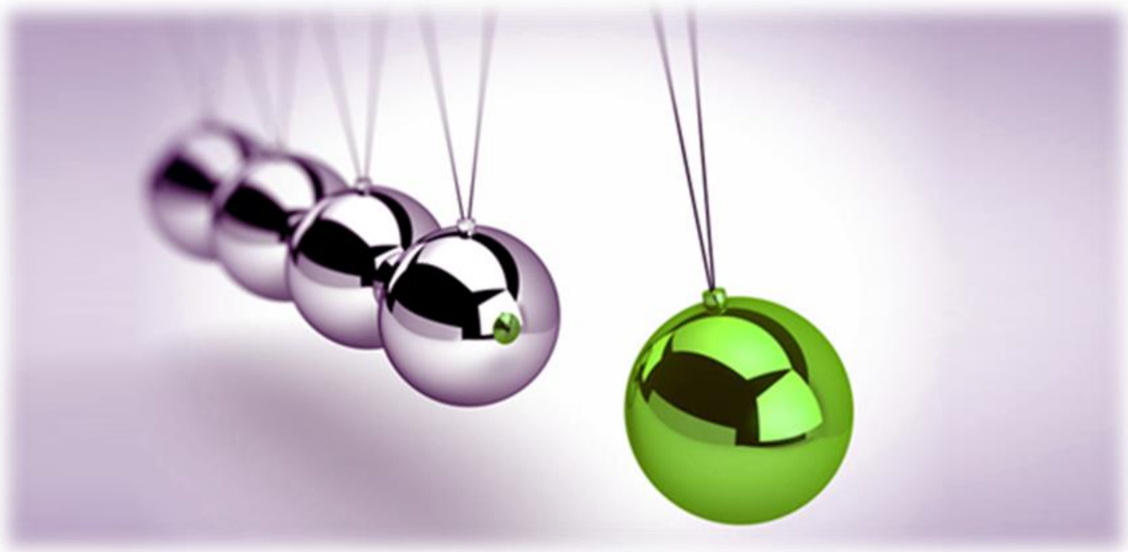
# PHYSICS I - MECHANICS
# WORK and KINETIC ENERGY



Doç. Dr. Ahmet Karatay
Ankara University
Engineering Faculty - Department of Engineering Physics

# CHAPTER 8. Work and Kinetic Energy

## Learning Objectives

8.1 Work

8.2 Kinetic Energy and the Work–Kinetic Energy Theorem

8.3 Work and Energy with Varying Forces

8.4 Situations Involving Kinetic Friction

8.5 Power

# What is the energy?

*capacity to do work*

Energy can exist as thermal (heat), mechanical, chemical, electrical, nuclear or other forms.

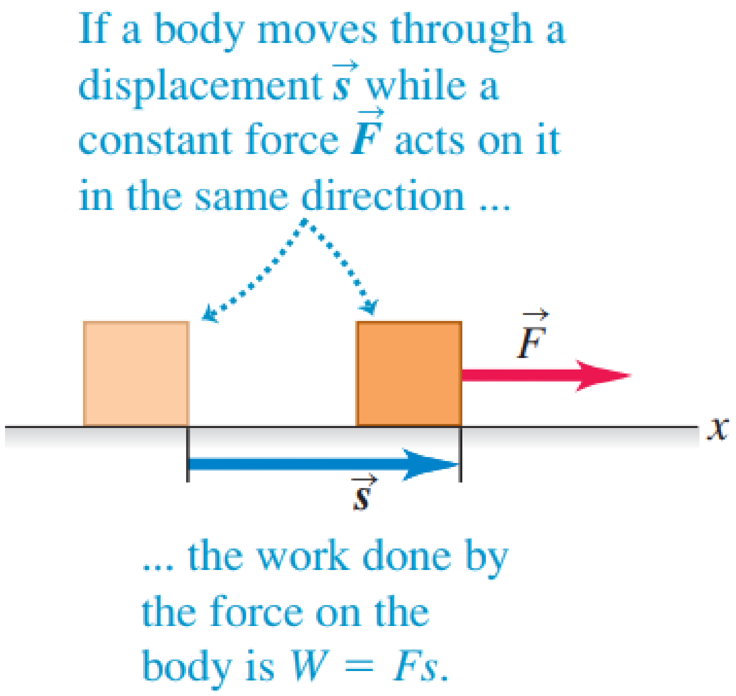
**For example,**
humans eat food for energy – chemical.
Computers use electrical energy – electricity.
Power plants may split atoms to generate energy – nuclear.

- Potential Energy – stored energy
- Kinetic Energy – energy of an object that it possesses due to motion

Every physical process that occurs in the Universe involves energy and energy transfers or transformations.

# 8.1 Work

The work, $W$, done on a system by an agent exerting a constant force on the system is the product of the magnitude $F$ of the force, the magnitude s of the displacement of the point of application of the force.




$$\boxed{W = F \cdot s}$$

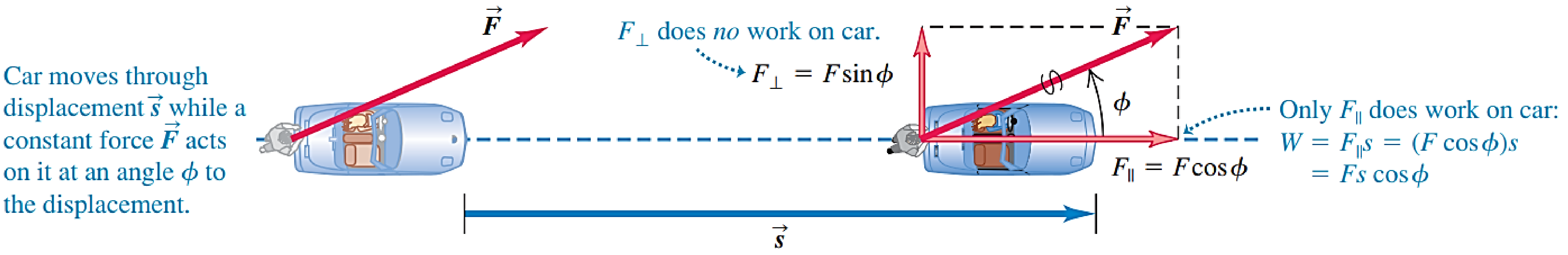
## Units of Work

- Work is a scalar quantity
- The unit of work is a joule (J)
  - 1 joule = 1 newton · 1 meter
  - J = N · m

