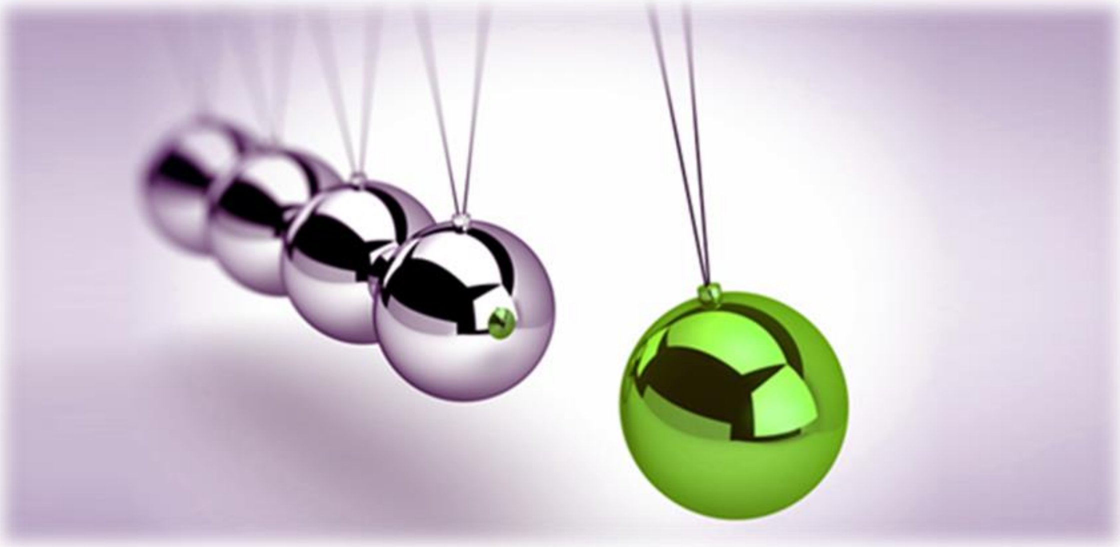
PHYSICS I - MECHANICS
CIRCULAR MOTION



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CHAPTER 7. Circular Motion

Learning Objectives

7.1 Dynamics of Circular Motion

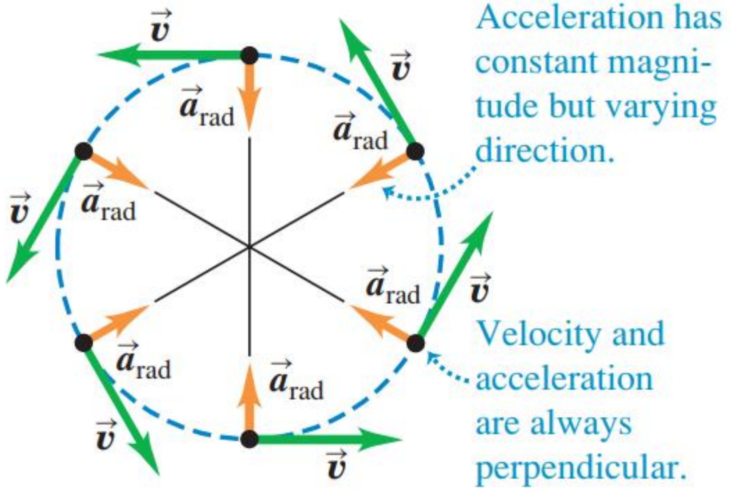
7.2 Newton’s Second Law Applied to Uniform Circular Motion

7.3 Newton’s Second Law Applied to Nonuniform Circular Motion

7.4 Motion in Accelerated Frames

7.5 Motion in the Presence of Resistive Forces

7.1 Dynamics of Circular Motion



A particle moves with a constant speed in a circular path of radius *r* with an acceleration:

$$a_{rad} = \frac{v^2}{r} \qquad a_{rad} = a_c \text{ (centripetal acceleration)}$$

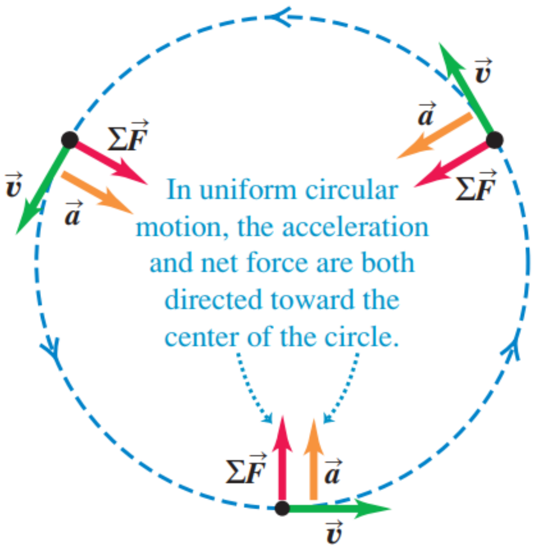
The centripetal acceleration, a_c is directed toward the center of the circle

The centripetal acceleration is always perpendicular to the velocity

$$a_{rad} = \frac{4r\pi^2}{T^2}$$

T, the time for one revolution

7.1 Dynamics of Circular Motion



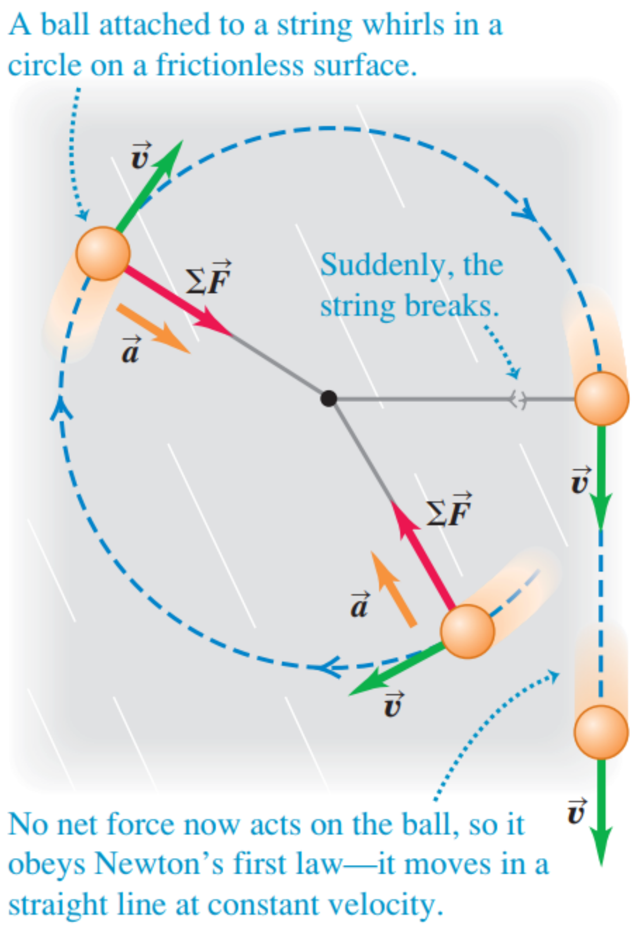
A force, $\vec{F}$, is associated with the centripetal acceleration

The force is also directed toward the center of the circle

Applying Newton's Second Law along the radial direction gives

$$\sum F = ma_c = m\frac{v^2}{r}$$

The magnitude of the acceleration is constant, so the magnitude of the net force must also be constant.

