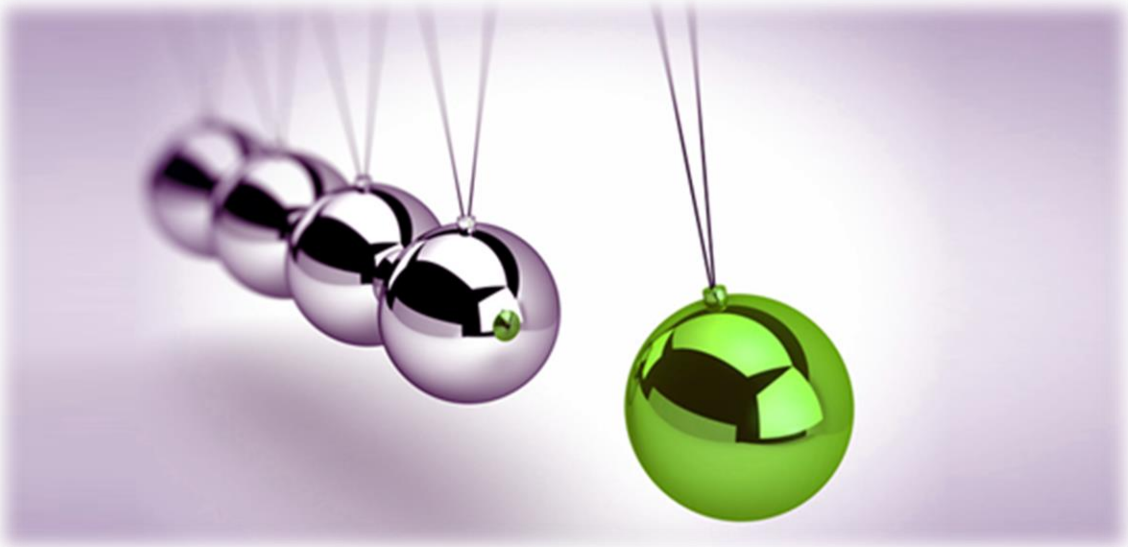
PHYSICS I - MECHANICS
VECTORS



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CHAPTER 2. VECTORS

Learning Objectives

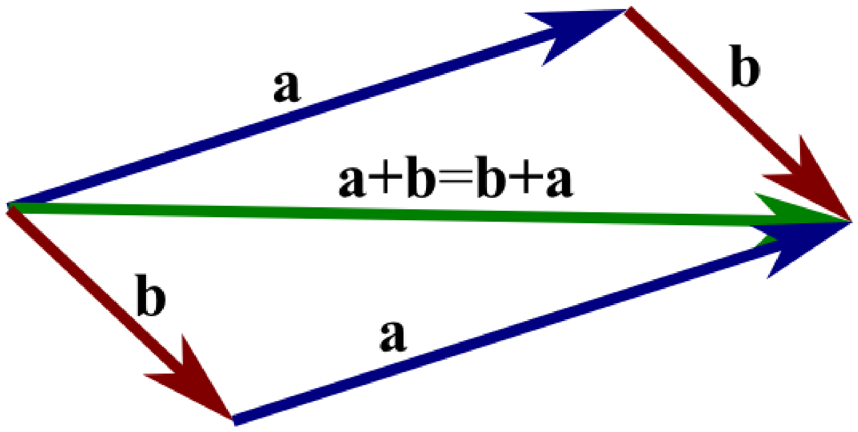
2.1 Coordinate Systems

2.2 Vector and Scalar Quantities

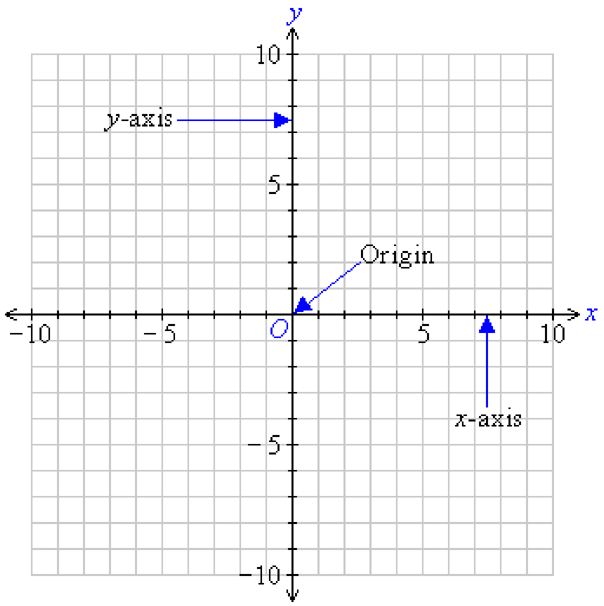
2.3 Some Properties of Vectors

2.4 Components of a Vector and Unit Vectors

2.5 Products of Vectors

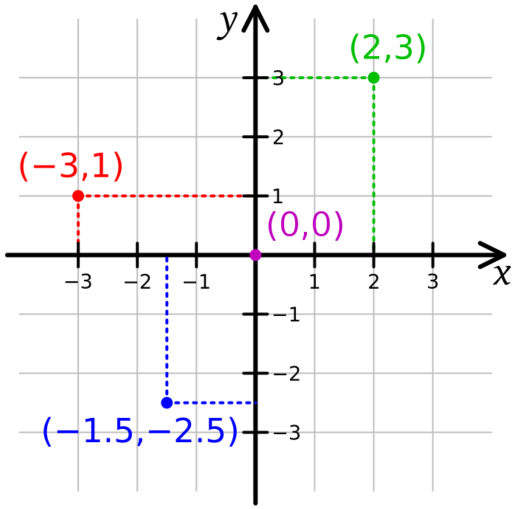


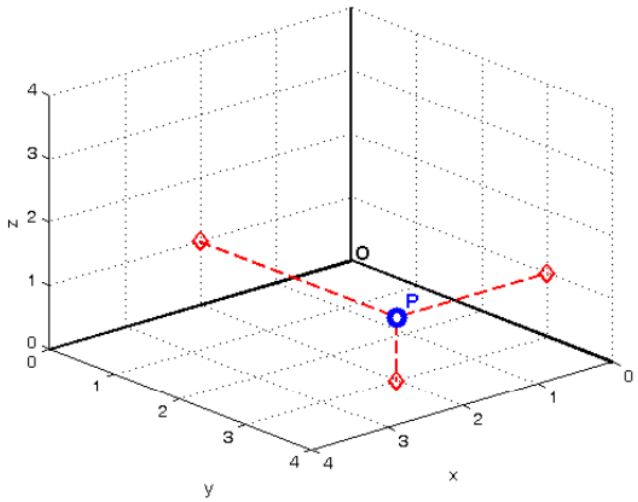
2.1 Coordinate Systems



- Used to describe the position of a point in space
- Coordinate system consists of
  - A fixed reference point called the origin
  - Specific axes with scales and labels
  - Instructions on how to label a point relative to the origin and the axes

A point in the plane is denoted as (x,y) or (x,y,z)





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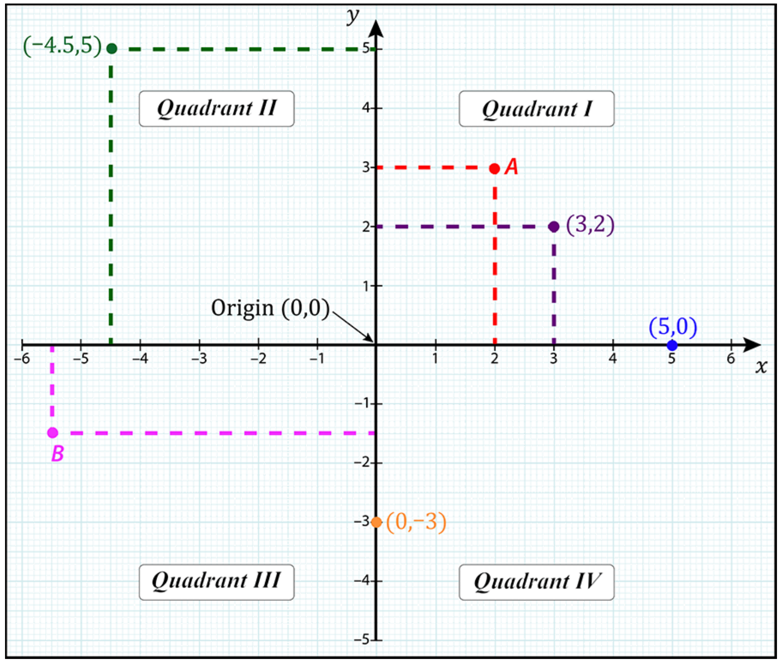
2.1 Coordinate Systems

