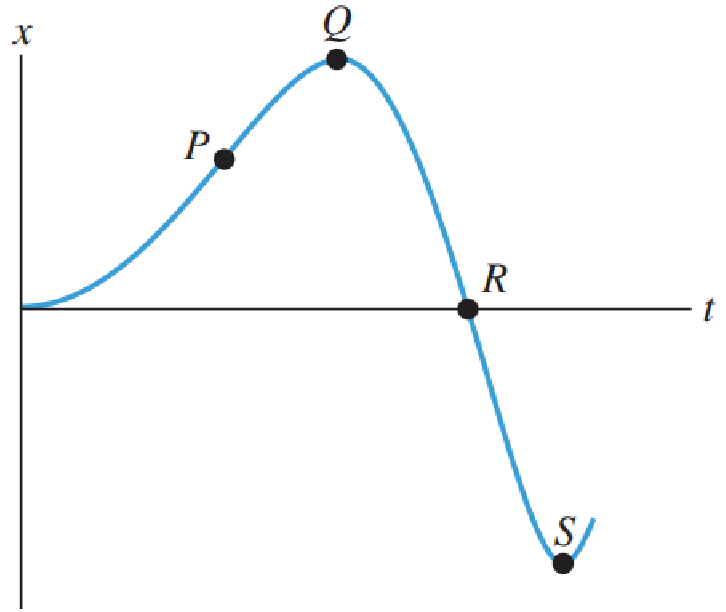
(a) The slope of the tangent at any point equals the velocity at that point.
(b) A motion diagram showing the position and velocity of the particle at each of the times labeled on the x-t graph.

3.2 Instantaneous Velocity and Speed

Example:

Figure is an *x*-*t* graph of the motion of a particle.

(a) Rank the values of the particle's *x*-velocity at the points *P*, *Q*, *R*, and *S* from most positive to most negative.
(b) At which points is positive?
(c) At which points is negative?
(d) At which points is zero?
(e) Rank the values of the particle's *speed* at the points *P*, *Q*, *R*, and *S* from fastest to slowest.



Answer:

a) P, Q and S (tie), R The x-velocity is
b) Positive when the slope of the x-t graph is positive (P),
c) negative when the slope is negative (R), and
d) zero when the slope is zero (Q and S).
e) R, P, Q and S (tie) The speed is greatest when the slope of the *x*-*t* graph is steepest (either positive or negative) and zero when the slope is zero.

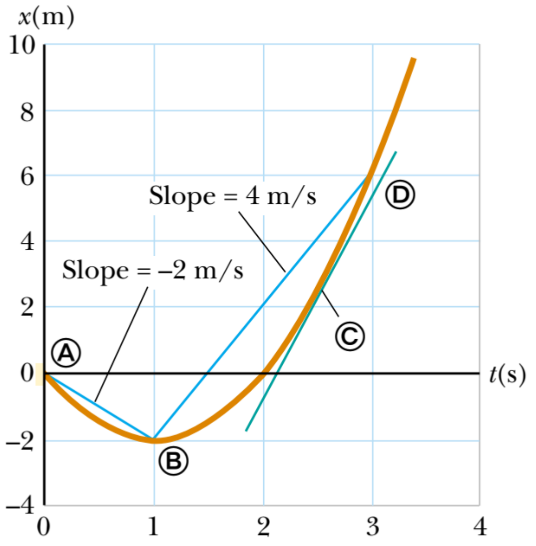
3.2 Instantaneous Velocity and Speed

Example:

A particle moves along the x axis. Its position varies with time according to the expression $x = -4t + 2t^2$ where x is in meters and t is in seconds. The position–time graph for this motion is shown in Figure below. Note that the particle moves in the negative x direction for the first second of motion, is momentarily at rest at the moment t = 1 s, and moves in the positive x direction at times t > 1 s.

(a) Determine the displacement of the particle in the time intervals t = 0 to t = 1 s and t = 1 s to t = 3 s.

$$t_i = t_A = 0 \qquad t_f = t_B = 1\text{ s} \qquad x = -4t + 2t^2$$



displacement between t =0 and t = 1 s

$$\begin{aligned}\Delta x_{A\to B} &= x_f - x_i = x_B - x_A\\ &= [-4(1) + 2(1)^2] - [-4(0) + 2(0)^2]\\ &= -2\text{ m}\end{aligned}$$

$$\begin{aligned}\Delta x_{B\to D} &= x_f - x_i = x_D - x_B\\ &= [-4(3) + 2(3)^2] - [-4(1) + 2(1)^2]\\ &= +8\text{ m}\end{aligned}$$