**Work = W , weight = w** Don't confuse uppercase *W* (work) with lowercase *w* (weight).

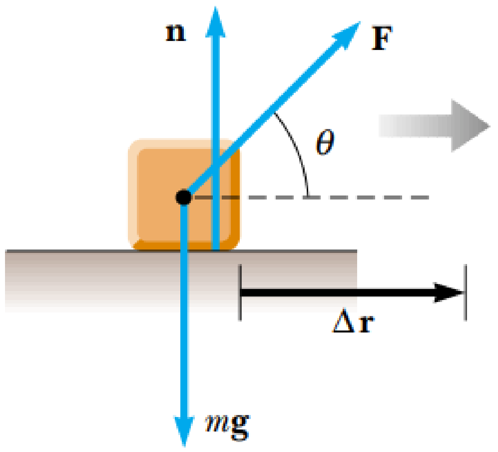
Though the symbols are similar, work and weight are different quantities.

# 8.1 Work




$$W = F\, s \cos \phi$$

- A force does no work on the object if the force does not move through a displacement
- The normal force and the gravitational force do no work on the object
  - $\cos\theta = \cos 90° = 0$
- The force $\vec{F}$ is the only force that does work on the object



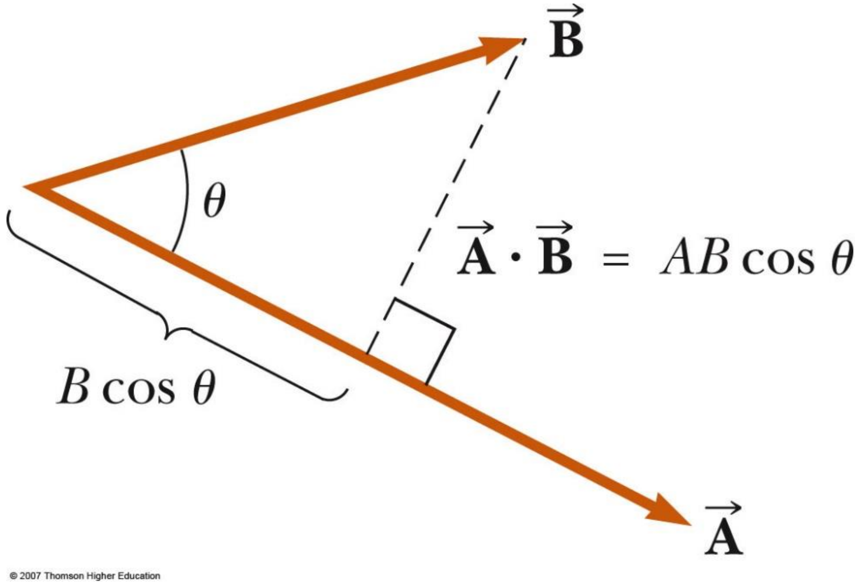
# 8.1 Work

- The scalar product of two vectors is written as $\vec{A} \cdot \vec{B}$
  - It is also called the dot product

- $\vec{A} \cdot \vec{B} = \mathrm{A\ B\ cos\theta}$

  - $\theta$ is the angle *between A* and *B*

- Applied to work, this means




$$W = F\Delta r\cos\theta = \vec{F} \cdot \Delta\vec{r}$$

$$\hat{\mathbf{i}} \cdot \hat{\mathbf{i}} = \hat{\mathbf{j}} \cdot \hat{\mathbf{j}} = \hat{\mathbf{k}} \cdot \hat{\mathbf{k}} = 1$$
$$\hat{\mathbf{i}} \cdot \hat{\mathbf{j}} = \hat{\mathbf{i}} \cdot \hat{\mathbf{k}} = \hat{\mathbf{j}} \cdot \hat{\mathbf{k}} = 0$$

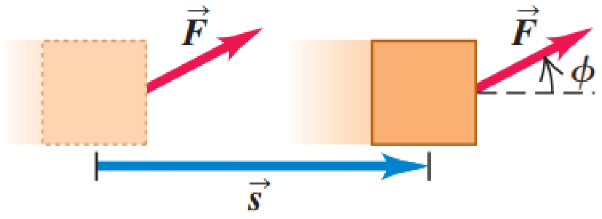
$$\mathbf{A} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}$$
$$\mathbf{B} = B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}}$$

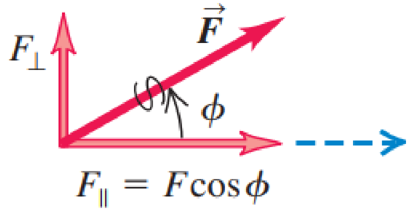
$$\mathbf{A} \cdot \mathbf{B} = A_xB_x + A_yB_y + A_zB_z$$

# 8.1 Work

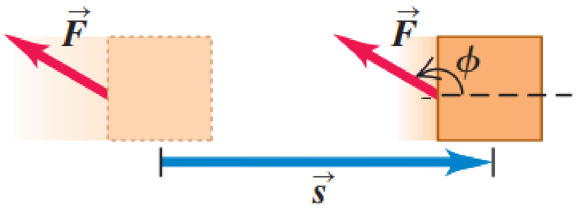
**Direction of Force (or Force Component)** | **Situation** | **Force Diagram**

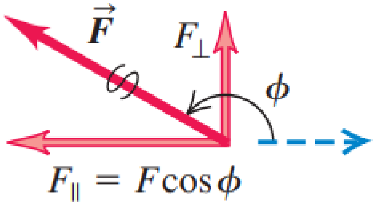
(a) **Force $\vec{F}$ has a component in direction of displacement:**
$W = F_{\parallel}s = (F\cos\phi)s$
Work is *positive*.