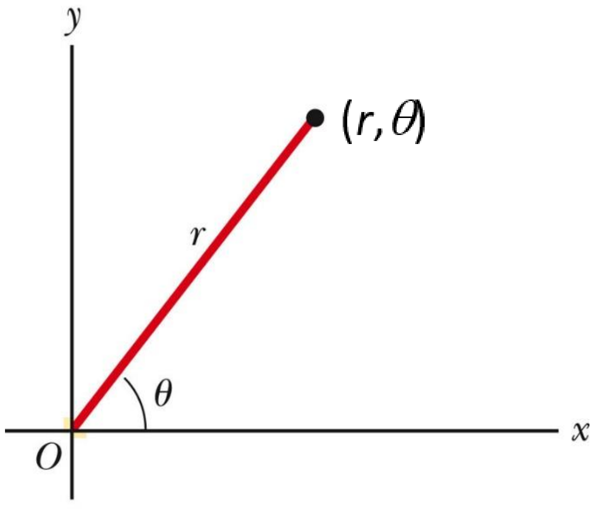
- Cartesian Coordinate System

- Also called rectangular coordinate system
- *x*- and *y*- axes intersect at the origin
- Points are labeled (x,y)



- Polar Coordinate System

- Origin and reference line are noted
- Point is distance r from the origin in the direction of angle θ, ccw (counterclockwise) to reference line
- Points are labeled (r,θ)



(a)

2.1 Coordinate Systems

Polar to Cartesian Coordinates

- Based on forming a right triangle from r and θ
- $x = r \cos \theta$
- $y = r \sin \theta$

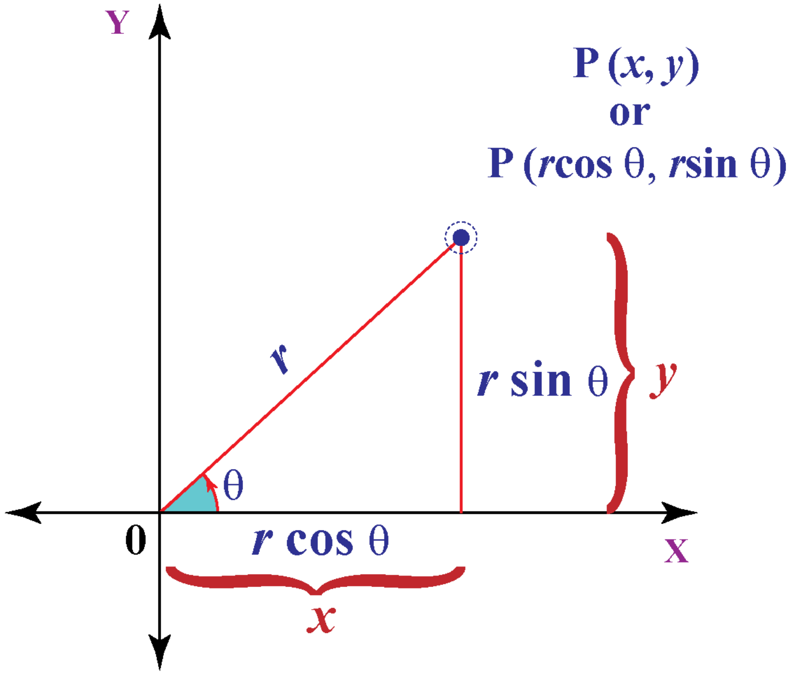
Cartesian to Polar Coordinates

- r is the hypotenuse and θ an angle

$$\tan\theta = \frac{y}{x}$$

$$r = \sqrt{x^2 + y^2}$$

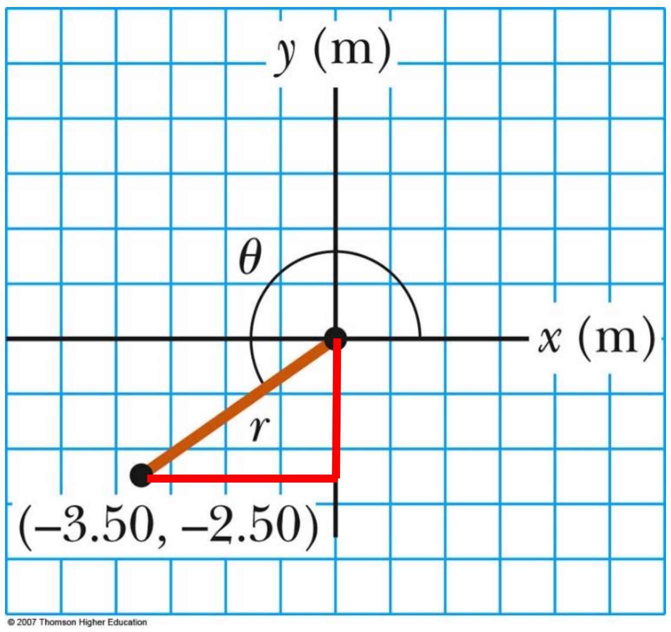
- θ must be ccw from positive x axis for these equations to be valid



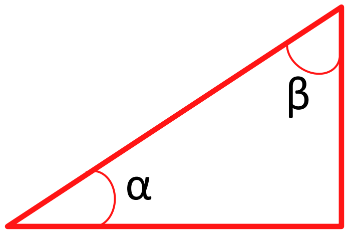
2.1 Coordinate Systems

Example:

The Cartesian coordinates of a point in the *xy* plane are (*x*,*y*) = (-3.50, -2.50) m, as shown in the figure. Find the polar coordinates of this point.



Solution:



$$\theta + \beta = 270°$$
$$\alpha + \beta = 90°$$

$$\theta + \beta = 270°$$
$$-\alpha - \beta = -90°$$

$$\theta - \alpha = 180° \Rightarrow \alpha = \theta - 180°$$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B} = \frac{tan\theta - tan180°}{1 + tan\theta \cdot tan180°} = tan\theta = tan\alpha$$

$$r = \sqrt{x^2 + y^2} = \sqrt{(-3.50\text{ m})^2 + (-2.50\text{ m})^2} = 4.30\text{ m}$$

$$\tan\theta = \frac{y}{x} = \frac{-2.50\text{ m}}{-3.50\text{ m}} = 0.714$$

$$\theta = 216°$$

2.2 Vector and Scalar Quantities

A ***scalar quantity*** is completely specified by a single value with an appropriate unit and has no direction.

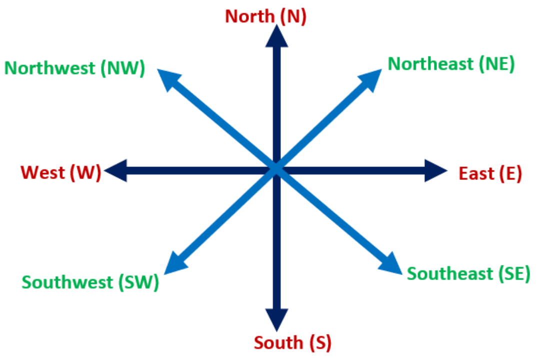
MAGNITUDE ONLY!!

Scaler Example	Magnitude
Speed	80 m/s
Distance	30 meters
Age	18 years

A ***vector quantity*** is completely described by a number and appropriate units plus a direction.

MAGNITUDE and DIRECTION!!

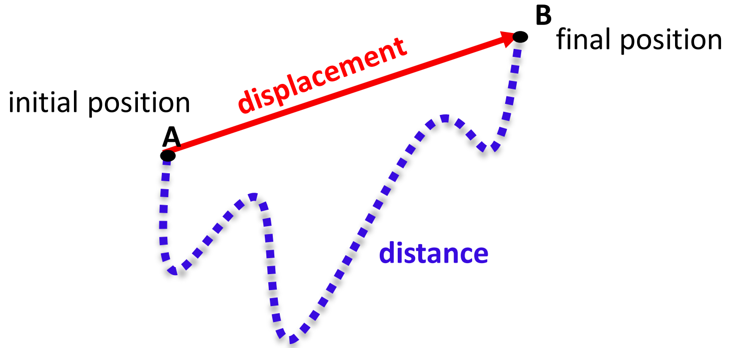
Vector Example	Magnitude and Direction
Velocity	35 m/s, North
Acceleration	10 m/s^2, South
Displacement	20 m, East



2.2 Vector and Scalar Quantities

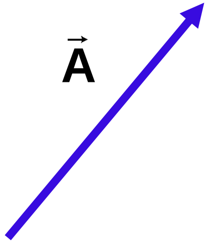
Vector Example

- A particle travels from A to B along the path shown by the dotted blue line
  - This is the ***distance*** traveled and is a scalar
- The ***displacement*** is the solid line from A to B
  - The displacement is independent of the path taken between the two points
  - Displacement is a vector



Vector Notation

- Text uses bold with arrow to denote a vector: $\vec{\boldsymbol{A}}$
- Also used for printing is simple bold print: ***A***
- When dealing with just the magnitude of a vector in print, an italic letter will be used: A or $|\vec{\boldsymbol{A}}|$
  - The magnitude of the vector has physical units
  - The magnitude of a vector is always a positive number
- When handwritten, use an arrow: $\vec{A}$