(b) Calculate the average velocity during these two time intervals

$$\Delta t = t_f - t_i = t_B - t_A = 1\text{ s.}$$

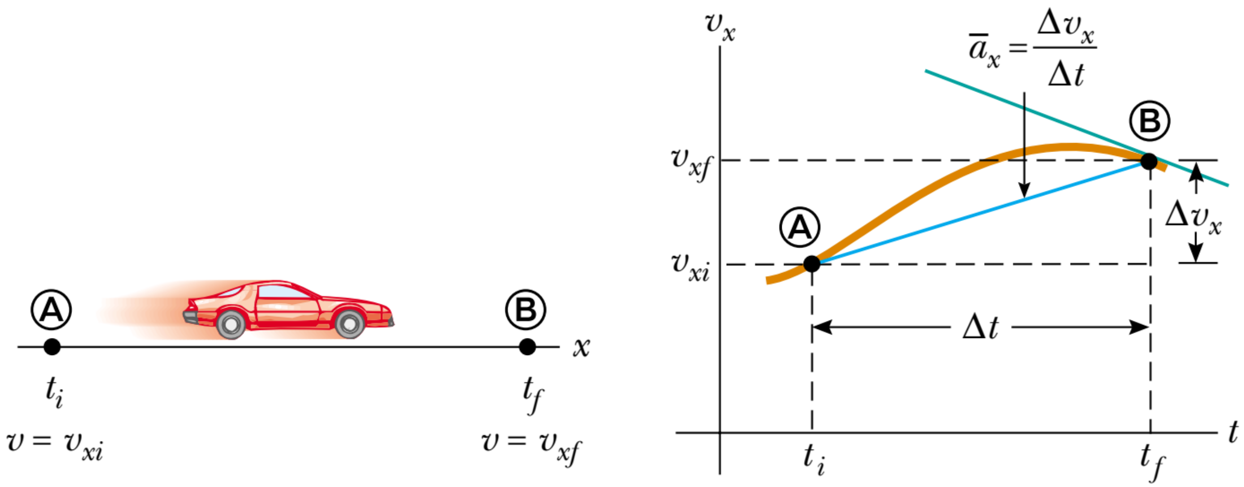
$$\bar{v}_{x(A\to B)} = \frac{\Delta x_{A\to B}}{\Delta t} = \frac{-2\text{ m}}{1\text{ s}} = -2\text{ m/s}$$

In the second time interval, $\Delta t = 2$ s; therefore,

$$\bar{v}_{x(B\to D)} = \frac{\Delta x_{B\to D}}{\Delta t} = \frac{8\text{ m}}{2\text{ s}} = +4\text{ m/s}$$

3.3 Accelaration

When the velocity of a particle changes with time, the particle is said to be ***accelerating*.**



Average Accelaration; $\bar{a}_x$ of the particle is defined as the *change* in velocity Δv_x divided by the time interval Δt during which that change occurs:

$$\bar{a}_x \equiv \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{t_f - t_i}$$

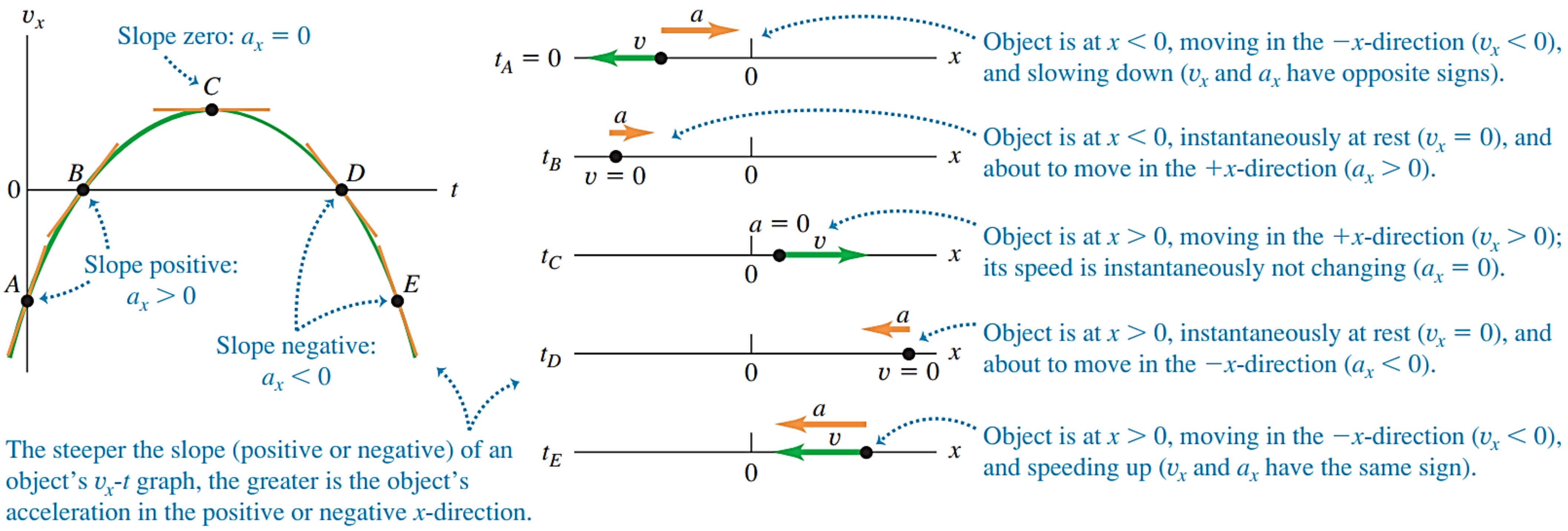
Instantaneous acceleration; as the limit of the average acceleration as Δt approaches zero.

