

# **PHYSICS I - MECHANICS**

## ***Problem Session-I***

1. Physics and Measurement
2. Vectors
3. Motion in One Dimension
4. Two Dimensional Motion

# 1. Physics and Measurements

## 1.1 Standards of Length, Mass and Time

The three fundamental physical quantities of mechanics are

Length	meters (m)
Mass	kilograms (kg)
Time	seconds (s)

Temperature (Kelvin), electric current (Ampere), luminous intensity (candela), amount of substance (mole)

### Prefixes

Multiples and Submultiples Prefixes Tables

Symbol	Name	Factor	Symbol	Name	Factor
Y	yotta	$10^{24}$	y	yocto	$10^{-24}$
Z	zetta	$10^{21}$	z	zepto	$10^{-21}$
E	exa	$10^{18}$	a	atto	$10^{-18}$
P	peta	$10^{15}$	f	femto	$10^{-15}$
T	tera	$10^{12}$	p	pico	$10^{-12}$
G	giga	$10^9$	n	nano	$10^{-9}$
M	mega	$10^6$	μ	micro	$10^{-6}$
k	kilo	$10^3$	m	milli	$10^{-3}$
h	hecto	$10^2$	c	centi	$10^{-2}$
da	deka	$10^1$	d	deci	$10^{-1}$

1) Express the following quantities using the prefixes.

(a)  $3 \times 10^{-4} \text{ m} = 0.3 \text{ mm}$

(b)  $5 \times 10^{-5} \text{ s} = 50 \mu\text{s}$

(c)  $72 \times 10^2 \text{ g} = 7.2 \text{ kg}$

(d)  $2 \times 10^9 \text{ Watt} = 2 \text{ GW}$

# 1. Physics and Measurements

## 1.3 Density and Atomic Mass

The density  $\rho$  (Greek letter rho) of any substance is defined as its *mass per unit volume*:

$$\rho \equiv \frac{m}{V}$$

**Atomic mass units (u)**       $1 \text{ u} = 1.660\ 538\ 7 \times 10^{-27} \text{ kg}$

- 2)** A small cube of iron is observed under a microscope. The edge of the cube is  $5.00 \times 10^{-6} \text{ cm}$  long. Find  
(a) the mass of the cube and  
(b) the number of iron atoms in the cube. The atomic mass of iron is 55.9 u, and its density is  $7.86 \text{ g/cm}^3$ .

(a)  $m = \rho L^3 = (7.86 \text{ g/cm}^3)(5.00 \times 10^{-6} \text{ cm})^3 = \boxed{9.83 \times 10^{-16} \text{ g}} = 9.83 \times 10^{-19} \text{ kg}$

(b)  $N = \frac{m}{m_0} = \frac{9.83 \times 10^{-19} \text{ kg}}{55.9 \text{ u}(1.66 \times 10^{-27} \text{ kg/1 u})} = \boxed{1.06 \times 10^7 \text{ atoms}}$

- 3)** What mass of a material with density  $\rho$  is required to make a hollow spherical shell having inner radius  $r_1$  and outer radius  $r_2$ ?

$$V = V_o - V_i = \frac{4}{3}\pi(r_2^3 - r_1^3)$$

$$\rho = \frac{m}{V}, \text{ so } m = \rho V = \rho \left( \frac{4}{3}\pi \right) (r_2^3 - r_1^3) = \boxed{\frac{4\pi\rho(r_2^3 - r_1^3)}{3}}.$$

# 1. Physics and Measurements

## 1.4 Dimensional Analysis

4) Which of the following equations are dimensionally correct?

a)  $v_f = v_i + ax$

b)  $y=(2 \text{ m}) \cos(kx)$ , where  $k=2 \text{ m}^{-1}$

$$\frac{L}{T} = \frac{L}{T} + \frac{L}{T^2} L \Rightarrow \frac{L}{T} + \frac{L^2}{T^2}$$

*This is incorrect*

$$L = L \cos\left(\frac{1}{L}L\right) \Rightarrow L = L$$

*This is correct.*

5) Which of the following equations are dimensionally correct?