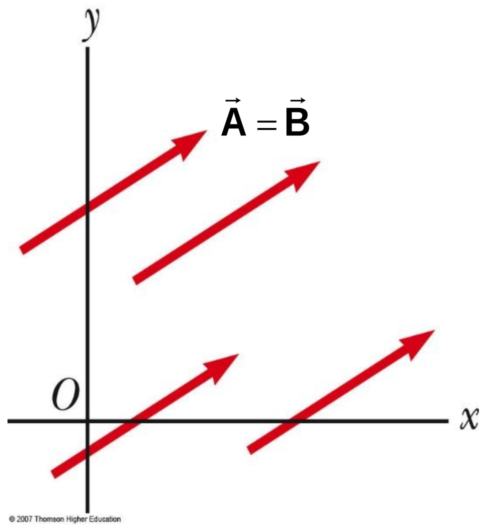
2.3 Some Properties of Vectors

Equality of Two Vectors

- Two vectors are ***equal*** if they have the same magnitude and the same direction

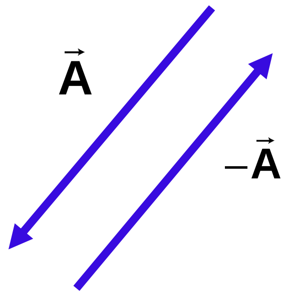
- $\vec{A} = \vec{B}$ if $A = B$ and they point along parallel lines

- All of the vectors shown are equal



Negativity of a vector

- The negative of the vector will have the same magnitude, but point in the opposite direction
  - Represented as - $\vec{A}$
  - $\vec{A} = -\vec{A}$



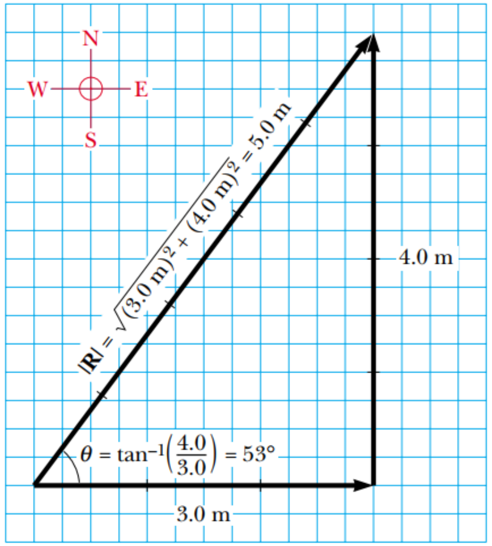
- The negative of a vector is defined as the vector that, when added to the original vector, gives a resultant of zero

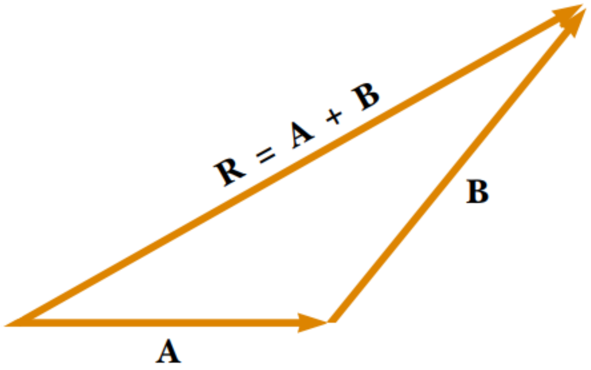
  - $\vec{A}$+(- $\vec{A}$)=0

2.3 Some Properties of Vectors

Adding Vectors Graphically

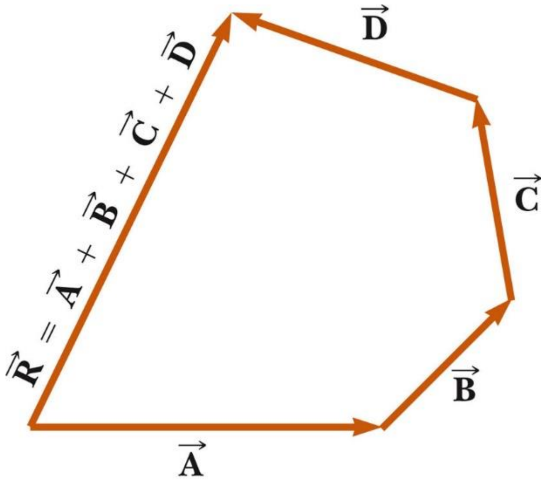
- Choose a scale
- Draw the first vector, $\vec{A}$, with the appropriate length and in the direction specified, with respect to a coordinate system
- draw vector $\vec{B}$ to the same scale with its tail starting from the tip of $\vec{A}$.





To add more than two vectors:

- Continue drawing the vectors “tip-to-tail”
- The resultant is drawn from the origin of $\vec{A}$ to the end of the last vector $\vec{D}$

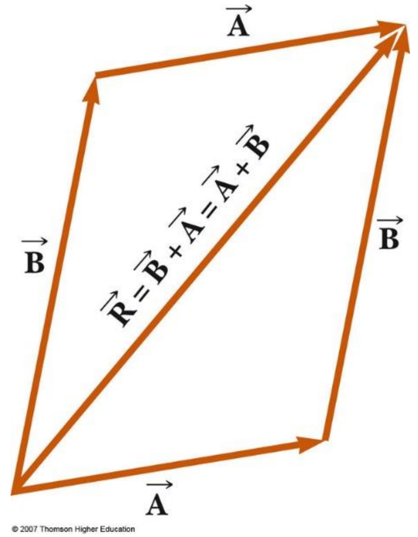


2.3 Some Properties of Vectors

Adding Vectors Rules

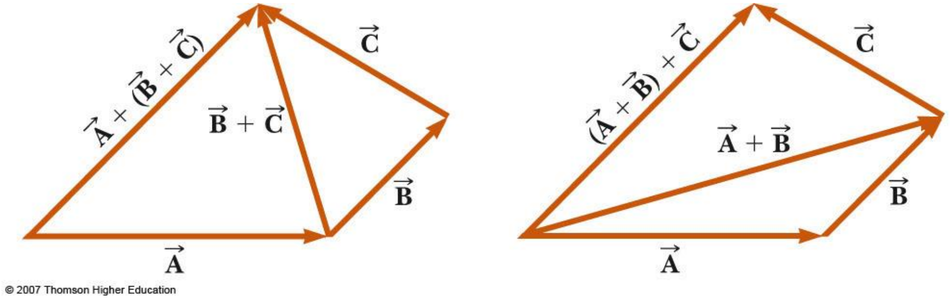
- When two vectors are added, the sum is independent of the order of the addition.
  - This is the ***Commutative Law of Addition***

$$\vec{\mathbf{A}} + \vec{\mathbf{B}} = \vec{\mathbf{B}} + \vec{\mathbf{A}}$$



- When adding three or more vectors, their sum is independent of the way in which the individual vectors are grouped
  - This is called the ***Associative Property of Addition***

$$\vec{\mathbf{A}} + \left(\vec{\mathbf{B}} + \vec{\mathbf{C}}\right) = \left(\vec{\mathbf{A}} + \vec{\mathbf{B}}\right) + \vec{\mathbf{C}}$$

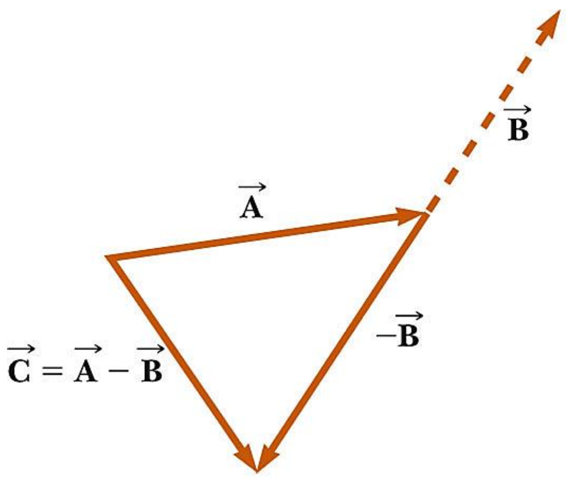


- When adding vectors, all of the vectors must have the same units
- All of the vectors must be of the same type of quantity
  - For example, you cannot add a displacement to a velocity

2.3 Some Properties of Vectors

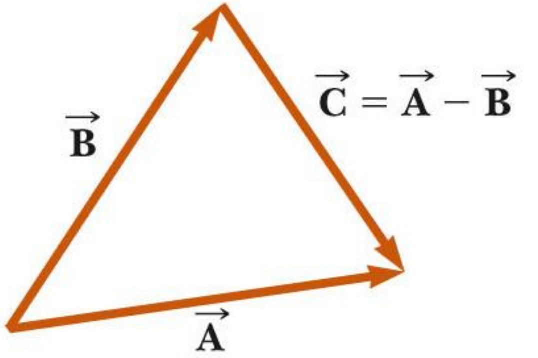
Subtracting Vectors

- If $\vec{A}$- $\vec{B}$, then use $\vec{A}$ +($-\vec{B}$)
- Continue with standard vector addition procedure



(a)

- Another way to look at subtraction is to find the vector that, added to the second vector gives you the first vector
  – As shown, the resultant vector points from the tip of the second to the tip of the first