$$a_x \equiv \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt}$$

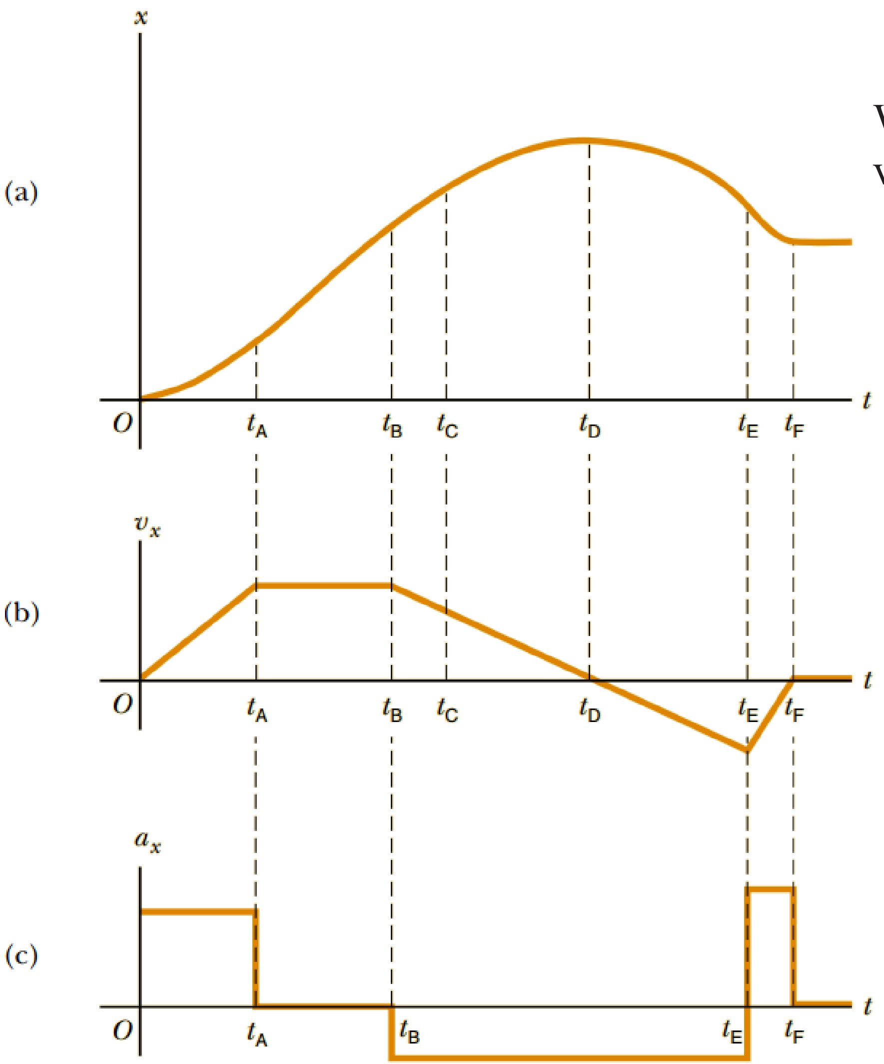
3.3 Accelaration

The v_x-t graph of the motion of a particular particle

Object's position, velocity, and acceleration on the x-axis

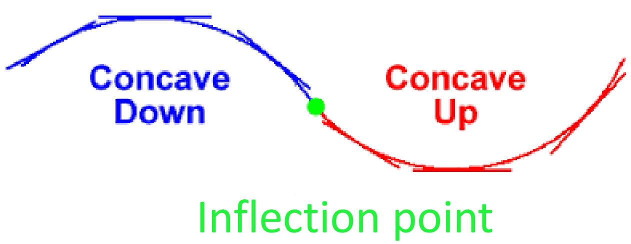


3.3 Accelaration



We can also learn about the acceleration of a body from a graph of its *position* versus time. Because $a_x = dvx/dt$ and $v_x = dx/dt$ we can write

$$\boxed{a_x = \frac{dv_x}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}}$$



3.3 Accelaration

Example:

The velocity of a particle moving along the *x* axis varies in time according to the expression $v_x = (40 - 5t^2)\ m/s$, where *t* is in seconds. Find the average acceleration in the time interval *t* = 0 to *t* = 2.0 s.

$$t_i = t_A = 0$$

$$v_{xA} = (40 - 5t_A^2)\ \text{m/s} = [40 - 5(0)^2]\ \text{m/s} = +40\ \text{m/s}$$

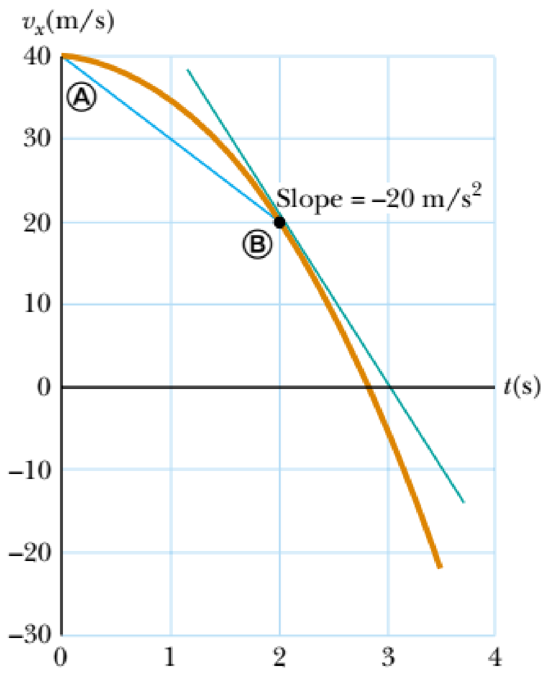
$$t_f = t_B = 2.0\ \text{s}$$

$$v_{xB} = (40 - 5t_B^2)\ \text{m/s} = [40 - 5(2.0)^2]\ \text{m/s} = +20\ \text{m/s}$$