$$x = \frac{1}{2}vt^2$$

$$L = \frac{L}{T}T^2 = LT$$

*This is incorrect*

$$x_f = x_i + v_{xi}t + \frac{1}{2}at^2$$

$$L = L + \frac{L}{T}T + \frac{L}{T^2}T^2 \Rightarrow L = L$$

*This is correct*

# 1. Physics and Measurements

## 1.5 Conversion of Units

**6)** A solid piece of lead has a mass of 23.94 g and a volume of 2.10 cm<sup>3</sup>. From these data, calculate the density of lead in SI units (kg/m<sup>3</sup>).

$$\rho = \frac{23.94 \text{ g}}{2.10 \text{ cm}^3} \left( \frac{1 \text{ kg}}{1000 \text{ g}} \right) \left( \frac{100 \text{ cm}}{1 \text{ m}} \right)^3 = \boxed{1.14 \times 10^4 \text{ kg/m}^3}$$

**7)** Suppose your hair grows at the rate 1/32 in. per day. Find the rate at which it grows in nanometers per second.

Apply the following conversion factors:

$$1 \text{ in} = 2.54 \text{ cm}, 1 \text{ d} = 86\,400 \text{ s}, 100 \text{ cm} = 1 \text{ m}, \text{ and } 10^9 \text{ nm} = 1 \text{ m}$$

$$\left( \frac{1}{32} \text{ in/day} \right) \frac{(2.54 \text{ cm/in})(10^{-2} \text{ m/cm})(10^9 \text{ nm/m})}{86\,400 \text{ s/day}} = \boxed{9.19 \text{ nm/s}}$$

# 1. Physics and Measurements

## 1.7 Significant Figures

1. Non-zero digits are always significant.

**24.7 meters**

**0.743 meter**

**714 meters**

2. Zeros appearing between nonzero digits are significant.

**7003 meters**

**40.79 meters**

**1.503 meters**

3. Trailing zeros follow a non zero digit and are significant only if there is a decimal point.

**0.00500**

**$2.30 \times 10^{-5}$**

**100.000**

**$4.500 \times 10^{12}$**

**0.03040**

**100**

**0.00000566**

# 1. Physics and Measurements

## 1.7 Significant Figures

4. When a number ends in zeroes that are not to the right of a decimal point, the zeroes are not necessarily significant:

➤ 190 miles may be 2 or 3 significant figures:

**19 × 10<sup>1</sup>** : 2 sig. fig.

**19.0 × 10<sup>1</sup>** : 3 sig. fig.

➤ 50,600 calories may be 3, 4, or 5 significant figures.

The potential ambiguity in the last rule can be avoided by the use of standard exponential, or "scientific," notation.

For example, depending on whether 3, 4, or 5 significant figures is correct, we could write 50,600 calories as:

- **5.06 × 10<sup>4</sup>** calories (3 significant figures)
- **5.060 × 10<sup>4</sup>** calories (4 significant figures), or
- **5.0600 × 10<sup>4</sup>** calories (5 significant figures).

# 1. Physics and Measurements

## 1.7 Significant Figures

**8)** Carry out the following arithmetic operations:

- (a) the sum of the measured values 756, 37.2, 0.83, and 2.5;
- (b) The product  $0.003\ 2 \times 356.3$ ;
- (c) the product  $5.620 \times \pi$ .

(a)      756.??  
              37.2?  
              0.83  
+ 2.5?  
796.53 = 797

(b)       $0.003\ 2(2 \text{ s.f.}) \times 356.3(4 \text{ s.f.}) = 1.140\ 16 = (2 \text{ s.f.})$  1.1

(c)       $5.620(4 \text{ s.f.}) \times \pi(> 4 \text{ s.f.}) = 17.656 = (4 \text{ s.f.})$  17.66

**9)** A carpet is to be installed in a room whose length is measured to be 12.71 m and whose width is measured to be 3.46 m. Find the area of the room.

If you multiply 12.71 m by 3.46 m on your calculator, you will see an answer of  $43.976\ 6 \text{ m}^2$ .

