PHYSICS I - MECHANICS

Problem Session-II

6. Some Applications of Newton's Laws

7. Circular Motion

8. Work and Kinetic Energy

9. Potential Energy and Energy Conservation

6. Some Applications of Newton's Laws

In this section we consider only equilibrium of a body that can be modeled as a particle.

Newton's First Law:

An object at rest will stay at rest, and an object in motion will stay in motion at constant velocity, unless acted upon by an unbalanced force.

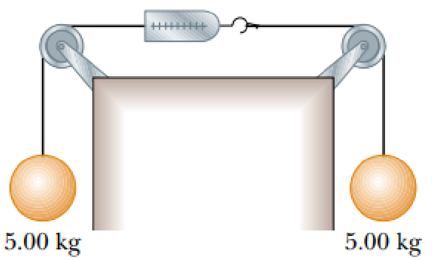
- ✓ In simpler terms, we can say that when no force acts on an object, the acceleration of the object is zero.
- ✓ If nothing acts to change the object's motion, then its velocity does not change.

$$\sum \vec{F} = 0 \qquad \text{(body in equilibrium)}$$

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \text{(body in equilibrium)}$$

6. Some Applications of Newton's Laws

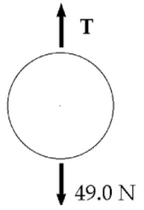
1) The systems shown are in equilibrium. If the spring scales are calibrated in newtons, what do they read? (Neglect the masses of the pulleys and strings, and assume the incline in part (c) is frictionless.)



(a)

(a) Isolate either mass

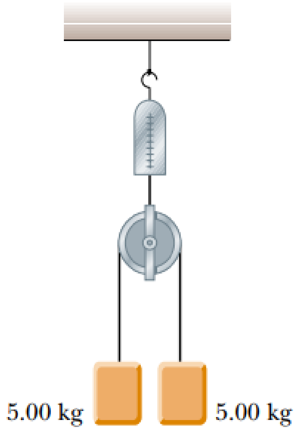
$$T + mg = ma = 0$$
$$|T| = |mg|.$$



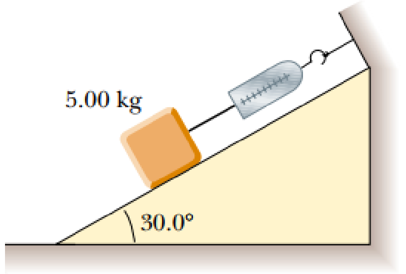
The scale reads the tension T,

so

$$T = mg = 5.00\ \text{kg}\left(9.80\ \text{m/s}^2\right) = \boxed{49.0\ \text{N}}.$$



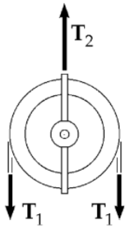
(b)



(c)

(b) Isolate the pulley

$$\mathbf{T}_2 + 2\mathbf{T}_1 = 0$$
$$T_2 = 2|T_1| = 2mg = \boxed{98.0\ \text{N}}.$$



(c) $\sum \mathbf{F} = \mathbf{n} + \mathbf{T} + m\mathbf{g} = 0$

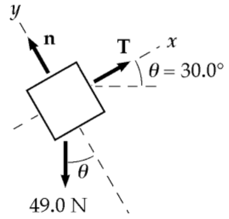
Take the component along the incline

$$\mathbf{n}_x + \mathbf{T}_x + m\mathbf{g}_x = 0$$

or

$$0 + T - mg\sin 30.0° = 0$$

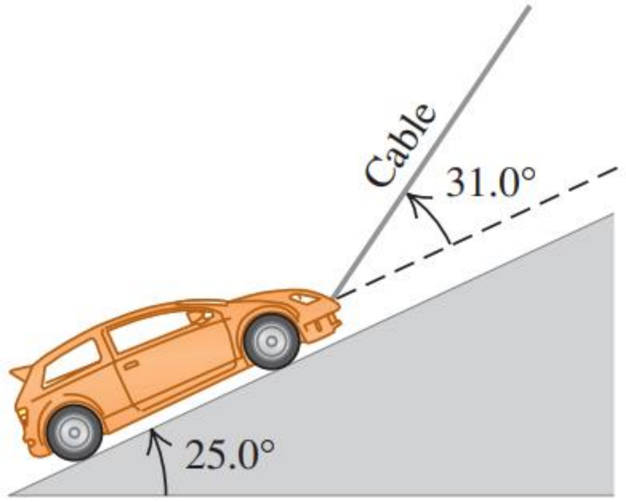
$$T = mg\sin 30.0° = \frac{mg}{2} = \frac{5.00(9.80)}{2}$$
$$= \boxed{24.5\ \text{N}}.$$

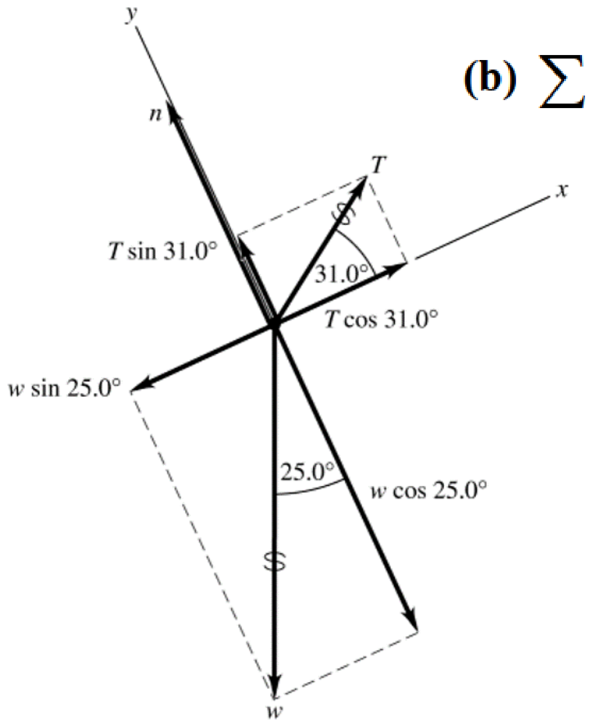


6. Some Applications of Newton's Laws

2) A 1130-kg car is held in place by a light cable on a very smooth (frictionless) ramp, as shown in figure. The cable makes an angle of 31.0° above the surface of the ramp, and the ramp itself rises at 25.0° above the horizontal.

(a) Draw a free-body diagram for the car.
(b) Find the tension in the cable.
(c) How hard does the surface of the ramp push on the car?





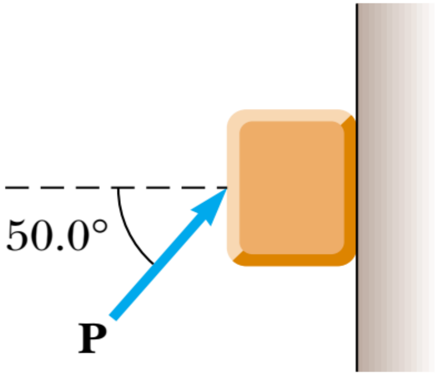
(b) $\sum F_x = 0$ gives $T\cos 31.0° - w\sin 25.0° = 0$ and $T = w\dfrac{\sin 25.0°}{\cos 31.0°} = (1130\ \text{kg})(9.80\ \text{m/s}^2)\dfrac{\sin 25.0°}{\cos 31.0°} = 5460\ \text{N}$

(c) $\sum F_y = 0$ gives $n + T\sin 31.0° - w\cos 25.0° = 0$ and

$n = w\cos 25.0° - T\sin 31.0° = (1130\ \text{kg})(9.80\ \text{m/s}^2)\cos 25.0° - (5460\ \text{N})\sin 31.0° = 7220\ \text{N}$

6. Some Applications of Newton's Laws

3) A block of mass 3.00 kg is pushed up against a wall by a force P that makes a 50.0° angle with the horizontal as shown in Figure below. The coefficient of static friction between the block and the wall is 0.250. Determine the possible values for the magnitude of P that allow the block to remain stationary.