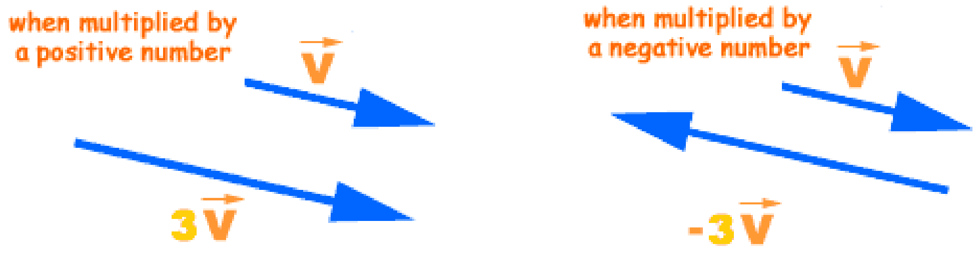
(b)

$$\vec{A} + \left(-\vec{B}\right) = \vec{C}$$

2.3 Some Properties of Vectors

Multiplying or Dividing a Vector by a Scalar

- The result of the multiplication or division of a vector by a scalar is a vector
- The magnitude of the vector is multiplied or divided by the scalar
- If the scalar is positive, the direction of the result is the same as of the original vector
- If the scalar is negative, the direction of the result is opposite that of the original vector



2.4 Components of a Vector and Unit Vectors

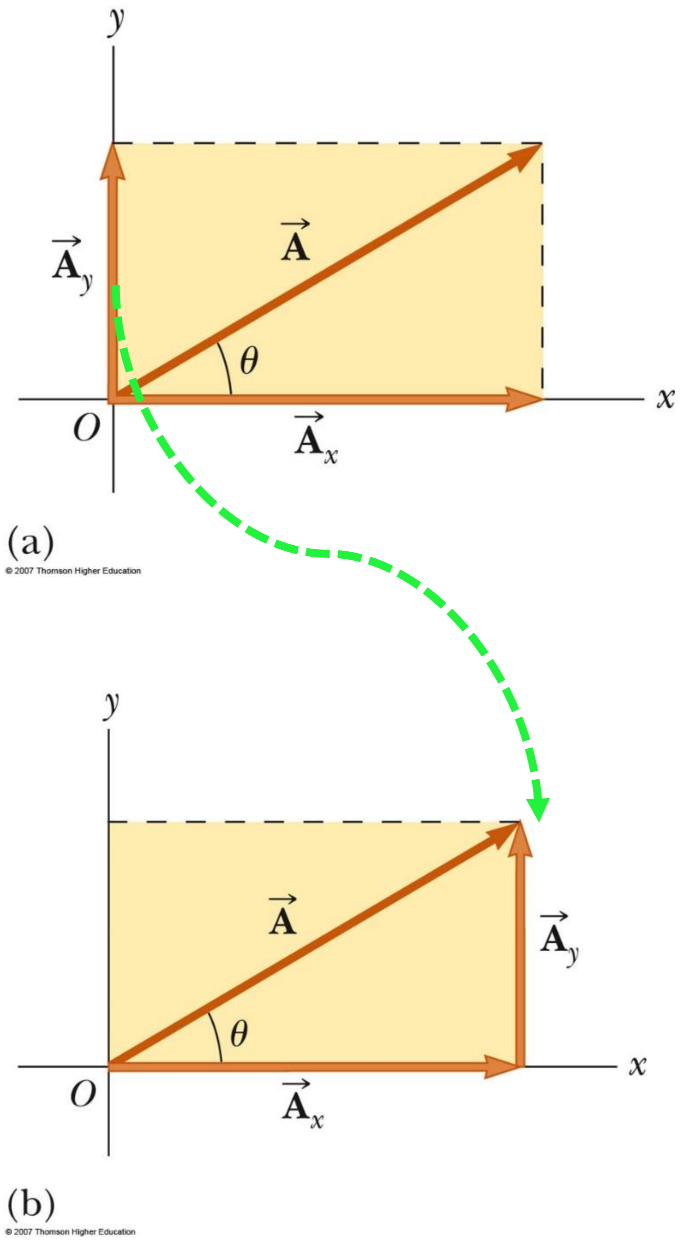
Components of a Vector

- Graphical addition is not recommended when
  - High accuracy is required
  - If you have a three-dimensional problem
- Component method is an alternative method
  - It uses projections of vectors along coordinate axes

- $\vec{A}_x$ and $\vec{A}_y$ are the ***component vectors*** of $\vec{A}$
  - They are vectors and follow all the rules for vectors

- The *y*-component is moved to the end of the *x*-component
  - These three vectors form a right triangle

- $\vec{A} = \vec{A}_x + \vec{A}_y$



2.4 Components of a Vector and Unit Vectors

Components of a Vector

- The x-component of a vector is the projection along the x-axis

$$A_x = A\cos\theta$$

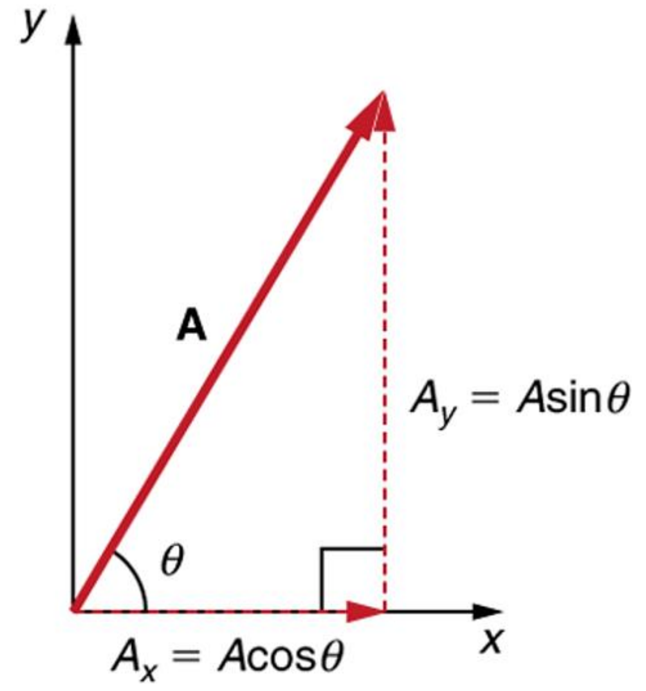
- The y-component of a vector is the projection along the y-axis

$$A_y = A\sin\theta$$

- This assumes the angle θ is measured with respect to the x-axis
  - If not, do not use these equations, use the sides of the triangle directly

- The components are the legs of the right triangle whose hypotenuse is the length of *A*

$$A = \sqrt{A_x^2 + A_y^2} \quad \text{and} \quad \theta = \tan^{-1}\frac{A_y}{A_x}$$

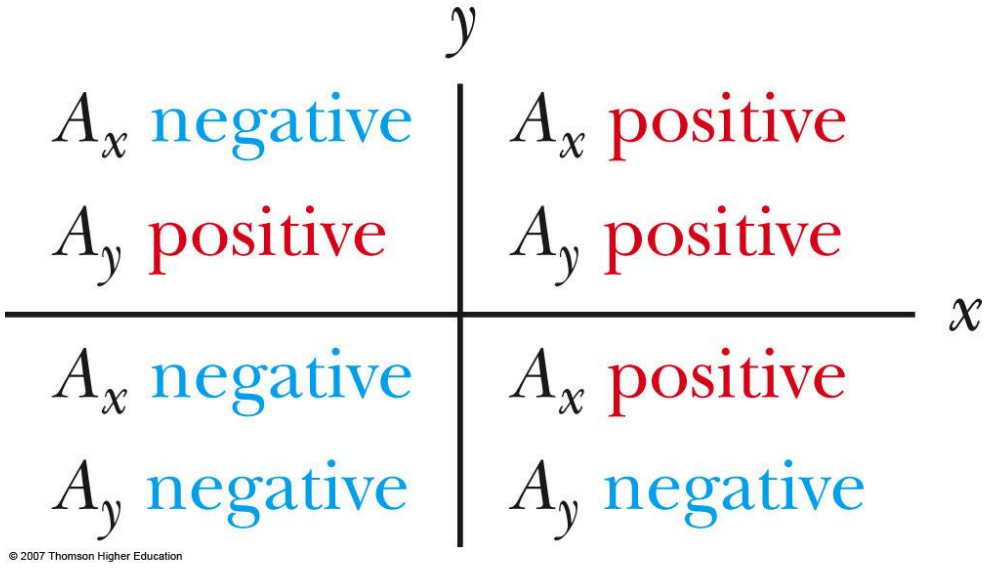


2.4 Components of a Vector and Unit Vectors

Components of a Vector

- The components can be positive or negative and will have the same units as the original vector