$$\Delta t = t_B - t_A = 2.0\ \text{s}$$

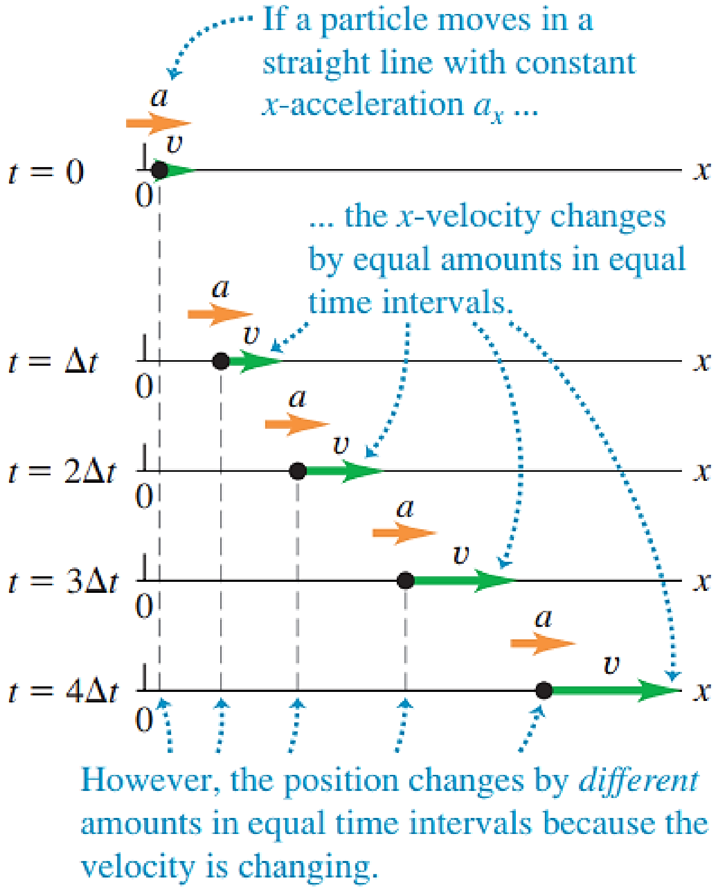
$$\bar{a}_x = \frac{v_{xf} - v_{xi}}{t_f - t_i} = \frac{v_{xB} - v_{xA}}{t_B - t_A} = \frac{(20 - 40)\ \text{m/s}}{(2.0 - 0)\ \text{s}}$$

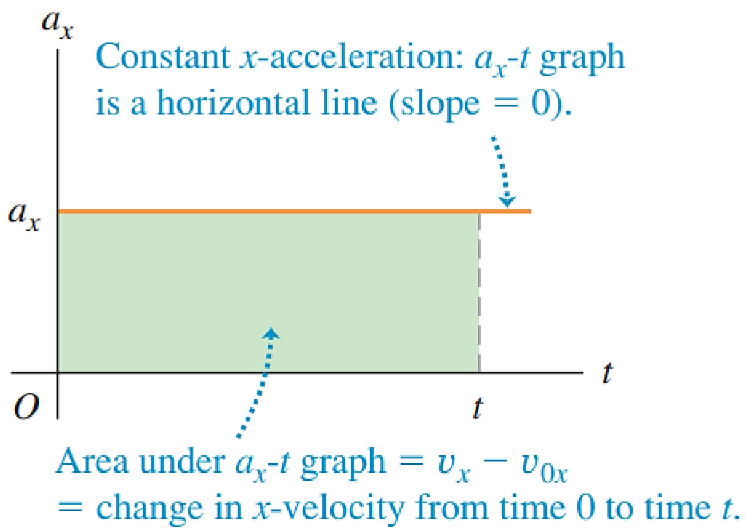
$$= \boxed{-10\ \text{m/s}^2}$$



3.4 One-Dimensional Motion with Constant Acceleration

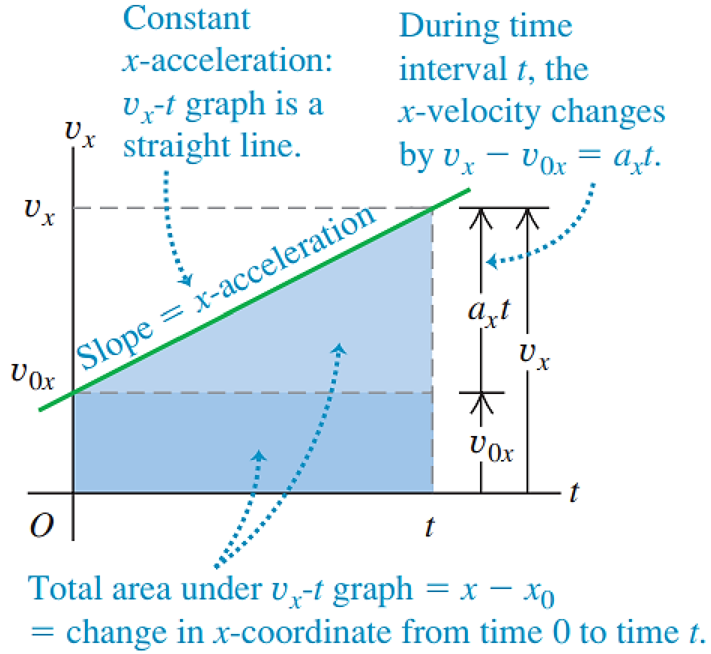
In the motion with constant acceleration, the velocity changes at the same rate throughout the motion. If the acceleration is constant, the average acceleration $\bar{a}_x$ over any time interval is numerically equal to the instantaneous acceleration a_x, take t_i=0.