(Case 1, impending upward motion)
Setting

$$\sum F_x = 0: \qquad P\cos 50.0° - n = 0$$

$$f_{s,\,\max} = \mu_s n: \quad f_{s,\,\max} = \mu_s P\cos 50.0° = 0.250(0.643)P = 0.161P$$

Setting

$$\sum F_y = 0: \quad P\sin 50.0° - 0.161P - 3.00(9.80) = 0$$

$$P_{\max} = \boxed{48.6\ \text{N}}$$

(Case 2, impending downward motion)
As in Case 1,

$$f_{s,\,\max} = 0.161P$$

Setting

$$\sum F_y = 0: \quad P\sin 50.0° + 0.161P - 3.00(9.80) = 0$$

$$P_{\min} = \boxed{31.7\ \text{N}}$$

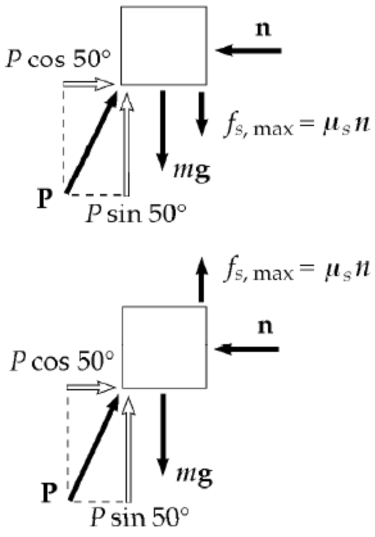


FIG. P5.46

6. Some Applications of Newton's Laws

In these problems, we apply Newton's second law to bodies on which the net force is not zero. These bodies are not in equilibrium and hence are accelerating.

Newton's second law :

If a net external force acts on a body, the body accelerates. The direction of acceleration is the same as the direction of the net force. The mass of the body times the acceleration of the body equals the net force Vector.

$$\sum \vec{F} = m\vec{a} \qquad \text{(Newton's second law of motion)}$$

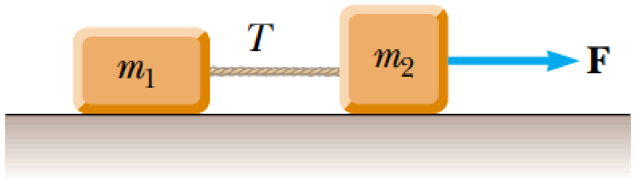
$$\sum F_x = ma_x \qquad \sum F_y = ma_y \qquad \text{(Newton's second law, component form)}$$

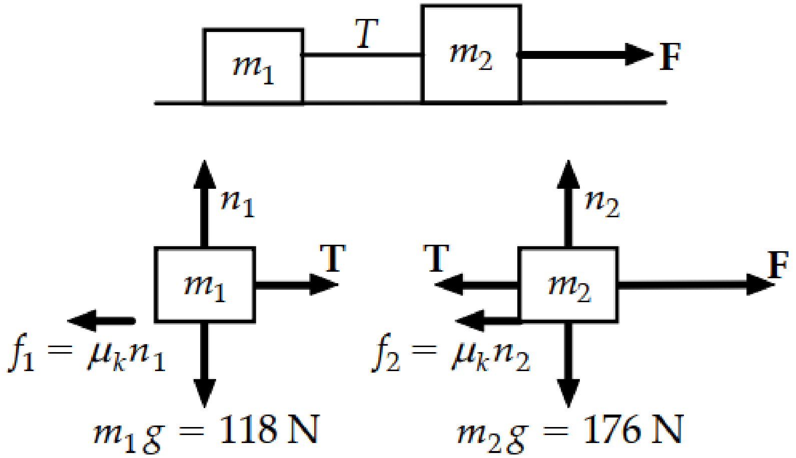
6. Some Applications of Newton's Laws

4) Two blocks connected by a rope of negligible mass are being dragged by a horizontal force **F.** Suppose that F=68.0 N, m_1 = 12.0 kg, m_2 = 18.0 kg, and the coefficient of kinetic friction between each block and the surface is 0.100.

(a) Draw a free-body diagram for each block.

(b) Determine the tension T and the magnitude of the acceleration of the system.





(b)

$$68.0 - T - \mu m_2 g = m_2 a \quad \text{(Block \#2)}$$

$$T - \mu m_1 g = m_1 a \quad \text{(Block \#1)}$$

Adding,

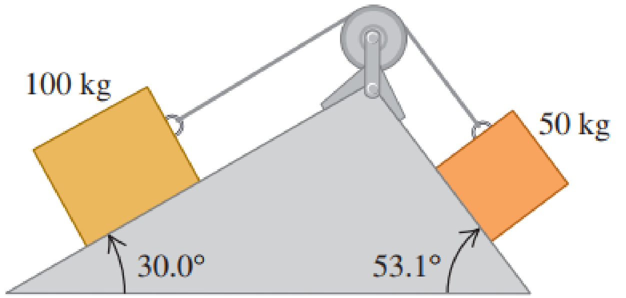
$$68.0 - \mu(m_1 + m_2)g = (m_1 + m_2)a$$

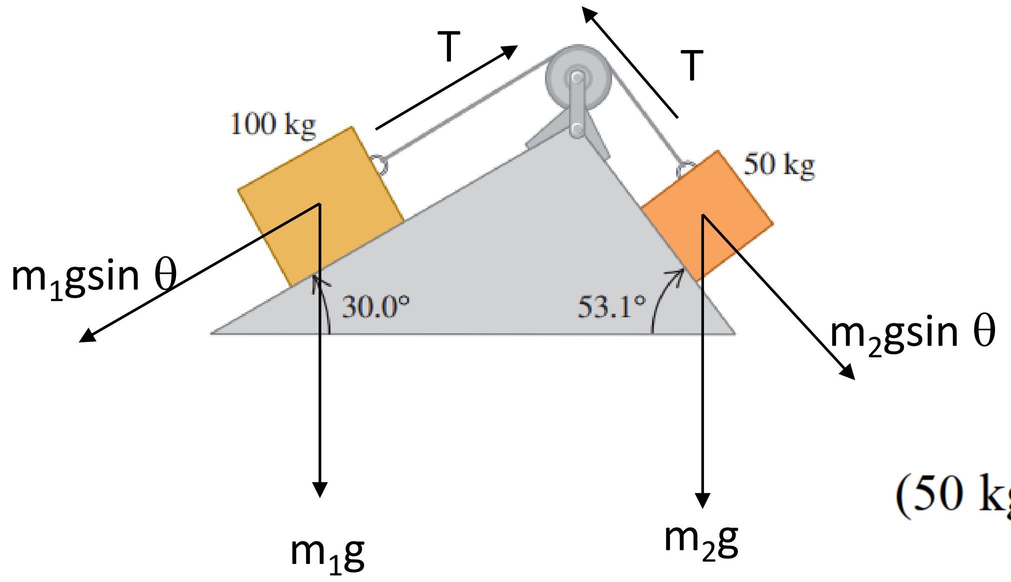
$$a = \frac{68.0}{(m_1 + m_2)} - \mu g = \boxed{1.29\ \text{m/s}^2}$$

$$T = m_1 a + \mu m_1 g = \boxed{27.2\ \text{N}}$$

6. Some Applications of Newton's Laws

5) Two blocks connected by a cord passing over a small, frictionless pulley rest on frictionless planes
(a) Which way will the system move when the blocks are released from rest?
(b) What is the acceleration of the blocks?
(c) What is the tension in the cord?