- The signs of the components will depend on the angle



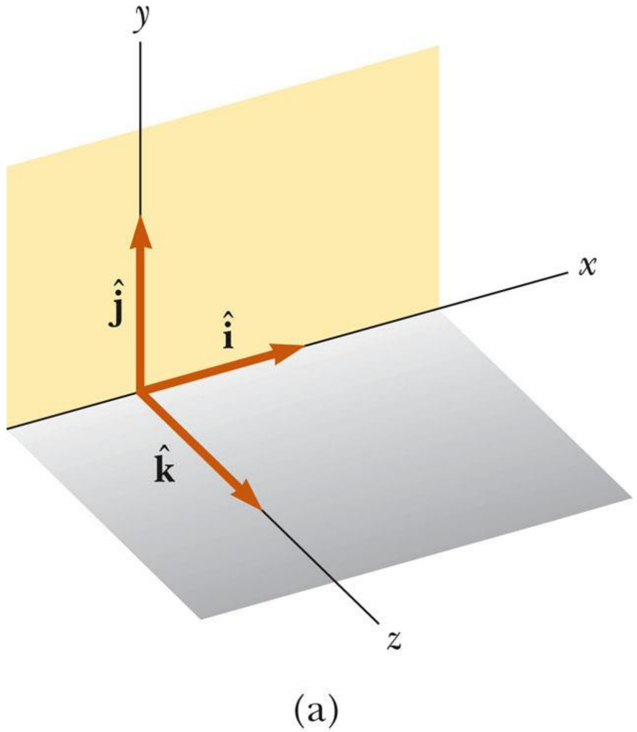
2.4 Components of a Vector and Unit Vectors

Unit Vectors

- Vector quantities often are expressed in terms of unit vectors.
- A ***unit vector*** is a dimensionless vector with a magnitude of exactly 1.
- Unit vectors are used to specify a direction and have no other physical significance
- The symbols

$$\hat{\text{i}}, \hat{\text{j}}, \text{and } \hat{\text{k}}$$

represent unit vectors



- They form a set of mutually perpendicular vectors in a right-handed coordinate system
- Remember,

$$\left|\hat{\mathbf{i}}\right| = \left|\hat{\mathbf{j}}\right| = \left|\hat{\mathbf{k}}\right| = 1$$

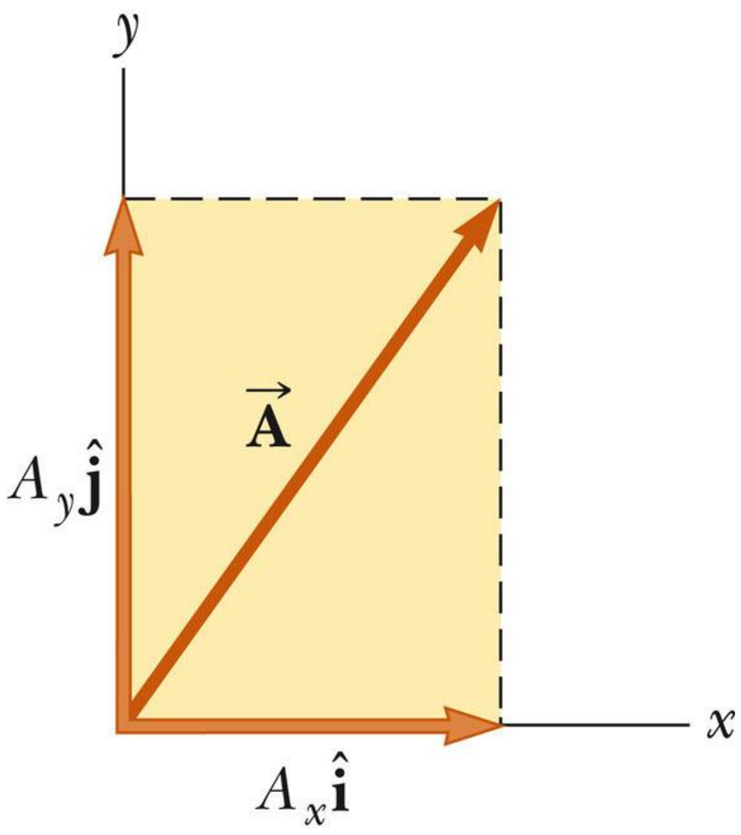
2.4 Components of a Vector and Unit Vectors

Unit Vectors Vector Notation

- $\vec{\boldsymbol{A}}_{\boldsymbol{x}}$ is the same as $A_x\hat{\boldsymbol{\imath}}$ and $\vec{\boldsymbol{A}}_{\boldsymbol{y}}$ is the same as $A_y\hat{\boldsymbol{\jmath}}$ etc.

- The complete vector can be expressed as

$$\vec{\boldsymbol{A}} = A_x\hat{\boldsymbol{\imath}} + A_y\hat{\boldsymbol{\jmath}}$$



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2.4 Components of a Vector and Unit Vectors

Adding Vectors with Unit Vectors

- Note the relationships among the components of the resultant and the components of the original vectors

- Using $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$
- Then

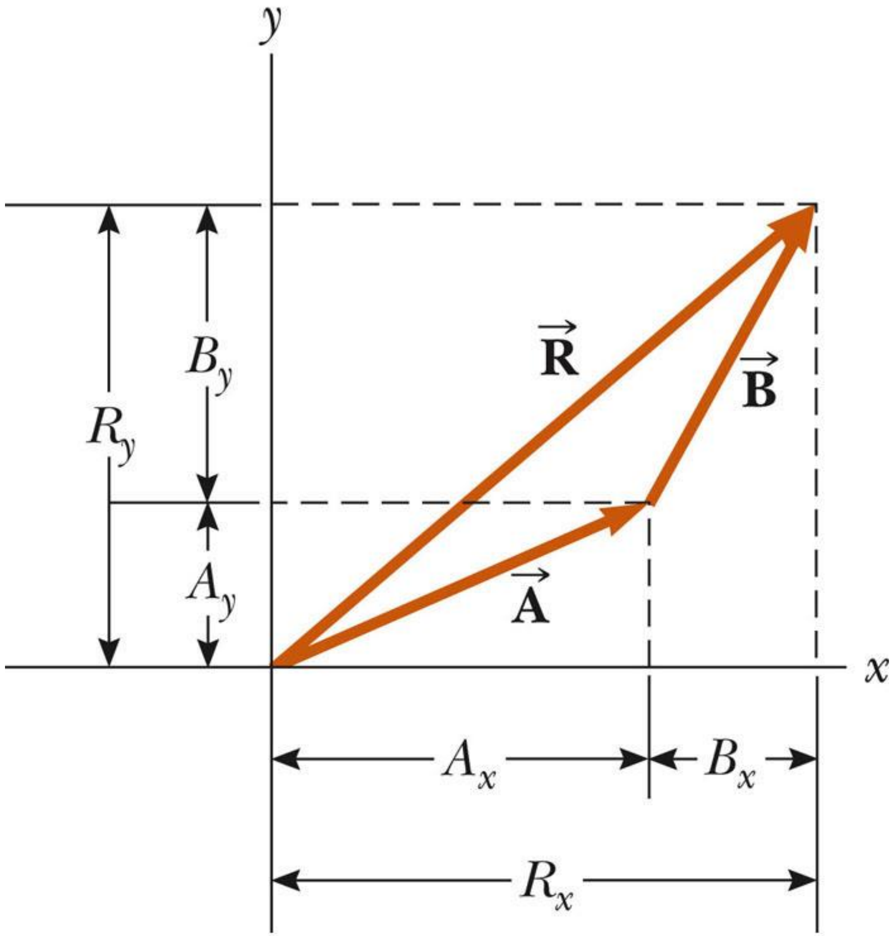
$$\vec{\mathbf{R}} = \left(A_x \hat{\mathbf{i}} + A_y \hat{\mathbf{j}}\right) + \left(B_x \hat{\mathbf{i}} + B_y \hat{\mathbf{j}}\right)$$

$$\vec{\mathbf{R}} = \left(A_x + B_x\right)\hat{\mathbf{i}} + \left(A_y + B_y\right)\hat{\mathbf{j}}$$

$$\vec{\mathbf{R}} = R_x \hat{\mathbf{i}} + R_y \hat{\mathbf{j}}$$

- and so $R_x = A_x + B_x$ and $R_y = A_y + B_y$

$$R = \sqrt{R_x^2 + R_y^2} \qquad \theta = \tan^{-1}\frac{R_y}{R_x}$$



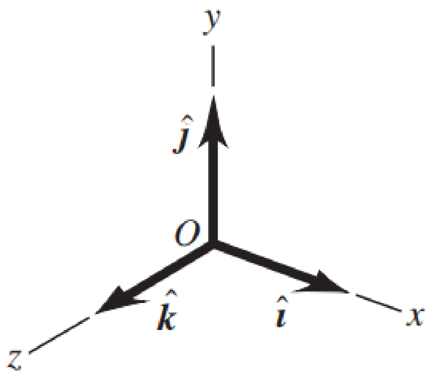
2.4 Components of a Vector and Unit Vectors

Adding Vectors with Unit Vectors

If the vectors do not all lie in the xy-plane, then we need a third component. We introduce a third unit vector $\hat{k}$ that points in the direction of the positive z-axis