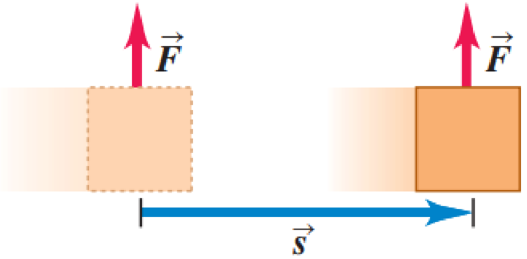



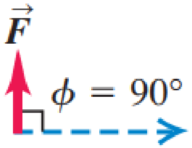

(b) **Force $\vec{F}$ has a component opposite to direction of displacement:**
$W = F_{\parallel}s = (F\cos\phi)s$
Work is *negative* (because $F\cos\phi$ is negative for $90° < \phi < 180°$).




(c) **Force $\vec{F}$ (or force component $F_{\perp}$) is perpendicular to direction of displacement:** The force (or force component) does *no* work on the object.



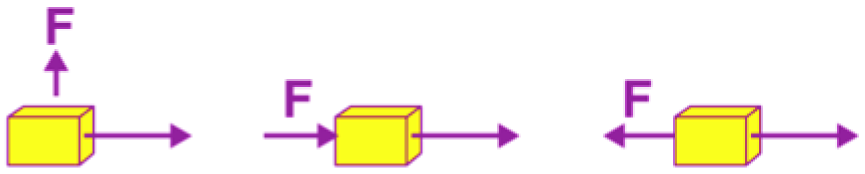

- ✓ When the force has a component in the *same direction* as the displacement between zero, the work $W$ is *positive.*
- ✓ When the force has a component *opposite* to the displacement $\cos\varphi$ is negative and the work is *negative.*
- ✓ When the force is *perpendicular* to the displacement, $\varphi = 90^{o}$ and the work done by the force is *zero.*

# 8.1 Work

## Work is an Energy Transfer

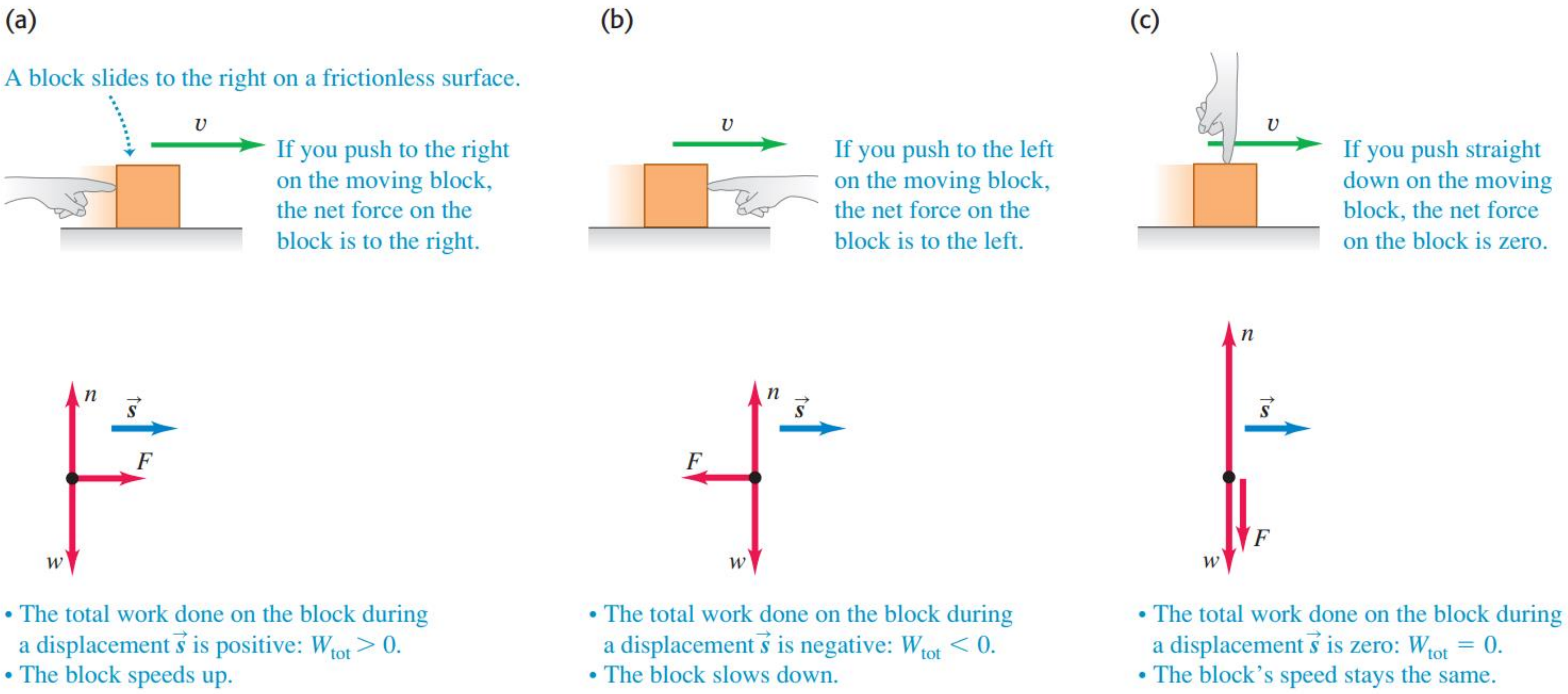
Work $W$ is energy transferred to or from an object by means of a force acting on the object.