(a) $T - (100\text{ kg})g\sin 30.0° = (100\text{ kg})a$

$$(50\text{ kg})g\sin 53.1° - T = (50\text{ kg})a$$

$$(50\text{ kg}\sin 53.1° - 100\text{ kg}\sin 30.0°)g = (50\text{ kg} + 100\text{ kg})a,\ \text{or } a = -0.067g$$

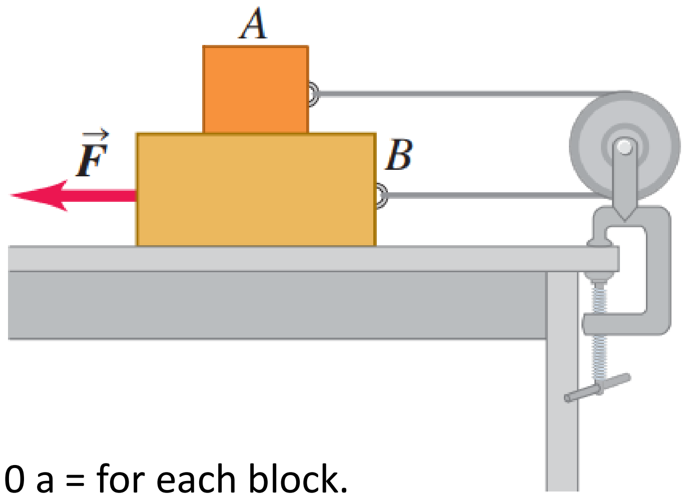
Since *a* comes out negative, the blocks will slide to the left; the 100-kg block will slide down. Of course, if coordinates had been chosen so that positive accelerations were to the left, *a* would be +0 067*g*.

(b) $a = 0.067(9.80\text{ m/s}^2) = 0.658\text{ m/s}^2$

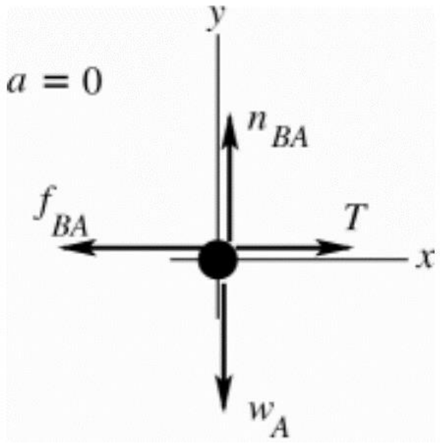
(c) T=424 N

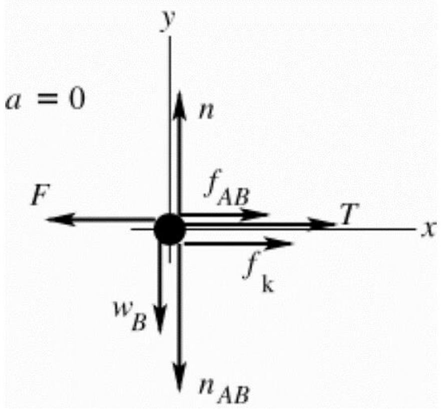
6. Some Applications of Newton's Laws

6) Block A weighs 1.90 N, and block B weighs 4.20 N. The coefficient of kinetic friction between all surfaces is 0.30. Find the magnitude of the horizontal force necessary to drag block B to the left at constant speed if A and B are connected by a light, flexible cord passing around a fixed, frictionless pulley.



Block B is pulled to the left at constant speed, so block A moves to the right at constant speed and 0 a = for each block.





$$\Sigma F_y = ma_y$$

$$n_{BA} - w_A = 0$$

$$n_{BA} = 1.90 \text{ N}$$

$$f_{BA} = \mu_k n_{BA} = (0.30)(1.90 \text{ N}) = 0.57 \text{ N}$$

$\Sigma F_x = ma_x$. $T - f_{BA} = 0$. $T = f_{BA} = 0.57$ N.

$\Sigma F_y = ma_y$: $n - w_B - n_{AB} = 0$. $n = w_B + n_{AB} = 4.20 \text{ N} + 1.90 \text{ N} = 6.10$ N. Then $f_k = \mu_k n = (0.30)(6.10 \text{ N}) = 1.83$ N. $\Sigma F_x = ma_x$: $f_{AB} + T + f_k - F = 0$. $F = T + f_{AB} + f_k = 0.57 \text{ N} + 0.57 \text{ N} + 1.83 \text{ N} = 3.0$ N.

6. Some Applications of Newton's Laws

7) A 8.00-kg block of ice, released from rest at the top of a 1.50-m-long frictionless ramp, slides downhill, reaching a speed of 2.50 m/s at the bottom.

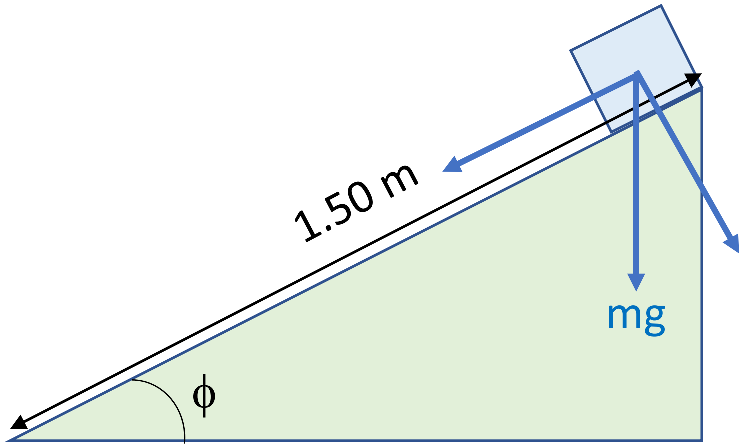
(a) What is the angle between the ramp and the horizontal?

(b) What would be the speed of the ice at the bottom if the motion were opposed by a constant friction force of 10.0 N parallel to the surface of the ramp?

(a) $x - x_0 = 1.50\text{ m}$, $v_{0x} = 0$. $v_x = 2.50\text{ m/s}$. $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives

$$a_x = \frac{v_x^2 - v_{0x}^2}{2(x - x_0)} = \frac{(2.50\text{ m/s})^2 - 0}{2(1.50\text{ m})} = 2.08\text{ m/s}^2.$$ $\Sigma F_x = ma_x$ gives $mg\sin\phi = ma$ and $\sin\phi = \frac{a}{g} = \frac{2.08\text{ m/s}^2}{9.80\text{ m/s}^2}$.

$\phi = 12.3°$.