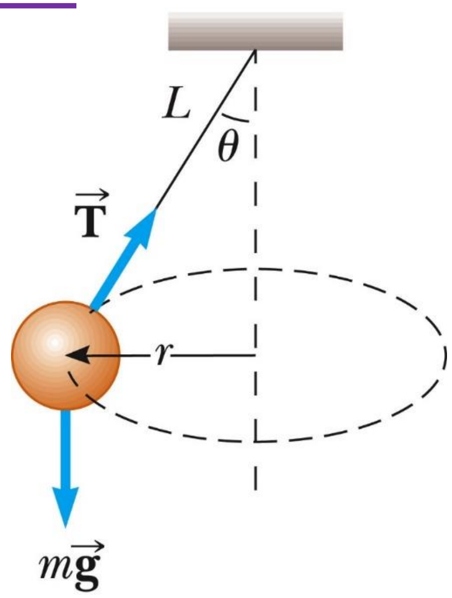


If the inward net force stops acting, the particle flies off in a straight line tangent to the circle.

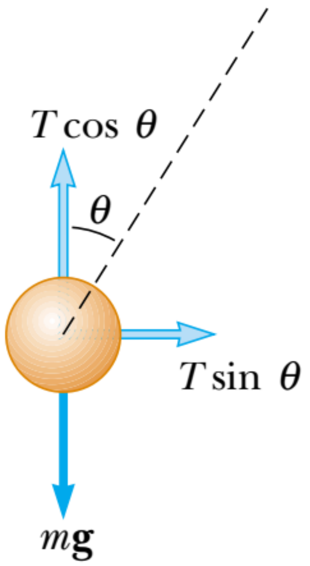
7.2 Newton's Second Law Applied to Uniform Circular Motion

Example: Conical Pendulum

A small object of mass *m* is suspended from a string of length *L*. The object revolves with constant speed *v* in a horizontal circle of radius *r.* Find an expression for *v*.



✓ *The object is in equilibrium in the vertical direction and undergoes uniform circular motion in the horizontal direction.*



$$\Sigma F_y = ma_y = 0$$

$$(1) \qquad T\cos\theta = mg$$

$$(2) \qquad \sum F = T\sin\theta = ma_c = \frac{mv^2}{r}$$

$$\sin\theta/\cos\theta = \tan\theta$$

$$\tan\theta = \frac{v^2}{rg}$$

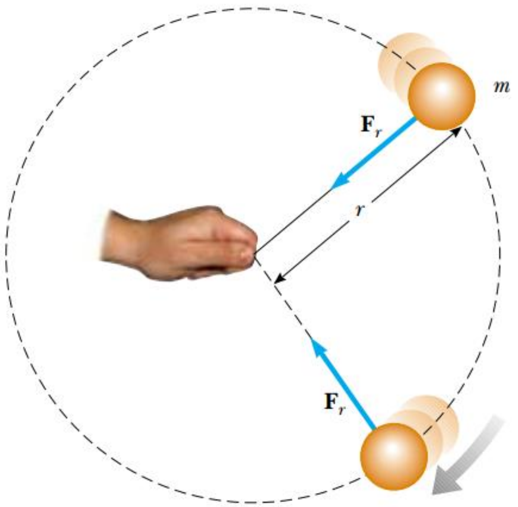
$$v = \sqrt{rg\tan\theta}$$

$$\boxed{v = \sqrt{Lg\sin\theta\tan\theta}}$$

7.2 Newton's Second Law Applied to Uniform Circular Motion

Example: Motion in a Horizontal Circle

A ball of mass 0.500 kg is attached to the end of a cord 1.50 m long. The ball is whirled in a horizontal circle as shown in Figure. If the cord can withstand a maximum tension of 50.0 N, what is the maximum speed at which the ball can be whirled before the cord breaks? Assume that the string remains horizontal during the motion.



The speed at which the object moves depends on the mass of the object and the tension in the cord
The centripetal force is supplied by the tension

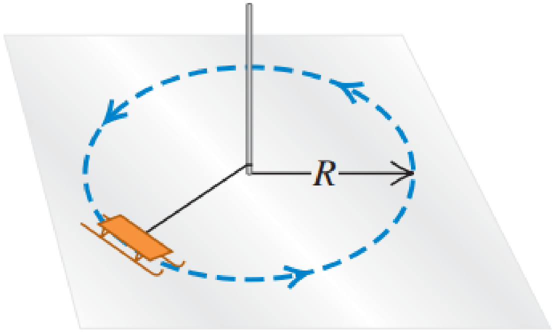
$$(1) \qquad T = m\frac{v^2}{r}$$

$$v = \sqrt{\frac{Tr}{m}}$$

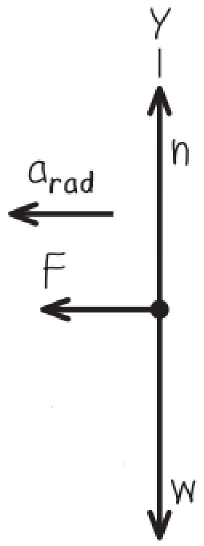
$$v_{\max} = \sqrt{\frac{T_{\max} r}{m}} = \sqrt{\frac{(50.0\ \mathrm{N})\ (1.50\ \mathrm{m})}{0.500\ \mathrm{kg}}} = \boxed{12.2\ \mathrm{m/s}}$$

7.2 Newton’s Second Law Applied to Uniform Circular Motion

Example: Force in uniform circular motion



A sled with a mass of 25.0 kg rests on a horizontal sheet of essentially frictionless ice.
It is attached by a 5.00-m rope to a post set in the ice. Once given a push, the sled revolves uniformly in a circle around the post. If the sled makes five complete revolutions every minute, find the force F exerted on it by the rope.