- If the work is done on a system and it is positive, energy is transferred to the system
- If the work done on the system is negative, energy is transferred from the system
- If a system interacts with its environment, this interaction can be described as a transfer of energy across the system boundary
  - This will result in a change in the amount of energy stored in the system



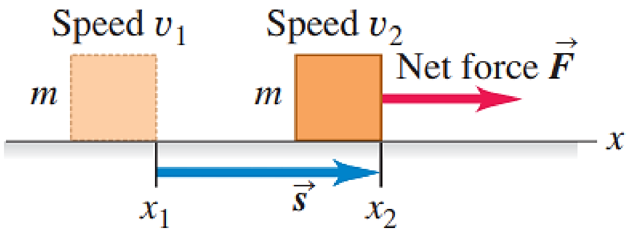

Zero work    Positive work    Negative work

# 8.2 Kinetic Energy and the Work–Kinetic Energy Theorem




The total work is also related to changes in the *speed* of the body.

# 8.2 Kinetic Energy and the Work–Kinetic Energy Theorem




The particle's acceleration is constant and given by Newton's second law: $F = ma$

Suppose the speed changes from $v_1$ to $v_2$ while the particle undergoes a displacement $s = x_2 - x_1$

$$v_2^2 = v_1^2 + 2a_x s$$

$$a_x = \frac{v_2^2 - v_1^2}{2s}$$

$$F = ma_x = m\frac{v_2^2 - v_1^2}{2s}$$

$$Fs = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

The quantity $\frac{1}{2}mv^2$ is called the **kinetic energy** *K* of the particle:

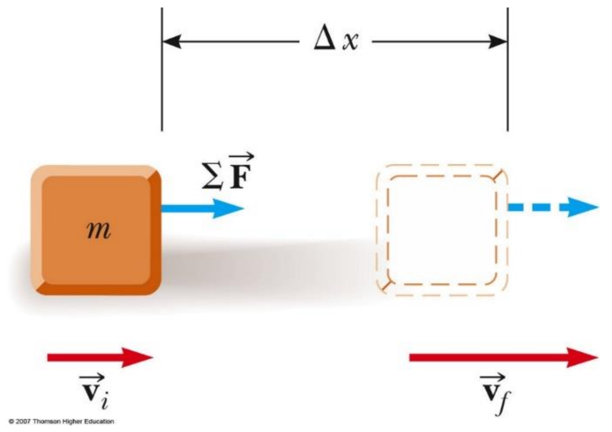
$$K = \tfrac{1}{2}mv^2 \qquad \text{(definition of kinetic energy)}$$

- *K* is the kinetic energy
- *m* is the mass of the particle
- *v* is the speed of the particle

- A change in kinetic energy is one possible result of doing work to transfer energy into a system

# 8.2 Kinetic Energy and the Work–Kinetic Energy Theorem

**The work done by the net force on a particle equals the change in the particle's kinetic energy:**



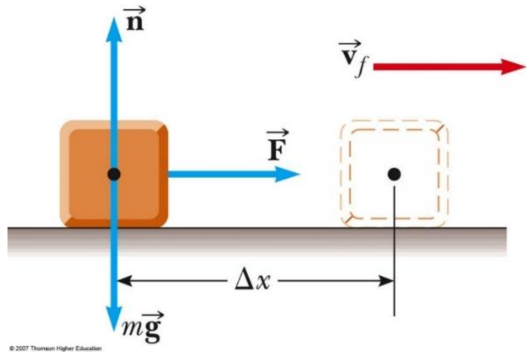

$$\Delta K = K_f - K_i = W$$

$$\left(\begin{array}{c}\text{change in the kinetic}\\ \text{energy of a particle}\end{array}\right) = \left(\begin{array}{c}\text{net work done on}\\ \text{the particle}\end{array}\right)$$

This result is the **work–energy theorem.**

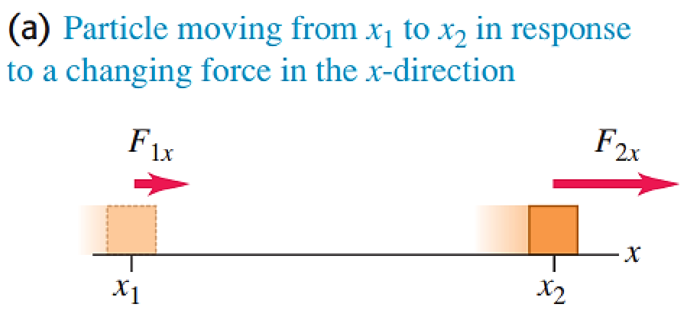
When work is done on a system and the only change in the system is in its speed, the work done by the net force equals the change in kinetic energy of the system.

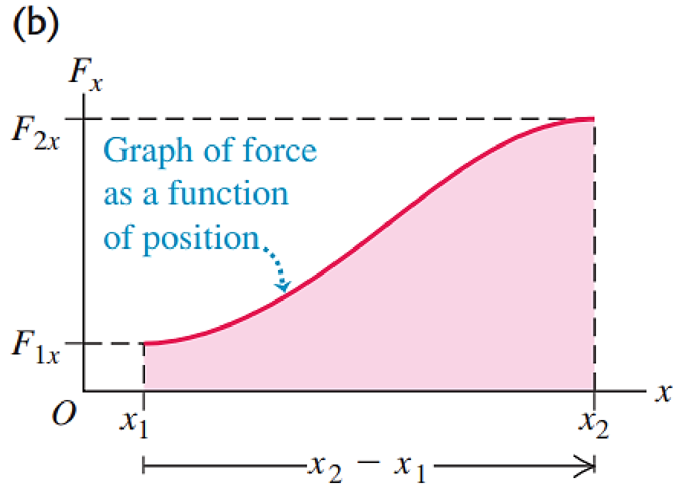
- The speed of the system increases if the work done on it is positive
- The speed of the system decreases if the net work is negative
- When the $W_{tot}=0$, kinetic energy stays the same and the speed is unchanged.
- The normal and gravitational forces do no work since they are perpendicular to the direction of the displacement



# 8.3 Work and Energy with Varying Forces

Suppose a particle moves along the x-axis from point $x_1$ to $x_2$ in response to a changing force in the x direction.