(b) $\Sigma F_x = ma_x$ gives $mg\sin\phi - f = ma$ and

$$a = \frac{mg\sin\phi - f}{m} = \frac{(8.00\text{ kg})(9.80\text{ m/s}^2)\sin 12.3° - 10.0\text{ N}}{8.00\text{ kg}} = 0.838\text{ m/s}^2.$$

Then $x - x_0 = 1.50\text{ m}$, $v_{0x} = 0$. $a_x = 0.838\text{ m/s}^2$ and $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives

$$v_x = \sqrt{2a_x(x - x_0)} = \sqrt{2(0.838\text{ m/s}^2)(1.50\text{ m})} = 1.59\text{ m/s}$$

7. Circular Motion

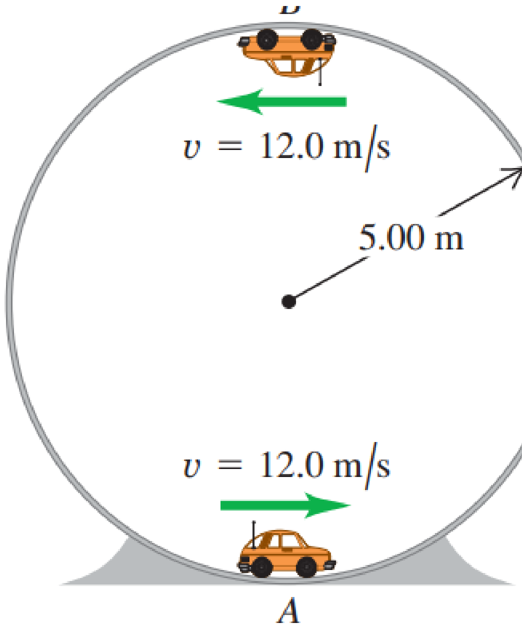
8) A small car with mass 0.800 kg travels at constant speed on the inside of a track that is a vertical circle with radius 5.00 m. If the normal force exerted by the track on the car when it is at the top of the track (point B) is 6.00 N, what is the normal force on the car when it is at the bottom of the track (point A)?

$$n_A - mg = ma$$

$$mg + n_B = ma$$

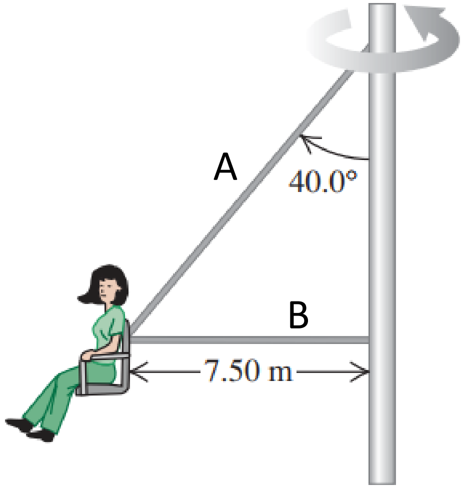
$$a = \frac{mg + n_B}{m} = \frac{(0.800\text{ kg})(9.8\text{ m/s}^2) + 6.00\text{ N}}{0.800\text{ kg}} = 17.3\text{ m/s}^2$$

$$n_A = m(g + a) = (0.800\text{ kg})(9.8\text{ m/s}^2 + 17.3\text{ m/s}^2) = 21.7\text{ N.}$$



7. Circular Motion

9) As shown in figure, the seat is connected to two cables one of which is horizontal. The seat swings in a horizontal circle at a rate of 32.0 rpm. If the seat weighs 255 N and an 825-N person is sitting in it, find the tension in each cable.



The total mass is (255 N + 825 N)/(9. 80 m/s^2) = 110.2 kg.

Since the rotation rate is 32.0 rev/min=0.5333 rev/s, the period T is $\frac{1}{0.5333\ \text{rev/s}} = 1.875\ \text{s}$.

$$\Sigma F_y = ma_y \text{ gives } T_A \cos 40.0° - mg = 0 \text{ and } T_A = \frac{mg}{\cos 40.0°} = \frac{255\ \text{N} + 825\ \text{N}}{\cos 40.0°} = 1410\ \text{N}.$$

$$\Sigma F_x = ma_x \text{ gives } T_A \sin 40.0° + T_B = ma_{\text{rad}} \text{ and}$$

$$T_B = m\frac{4\pi^2 R}{T^2} - T_A \sin 40.0° = (110.2\ \text{kg})\frac{4\pi^2 (7.50\ \text{m})}{(1.875\ \text{s})^2} - (1410\ \text{N})\sin 40.0° = 8370\ \text{N}$$

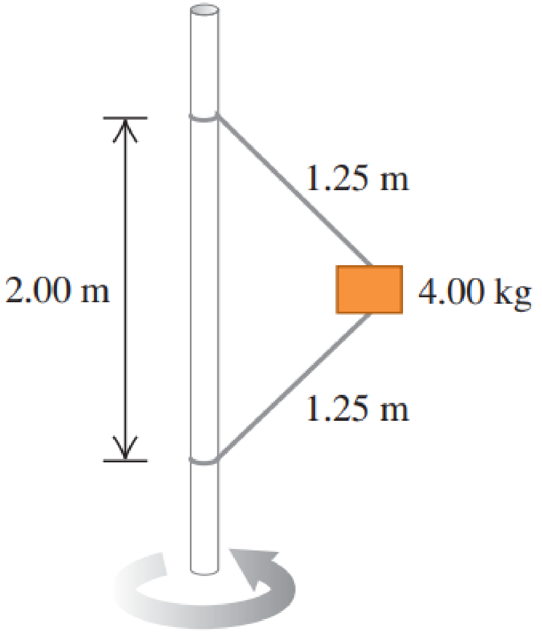
$$a_{rad} = \frac{4r\pi^2}{T^2}$$

7. Circular Motion

10) The 4.00-kg block is attached to a vertical rod by means of two strings. When the system rotates about the axis of the rod, the strings are extended as shown in the diagram and the tension in the upper string is 80.0 N.

(a) What is the tension in the lower cord?

(b) How many revolutions per minute does the system make?



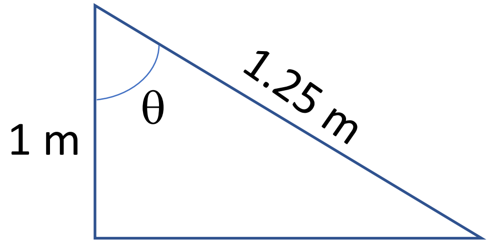
SET UP: The block moves in a horizontal circle of radius $r = \sqrt{(1.25\text{ m})^2 - (1.00\text{ m})^2} = 0.75\text{ m}$. Each string makes an angle θ with the vertical. $\cos\theta = \dfrac{1.00\text{ m}}{1.25\text{ m}}$, so $\theta = 36.9°$.

(a) $\Sigma F_y = ma_y$ gives $T_\text{u}\cos\theta - T_\text{l}\cos\theta - mg = 0$.

$$T_\text{l} = T_\text{u} - \frac{mg}{\cos\theta} = 80.0\text{ N} - \frac{(4.00\text{ kg})(9.80\text{ m/s}^2)}{\cos 36.9°} = 31.0\text{ N}.$$

(b) $\Sigma F_x = ma_x$ gives $(T_\text{u} + T_\text{l})\sin\theta = m\dfrac{v^2}{r}$.

$$v = \sqrt{\frac{r(T_\text{u} + T_\text{l})\sin\theta}{m}} = \sqrt{\frac{(0.75\text{ m})(80.0\text{ N} + 31.0\text{ N})\sin 36.9°}{4.00\text{ kg}}} = 3.53\text{ m/s}.$$ The number of revolutions per second is $\dfrac{v}{2\pi r} = \dfrac{3.53\text{ m/s}}{2\pi(0.75\text{ m})} = 0.749\text{ rev/s} = 44.9\text{ rev/min}$.