$$a_x = \frac{v_{2x} - v_{1x}}{t_2 - t_1}$$

$$a_x = \frac{v_{xf} - v_{xi}}{t - 0}$$



$$v_{xf} = v_{xi} + a_x t \qquad \text{(for constant } a_x\text{)}$$

3.4 One-Dimensional Motion with Constant Acceleration

Arithmetic mean of the initial velocity v_{xi} and the final velocity v_{xf}

$$\bar{v}_x = \frac{v_{xi} + v_{xf}}{2} \qquad \text{(for constant } a_x\text{)}$$

$$\Delta x \equiv x_f - x_i \qquad\qquad \Delta t = t_f - t_i = t - 0 = t,$$

$$x_f - x_i = \bar{v}t = \tfrac{1}{2}(v_{xi} + v_{xf})\,t$$

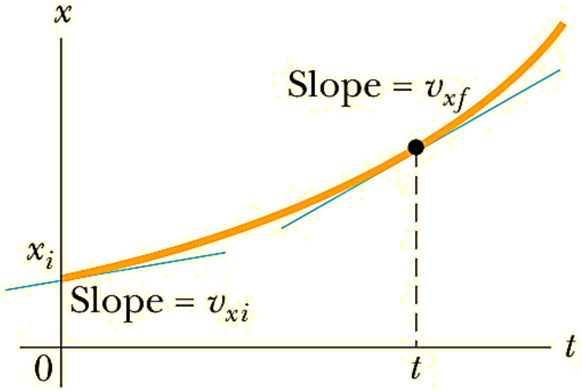
$$x_f = x_i + \tfrac{1}{2}(v_{xi} + v_{xf})\,t \qquad \text{(for constant } a_x\text{)}$$

$$v_{xf} = v_{xi} + a_x t \quad \Longrightarrow \quad x_f = x_i + \tfrac{1}{2}\left[v_{xi} + (v_{xi} + a_x t)\right]t$$

$$x_f = x_i + v_{xi}t + \tfrac{1}{2}a_x t^2 \qquad \text{(for constant } a_x\text{)}$$

This equation provides the final position of the particle at time *t* in terms of the initial velocity and the acceleration.

3.4 One-Dimensional Motion with Constant Acceleration



$$x_f = x_i + \frac{1}{2}(v_{xi} + v_{xf})\left(\frac{v_{xf} - v_{xi}}{a_x}\right) = \frac{v_{xf}^2 - v_{xi}^2}{2a_x}$$

$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i) \qquad \text{(for constant } a_x\text{)}$$

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2 \rightarrow \left.\begin{aligned} v_{xf} &= v_{xi} = v_x \\ x_f &= x_i + v_x t \end{aligned}\right\} \quad \text{when } a_x = 0$$

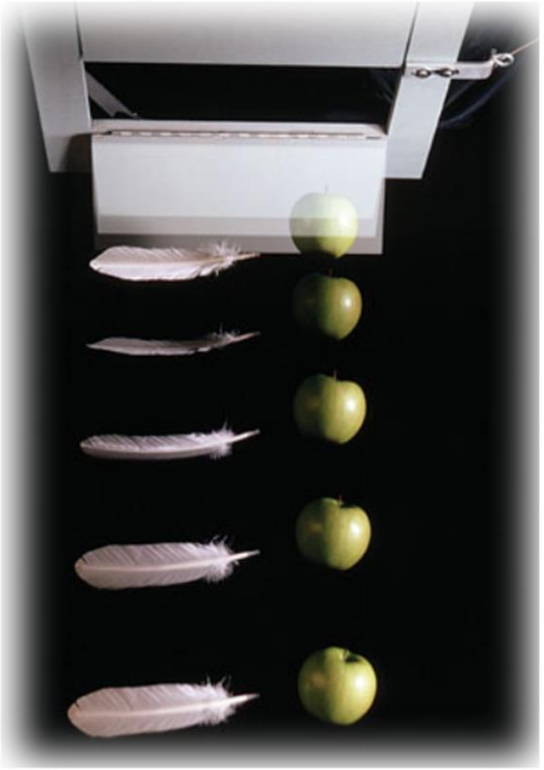
Kinematic Equations for Motion of a Particle Under Constant Acceleration	
Equation	**Information Given by Equation**
$v_{xf} = v_{xi} + a_x t$	Velocity as a function of time
$x_f = x_i + \frac{1}{2}(v_{xi} + v_{xf})t$	Position as a function of velocity and time
$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$	Position as a function of time
$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$	Velocity as a function of position

3.5 Freely Falling Objects

It is well known that, in the absence of air resistance, all objects dropped near the Earth's surface fall toward the Earth with the same constant acceleration under the influence of the Earth's gravity.

A freely falling object is any object moving freely under the influence of gravity alone, regardless of its initial motion.

Any freely falling object experiences an acceleration directed *downward,* regardless of its initial motion.