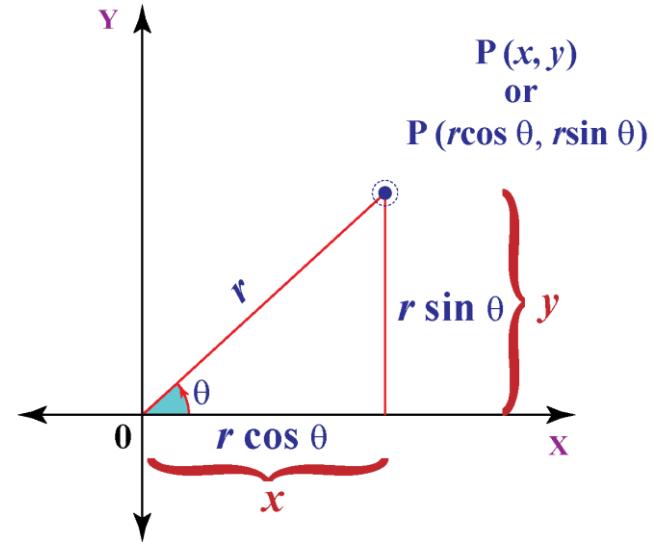
3.46 m, so we should express our final answer as  $44.0 \text{ m}^2$ .

## 2. Vectors

### 2.1 Coordinate Systems

#### Polar to Cartesian Coordinates

- Based on forming a right triangle from  $r$  and  $\theta$
- $x = r \cos \theta$
- $y = r \sin \theta$



#### Cartesian to Polar Coordinates

- $r$  is the hypotenuse and  $\theta$  an angle

$$\tan \theta = \frac{y}{x}$$

$$r = \sqrt{x^2 + y^2}$$

- $\theta$  must be ccw from positive x axis for these equations to be valid

**10)** If the rectangular coordinates of a point are given by  $(2, y)$  and its polar coordinates are  $(r, 30^\circ)$ , determine  $y$  and  $r$ .

We have  $2.00 = r \cos 30.0^\circ$

$$r = \frac{2.00}{\cos 30.0^\circ} = \boxed{2.31}$$

and  $y = r \sin 30.0^\circ = 2.31 \sin 30.0^\circ = \boxed{1.15}$ .

## 2. Vectors

### 2.4 Components of a Vector and Unit Vectors

- 11) Vector  $\vec{A}$  is in the direction  $34^\circ$  clockwise from the  $-y$  axis. The  $x$ -component of  $\vec{A}$  is  $A_x = -16$  m. (a) What is the  $y$ -component of  $\vec{A}$ ? (b) What is the magnitude of  $\vec{A}$ ?

$$(a) \tan 34.0^\circ = \frac{|A_x|}{|A_y|}$$

$$|A_y| = \frac{|A_x|}{\tan 34.0^\circ} = \frac{16.0 \text{ m}}{\tan 34.0^\circ} = 23.72 \text{ m}$$

$$A_y = -23.7 \text{ m.}$$

$$(b) A = \sqrt{A_x^2 + A_y^2} = 28.6 \text{ m.}$$

- 12) Consider the two vectors  $\mathbf{A} = 3\hat{i} - 2\hat{j}$  and  $\mathbf{B} = -\hat{i} - 4\hat{j}$ .

Calculate (a)  $\mathbf{A} + \mathbf{B}$ , (b)  $\mathbf{A} - \mathbf{B}$ , (c)  $|\mathbf{A} + \mathbf{B}|$ , (d)  $|\mathbf{A} - \mathbf{B}|$ , and (e) the directions of  $\mathbf{A} + \mathbf{B}$  and  $\mathbf{A} - \mathbf{B}$ .

$$(a) (\mathbf{A} + \mathbf{B}) = (3\hat{i} - 2\hat{j}) + (-\hat{i} - 4\hat{j}) = \boxed{2\hat{i} - 6\hat{j}}$$

$$(b) (\mathbf{A} - \mathbf{B}) = (3\hat{i} - 2\hat{j}) - (-\hat{i} - 4\hat{j}) = \boxed{4\hat{i} + 2\hat{j}}$$

$$(c) |\mathbf{A} + \mathbf{B}| = \sqrt{2^2 + 6^2} = \boxed{6.32}$$

$$(d) |\mathbf{A} - \mathbf{B}| = \sqrt{4^2 + 2^2} = \boxed{4.47}$$

$$(e) \theta_{|\mathbf{A} + \mathbf{B}|} = \tan^{-1}\left(-\frac{6}{2}\right) = -71.6^\circ = \boxed{288^\circ}$$

$$\theta_{|\mathbf{A} - \mathbf{B}|} = \tan^{-1}\left(\frac{2}{4}\right) = \boxed{26.6^\circ}$$

## 2. Vectors

### 2.4 Components of a Vector and Unit Vectors

**13)** A vector is given by  $\mathbf{R} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ . Find

- (a) the magnitudes of the  $x$ ,  $y$ , and  $z$  components,
- (b) the magnitude of  $\mathbf{R}$ , and
- (c) the angles between  $\mathbf{R}$  and the  $x$ ,  $y$ , and  $z$  axes.