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$$
$$\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}$$

$$\vec{R} = (A_x + B_x)\hat{\imath} + (A_y + B_y)\hat{\jmath} + (A_z + B_z)\hat{k}$$
$$= R_x\hat{\imath} + R_y\hat{\jmath} + R_z\hat{k}$$



2.5 Products of Vectors

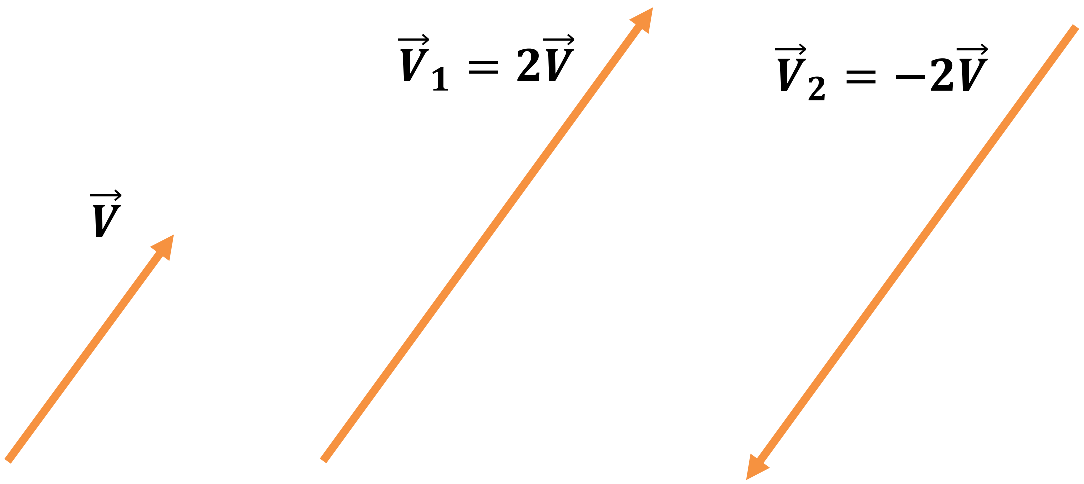
Multiplication by a scalar

A vector $\vec{V}$ can be multiplied by a scalar c

$$\vec{V}_1 = \mathrm{c}\,\vec{V}$$

$\vec{V}_1$= vector with magnitude c**V** the same direction as **V**. $\vec{V}_1 = c\vec{V}$

If c is negative, the result is in the opposite direction $\vec{V}_2 = -c\vec{V}$



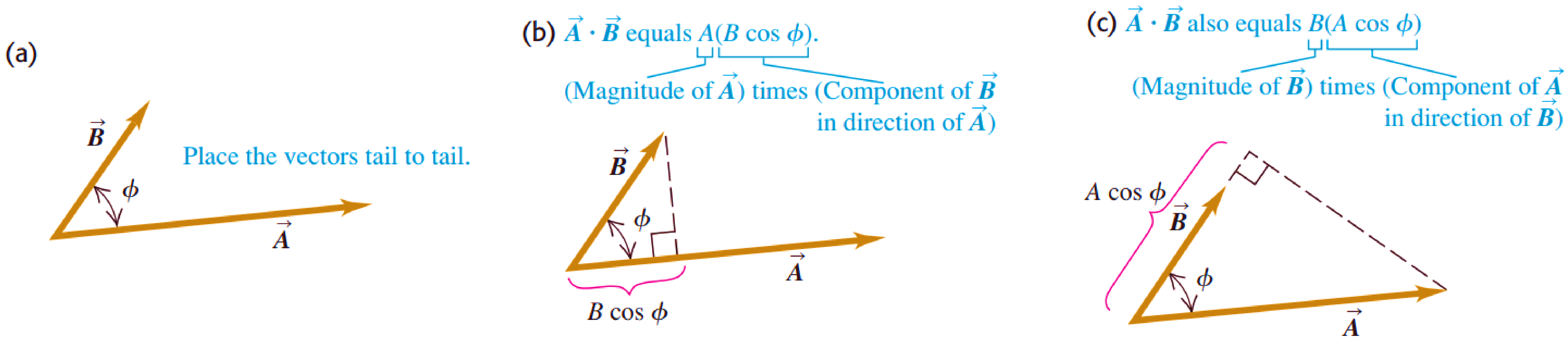
2.5 Products of Vectors

i) Scalar Product

The **scalar product** of two vectors $\vec{\boldsymbol{A}}$ and $\vec{\boldsymbol{B}}$ is denoted by $\vec{\boldsymbol{A}} \cdot \vec{\boldsymbol{B}}$ .Because of this notation, the scalar product is also called the **dot product.**

Although $\vec{\boldsymbol{A}}$ and $\vec{\boldsymbol{B}}$ are vectors, the quantity $\vec{\boldsymbol{A}} \cdot \vec{\boldsymbol{B}}$ is a scalar.

To define a scalar product $\vec{\boldsymbol{A}} \cdot \vec{\boldsymbol{B}}$ we draw the two vectors $\vec{\boldsymbol{A}}$ and $\vec{\boldsymbol{B}}$ with their tails at the sama point.



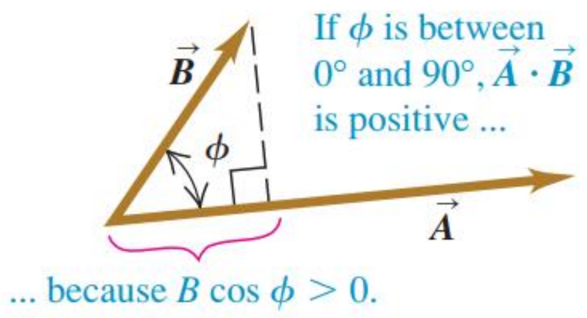
$$\boxed{\vec{A} \cdot \vec{B} = AB\cos\phi = |\vec{A}||\vec{B}|\cos\phi \quad \text{(definition of the scalar (dot) product)}}$$

2.5 Products of Vectors

i) Scalar Product

The scalar product is a scalar quantity, not a vector, and it may be positive, negative, or zero.

(a)



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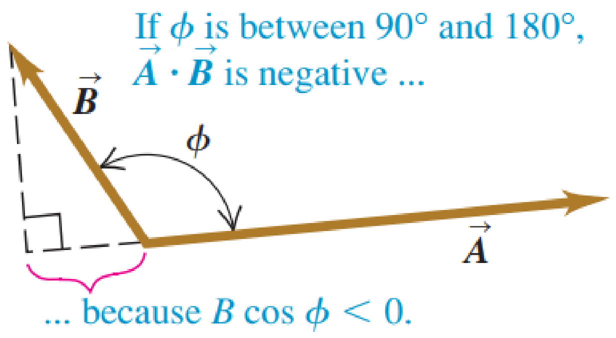

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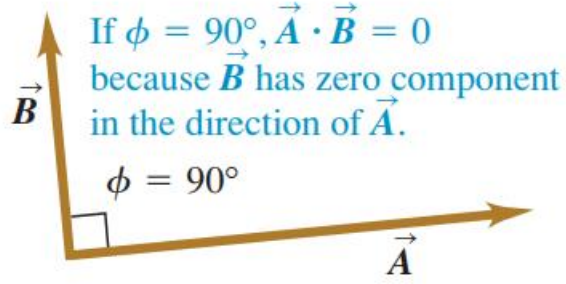


When ϕ is between 0° and 90°, cos ϕ >0 and the scalar product is positive

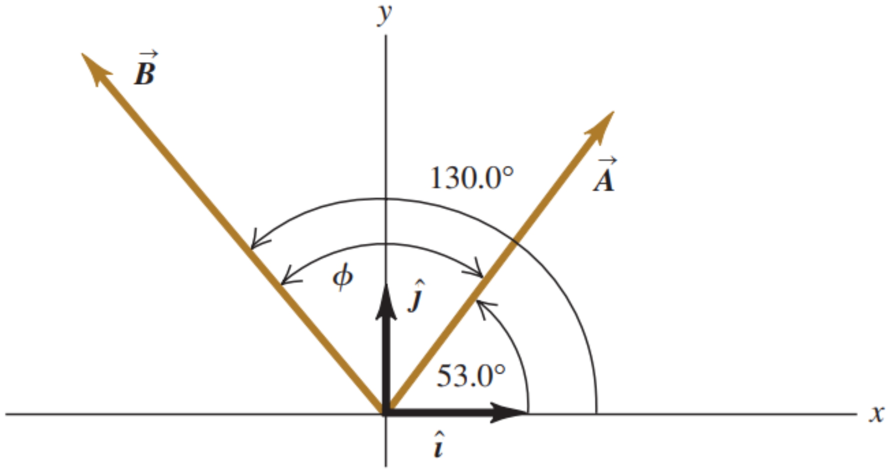
When ϕ is between 90° and 180°, cos ϕ < 0 and the scalar product is negative

When ϕ= 90° cos ϕ = 0 and the scalar product is zero

2.5 Products of Vectors

i) Scalar Product

Example



Find the scalar product of the two vectors in Figure.
The magnitudes of the vectors are A=4.00 and B=5.00.