$$\sum F_x = F = ma_{\text{rad}}$$

$$T = (60.0\ \text{s})/(5\ \text{rev}) = 12.0\ \text{s}$$

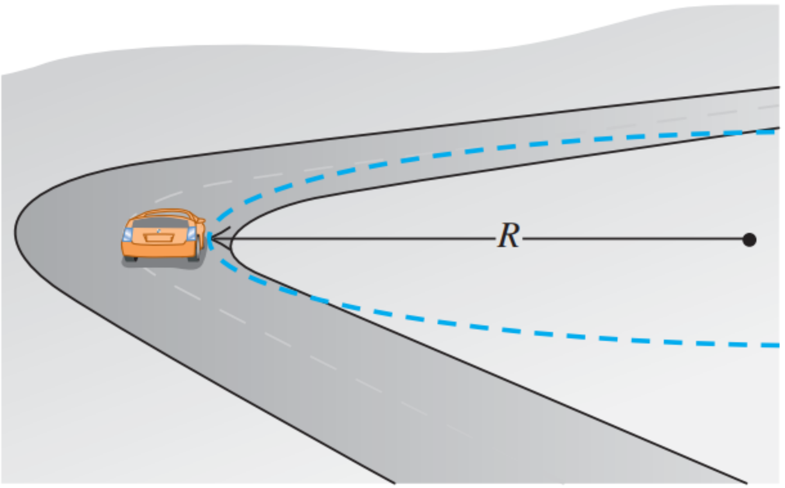
$$a_{\text{rad}} = \frac{4\pi^2 R}{T^2} = \frac{4\pi^2(5.00\ \text{m})}{(12.0\ \text{s})^2} = 1.37\ \text{m/s}^2$$

$$F = ma_{\text{rad}} = (25.0\ \text{kg})(1.37\ \text{m/s}^2)$$
$$= 34.3\ \text{kg}\cdot\text{m/s}^2 = 34.3\ \text{N}$$

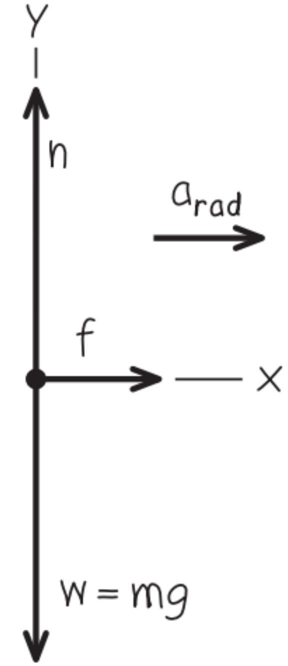
7.2 Newton’s Second Law Applied to Uniform Circular Motion

Example: Rounding a flat curve

The sports car is rounding a flat, unbanked curve with radius R. If the coefficient of static friction between tires and road is μ_s what is the maximum speed v_{max} at which the driver can take the curve without sliding?



Free-body diagram for car



$$a_{rad} = \frac{v^2}{r}$$

$$\sum F_x = f = ma_{\text{rad}} = m\frac{v^2}{R}$$

$$\sum F_y = n + (-mg) = 0$$

$$f_{\text{max}} = \mu_s n = \mu_s mg$$

$$\mu_s mg = m\frac{v_{\text{max}}^2}{R} \quad \text{so} \quad v_{\text{max}} = \sqrt{\mu_s g R}$$

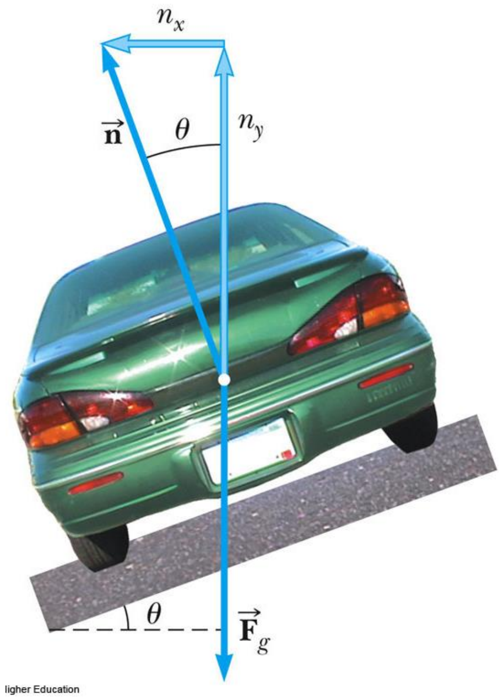
As an example, if $\mu_s = 0.96$ and $R = 230\text{ m}$, we have

$$v_{\text{max}} = \sqrt{(0.96)(9.8\text{ m/s}^2)(230\text{ m})} = 47\text{ m/s}$$

7.2 Newton's Second Law Applied to Uniform Circular Motion

Example: Rounding a Banked Curve

A car moving at the designated speed can negotiate the curve even when the road is covered with ice. Such a ramp is usually banked; this means the roadway is tilted toward the inside of the curve. Suppose the designated speed for the ramp is to be 13.4 m/s and the radius of the curve is 50.0 m. At what angle should the curve be banked?



$$n_x = n \sin \theta$$

(1) $$\sum F_r = n \sin \theta = \frac{mv^2}{r}$$

$$\sum F_y = 0$$

(2) $$n \cos \theta = mg$$

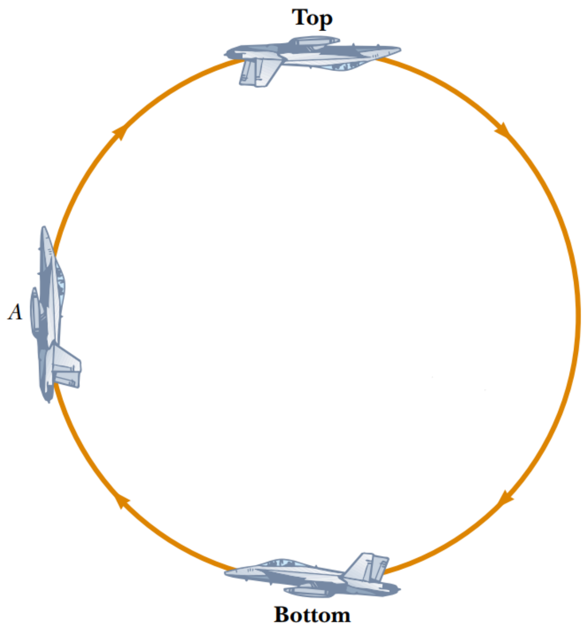
(3) $$\tan \theta = \frac{v^2}{rg}$$

$$\theta = \tan^{-1}\left(\frac{(13.4 \text{ m/s})^2}{(50.0 \text{ m})(9.80 \text{ m/s}^2)}\right) = 20.1°$$