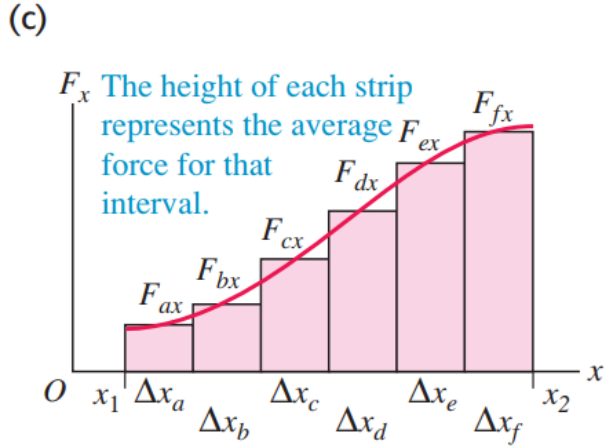




To find the work done by this force, we divide the total displacement into small segments $\Delta x_a$, $\Delta x_b$ and so on.

The work done by the force in the total displacement from $x_1$ to $x_2$ is approximately

$$W = F_{ax}\Delta x_a + F_{bx}\Delta x_b + \cdots$$

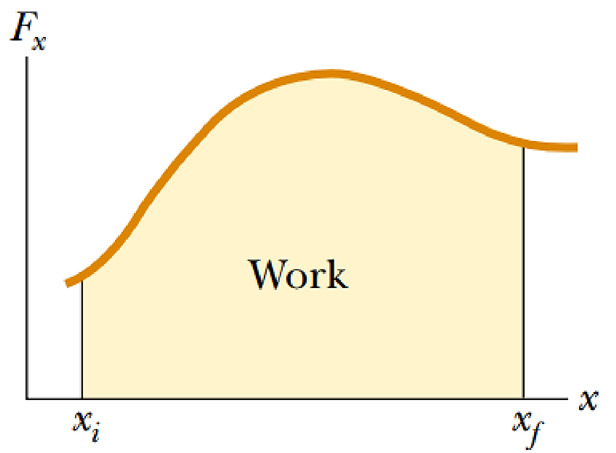
$$W \approx \sum_{x_i}^{x_f} F_x \Delta x$$



# 8.3 Work and Energy with Varying Forces

If the size of the displacements is allowed to approach zero, the number of terms in the sum increases without limit but the value of the sum approaches a definite value equal to the area bounded by the *Fx* curve and the x axis:

$$\lim_{\Delta x \to 0} \sum_{x_i}^{x_f} F_x \Delta x = \int_{x_i}^{x_f} F_x \, dx$$




- If more than one force acts on a system *and the system can be modeled as a particle*, the total work done on the system is the work done by the net force

$$\sum W = W_{net} = \int_{x_i}^{x_f} \left(\sum F_x\right) dx$$

- In the general case of a net force whose magnitude and direction may vary

$$\sum W = W_{net} = \int_{x_i}^{x_f} \left(\sum \vec{\mathbf{F}}\right) d\vec{\mathbf{r}}$$

- If the system cannot be modeled as a particle, then the total work is equal to the algebraic sum of the work done by the individual forces

$$W_{\text{net}} = \sum W_{\text{by individual forces}}$$

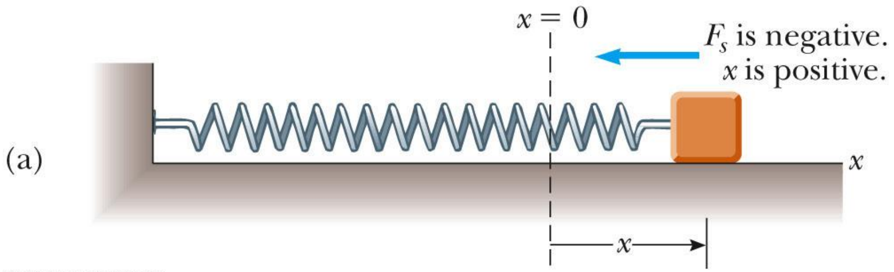
*Remember work is a scalar, so this is the algebraic sum.*

# 8.3 Work and Energy with Varying Forces

## Work Done By A Spring

- A model of a common physical system for which the force varies with position

- The block is on a horizontal, frictionless surface is connected to a spring.





- The force exerted by the spring is

$$F_s = - kx$$

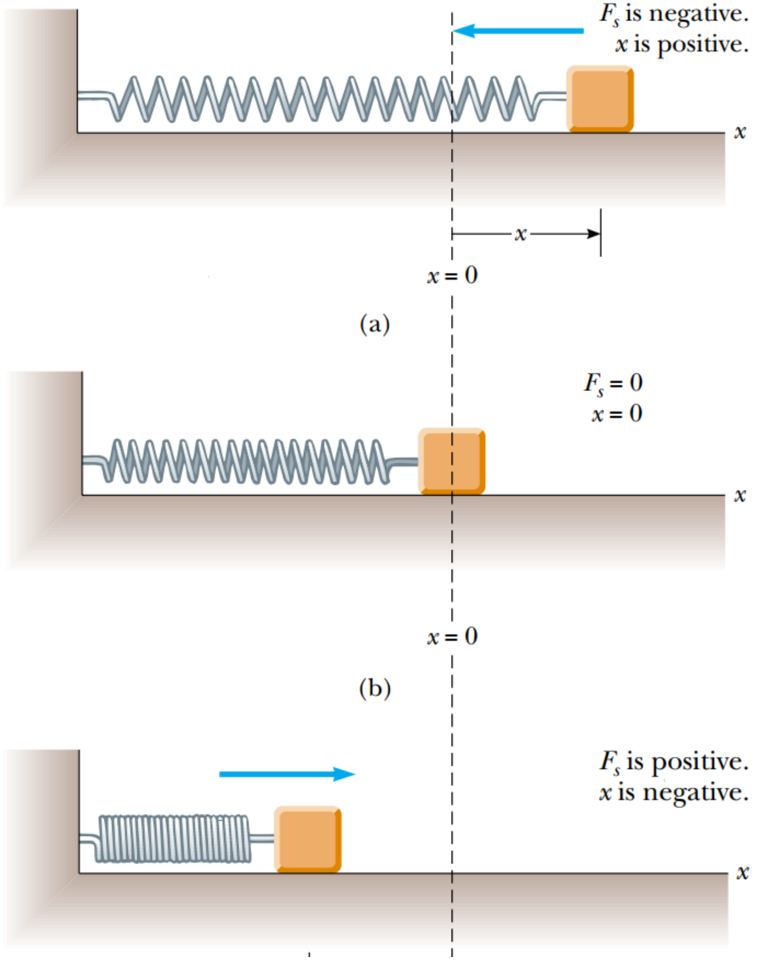
  – $x$ is the position of the block with respect to the equilibrium position ($x = 0$)
  – $k$ is called the spring constant or force constant and measures the stiffness of the spring and has unit of N/m.