We shall denote the magnitude of the *free-fall acceleration* by the symbol g

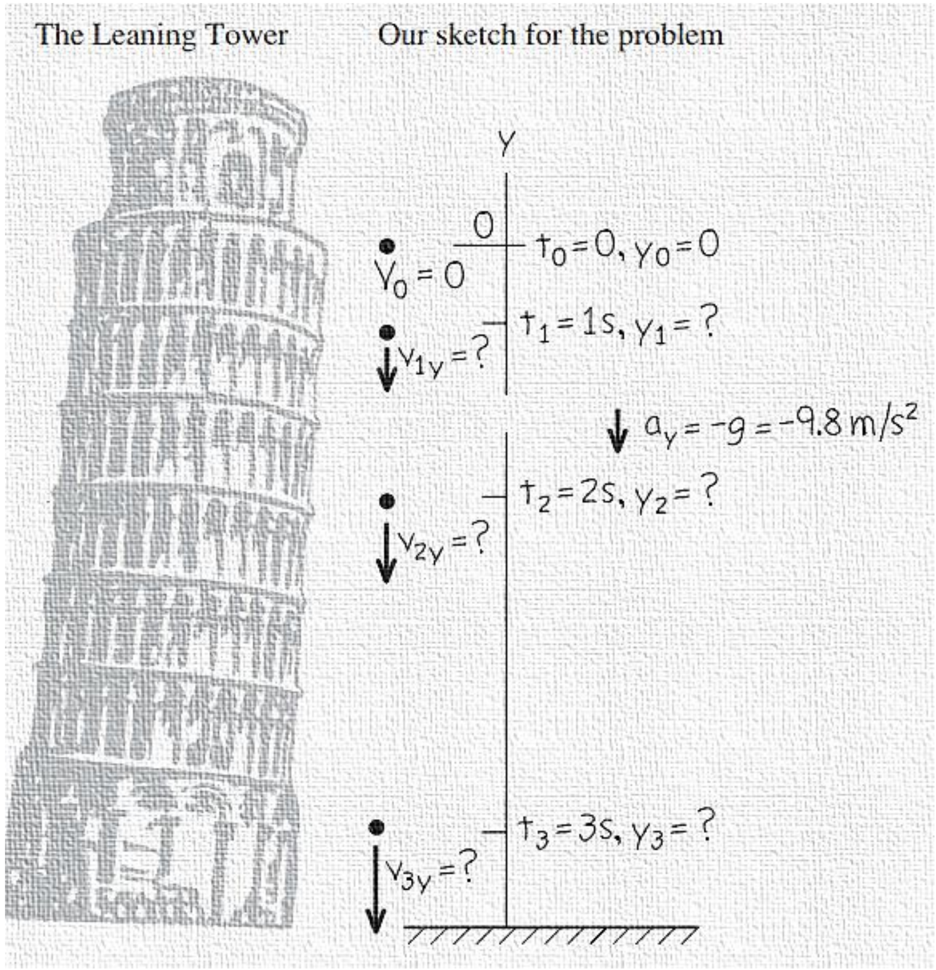
For freely falling object, we always choose

$$\boxed{a_y = -g = -9.80 \text{ m/s}^2}$$

3.5 Freely Falling Objects

Example:

A one-euro coin is dropped from the Leaning Tower of Pisa and falls freely from rest. What are its position and velocity after 1.0 s, 2.0 s, and 3.0 s?



$$y = y_0 + v_{0y}t + \tfrac{1}{2}a_y t^2 = 0 + 0 + \tfrac{1}{2}(-g)t^2 = (-4.9\ \mathrm{m/s^2})t^2$$

$$v_y = v_{0y} + a_y t = 0 + (-g)t = (-9.8\ \mathrm{m/s^2})t$$

When t=1.0 s, y=(-4.9 m/s^2) (1.0 s)^2= -4.9 m and $v_y = (-9.8\mathrm{m/s^2})(1.0\ s) = -9.8$ m/s; after 1s, the coin is 4.9 m below the origin (*y* is negative) and has a downward velocity (is negative) with magnitude 9.8 m/s.

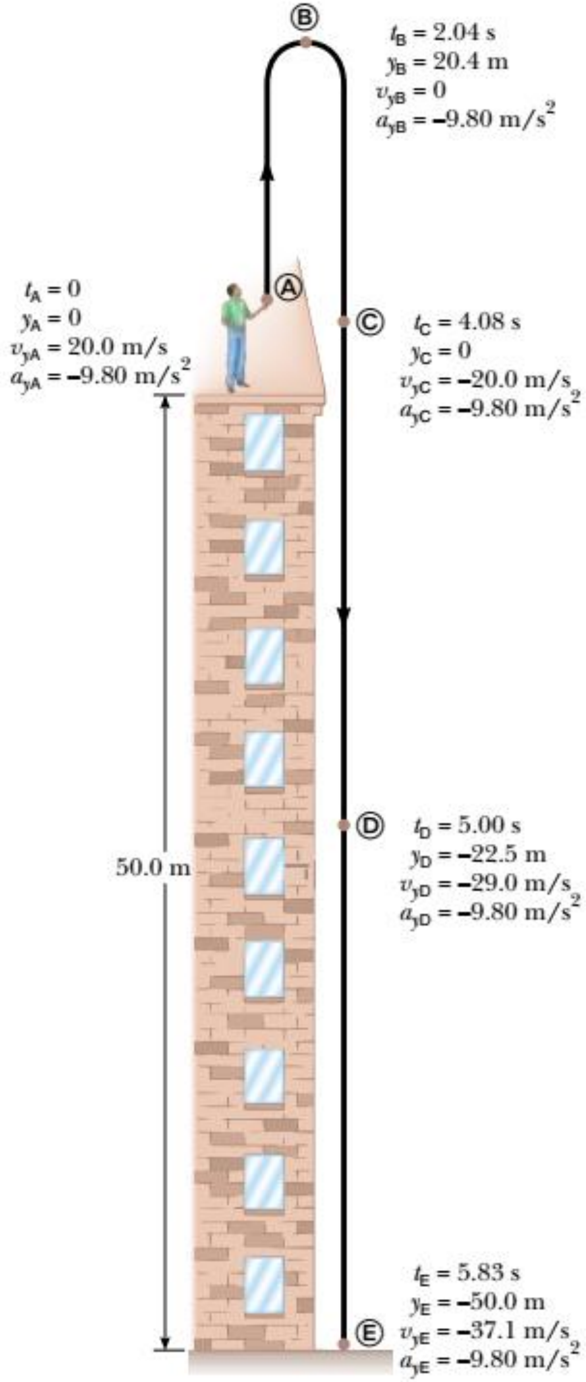
We can find the positions and *y*-velocities at 2.0 s and 3.0 s in the same way. The results are y=-20 m v_y =-20 m/s at t=2.0 s, and y=-44 m and v_y =-29 m/s at t=3.0 s.