- ✓ If the car rounds the curve at less than the design speed, friction is necessary to keep it from sliding down the bank
- ✓ If the car rounds the curve at more than the design speed, friction is necessary to keep it from sliding up the bank

7.2 Newton's Second Law Applied to Uniform Circular Motion

Example: Loop-the-Loop

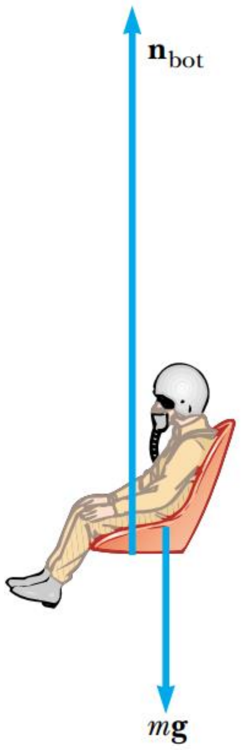


A pilot of mass m in a jet aircraft executes a loop-the-loop, as shown in figure. In this case, the aircraft moves in a vertical circle of radius 2.70 km at a constant speed of 225 m/s. Determine the force exerted by the seat on the pilot

(A) at the bottom of the loop and

(B) at the top of the loop. Express your answers in terms of the weight of the pilot mg.

(A)



$$\sum F = n_{\text{bot}} - mg = m\frac{v^2}{r}$$

$$n_{\text{bot}} = mg + m\frac{v^2}{r} = mg\left(1 + \frac{v^2}{rg}\right)$$

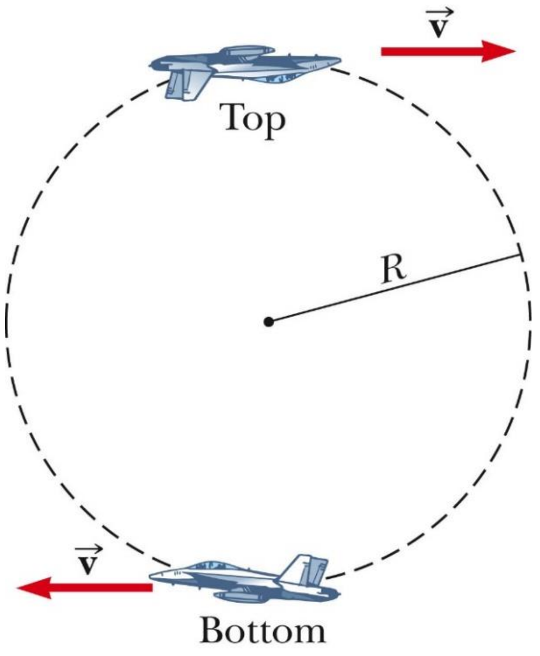
$$n_{\text{bot}} = mg\left(1 + \frac{(225\ \text{m/s})^2}{(2.70 \times 10^3\ \text{m})(9.80\ \text{m/s}^2)}\right) = \boxed{2.91\,mg}$$

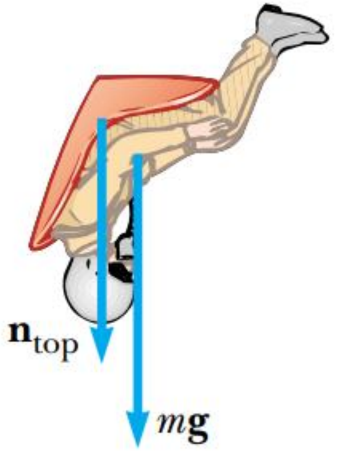
The magnitude of the force $\mathbf{n}_{bot}$ *exerted by the seat on the pilot is greater than the weight of the pilot by a factor of 2.91.*

7.2 Newton's Second Law Applied to Uniform Circular Motion

Example: Loop-the-Loop (cont.)

(B)





$$\sum F = n_{\text{top}} + mg = m\,\frac{v^2}{r}$$

$$n_{\text{top}} = m\,\frac{v^2}{r} - mg = mg\left(\frac{v^2}{rg} - 1\right)$$

$$n_{\text{top}} = mg\left(\frac{(225\ \text{m/s})^2}{(2.70\times10^3\ \text{m})(9.80\ \text{m/s}^2)} - 1\right)$$

$$= \boxed{0.913mg}$$

In this case, the magnitude of the force exerted by the seat on the pilot is less than his true weight by a factor of 0.913, and the pilot feels lighter.

7.3 Newton's Second Law Applied to Nonuniform Circular Motion

Nonuniform Circular Motion

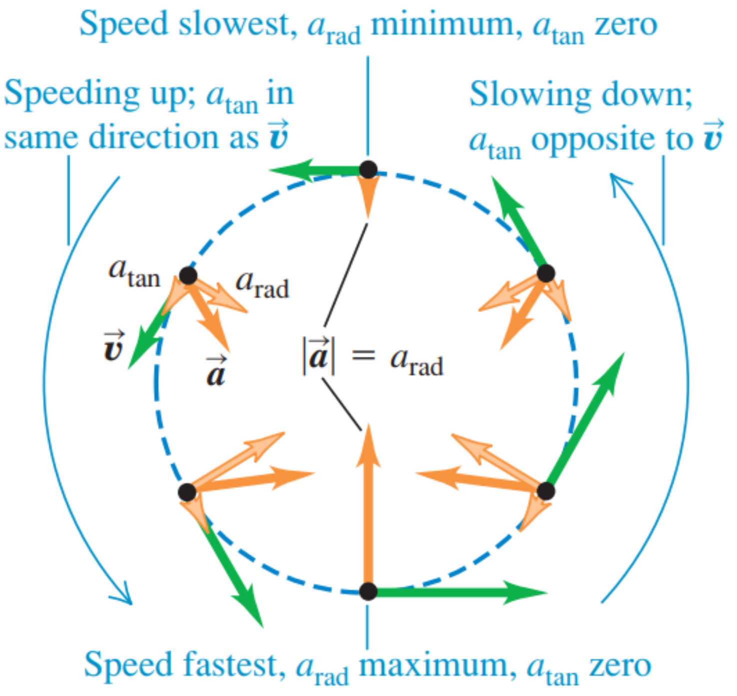
In circular motion the magnitude of the velocity could also be changing. If the speed varies, we call the motion **nonuniform circular motion.**

In this case, the velocity vector is always tangent to the path.