7. Circular Motion

11) A small, spherical bead of mass 3.00 g is released from rest at t =0 in a bottle of liquid shampoo. The terminal speed is observed to be v_T=2.00 cm/s. Find

(a) the value of the constant b

(b) the time at which the bead reaches $0.632v_T$, and

(c) the value of the resistive force when the bead reaches terminal speed.

$$\boxed{v_T = \frac{mg}{b}}$$

$$\boxed{v = \frac{mg}{b}\left(1 - e^{-bt/m}\right) = v_T\left(1 - e^{-t/\tau}\right)}$$

(a) At terminal velocity, $R = v_T b = mg$

$$\therefore b = \frac{mg}{v_T} = \frac{(3.00\times10^{-3}\ \text{kg})(9.80\ \text{m/s}^2)}{2.00\times10^{-2}\ \text{m/s}} = \boxed{1.47\ \text{N}\cdot\text{s/m}}$$

(b) In the equation describing the time variation of the velocity, we have

$v = v_T\left(1 - e^{-bt/m}\right)$ $\qquad v = 0.632v_T$ when $e^{-bt/m} = 0.368$

or at time $\qquad t = -\left(\frac{m}{b}\right)\ln(0.368) = \boxed{2.04\times10^{-3}\ \text{s}}$

(c) At terminal velocity, $\qquad R = v_T b = mg = \boxed{2.94\times10^{-2}\ \text{N}}$

7. Circular Motion

12) A sky diver of mass 80.0 kg jumps from a slow-moving aircraft and reaches a terminal speed of 50.0 m/s.
(a) What is the acceleration of the sky diver when her speed is 30.0 m/s? What is the drag force on the diver when her speed is
(b) 50.0 m/s? (c) 30.0 m/s?

$$m = 80.0\ \text{kg},\ v_T = 50.0\ \text{m/s},\ mg = \frac{D\rho A v_T^2}{2}\ \therefore\ \frac{D\rho A}{2} = \frac{mg}{v_T^2} = 0.314\ \text{kg/m}$$

(a) At $v = 30.0\ \text{m/s}$

$$a = g - \frac{\frac{D\rho A v^2}{2}}{m} = 9.80 - \frac{(0.314)(30.0)^2}{80.0} = \boxed{6.27\ \text{m/s}^2\ \text{downward}}$$

(b) At $v = 50.0\ \text{m/s}$, terminal velocity has been reached.

$$\sum F_y = 0 = mg - R$$

$$\Rightarrow R = mg = (80.0\ \text{kg})\left(9.80\ \text{m/s}^2\right) = \boxed{784\ \text{N directed up}}$$

(c) At $v = 30.0\ \text{m/s}$

$$\frac{D\rho A v^2}{2} = (0.314)(30.0)^2 = \boxed{283\ \text{N}}\ \text{upward}$$

8. Work and Kinetic Energy

13) You apply a constant force $\vec{\boldsymbol{F}} = (30\ N)\hat{\boldsymbol{\imath}} - (40\ N)\hat{\boldsymbol{\jmath}}$to the cart as it undergoes a displacement $\vec{\boldsymbol{s}} = (-9.0\ m)\hat{\boldsymbol{\imath}} - (3\ m)\hat{\boldsymbol{\jmath}}$ How much work does the force you apply do on the cart?

$$\hat{\boldsymbol{\imath}} \cdot \hat{\boldsymbol{\imath}} = \hat{\boldsymbol{\jmath}} \cdot \hat{\boldsymbol{\jmath}} = 1 \text{ and } \hat{\boldsymbol{\imath}} \cdot \hat{\boldsymbol{\jmath}} = \hat{\boldsymbol{\jmath}} \cdot \hat{\boldsymbol{\imath}} = 0$$

EXECUTE: The work you do is $\vec{\boldsymbol{F}} \cdot \vec{\boldsymbol{s}} = ((30\text{ N})\hat{\boldsymbol{\imath}} - (40\text{ N})\hat{\boldsymbol{\jmath}}) \cdot ((-9.0\text{ m})\hat{\boldsymbol{\imath}} - (3.0\text{ m})\hat{\boldsymbol{\jmath}})$

$$\vec{\boldsymbol{F}} \cdot \vec{\boldsymbol{s}} = (30\text{ N})(-9.0\text{ m}) + (-40\text{ N})(-3.0\text{ m}) = -270\text{ N}\cdot\text{m} + 120\text{ N}\cdot\text{m} = -150\text{ J}.$$

14) If it takes 4.00 J of work to stretch a Hooke's-law spring 10.0 cm from its unstressed length, determine the extra work required to stretch it an additional 10.0 cm

$$4.00\text{ J} = \frac{1}{2}k(0.100\text{ m})^2$$

$\therefore k = 800$ N/m and to stretch the spring to 0.200 m requires

$$\Delta W = \frac{1}{2}(800)(0.200)^2 - 4.00\text{ J} = \boxed{12.0\text{ J}}$$

8. Work and Kinetic Energy

15) A block of mass 1.6 kg is attached to a horizontal spring that has a force constant of 1.0 x10^3 N/m. The spring is compressed 2.0 cm and is then released from rest.

(A) Calculate the speed of the block as it passes through the equilibrium position x=0 if the surface is frictionless.

$$x_{\max} = x_i = -2.0 \text{ cm} = -2.0 \times 10^{-2} \text{ m}:$$

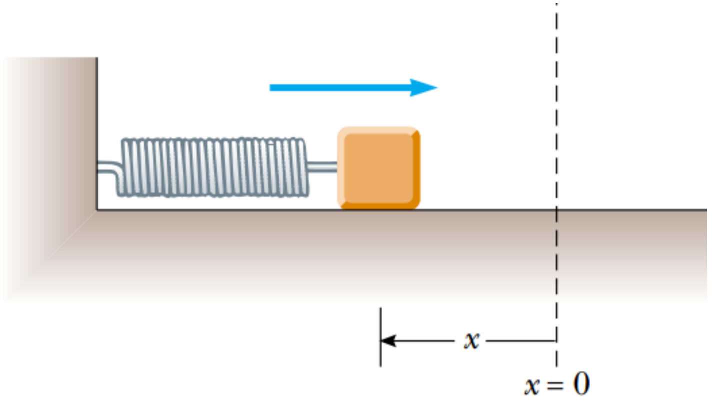
$$W_s = \tfrac{1}{2}kx_{\max}^2 = \tfrac{1}{2}(1.0 \times 10^3 \text{ N/m})(-2.0 \times 10^{-2} \text{ m})^2 = 0.20 \text{ J}$$

$$W_s = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2$$

$$v_f = \sqrt{v_i^2 + \frac{2}{m}W_s}$$

$$= \sqrt{0 + \frac{2}{1.6 \text{ kg}}(0.20 \text{ J})}$$

$$= \boxed{0.50 \text{ m/s}}$$



(B) Calculate the speed of the block as it passes through the equilibrium position if a constant friction force of 4.0 N retards its motion from the moment it is released.

$$-f_k d = -(4.0 \text{ N})(2.0 \times 10^{-2} \text{ m}) = -0.080 \text{ J}$$

$$\Delta K = -f_k d + \sum W_{\text{other forces}}$$

$$K_f = K_i - f_k d + \sum W_{\text{other forces}}$$

$$K_f = 0.20 \text{ J} - 0.080 \text{ J} = 0.12 \text{ J} = \tfrac{1}{2}mv_f^2$$

$$v_f = \sqrt{\frac{2K_f}{m}} = \sqrt{\frac{2(0.12 \text{ J})}{1.6 \text{ kg}}} = \boxed{0.39 \text{ m/s}}$$

8. Work and Kinetic Energy

16) A 2.00-kg block is attached to a spring of force constant 500 N/m. The block is pulled 5.00 cm to the right of equilibrium and released from rest. Find the speed of the block as it passes through equilibrium if (a) the horizontal surface is frictionless and (b) the coefficient of friction between block and surface is 0.350.