(a)  $R_x = \boxed{2.00}$ ,  $R_y = \boxed{1.00}$ ,  $R_z = \boxed{3.00}$

(b)  $|\mathbf{R}| = \sqrt{R_x^2 + R_y^2 + R_z^2} = \sqrt{4.00 + 1.00 + 9.00} = \sqrt{14.0} = \boxed{3.74}$

(c)  $\cos \theta_x = \frac{R_x}{|\mathbf{R}|} \Rightarrow \theta_x = \cos^{-1}\left(\frac{R_x}{|\mathbf{R}|}\right) = \boxed{57.7^\circ \text{ from } +x}$

$$\cos \theta_y = \frac{R_y}{|\mathbf{R}|} \Rightarrow \theta_y = \cos^{-1}\left(\frac{R_y}{|\mathbf{R}|}\right) = \boxed{74.5^\circ \text{ from } +y}$$

$$\cos \theta_z = \frac{R_z}{|\mathbf{R}|} \Rightarrow \theta_z = \cos^{-1}\left(\frac{R_z}{|\mathbf{R}|}\right) = \boxed{36.7^\circ \text{ from } +z}$$

## 2. Vectors

### 2.5 Products of Vectors

#### i) **Scalar Product**

$$\vec{A} \cdot \vec{B} = AB \cos \phi = |\vec{A}| |\vec{B}| \cos \phi$$

(definition of the scalar  
(dot) product)

#### ii) **Vector Product**

$$\vec{C} = \vec{A} \times \vec{B} \Rightarrow C = AB \sin \phi$$

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

$$\vec{B} \times \vec{A} = -\vec{A} \times \vec{B}$$

$$\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = \mathbf{0}$$

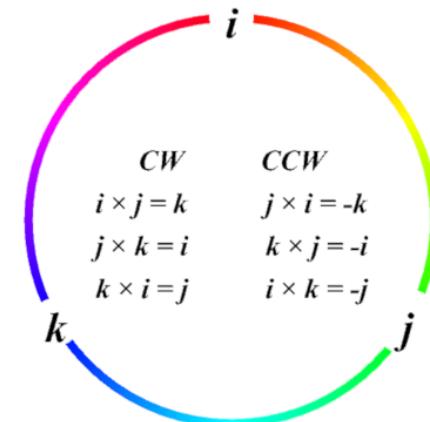
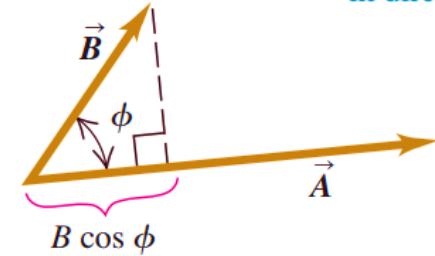
$$\hat{i} \times \hat{j} = -\hat{j} \times \hat{i} = \hat{k}$$

$$\hat{j} \times \hat{k} = -\hat{k} \times \hat{j} = \hat{i}$$

$$\hat{k} \times \hat{i} = -\hat{i} \times \hat{k} = \hat{j}$$

(b)  $\vec{A} \cdot \vec{B}$  equals

$A(B \cos \phi)$ .  
(Magnitude of  $\vec{A}$ ) times (Component of  $\vec{B}$   
in direction of  $\vec{A}$ )



## 2. Vectors

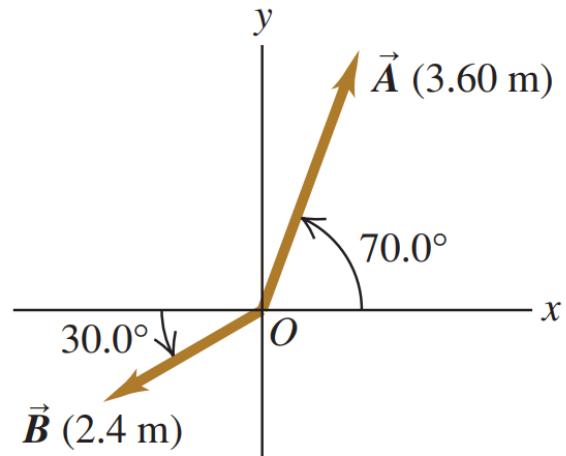
### 2.5 Products of Vectors

14) For the two vectors  $\vec{A}$  and  $\vec{B}$  in figure

- find the scalar product  $\vec{A} \cdot \vec{B}$
- find the magnitude and direction of the vector product  $\vec{A} \times \vec{B}$

The angle between the vectors is  $20^\circ + 90^\circ + 30^\circ = 140^\circ$ .