3.5 Freely Falling Objects

Example:

A stone thrown from the top of a building is given an initial velocity of 20.0 m/s straight upward. The building is 50.0 m high, and the stone just misses the edge of the roof on its way down, as shown in Figure. Using $t_A = 0$ as the time the stone leaves the thrower's hand at position **A**,

(a) determine the time at which the stone reaches its maximum height,
(b) the maximum height,
(c) the time at which the stone returns to the height from which it was thrown,
(d) the velocity of the stone at this instant, and
(e) the velocity and position of the stone at $t = 5.00$ s.



Solution:

(a) $v_{yB} = v_{yA} + a_y t$ $\qquad v_{yB} = 0$ $\qquad t_A = 0$

$$0 = 20.0\text{ m/s} + (-9.80\text{ m/s}^2)t$$

$$t = t_B = \frac{20.0\text{ m/s}}{9.80\text{ m/s}^2} = \boxed{2.04\text{ s}}$$

(b) $x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$ $\qquad y_A = 0$

$$y_{max} = y_B = y_A + v_{xA}t + \frac{1}{2}a_y t^2$$

$$y_B = 0 + (20.0\text{ m/s})(2.04\text{ s}) + \frac{1}{2}(-9.80\text{ m/s}^2)(2.04\text{ s})^2$$

$$= \boxed{20.4\text{ m}}$$

3.5 Freely Falling Objects

(c) The motion from Ⓐ to Ⓒ is symmetric.

$$x_f = x_i + v_{xi}t + \frac{1}{2} a_x t^2 \qquad y_C = 0$$

$$y_C = y_A + v_{yA}t + \frac{1}{2} a_y t^2 \qquad t(20.0 - 4.90t) = 0$$

$$0 = 0 + 20.0t - 4.90t^2$$

$$t = 0 \qquad t = 4.08 \text{ s},$$

(d)

$$v_{xf} = v_{xi} + a_x t$$

$$v_{yC} = v_{yA} + a_y t = 20.0 \text{ m/s} + (-9.80 \text{ m/s}^2)(4.08 \text{ s})$$

$$= \boxed{-20.0 \text{ m/s}}$$

(e)

$$5.00 \text{ s} - 2.04 \text{ s} = 2.96 \text{ s}$$

$$v_{xf} = v_{xi} + a_x t \qquad t = 2.96 \text{ s}$$

$$v_{yD} = v_{yB} + a_y t = 0 \text{ m/s} + (-9.80 \text{ m/s}^2)(2.96 \text{ s})$$

$$= \boxed{-29.0 \text{ m/s}}$$

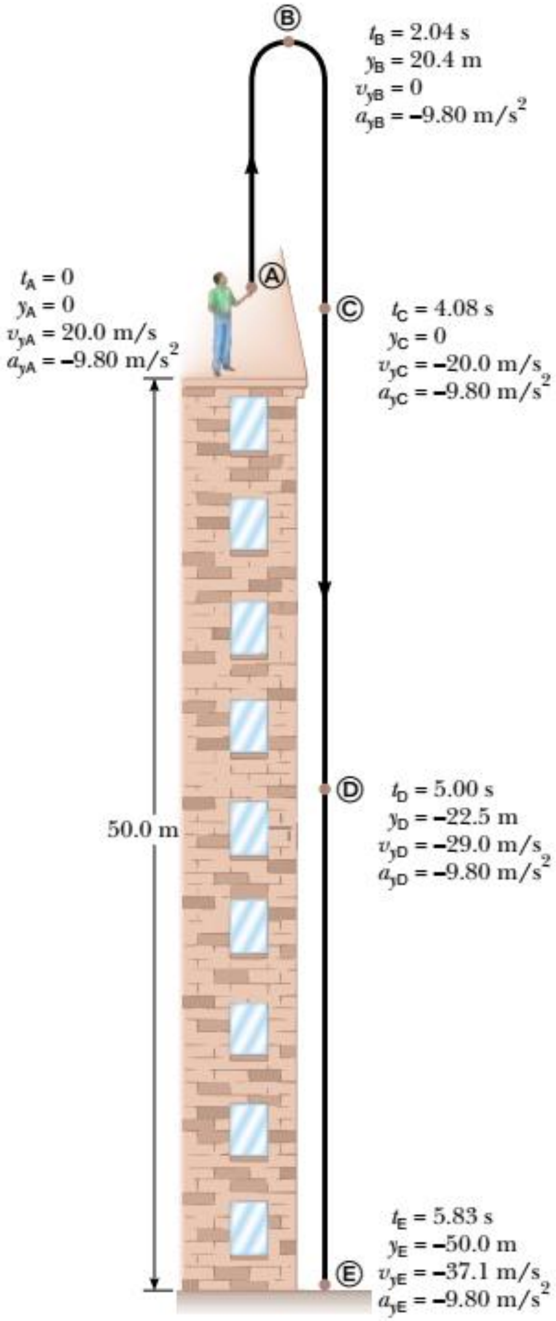
$$x_f = x_i + v_{xi}t + \frac{1}{2} a_x t^2$$

