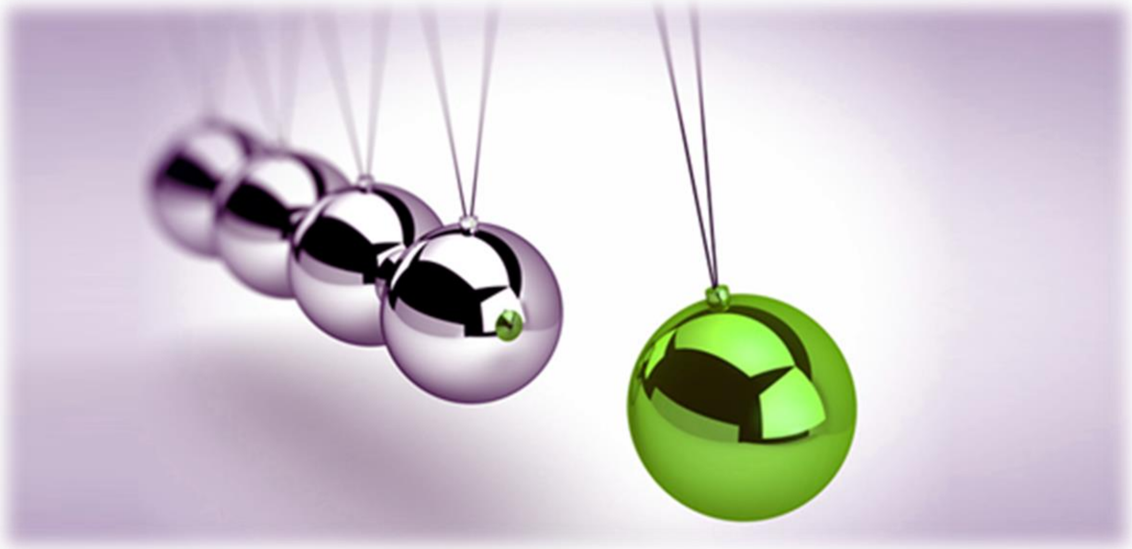
PHYSICS I - MECHANICS
TWO DIMESIONAL MOTION



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CHAPTER 4. TWO DIMESIONAL MOTION