(a)  $\vec{A} \cdot \vec{B} = (3.60 \text{ m})(2.40 \text{ m})\cos 140^\circ = -6.62 \text{ m}^2$



(b)  $(3.60 \text{ m})(2.40 \text{ m})\sin 140^\circ = 5.55 \text{ m}^2$

direction, from the right-hand rule, is out of the page (the  $+z$ -direction)

### 3. Motion in One Dimension

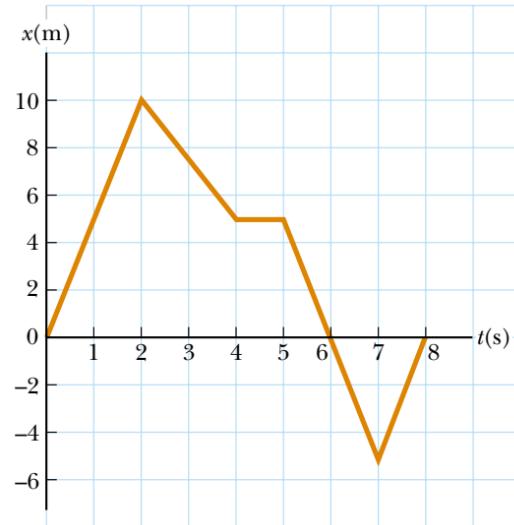
**15)** The position versus time for a certain particle moving along the  $x$  axis is shown in Figure given below. Find the average velocity in the time intervals

- (a) 2 s to 4 s, (b) 4 s to 7 s, (c) 0 to 8 s

(a)  $\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{5 \text{ m} - 10 \text{ m}}{4 \text{ s} - 2 \text{ s}} = \boxed{-2.5 \text{ m/s}}$

(b)  $\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{-5 \text{ m} - 5 \text{ m}}{7 \text{ s} - 4 \text{ s}} = \boxed{-3.3 \text{ m/s}}$

(c)  $\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{0 - 0}{8 - 0} = \boxed{0 \text{ m/s}}$



**16)** A position-time graph for a particle moving along the  $x$  axis is shown in Figure given below.

- (a) Find the average velocity in the time interval  $t = 1.50 \text{ s}$  to  $t = 4.00 \text{ s}$ .  
(b) Determine the instantaneous velocity at  $t = 2.00 \text{ s}$  by measuring the slope of the tangent line shown in the graph.  
(c) At what value of  $t$  is the velocity zero?

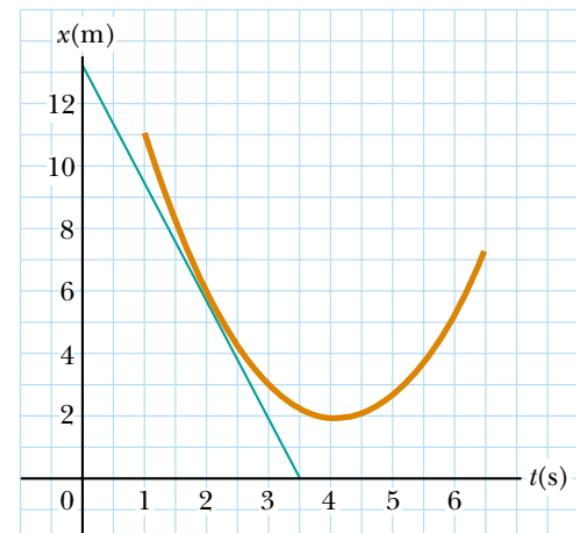
- (a) at  $t_i = 1.5 \text{ s}$ ,  $x_i = 8.0 \text{ m}$  (Point A)  
at  $t_f = 4.0 \text{ s}$ ,  $x_f = 2.0 \text{ m}$  (Point B)

$$\bar{v} = \frac{x_f - x_i}{t_f - t_i} = \frac{(2.0 - 8.0) \text{ m}}{(4 - 1.5) \text{ s}} = -\frac{6.0 \text{ m}}{2.5 \text{ s}} = \boxed{-2.4 \text{ m/s}}$$