$t_D = 5.00$ s (with respect to $t_A = 0$)

$$y_D = y_C + v_{yC}t + \frac{1}{2} a_y t^2$$

$$= 0 + (-20.0 \text{ m/s})(5.00 \text{ s} - 4.08 \text{ s}) + \frac{1}{2}(-9.80 \text{ m/s}^2)(5.00 \text{ s} - 4.08 \text{ s})^2$$

$$= \boxed{-22.5 \text{ m}}$$



3.6 Graphical Integration in Motion Analysis

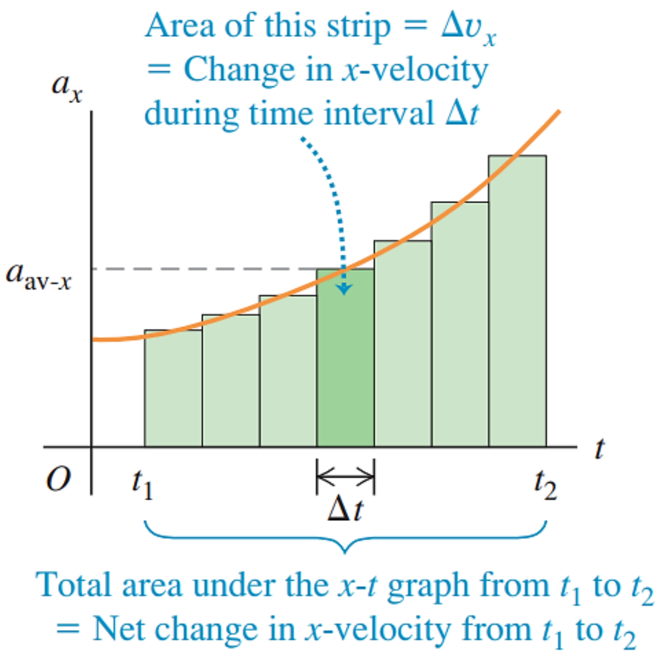
The average acceleration during Δt is $a_{av\text{-}x}$ and the change in velocity Δv_x during Δt is

$$\Delta v_x = a_{\text{av-}x}\,\Delta t$$

If v_{1x} is the velocity of the body at time t_1 and v_{2x} is the velocity at time t_2 then

$$v_{2x} - v_{1x} = \int_{v_{1x}}^{v_{2x}} dv_x = \int_{t_1}^{t_2} a_x\,dt$$

The change in the *x*-velocity v_x is the time integral of the acceleration a_x.



We can carry out exactly the same procedure with the curve of velocity versus time

$$x_2 - x_1 = \int_{x_1}^{x_2} dx = \int_{t_1}^{t_2} v_x\,dt$$

If $t_1 = 0$ and t_2 is any later time t, and if x_0 and v_{0x} are the position and velocity, respectively, at time t=0 then we can rewrite

$$v_x = v_{0x} + \int_0^t a_x\,dt \qquad\qquad x = x_0 + \int_0^t v_x\,dt$$

3.6 Graphical Integration in Motion Analysis

Example:

A driver is driving along a straight highway in a car. At t=0, when the car is moving at 10 m/s in the positive *x*-direction, it passes a signpost at x=50 m. The car's acceleration as a function of time is

$$a_x = 2.0\,m/s^2 - (0.10\ m/s^3)t$$

a) Find the car's velocity and position *x* as functions of time.
b) When is the car's velocity greatest?
c) What is that maximum velocity?
d) Where is the car when it reaches that maximum velocity?

Answer:

a)

$$v_x = v_{0x} + \int_0^t a_x\,dt \longrightarrow v_x = 10\text{ m/s} + \int_0^t [2.0\text{ m/s}^2 - (0.10\text{ m/s}^3)t]\,dt$$

$$= 10\text{ m/s} + (2.0\text{ m/s}^2)t - \tfrac{1}{2}(0.10\text{ m/s}^3)t^2$$

$$x = x_0 + \int_0^t v_x\,dt \longrightarrow x = 50\text{ m} + \int_0^t [10\text{ m/s} + (2.0\text{ m/s}^2)t - \tfrac{1}{2}(0.10\text{ m/s}^3)t^2]\,dt$$

$$= 50\text{ m} + (10\text{ m/s})t + \tfrac{1}{2}(2.0\text{ m/s}^2)t^2 - \tfrac{1}{6}(0.10\text{ m/s}^3)t^3$$

3.6 Graphical Integration in Motion Analysis

Answer (cont.):

b) The maximum value of v_x occurs when the velocity stops increasing and begins to decrease. At that instant, $dv_x/d_t = a_x = 0$. So, we set the expression for a_x equal to zero and solve for t:

$$0 = 2.0\ \mathrm{m/s^2} - (0.10\ \mathrm{m/s^3})t$$

$$t = \frac{2.0\ \mathrm{m/s^2}}{0.10\ \mathrm{m/s^3}} = 20\ \mathrm{s}$$

c) We find the maximum velocity by substituting t=20 s, the time from part b) when velocity is maximum, into the equation for v_x from part a):

$$v_{\text{max-}x} = 10\ \mathrm{m/s} + (2.0\ \mathrm{m/s^2})(20\ \mathrm{s}) - \tfrac{1}{2}(0.10\ \mathrm{m/s^3})(20\ \mathrm{s})^2$$

$$= 30\ \mathrm{m/s}$$

d) To find the car's position at the time that we found in part b), we substitute t=20 s into the expression for x from part a):

$$x = 50\ \mathrm{m} + (10\ \mathrm{m/s})(20\ \mathrm{s}) + \tfrac{1}{2}(2.0\ \mathrm{m/s^2})(20\ \mathrm{s})^2$$

$$- \tfrac{1}{6}(0.10\ \mathrm{m/s^3})(20\ \mathrm{s})^3$$

$$= 517\ \mathrm{m}$$