The angle between the two vectors is $\phi = 130.0^{o} - 53.0^{o} = 77.0^{o}$

$$\vec{A} \cdot \vec{B} = AB\cos\phi = (4.00)(5.00)\cos 77.0^\circ = 4.50$$

$$A_x = (4.00)\cos 53.0^\circ = 2.407$$
$$A_y = (4.00)\sin 53.0^\circ = 3.195$$
$$B_x = (5.00)\cos 130.0^\circ = -3.214$$
$$B_y = (5.00)\sin 130.0^\circ = 3.830$$

$$\begin{aligned}\vec{A} \cdot \vec{B} &= A_xB_x + A_yB_y + A_zB_z \\ &= (2.407)(-3.214) + (3.195)(3.830) + (0)(0) = 4.50\end{aligned}$$

2.5 Products of Vectors

ii) Vector Product

The **vector product** of two vectors $\vec{A}$ and $\vec{B}$ also called the **cross product,** is denoted by $\vec{A} \times \vec{B}$.

We define the vector product to be a vector quantity with a direction perpendicular to a plane and a magnitude equal to $A\,B\,\sin\phi$.

$$\boxed{\vec{C} = \vec{A} \times \vec{B}}$$

magnitude of the vector (cross) product $\vec{A}$ and $\vec{B}$

$$\boxed{C = AB\sin\phi}$$

When $\vec{A}$ and $\vec{B}$ are parallel or antiparallel, $\phi = 0$ or $180°$ and $C=0$. That is, *the vector product of two parallel or antiparallel vectors is always zero.* In particular, *the vector product of any vector with itself is zero.*

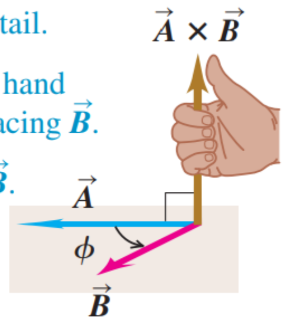
The magnitude of $\vec{A} \times \vec{B}$, which can be written as $|\vec{A} \times \vec{B}|$, is maximum when $\vec{A}$ and $\vec{B}$ are perpendicular to each other.

2.5 Products of Vectors

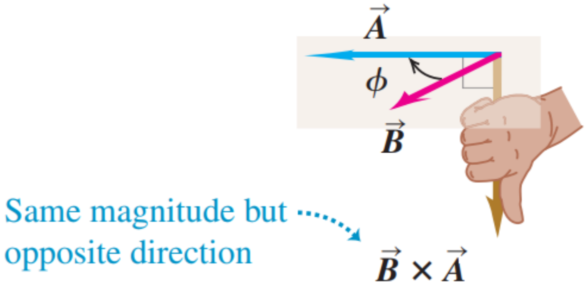
ii) Vector Product

(a) Using the right-hand rule to find the direction of $\vec{A} \times \vec{B}$

1. Place $\vec{A}$ and $\vec{B}$ tail to tail.
2. Point fingers of right hand along $\vec{A}$, with palm facing $\vec{B}$.
3. Curl fingers toward $\vec{B}$.
4. Thumb points in direction of $\vec{A} \times \vec{B}$.



(b) $\vec{B} \times \vec{A} = -\vec{A} \times \vec{B}$ (the vector product is anticommutative)



$$\vec{A}\,X\,\vec{B} = -\vec{B}\,X\,\vec{A}$$

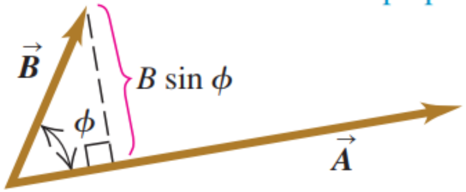
$$\vec{B}\,X\,\vec{A} = -\vec{A}\,X\,\vec{B}$$

A geometrical interpretation of the magnitude of the vector product:

(a)

(Magnitude of $\vec{A} \times \vec{B}$) equals $A(B \sin\phi)$.

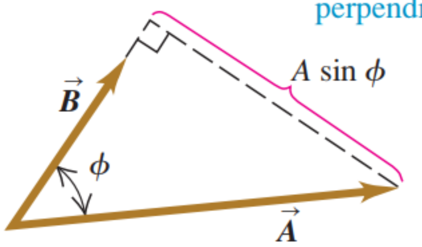
(Magnitude of $\vec{A}$) times (Component of $\vec{B}$ perpendicular to $\vec{A}$)



(b)

(Magnitude of $\vec{A} \times \vec{B}$) also equals $B(A \sin\phi)$.

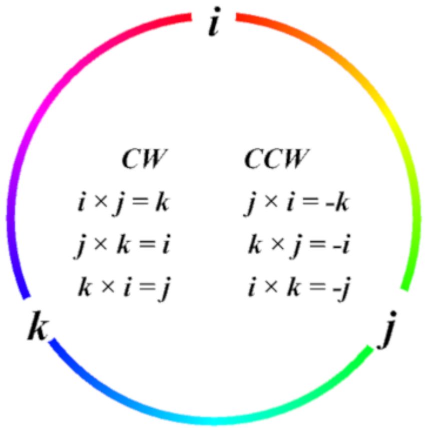
(Magnitude of $\vec{B}$) times (Component of $\vec{A}$ perpendicular to $\vec{B}$)



2.5 Products of Vectors

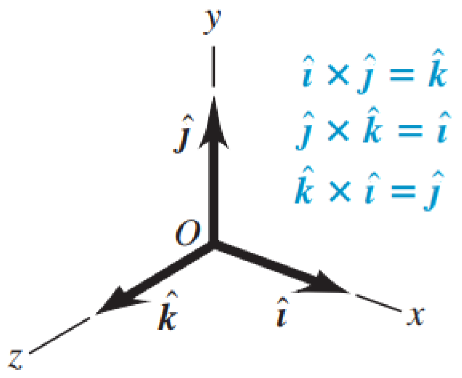
ii) Vector Product - Calculating the Vector Product Using Components

The vector product of any vector with itself is zero, so



$$\hat{\imath} \times \hat{\imath} = \hat{\jmath} \times \hat{\jmath} = \hat{k} \times \hat{k} = 0$$

$$\hat{\imath} \times \hat{\jmath} = -\hat{\jmath} \times \hat{\imath} = \hat{k}$$
$$\hat{\jmath} \times \hat{k} = -\hat{k} \times \hat{\jmath} = \hat{\imath}$$
$$\hat{k} \times \hat{\imath} = -\hat{\imath} \times \hat{k} = \hat{\jmath}$$



Next we express $\vec{A}$ and $\vec{B}$ in terms of their components and the corresponding unit vectors, and we expand the expression for the vector product:

$$\begin{aligned}
\vec{A} \times \vec{B} &= (A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}) \times (B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}) \\
&= A_x\hat{\imath} \times B_x\hat{\imath} + A_x\hat{\imath} \times B_y\hat{\jmath} + A_x\hat{\imath} \times B_z\hat{k} \\
&\quad + A_y\hat{\jmath} \times B_x\hat{\imath} + A_y\hat{\jmath} \times B_y\hat{\jmath} + A_y\hat{\jmath} \times B_z\hat{k} \\
&\quad + A_z\hat{k} \times B_x\hat{\imath} + A_z\hat{k} \times B_y\hat{\jmath} + A_z\hat{k} \times B_z\hat{k}
\end{aligned}$$

2.5 Products of Vectors

ii) Vector Product - Calculating the Vector Product Using Components (cont.)

Evaluating these by using the multiplication table for the unit vectors and then grouping the terms, we get

$$\vec{A} \times \vec{B} = (A_yB_z - A_zB_y)\hat{\imath} + (A_zB_x - A_xB_z)\hat{\jmath} + (A_xB_y - A_yB_x)\hat{k}$$

$$\vec{C} = \vec{A} \text{ X } \vec{B}$$

$$C_x = A_yB_z - A_zB_y \qquad C_y = A_zB_x - A_xB_z \qquad C_z = A_xB_y - A_yB_x$$

$$(\text{components of } \vec{C} = \vec{A} \times \vec{B})$$

The vector product can also be expressed in determinant form as

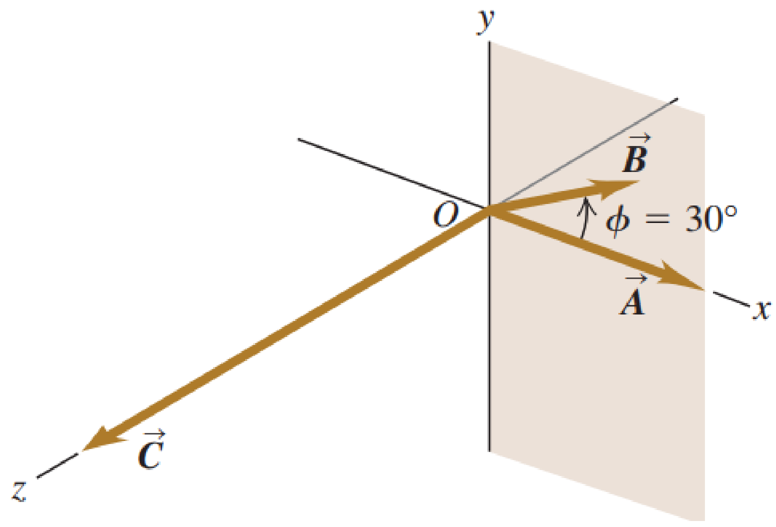
$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

2.5 Products of Vectors

ii) Vector Product - Calculating the Vector Product Using Components

Example

Vector $\vec{A}$ has magnitude 6 units and is in the direction of the $+x$ axis. Vector $\vec{B}$ has magnitude 4 units and lies in the xy-plane, making an angle of 30° with the $+x$ axis. Find the vector product $\vec{C} = \vec{A} \times \vec{B}$.