- (b) The slope of the tangent line is found from points C and D. ( $t_C = 1.0 \text{ s}$ ,  $x_C = 9.5 \text{ m}$ ) and ( $t_D = 3.5 \text{ s}$ ,  $x_D = 0$ ),

$$v \approx \boxed{-3.8 \text{ m/s}}.$$

- (c) The velocity is zero when  $x$  is a minimum. This is at  $t \approx \boxed{4 \text{ s}}$



### 3. Motion in One Dimension

17) The graph in figure shows the velocity of a car plotted as a function of time.

- (a) Find the instantaneous acceleration at  $t=3$  s, at  $t=7$  s and at  $t=11$  s
- (b) How far does the car go in the first 5 s? The first 9 s?

(a) (a) **IDENTIFY and SET UP:** The acceleration  $a_x$  at time  $t$  is the slope of the tangent to the  $v_x$  versus  $t$  curve at time  $t$ .

**EXECUTE:** At  $t = 3$  s, the  $v_x$  versus  $t$  curve is a horizontal straight line, with zero slope. Thus  $a_x = 0$ .

$$\text{At } t = 7 \text{ s, the } v_x \text{ versus } t \text{ curve is a straight-line segment with slope } \frac{45 \text{ m/s} - 20 \text{ m/s}}{9 \text{ s} - 5 \text{ s}} = 6.3 \text{ m/s}^2.$$

$$\text{Thus } a_x = 6.3 \text{ m/s}^2.$$

$$\text{At } t = 11 \text{ s, the curve is again a straight-line segment, now with slope } \frac{-0 - 45 \text{ m/s}}{13 \text{ s} - 9 \text{ s}} = -11.2 \text{ m/s}^2.$$

$$\text{Thus } a_x = -11.2 \text{ m/s}^2.$$

(b)  $v_{0x} = 20 \text{ m/s}$   $a_x = 0$   $t = 5 \text{ s}$   $x - x_0 = ?$

$$x - x_0 = v_{0x}t \quad (a_x = 0 \text{ so no } \frac{1}{2}a_x t^2 \text{ term})$$

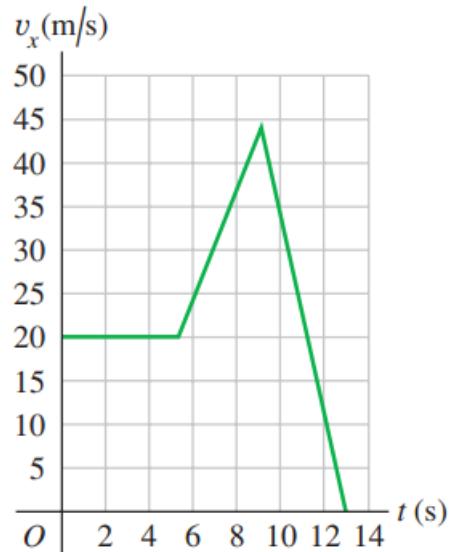
$$x - x_0 = (20 \text{ m/s})(5 \text{ s}) = 100 \text{ m; this is the distance the officer travels in the first 5 seconds.}$$

$$v_{0x} = 20 \text{ m/s}$$
  $a_x = 6.25 \text{ m/s}^2$   $t = 4 \text{ s}$   $x_0 = 100 \text{ m}$   $x - x_0 = ?$

$$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$$

$$x - x_0 = (20 \text{ m/s})(4 \text{ s}) + \frac{1}{2}(6.25 \text{ m/s}^2)(4 \text{ s})^2 = 80 \text{ m} + 50 \text{ m} = 130 \text{ m.}$$

$$\text{Thus } x - x_0 + 130 \text{ m} = 100 \text{ m} + 130 \text{ m} = 230 \text{ m.}$$



### 3. Motion in One Dimension

- 18)** A particle starts from rest and accelerates as shown in Figure. Determine (a) the particle's speed at  $t = 10.0$  s and at  $t = 20.0$  s, and (b) the distance traveled in the first 20.0 s.