(a)

$$W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2 = \frac{1}{2}(500)\left(5.00\times10^{-2}\right)^2 - 0 = 0.625\text{ J}$$

$$W_s = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}mv_f^2 - 0$$

so $$v_f = \sqrt{\frac{2\left(\sum W\right)}{m}} = \sqrt{\frac{2(0.625)}{2.00}}\text{ m/s} = \boxed{0.791\text{ m/s}}$$

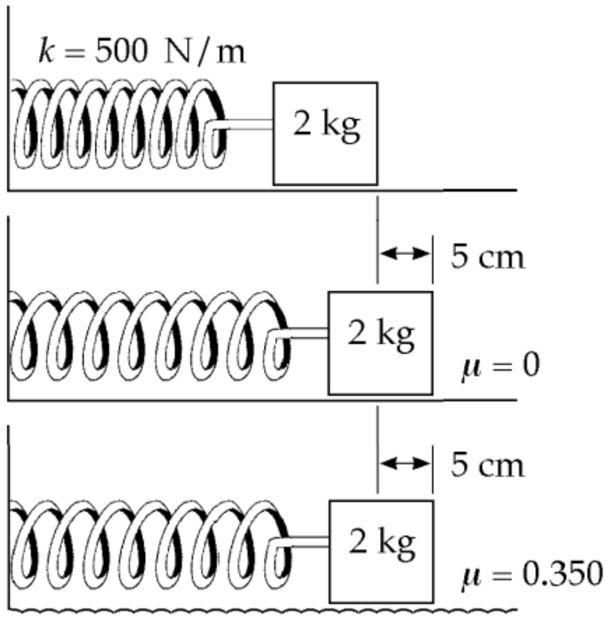
(b)

$$\frac{1}{2}mv_i^2 - f_k\Delta x + W_s = \frac{1}{2}mv_f^2$$

$$0 - (0.350)(2.00)(9.80)(0.050\,0)\text{ J} + 0.625\text{ J} = \frac{1}{2}mv_f^2$$

$$0.282\text{ J} = \frac{1}{2}(2.00\text{ kg})v_f^2$$

$$v_f = \sqrt{\frac{2(0.282)}{2.00}}\text{ m/s} = \boxed{0.531\text{ m/s}}$$



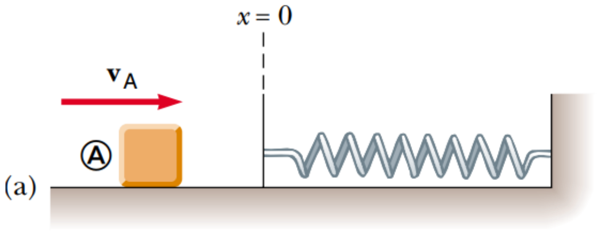
$$\Delta K = -f_k d + \sum W_{\text{other forces}}$$

$$K_f = K_i - f_k d + \sum W_{\text{other forces}}$$

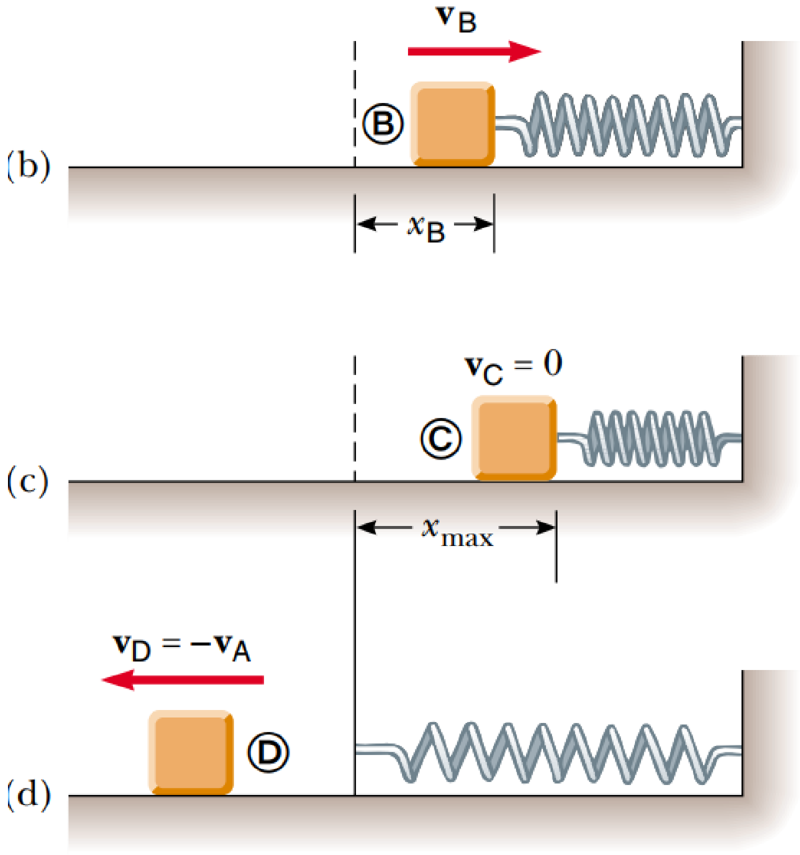
9. Potential Energy and Energy Conservation

17) A block having a mass of 0.80 kg is given an initial velocity v_A =1.2 m/s to the right and collides with a spring of negligible mass and force constant k=50 N/m, as shown in figure.

A) Assuming the surface to be frictionless, calculate the maximum compression of the spring after the collision.



$E = \frac{1}{2} m v_A{}^2$



$E = \frac{1}{2} m v_B{}^2 + \frac{1}{2} k x_B{}^2$

$E = \frac{1}{2} k x_{max}^2$

$E = \frac{1}{2} m v_D{}^2 = \frac{1}{2} m v_A{}^2$

$$E_C = E_A$$

$$K_C + U_{sC} = K_A + U_{sA}$$

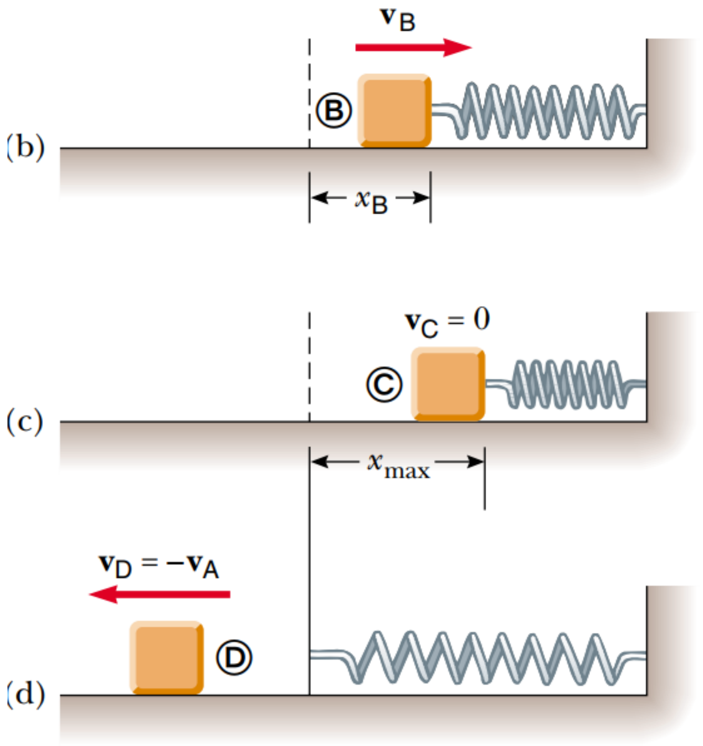
$$0 + \tfrac{1}{2} k x_{max}^2 = \tfrac{1}{2} m v_A{}^2 + 0$$

$$x_{max} = \sqrt{\frac{m}{k}}\, v_A = \sqrt{\frac{0.80\ \text{kg}}{50\ \text{N/m}}}\,(1.2\ \text{m/s})$$

$$= 0.15\ \text{m}$$

9. Potential Energy and Energy Conservation

B) Suppose a constant force of kinetic friction acts between the block and the surface, with μ_k =0.50. If the speed of the block at the moment it collides with the spring is v_A =1.2 m/s, what is the maximum compression x_C in the spring?