The acceleration vector **a** has two components

–Along the Radius a_r
–Perpendicular to the radius a_t

$$a_{\text{rad}} = \frac{v^2}{R} \quad \text{and} \quad a_{\tan} = \frac{d|\vec{v}|}{dt} \quad \text{(nonuniform circular motion)}$$



✓ The *tangential acceleration* causes the change in the speed of the particle.

✓ The *radial acceleration* comes from a change in the direction of the velocity vector.

$$\boldsymbol{a} = \boldsymbol{a_{rad}} + \boldsymbol{a_t}$$

7.3 Newton's Second Law Applied to Nonuniform Circular Motion

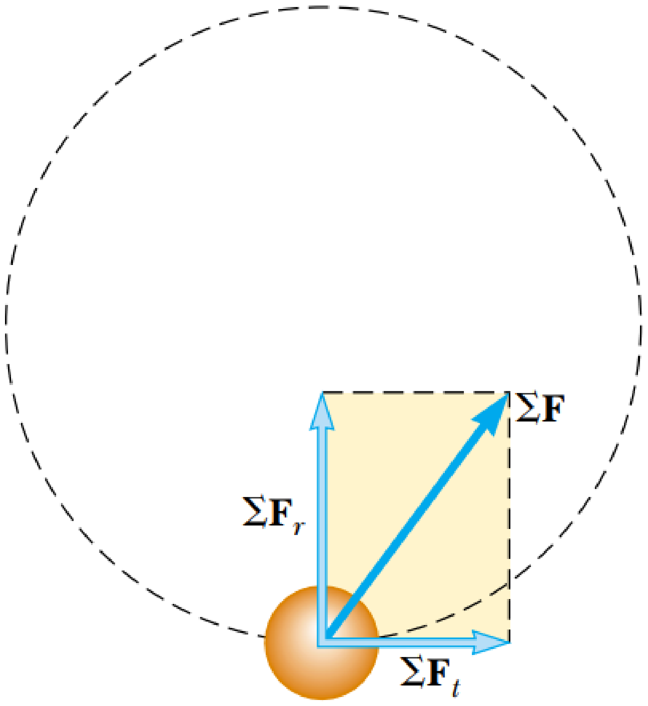
Nonuniform Circular Motion

The acceleration and force have radial and tangential components

$\overrightarrow{F_r}$ produces the centripetal acceleration and is directed toward the center of the circle

$\overrightarrow{F_t}$ is tangent to the circle and is responsible for the tangential acceleration, which represents a change in the speed of the particle with time.

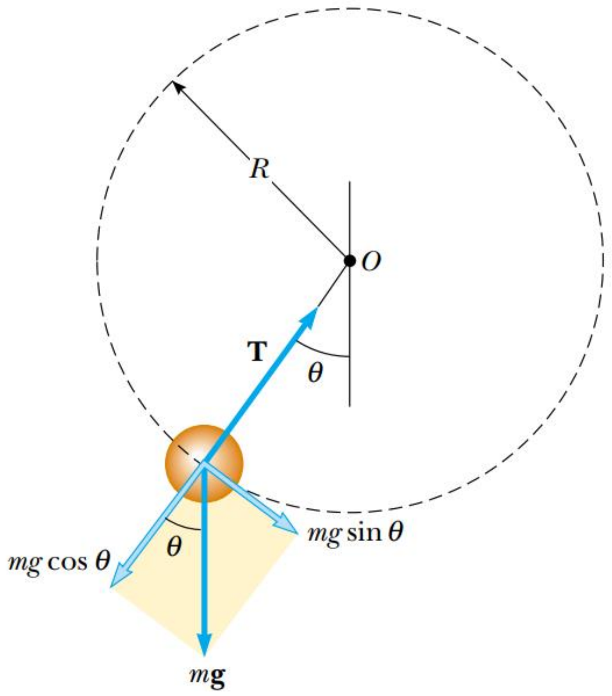
$$\Sigma \mathbf{F} = \Sigma \mathbf{F}_r + \Sigma \mathbf{F}_t$$

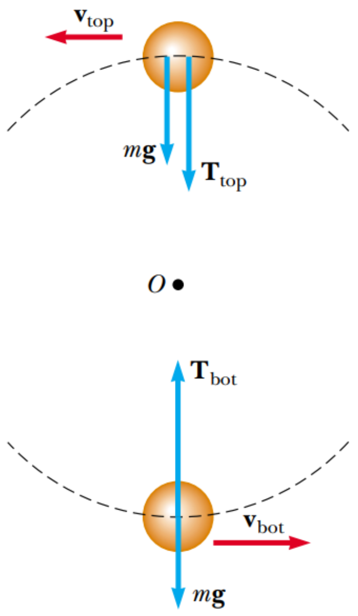


7.3 Newton's Second Law Applied to Nonuniform Circular Motion

Example: Vertical Circle with Non-Uniform Speed

A small sphere of mass *m* is attached to the end of a cord of length *R* and set into motion in a *vertical* circle about a fixed point *O*. Determine the tension in the cord at any instant when the speed of the sphere is *v* and the cord makes an angle θ with the vertical.





- The tension at the bottom is a maximum

$$T = mg\left(\frac{v_{bot}^2}{Rg} + 1\right)$$

- The tension at the top is a minimum

$\theta = 180°$, we have $\cos 180° = -1$

$$T = mg\left(\frac{v_{top}^2}{Rg} - 1\right)$$

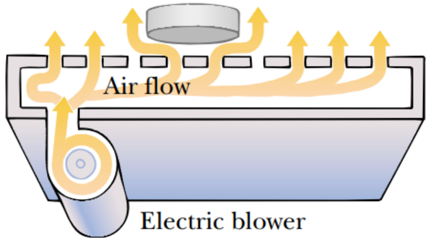
$$\sum F_t = mg \sin\theta = ma_t$$

$$a_t = g \sin\theta$$

$$\sum F_r = T - mg\cos\theta = \frac{mv^2}{R}$$

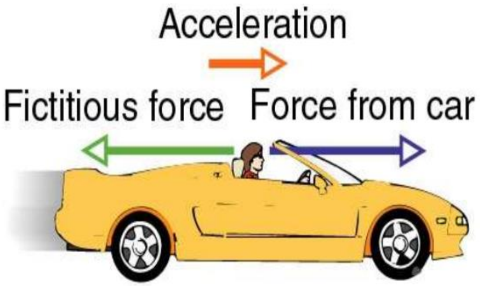
$$T = m\left(\frac{v^2}{R} + g\cos\theta\right)$$

7.4 Motion in Accelerated Frames



On an air hockey table, air blown through holes in the surface allow the puck to move almost without friction. If the table which is on the train is not accelerating, a puck placed on the table will remain at rest. If the train is accelerating, the puck accelerates from rest toward the back of the train.