(a) Acceleration is constant over the first ten seconds, so at the end,

$$v_f = v_i + at = 0 + (2.00 \text{ m/s}^2)(10.0 \text{ s}) = \boxed{20.0 \text{ m/s}}.$$

Then  $a = 0$  so  $v$  is constant from  $t = 10.0$  s to  $t = 15.0$  s. And over the last five seconds the velocity changes to

$$v_f = v_i + at = 20.0 \text{ m/s} + (3.00 \text{ m/s}^2)(5.00 \text{ s}) = \boxed{5.00 \text{ m/s}}.$$

(b) In the first ten seconds,

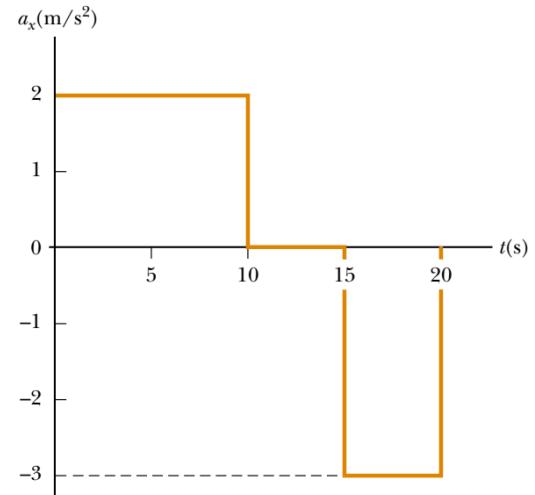
$$x_f = x_i + v_i t + \frac{1}{2} a t^2 = 0 + 0 + \frac{1}{2} (2.00 \text{ m/s}^2)(10.0 \text{ s})^2 = 100 \text{ m}.$$

Over the next five seconds the position changes to

$$x_f = x_i + v_i t + \frac{1}{2} a t^2 = 100 \text{ m} + (20.0 \text{ m/s})(5.00 \text{ s}) + 0 = 200 \text{ m}.$$

And at  $t = 20.0$  s,

$$x_f = x_i + v_i t + \frac{1}{2} a t^2 = 200 \text{ m} + (20.0 \text{ m/s})(5.00 \text{ s}) + \frac{1}{2} (-3.00 \text{ m/s}^2)(5.00 \text{ s})^2 = \boxed{262 \text{ m}}.$$



#### Equation

$$\begin{aligned} v_{xf} &= v_{xi} + a_x t \\ x_f &= x_i + \frac{1}{2}(v_{xi} + v_{xf})t \\ x_f &= x_i + v_{xi}t + \frac{1}{2}a_x t^2 \\ v_{xf}^2 &= v_{xi}^2 + 2a_x(x_f - x_i) \end{aligned}$$

### 3. Motion in One Dimension

**19)** A particle moves along the  $x$  axis according to the equation

$x = 2.00 + 3.00t - 1.00t^2$ , where  $x$  is in meters and  $t$  is in seconds. At  $t = 3.00$  s, find

- (a) the position of the particle,
- (b) its velocity, and
- (c) its acceleration.

$$x = 2.00 + 3.00t - t^2, v = \frac{dx}{dt} = 3.00 - 2.00t, a = \frac{dv}{dt} = -2.00$$

At  $t = 3.00$  s :

(a)  $x = (2.00 + 9.00 - 9.00) \text{ m} = \boxed{2.00 \text{ m}}$

(b)  $v = (3.00 - 6.00) \text{ m/s} = \boxed{-3.00 \text{ m/s}}$

(c)  $a = \boxed{-2.00 \text{ m/s}^2}$

### 3. Motion in One Dimension

**20)** The height of a helicopter above the ground is given by  $h = 3.00t^3$ , where  $h$  is in meters and  $t$  is in seconds. After 2.00 s, the helicopter releases a box. How long after its release does the box reach the ground?

$y = 3.00t^3$ : At  $t = 2.00$  s,  $y = 3.00(2.00)^3 = 24.0$  m and

$$v_y = \frac{dy}{dt} = 9.00t^2 = 36.0 \text{ m/s} \uparrow.$$

$$y_b = y_{bi} + v_i t - \frac{1}{2} g t^2 = 24.0 + 36.0t - \frac{1}{2}(9.80)t^2.$$

Setting  $y_b = 0$ ,

$$0 = 24.0 + 36.0t - 4.90t^2.$$

Solving for  $t$ , (only positive values of  $t$  count),  $t = 7.96$  s.