The magnitude of the vector product is $A\,B\,\sin\phi = (6)(4)(\sin 30°) = 12$

$$\vec{C} = \vec{A} \times \vec{B} = 12\hat{k}$$

$$A_x = 6 \qquad A_y = 0 \qquad A_z = 0$$

$$B_x = 4\cos 30° = 2\sqrt{3} \qquad B_y = 4\sin 30° = 2 \qquad B_z = 0$$

$$C_x = (0)(0) - (0)(2) = 0$$

$$C_y = (0)(2\sqrt{3}) - (6)(0) = 0$$

$$C_z = (6)(2) - (0)(2\sqrt{3}) = 12$$

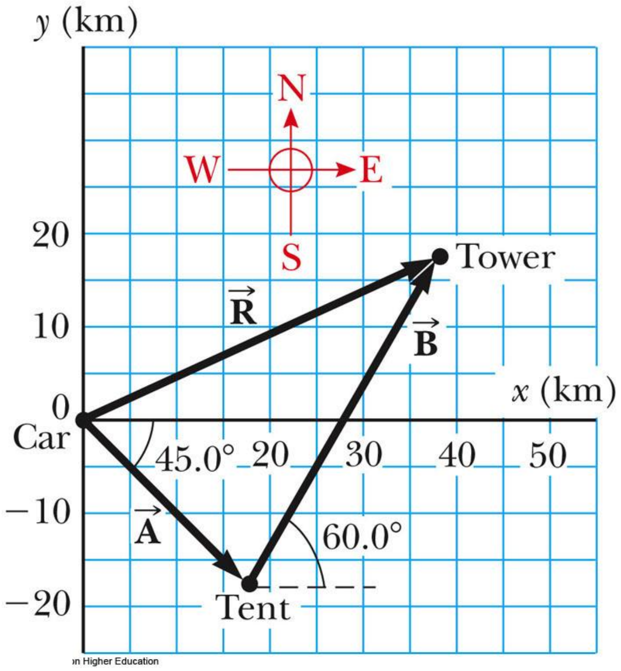
$$\vec{C} = 12\hat{k}$$

Homework-I

A hiker begins a trip by first walking 25.0 km southeast from her car. She stops and sets up her tent for the night. On the second day, she walks 40.0 km in a direction 60.0° north of east, at which point she discovers a forest ranger's tower.

(A) Determine the components of the hiker's displacement for each day.

(B) Determine the components of the hiker's resultant displacement $\vec{R}$ for the trip. Find an expression for $\vec{R}$ in terms of unit vectors.



$$A_x = A\cos(-45.0°) = (25.0\text{ km})(0.707) = \boxed{17.7\text{ km}}$$

$$A_y = A\sin(-45.0°) = (25.0\text{ km})(-0.707) = \boxed{-17.7\text{ km}}$$

$$B_x = B\cos 60.0° = (40.0\text{ km})(0.500) = \boxed{20.0\text{ km}}$$

$$B_y = B\sin 60.0° = (40.0\text{ km})(0.866) = \boxed{34.6\text{ km}}$$

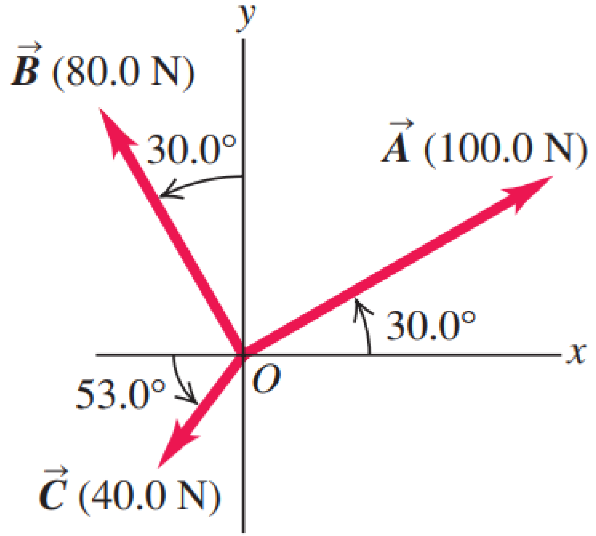
$$R_x = A_x + B_x = 17.7\text{ km} + 20.0\text{ km} = \boxed{37.7\text{ km}}$$

$$R_y = A_y + B_y = -17.7\text{ km} + 34.6\text{ km} = \boxed{16.9\text{ km}}$$

In unit–vector form, we can write the total displacement as

$$\mathbf{R} = \boxed{(37.7\hat{\mathbf{i}} + 16.9\hat{\mathbf{j}})\text{ km}}$$

Homework-II



Three horizontal ropes pull on a large stone stuck in the ground, producing the vector forces $\vec{A}$, $\vec{B}$ and $\vec{C}$ shown in figure. Find the magnitude and direction of a fourth force on the stone that will make the vector sum of the four forces zero.

IDENTIFY: Let $\vec{D}$ be the fourth force. Find $\vec{D}$ such that $\vec{A}+\vec{B}+\vec{C}+\vec{D}=0$, so $\vec{D}=-\left(\vec{A}+\vec{B}+\vec{C}\right)$.

SET UP: Use components and solve for the components D_x and D_y of $\vec{D}$.

EXECUTE: $A_x=+A\cos 30.0°=+86.6\text{N}$, $A_y=+A\cos 30.0°=+50.00\text{N}$.

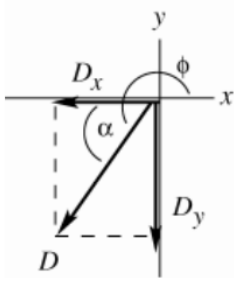
$B_x=-B\sin 30.0°=-40.00\text{N}$, $B_y=+B\cos 30.0°=+69.28\text{N}$.

$C_x=+C\cos 53.0°=-24.07\text{N}$, $C_y=-C\sin 53.0°=-31.90\text{N}$.

Then $D_x=-22.53\text{ N}$, $D_y=-87.34\text{N}$ and $D=\sqrt{D_x^2+D_y^2}=90.2\text{ N}$. $\tan\alpha=\left|D_y/D_x\right|=87.34/22.53$. $\alpha=75.54°$.

$\phi=180°+\alpha=256°$, counterclockwise from the $+x$-axis.

EVALUATE: As shown in Figure 1.68, since D_x and D_y are both negative, $\vec{D}$ must lie in the third quadrant.



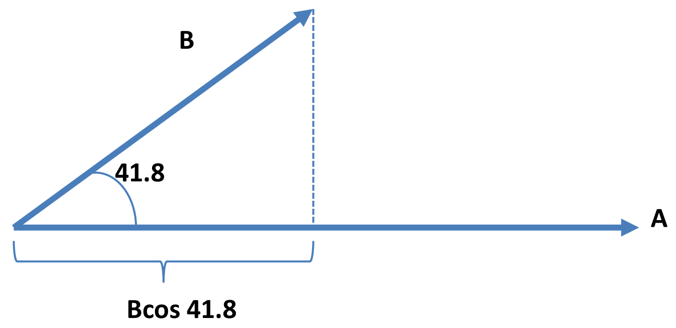
Homework-III

Vector $\vec{A}$ has magnitude 6.00 m and Vector $\vec{B}$ has magnitude 3.00 m. The vector product between these two vectors has Magnitude 12.0 m^2 . What are the two possible values for the scalar product of these two vectors? For each value of $\vec{A}\cdot \vec{B}$, draw a sketch that shows $\vec{A}$ and $\vec{B}$ and explain why the vector products in the two sketches are the same but the scalar products differ.

$$A \times B = ||A|||B||sin\theta.$$

Let's find the angle θ between the two vectors:

$$\theta = sin^{-1}(\frac{A \times B}{||A|||B||}) = sin^{-1}(\frac{12\ m^2}{6\ m \cdot 3\ m}) = 41.8^\circ.$$



If our vectors located in the second quadrant, for example, the angle between them will be:

$$\theta = 180^\circ - 41.8^\circ = 138.2^\circ.$$

Then, we can find the two possible values for the scalar product of these two vectors:

$$A \cdot B = ||6\ m|||3\ m||cos41.8^\circ = 13.42\ m^2,$$
$$A \cdot B = ||6\ m|||3\ m||cos138.2^\circ = -13.42\ m^2.$$