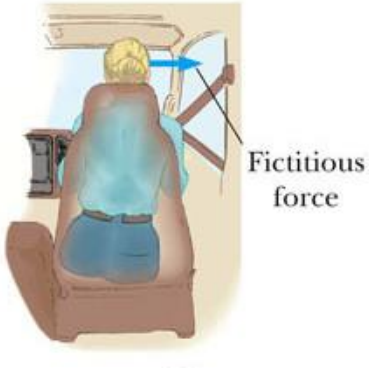
A ***fictitious force*** results from an accelerated frame of reference.



A fictitious force appears to act on an object in the same way as a real force, but you cannot identify a second object for the fictitious force.

Remember that real forces are always interactions between two objects.

- Although fictitious forces are not real forces, they can have real effects



- Examples:
  - Objects in the car do slide
  - You feel pushed to the outside of a rotating platform
  - The Coriolis force is responsible for the rotation of weather systems, including hurricanes, and ocean currents

7.4 Motion in Accelerated Frames

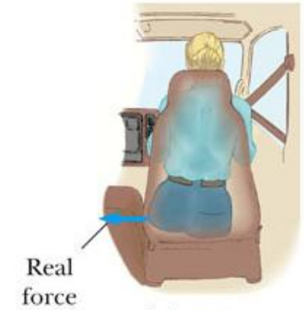
- ✓ From the frame of the passenger (b), a force appears to push her toward the door
- ✓ From the frame of the Earth, the car applies a leftward force on the passenger
- ✓ The outward force is often called a ***centrifugal*** force

  It is a fictitious force due to the centripetal acceleration associated with the car's change in direction

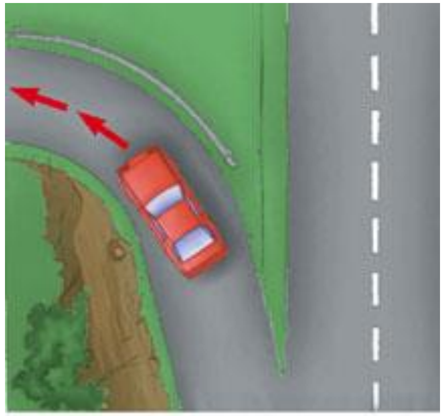
- ✓ In actuality, friction supplies the force to allow the passenger to move with the car

  If the frictional force is not large enough, the passenger continues on her initial path according to Newton's First Law

(b)



(c)



(a)

7.4 Motion in Accelerated Frames

Example: Fictitious Forces in Linear Systems

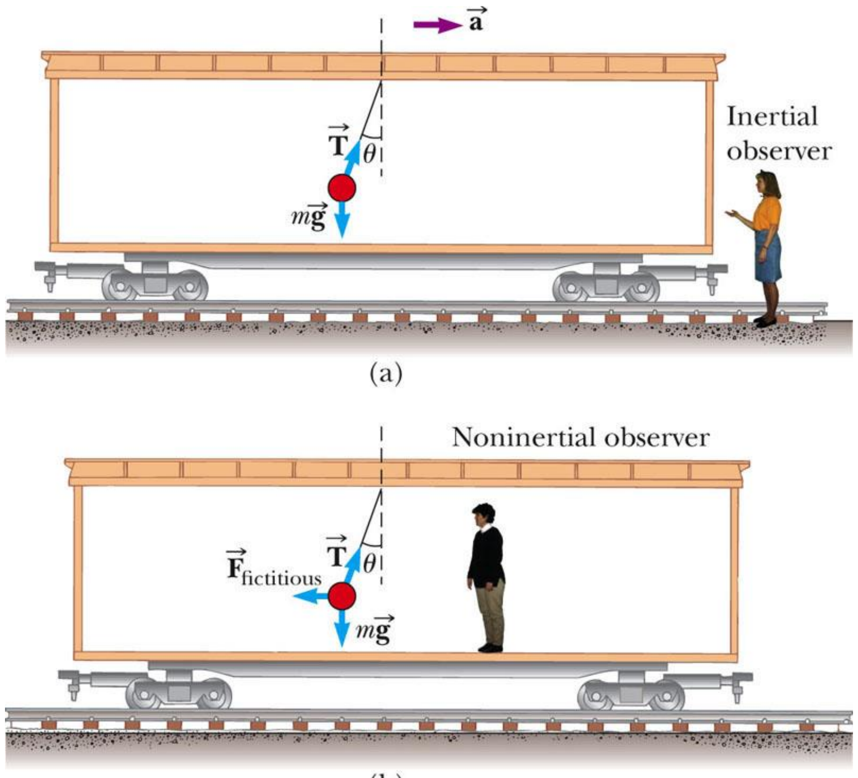
A small sphere of mass m is hung by a cord from the ceiling of a boxcar that is accelerating to the right, as shown in figure. The noninertial observer in Figure b claims that a force, which we know to be fictitious, must act in order to cause the observed deviation of the cord from the vertical. How is the magnitude of this force related to the acceleration of the boxcar measured by the inertial observer in Figure a.