- This is called ***Hooke's Law***

# 8.3 Work and Energy with Varying Forces

## Work Done By A Spring

- When *x* is positive (spring is stretched), *F* is negative
- When *x* is 0 (at the equilibrium position), *F* is 0
- When *x* is negative (spring is compressed), *F* is positive




- The force exerted by the spring is always directed opposite to the displacement from equilibrium
- The spring force is sometimes called the *restoring force*

# 8.3 Work and Energy with Varying Forces

## Work Done By A Spring

- The work as the block moves from $x_i = -x_{max}$ to $x_f = 0$

$$W_s = \int_{x_i}^{x_f} F_x dx = \int_{-x_{max}}^{0} (-kx)\,dx = \frac{1}{2}kx_{max}^2$$

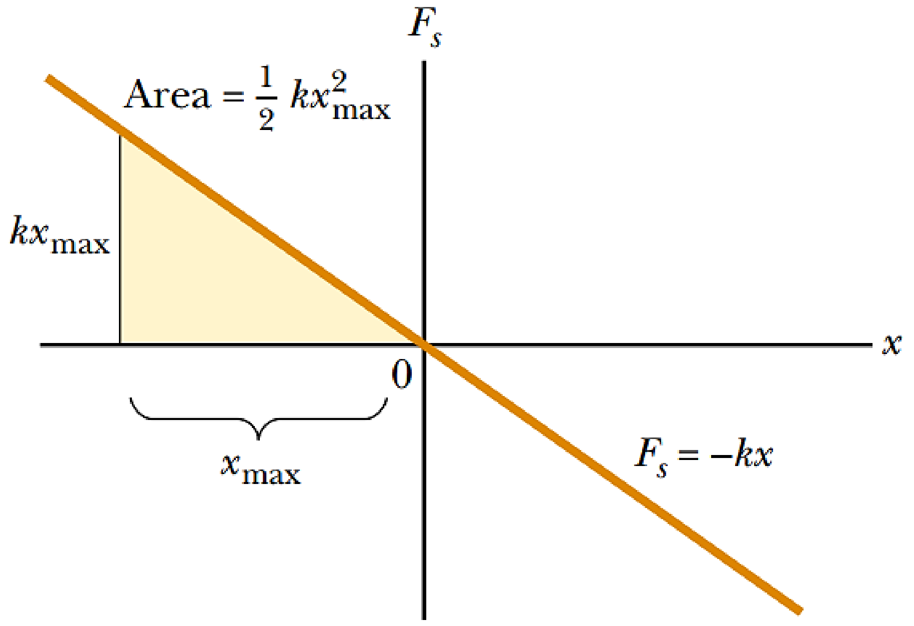
- The work done as the block moves from $x_i = 0$ to $x_f = +x_{max}$

$$W_s = -\frac{1}{2}kx_{max}^2$$

- The total work done by the spring on the block is

$$W_s = \int_{x_i}^{x_f} (-kx)\,dx = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$

  – If the motion ends where it begins, W = 0



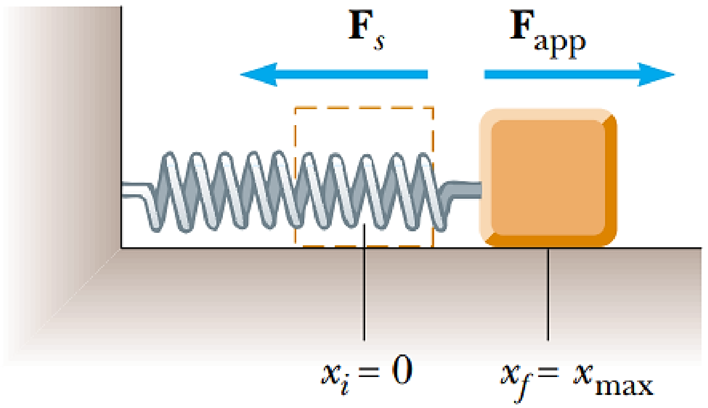
# 8.3 Work and Energy with Varying Forces

## Spring with an Applied Force

- Suppose an external agent, $F_{app}$, stretches the spring

- The applied force is equal and opposite to the spring force

$$F_{app} = -(-kx) = kx$$




- The work done by the applied force is

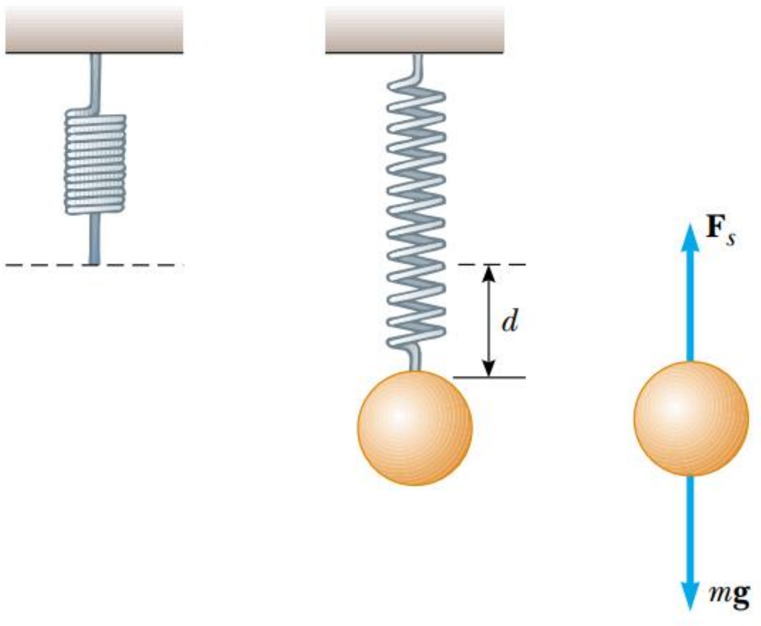
$$W_{F_{app}} = \int_0^{x_{max}} F_{app}\, dx = \int_0^{x_{max}} kx\, dx = \frac{1}{2}kx_{max}^2$$