$E = \frac{1}{2} m v_B^2 + \frac{1}{2} k x_B^2$

$E = \frac{1}{2} k x_{max}^2$

$E = \frac{1}{2} m v_D^2 = \frac{1}{2} m v_A^2$

$$f_k = \mu_k n = \mu_k mg = 0.50(0.80 \text{ kg})(9.80 \text{ m/s}^2) = 3.92 \text{ N}$$

$$\Delta E_{mech} = -f_k x_C = (-3.92 x_C)$$

$$\Delta E_{mech} = E_f - E_i = (0 + \tfrac{1}{2} k x_C^2) - (\tfrac{1}{2} m v_A^2 + 0) = -f_k x_C$$

$$\tfrac{1}{2}(50) x_C^2 - \tfrac{1}{2}(0.80)(1.2)^2 = -3.92 x_C$$

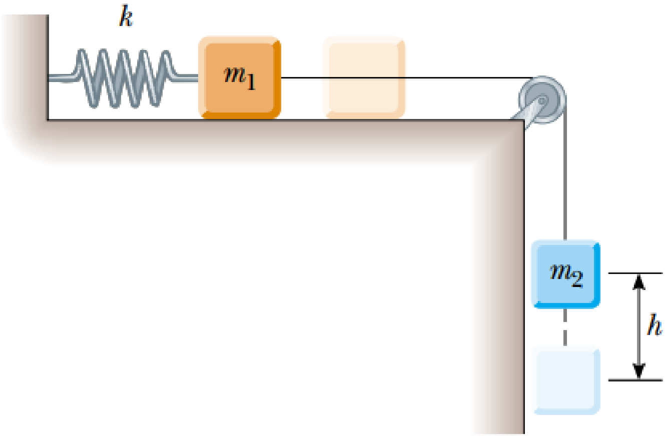
$$25 x_C^2 + 3.92 x_C - 0.576 = 0$$

$$x_C = 0.092 \text{ m}$$

$$\cancel{x_C = -0.25 \text{ m}}$$

9. Potential Energy and Energy Conservation

18) Two blocks are connected by a light string that passes over a frictionless pulley. The block of mass m_1 lies on a horizontal surface and is connected to a spring of force constant k. The system is released from rest when the spring is unstretched. If the hanging block of mass m_2 falls a distance h before coming to rest, calculate the coefficient of kinetic friction between the block of mass m_1 and the surface.



$$\Delta K = 0$$

(1) $$\Delta E_{\text{mech}} = \Delta U_g + \Delta U_s$$

$$\Delta U_g = U_{gf} - U_{gi}$$

$$\Delta U_s = U_{sf} - U_{si}$$

(2) $$\Delta E_{\text{mech}} = -f_k h = -\mu_k m_1 g h$$

(3) $$\Delta U_g = U_{gf} - U_{gi} = 0 - m_2 g h$$

(4) $$\Delta U_s = U_{sf} - U_{si} = \frac{1}{2} k h^2 - 0$$

$$-\mu_k m_1 g h = -m_2 g h + \frac{1}{2} k h^2$$

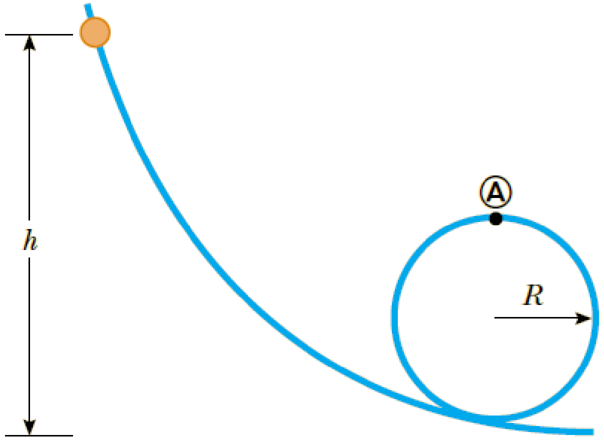
$$\mu_k = \frac{m_2 g - \frac{1}{2} k h}{m_1 g}$$

9. Potential Energy and Energy Conservation

19) A bead slides without friction around a loop-the-loop. The bead is released from a height h = 3.50R.

A) What is its speed at point $\boxed{A}$

B) How large is the normal force on it at point $\boxed{A}$ if its mass is 5.00 g?



$$U_i + K_i = U_f + K_f: \qquad mgh + 0 = mg(2R) + \frac{1}{2}mv^2$$

$$g(3.50R) = 2g(R) + \frac{1}{2}v^2$$

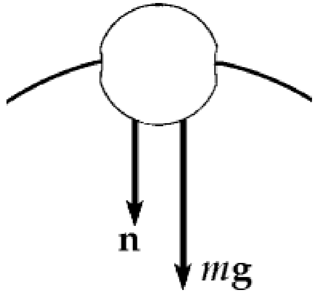
$$\boxed{v = \sqrt{3.00gR}}$$

$$\sum F = m\frac{v^2}{R}: \qquad n + mg = m\frac{v^2}{R}$$

$$n = m\left[\frac{v^2}{R} - g\right] = m\left[\frac{3.00gR}{R} - g\right] = 2.00mg$$

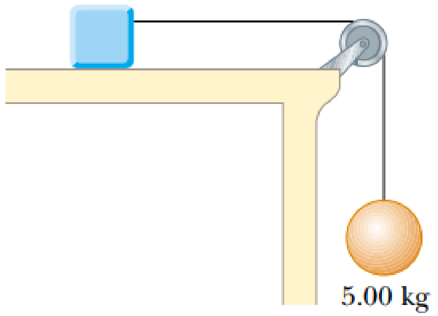
$$n = 2.00\left(5.00\times10^{-3}\text{ kg}\right)\left(9.80\text{ m/s}^2\right)$$

$$= \boxed{0.098\,0\text{ N downward}}$$



9. Potential Energy and Energy Conservation

20) The coefficient of friction between the 3.00-kg block and the surface is 0.400. The system starts from rest. What is the speed of the 5.00-kg ball when it has fallen 1.50 m?