- The inertial observer (a) at rest sees

$$\sum F_x = T\sin\theta = ma$$
$$\sum F_y = T\cos\theta - mg = 0$$

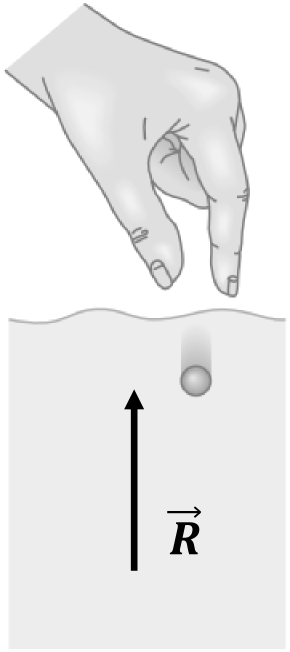
- The noninertial observer (b) sees

$$\sum F'_x = T\sin\theta - F_{fictitious} = ma$$
$$\sum F'_y = T\cos\theta - mg = 0$$



7.5 Motion with Resistive Forces

- Motion can be through a medium
  - Either a liquid or a gas
- The medium exerts a *resistive force,* $\vec{R}$, on an object moving through the medium
- The magnitude of $\vec{R}$, depends on the medium
- The direction of $\vec{R}$ is opposite the direction of motion of the object relative to the medium
- $\vec{R}$ nearly always increases with increasing speed



- The magnitude of $\vec{R}$ can depend on the speed in complex ways
- We will discuss only two situations:
  - $\vec{R}$ is proportional to *v*
    - Good approximation for slow motions or small objects
  - $\vec{R}$ is proportional to v^2
    - Good approximation for large objects

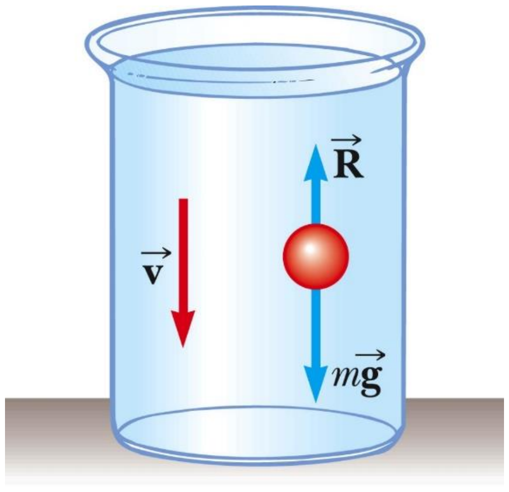
7.5 Motion with Resistive Forces

Resistive Force Proportional to Object Speed

- The resistive force can be expressed as

$$\vec{\mathbf{R}} = -b\vec{\mathbf{v}}$$

- b depends on the property of the medium, and on the shape and dimensions of the object
- The negative sign indicates $\vec{R}$ is in the opposite direction to $\vec{v}$



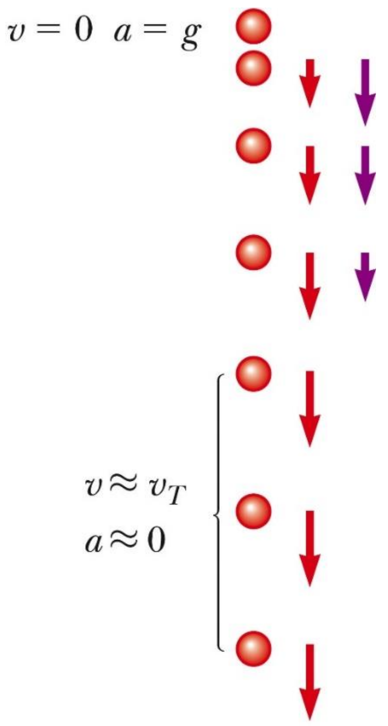
(a)

- Assume a small sphere of mass m is released from rest in a liquid
- Forces acting on it are
  - Resistive force
  - Gravitational force
- Analyzing the motion results in

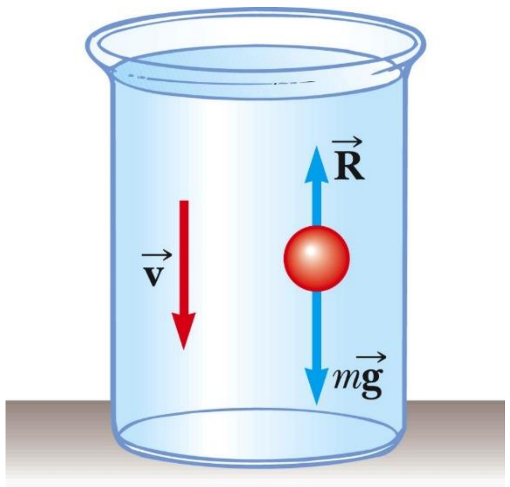
$$mg - bv = ma = m\frac{dv}{dt}$$

$$a = \frac{dv}{dt} = g - \frac{b}{m}v$$

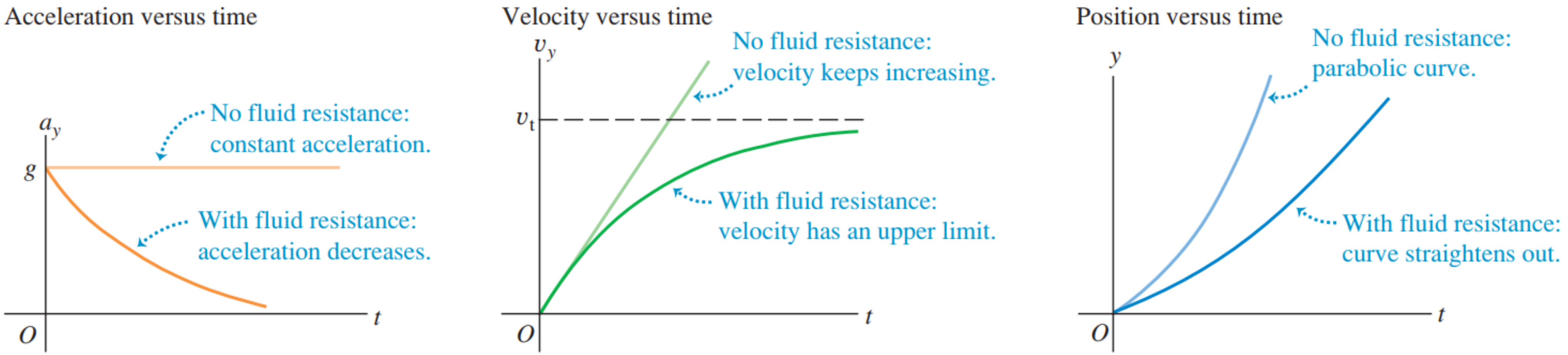
7.5 Motion with Resistive Forces



- Initially, $v = 0$ and $dv/dt = g$
- As t increases, R increases and a decreases
- The acceleration approaches 0 when $R \to mg$
- At this point, v approaches the ***terminal speed*** of the object



(a)