## 4. Two Dimensional Motion

**21)** A particle initially located at the origin has an acceleration of  $\mathbf{a} = 3.00\hat{\mathbf{j}} \text{ m/s}^2$  and an initial velocity of  $\mathbf{v}_i = 500\hat{\mathbf{i}} \text{ m/s}$ . Find (a) the vector position and velocity at any time  $t$  and (b) the coordinates and speed of the particle at  $t = 2.00 \text{ s}$ .

$$\mathbf{a} = 3.00\hat{\mathbf{j}} \text{ m/s}^2; \mathbf{v}_i = 500\hat{\mathbf{i}} \text{ m/s}; \mathbf{r}_i = 0\hat{\mathbf{i}} + 0\hat{\mathbf{j}}$$

(a)  $\mathbf{r}_f = \mathbf{r}_i + \mathbf{v}_i t + \frac{1}{2} \mathbf{a} t^2 = \boxed{\left[ 5.00t\hat{\mathbf{i}} + \frac{1}{2} 3.00t^2\hat{\mathbf{j}} \right] \text{ m}}$

$$\mathbf{v}_f = \mathbf{v}_i + \mathbf{a} t = \boxed{(5.00\hat{\mathbf{i}} + 3.00t\hat{\mathbf{j}}) \text{ m/s}}$$

(b)  $t = 2.00 \text{ s}, \mathbf{r}_f = 5.00(2.00)\hat{\mathbf{i}} + \frac{1}{2}(3.00)(2.00)^2\hat{\mathbf{j}} = (10.0\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}}) \text{ m}$   
so  $x_f = \boxed{10.0 \text{ m}}, y_f = \boxed{6.00 \text{ m}}$

$$\mathbf{v}_f = 5.00\hat{\mathbf{i}} + 3.00(2.00)\hat{\mathbf{j}} = (5.00\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}}) \text{ m/s}$$

$$v_f = |\mathbf{v}_f| = \sqrt{v_{xf}^2 + v_{yf}^2} = \sqrt{(5.00)^2 + (6.00)^2} = \boxed{7.81 \text{ m/s}}$$

## 4. Two Dimensional Motion

- 22)** A moderate wind accelerates a pebble over a horizontal  $xy$  plane with a constant acceleration  $\vec{a} = (5.00 \text{ m/s}^2)\hat{i} + (5.00 \text{ m/s}^2)\hat{j}$ . At time  $t = 0$ , the velocity is  $(4.00 \text{ m/s})\hat{i}$ . What are the  
(a) magnitude and  
(b) angle of its velocity when it has been displaced by 12.0 m parallel to the  $x$  axis?

We find  $t$  by solving  $\Delta x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$ :

$$12.0 \text{ m} = 0 + (4.00 \text{ m/s})t + \frac{1}{2}(5.00 \text{ m/s}^2)t^2$$

where we have used  $\Delta x = 12.0 \text{ m}$ ,  $v_x = 4.00 \text{ m/s}$ , and  $a_x = 5.00 \text{ m/s}^2$ . We use the quadratic formula and find  $t = 1.53 \text{ s}$ .

Therefore, its velocity (when  $\Delta x = 12.00 \text{ m}$ ) is

$$\begin{aligned}\vec{v} &= \vec{v}_0 + \vec{a}t = (4.00 \text{ m/s})\hat{i} + (5.00 \text{ m/s}^2)(1.53 \text{ s})\hat{i} + (7.00 \text{ m/s}^2)(1.53 \text{ s})\hat{j} \\ &= (11.7 \text{ m/s})\hat{i} + (10.7 \text{ m/s})\hat{j}.\end{aligned}$$

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Thus, the magnitude of  $\vec{v}$  is  $|\vec{v}| = \sqrt{(11.7 \text{ m/s})^2 + (10.7 \text{ m/s})^2} = 15.8 \text{ m/s}$ .

- (b) The angle of  $\vec{v}$ , measured from  $+x$ , is

$$\tan^{-1} \left( \frac{10.7 \text{ m/s}}{11.7 \text{ m/s}} \right) = 42.6^\circ.$$