$$W_{F_{app}} = \int_{x_i}^{x_f} F_{app}\, dx = \int_{x_i}^{x_f} kx\, dx = \frac{1}{2}kx_f^{\,2} - \frac{1}{2}kx_i^{\,2}$$

# 8.3 Work and Energy with Varying Forces

## Example: Spring with an Applied Force

If a spring is stretched 2.0 cm by a suspended object having a mass of 0.55 kg, what is the force constant of the spring?




$$|\mathbf{F}_s| = kd = mg,$$

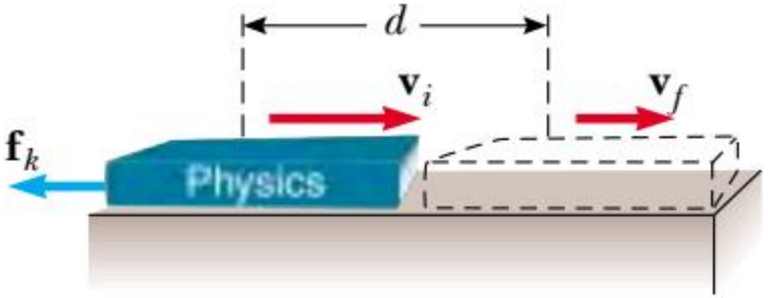
$$k = \frac{mg}{d} = \frac{(0.55\ \text{kg})(9.80\ \text{m/s}^2)}{2.0 \times 10^{-2}\ \text{m}} = \boxed{2.7 \times 10^2\ \text{N/m}}$$

How much work is done by the spring as it stretches through this distance?

$$W_s = 0 - \tfrac{1}{2}kd^2 = -\tfrac{1}{2}(2.7 \times 10^2\ \text{N/m})(2.0 \times 10^{-2}\ \text{m})^2$$

$$= \boxed{-5.4 \times 10^{-2}\,\text{J}}$$

# 8.4 Situations Involving Kinetic Friction




by a displacement Δx of the book

$$\left(\sum F_x\right)\Delta x = (ma_x)\Delta x$$

For a particle under constant acceleration, we know that the following relationships

$$a_x = \frac{v_f - v_i}{t} \qquad \Delta x = \tfrac{1}{2}(v_i + v_f)\,t$$

$$\left(\sum F_x\right)\Delta x = m\left(\frac{v_f - v_i}{t}\right)\tfrac{1}{2}(v_i + v_f)\,t$$

$$\left(\sum F_x\right)\Delta x = \tfrac{1}{2}m{v_f}^2 - \tfrac{1}{2}m{v_i}^2$$

The net force on the book is the kinetic friction force **f***k*, which is directed opposite to the displacement Δ*x*

$$\left(\sum F_x\right)\Delta x = -f_k\,\Delta x = \tfrac{1}{2}m{v_f}^2 - \tfrac{1}{2}m{v_i}^2 = \Delta K$$

$$-f_k\,\Delta x = \Delta K$$

$$\boxed{-f_k\,d = \Delta K}$$

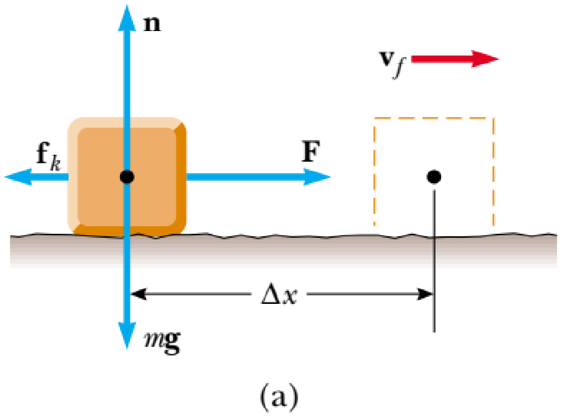
$$\Delta K = -f_k\,d + \sum W_{\text{other forces}}$$

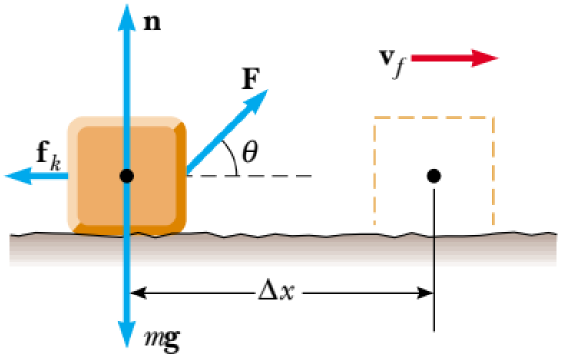
$$K_f = K_i - f_k\,d + \sum W_{\text{other forces}}$$

# 8.4 Situations Involving Kinetic Friction

## Example:

A 6.0-kg block initially at rest is pulled to the right along a horizontal surface by a constant horizontal force of 12 N. Find the speed of the block after it has moved 3.0 m if the surfaces in contact have a coefficient of kinetic friction of 0.15.