7.5 Motion with Resistive Forces

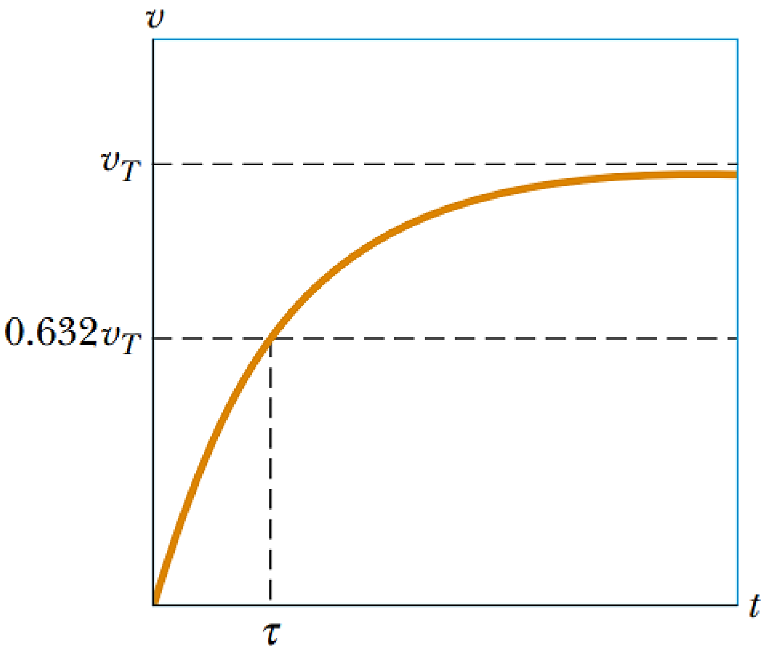
Terminal Speed

- To find the terminal speed, let $a = 0$

$$mg - bv = ma = m\frac{dv}{dt}$$

$$a = \frac{dv}{dt} = g - \frac{b}{m}v$$

$$v_T = \frac{mg}{b}$$



Speed–time graph for the sphere

- Solving the differential equation gives

$$v = 0 \text{ at } t = 0$$

$$v = \frac{mg}{b}\left(1 - e^{-bt/m}\right) = v_T\left(1 - e^{-t/\tau}\right)$$

- τ is the ***time constant*** and $\tau = m/b$

7.5 Motion with Resistive Forces

Example: Sphere Falling in Oil

A small sphere of mass 2.00 g is released from rest in a large vessel filled with oil, where it experiences a resistive force proportional to its speed. The sphere reaches a terminal speed of 5.00 cm/s. Determine the time constant and the time at which the sphere reaches 90.0% of its terminal speed.

$$v_T = \frac{mg}{b}$$

$$b = \frac{mg}{v_T} = \frac{(2.00\ \mathrm{g})(980\ \mathrm{cm/s^2})}{5.00\ \mathrm{cm/s}} = 392\ \mathrm{g/s}$$

$$\tau = \frac{m}{b} = \frac{2.00\ \mathrm{g}}{392\ \mathrm{g/s}} = \boxed{5.10 \times 10^{-3}\ \mathrm{s}}$$

To find the time t at which the sphere reaches a speed of $0.900 v_T$ we set $v = 0.900\, v_T$

$$0.900 v_T = v_T(1 - e^{-t/\tau})$$

$$1 - e^{-t/\tau} = 0.900$$

$$e^{-t/\tau} = 0.100$$

$$-\frac{t}{\tau} = \ln(0.100) = -2.30$$

$$t = 2.30\tau = 2.30(5.10 \times 10^{-3}\ \mathrm{s})$$

$$= 11.7 \times 10^{-3}\ \mathrm{s} = \boxed{11.7\ \mathrm{ms}}$$

7.5 Motion with Resistive Forces

Air Drag at High Speeds

- For objects moving at high speeds through air, the resistive force is approximately equal to the square of the speed

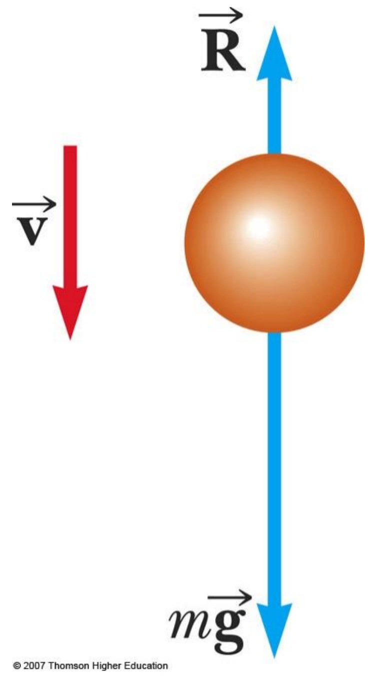
$R = ½\ D\rho A v^2$

  - D is a dimensionless empirical quantity called the *drag coefficient*
  - r is the density of air
  - A is the cross-sectional area of the object
  - v is the speed of the object

$$R = \frac{1}{2} D\rho A v^2$$

$$\sum F = mg - \frac{1}{2} D\rho A v^2 = ma$$

$$a = g - \left(\frac{D\rho A}{2m}\right) v^2$$



- The terminal speed will occur when the acceleration goes to zero
- Solving the previous equation gives

$$g - \left(\frac{D\rho A}{2m}\right) v_T{}^2 = 0$$

$$v_T = \sqrt{\frac{2mg}{D\rho A}}$$

7.5 Motion with Resistive Forces

Example: Resistive Force Exerted on a Baseball

A pitcher hurls a 0.145-kg baseball past a batter at 40.2 m/s. Find the resistive force acting on the ball at this speed.

$$D = \frac{2mg}{v_T{}^2 \rho A} = \frac{2(0.145\ \mathrm{kg})(9.80\ \mathrm{m/s^2})}{(43\ \mathrm{m/s})^2(1.20\ \mathrm{kg/m^3})(4.2 \times 10^{-3}\ \mathrm{m^2})}$$
$$= 0.305$$

$$R = \tfrac{1}{2} D\rho A v^2$$
$$= \tfrac{1}{2}(0.305)(1.20\ \mathrm{kg/m^3})(4.2 \times 10^{-3}\ \mathrm{m^2})(40.2\ \mathrm{m/s})^2$$
$$= \boxed{1.2\ \mathrm{N}}$$

Terminal Speed for Various Objects Falling Through Air

Object	Mass (kg)	Cross-Sectional Area (m^2)	v_T (m/s)
Sky diver	75	0.70	60
Baseball (radius 3.7 cm)	0.145	4.2×10^{-3}	43
Golf ball (radius 2.1 cm)	0.046	1.4×10^{-3}	44
Hailstone (radius 0.50 cm)	4.8×10^{-4}	7.9×10^{-5}	14
Raindrop (radius 0.20 cm)	3.4×10^{-5}	1.3×10^{-5}	9.0