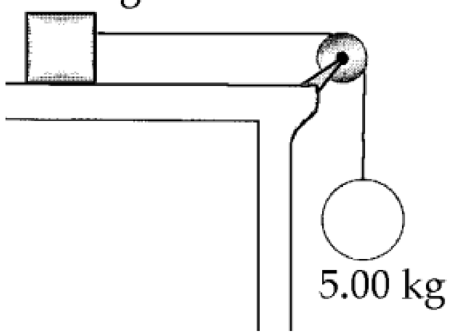
$$U_i + K_i + \Delta E_{\text{mech}} = U_f + K_f: \qquad m_2gh - fh = \frac{1}{2}m_1v^2 + \frac{1}{2}m_2v^2$$

$$f = \mu n = \mu m_1 g$$

$$m_2gh - \mu m_1gh = \frac{1}{2}(m_1 + m_2)v^2$$

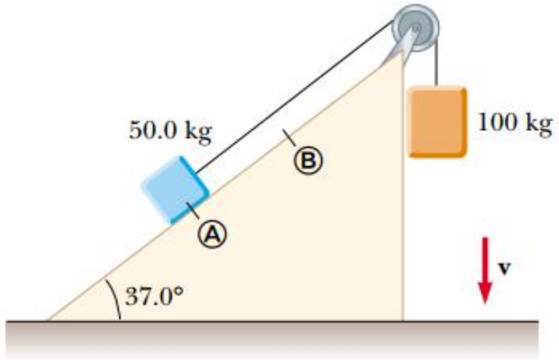
$$v^2 = \frac{2(m_2 - \mu m_1)(hg)}{m_1 + m_2}$$

$$v = \sqrt{\frac{2(9.80\ \text{m/s}^2)(1.50\ \text{m})[5.00\ \text{kg} - 0.400(3.00\ \text{kg})]}{8.00\ \text{kg}}} = \boxed{3.74\ \text{m/s}}$$



9. Potential Energy and Energy Conservation

21) A 50.0-kg block and a 100-kg block are connected by a string as in figure. The pulley is frictionless and of negligible mass. The coefficient of kinetic friction between the 50.0 kg block and incline is 0.250. Determine the change in the kinetic energy of the 50.0-kg block as it moves from (A) to (B), a distance of 20.0 m.



$$\sum F_y = n - mg\cos 37.0° = 0$$

$$\therefore n = mg\cos 37.0° = 400\text{ N}$$

$$f = \mu n = 0.250(400\text{ N}) = 100\text{ N}$$

$$-f\Delta x = \Delta E_{\text{mech}}$$

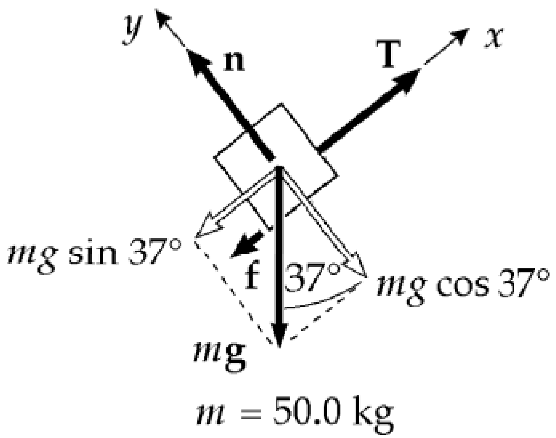
$$(-100)(20.0) = \Delta U_A + \Delta U_B + \Delta K_A + \Delta K_B$$

$$\Delta U_A = m_A g(h_f - h_i) = (50.0)(9.80)(20.0\sin 37.0°) = 5.90\times 10^3$$

$$\Delta U_B = m_B g(h_f - h_i) = (100)(9.80)(-20.0) = -1.96\times 10^4$$

$$\Delta K_A = \frac{1}{2}m_A\left(v_f^2 - v_i^2\right)$$

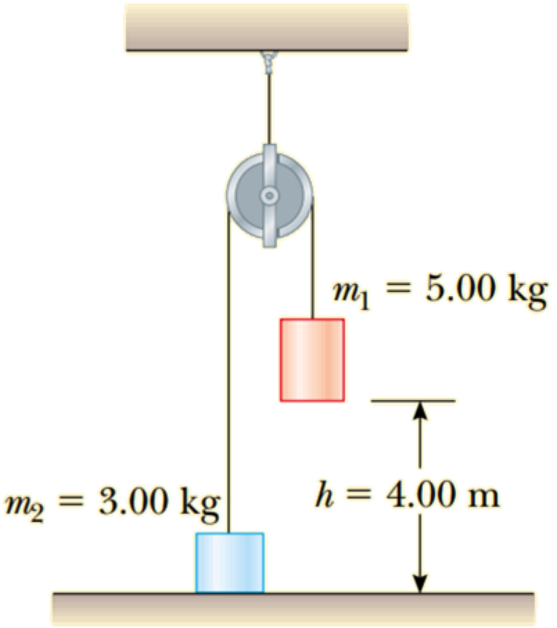
$$\Delta K_B = \frac{1}{2}m_B\left(v_f^2 - v_i^2\right) = \frac{m_B}{m_A}\Delta K_A = 2\Delta K_A$$



Adding and solving, $\Delta K_A = \boxed{3.92\text{ kJ}}$.

9. Potential Energy and Energy Conservation

22) Two objects are connected by a light string passing over a light frictionless pulley as shown in Figure. The object of mass 5.00 kg is released from rest. Using the principle of conservation of energy,
(a) Determine the speed of the 3.00-kg object just as the 5.00-kg object hits the ground.
(b) Find the maximum height to which the 3.00-kg object rises.



(a) $$(5.00\text{ kg})g(4.00\text{ m}) = (3.00\text{ kg})g(4.00\text{ m}) + \frac{1}{2}(5.00 + 3.00)v^2$$

$$v = \sqrt{19.6} = \boxed{4.43\text{ m/s}}$$

$$\frac{1}{2}(3.00)v^2 = mg\,\Delta y = 3.00 g \Delta y$$

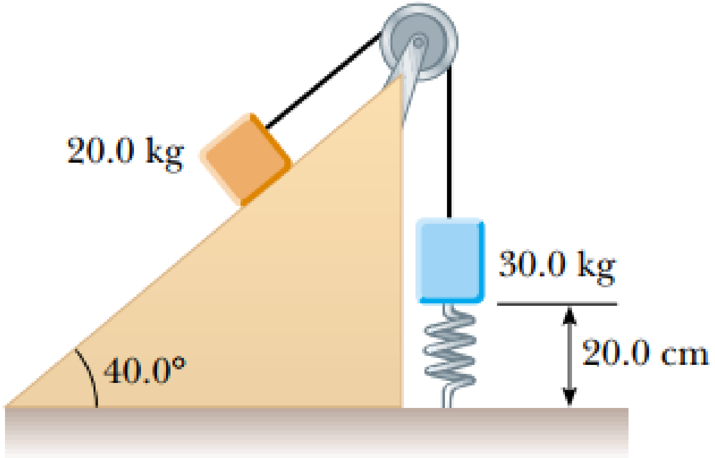
$$\Delta y = 1.00\text{ m}$$

$$y_{\max} = 4.00\text{ m} + \Delta y = \boxed{5.00\text{ m}}$$

9. Potential Energy and Energy Conservation

23) A 20.0-kg block is connected to a 30.0-kg block by a string that passes over a light frictionless pulley. The 30.0-kg block is connected to a spring that has negligible mass and a force constant of 250 N/m.

The spring is unstretched when the system is as shown in the figure, and the incline is frictionless. The 20.0-kg block is pulled 20.0 cm down the incline (so that the 30.0-kg block is 40.0 cm above the floor) and released from rest. Find the speed of each block when the 30.0-kg block is 20.0 cm above the floor (that is, when the spring is unstretched).



$$(K+U)_i = (K+U)_f$$

$$0 + (30.0\ \text{kg})(9.80\ \text{m/s}^2)(0.200\ \text{m}) + \frac{1}{2}(250\ \text{N/m})(0.200\ \text{m})^2$$

$$= \frac{1}{2}(50.0\ \text{kg})v^2 + (20.0\ \text{kg})(9.80\ \text{m/s}^2)(0.200\ \text{m})\sin 40.0°$$

$$58.8\ \text{J} + 5.00\ \text{J} = (25.0\ \text{kg})v^2 + 25.2\ \text{J}$$

$$\boxed{v = 1.24\ \text{m/s}}$$