$$W = F\Delta x = (12\text{ N})(3.0\text{ m}) = 36\text{ J}$$

$$f_k = \mu_k n = \mu_k mg = (0.15)(6.0\text{ kg})(9.80\text{ m/s}^2) = 8.82\text{ N}$$

$$\Delta K_{\text{friction}} = -f_k d = -(8.82\text{ N})(3.0\text{ m}) = -26.5\text{ J}$$

$$\tfrac{1}{2}mv_f{}^2 = \tfrac{1}{2}mv_i{}^2 - f_k d + \sum W_{\text{other forces}}$$

$$v_f = \sqrt{v_i{}^2 + \frac{2}{m}\left(-f_k d + \sum W_{\text{other forces}}\right)}$$

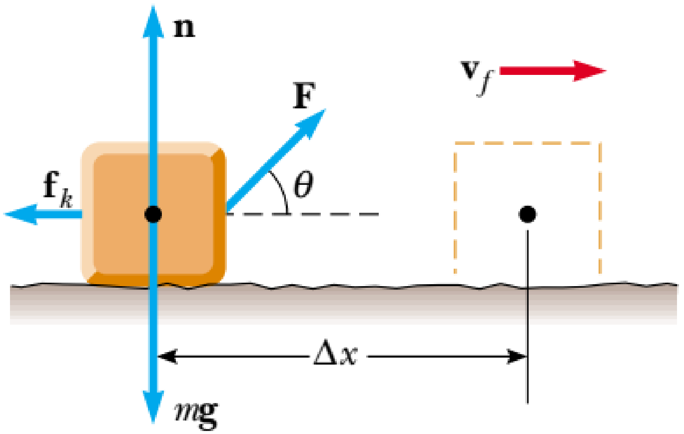
$$= \sqrt{0 + \frac{2}{6.0\text{ kg}}(-26.5\text{ J} + 36\text{ J})}$$

$$= \boxed{1.8\text{ m/s}}$$

# 8.4 Situations Involving Kinetic Friction

## Example:

Suppose the force **F** is applied at an angle θ as shown in Figure. At what angle should the force be applied to achieve the largest possible speed after the block has moved 3.0 m to the right?




$$W = F\Delta x \cos\theta = Fd\cos\theta$$

$$\sum F_y = n + F\sin\theta - mg = 0$$

$$n = mg - F\sin\theta$$

$$K_f = -f_k d + \sum W_{\text{other forces}}$$
$$= -\mu_k n d + Fd\cos\theta$$
$$= -\mu_k(mg - F\sin\theta)d + Fd\cos\theta$$

$$\frac{d(K_f)}{d\theta} = -\mu_k(0 - F\cos\theta)d - Fd\sin\theta = 0$$

$$\mu_k\cos\theta - \sin\theta = 0$$

$$\tan\theta = \mu_k$$

For $\mu_k = 0.15$, we have,

$$\theta = \tan^{-1}(\mu_k) = \tan^{-1}(0.15) = \boxed{8.5^\circ}$$

# 8.5 Power

We need to know how quickly work is done. We describe this in terms of power. Power is the time rate of energy transfer. Using work as the energy transfer method, this can also be written

$$\overline{\mathscr{P}} \equiv \frac{W}{\Delta t}$$

The instantaneous power is the limiting value of the average power as $\Delta$t approaches zero. The ***instantaneous power*** is defined as

$$\mathscr{P} \equiv \lim_{\Delta t \to 0} \frac{W}{\Delta t} = \frac{dW}{dt}$$

In general, power is defined for any type of energy transfer. Therefore, the most general expression for power is:

$$\mathscr{P} = \frac{dE}{dt}$$

## Units of Power

- The SI unit of power is called the watt.
  1 watt = 1 joule / second = 1 kg · m$^2$ / s$^3$
- A unit of power in the US Customary system is horsepower.
  1 hp = 746 W
- Units of power can also be used to express units of work or energy.
  1 kWh = (1000 W)(3600 s) = 3.6 x10$^6$ J

## 8.5 Power

In mechanics we can also express power in terms of force and velocity. The average power is

$$P_{av} = \frac{F_{\parallel}\Delta s}{\Delta t} = F_{\parallel}\frac{\Delta s}{\Delta t} = F_{\parallel}v_{av}$$

Instantaneous power P is the limit of this expression as $\Delta t \to 0$

$$P = F_{\parallel}v$$

$$P = \vec{F} \cdot \vec{v}$$ (instantaneous rate at which force $\vec{F}$ does work on a particle)

# 8.5 Power

## Example:

Each of the four jet engines on an Airbus A380 airliner develops a thrust (a forward force on the airliner) of 322,000 N. When the airplane is flying at 250 m/s (900 km/h, or roughly 560 mi/h) , what horsepower does each engine develop?

$$P = F_{\|}v = (3.22 \times 10^5 \text{ N})(250 \text{ m/s}) = 8.05 \times 10^7 \text{ W}$$

$$= (8.05 \times 10^7 \text{ W})\frac{1 \text{ hp}}{746 \text{ W}} = 108{,}000 \text{ hp}$$

## Example:

A 50.0-kg marathon runner runs up the stairs to the top of Chicago's 443-m-tall Willis Tower, the tallest building in the United States. To lift herself to the top in 15.0 minutes, what must be her average power output? Express your answer in watts, in kilowatts, and in horsepower.

$$W = mgh = (50.0 \text{ kg})(9.80 \text{ m/s}^2)(443 \text{ m})$$

$$= 2.17 \times 10^5 \text{ J}$$

$$15.0 \text{ min} = 900 \text{ s}$$

$$P_{\text{av}} = \frac{2.17 \times 10^5 \text{J}}{900 \text{ s}} = 241 \text{ W} = 0.241 \text{ kW} = 0.323 \text{ hp}$$

$$P_{\text{av}} = F_{\|}v_{\text{av}} = (mg)v_{\text{av}}$$

$$= (50.0 \text{ kg})(9.80 \text{ m/s}^2)(0.492 \text{ m/s}) = 241 \text{ W}$$

$$(443 \text{ m})/(900 \text{ s}) = 0.492 \text{ m/s}$$