Learning Objectives

4.1 The Position, Velocity, and Acceleration Vectors

4.2 Two-Dimensional Motion with Constant Acceleration

4.3 Projectile Motion

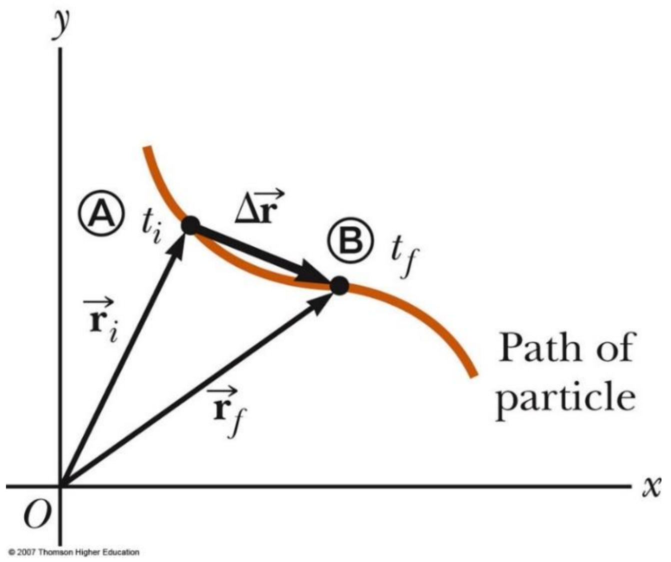
4.4 Motion in a Circle

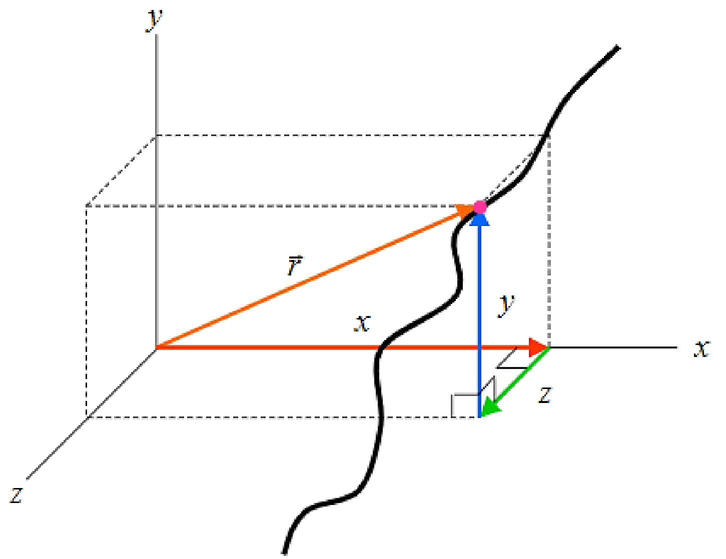
4.5 Relative Velocity and Relative Acceleration

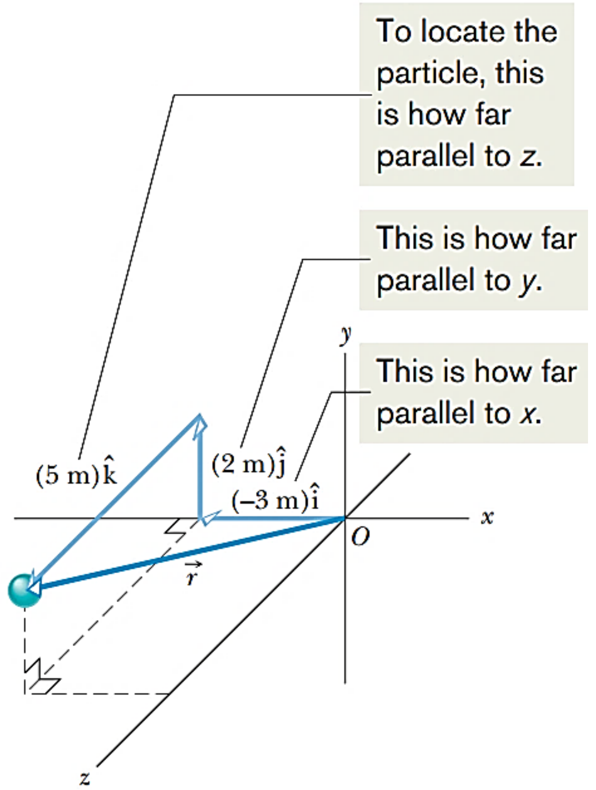
4.1 The Position, Velocity, and Acceleration Vectors

Position and Displacement

- The **position** of an object is described by its **position vector *r***







$$\boldsymbol{r} = x\hat{\imath} + y\hat{\jmath}$$

$$\boldsymbol{r} = x\hat{\boldsymbol{\imath}} + y\hat{\boldsymbol{\jmath}} + z\hat{\boldsymbol{k}}$$

$$\boldsymbol{r} = (-3m)\hat{\boldsymbol{\imath}} + (2m)\hat{\boldsymbol{\jmath}} + (5m)\hat{\boldsymbol{k}}$$

- The **displacement** of the object is defined as the ***change in its position***

$$\Delta\boldsymbol{r} = \boldsymbol{r}_f - \boldsymbol{r}_i$$

$$\Delta\boldsymbol{r} = (\boldsymbol{x}_f - \boldsymbol{x}_i)\hat{\boldsymbol{\imath}} + (\boldsymbol{y}_f - \boldsymbol{y}_i)\hat{\boldsymbol{\jmath}} + (\boldsymbol{z}_f - \boldsymbol{z}_i)\hat{\boldsymbol{k}}$$

$$\Delta\boldsymbol{r} = \Delta x\hat{\boldsymbol{\imath}} + \Delta y\hat{\boldsymbol{\jmath}} + \Delta z\hat{\boldsymbol{k}}$$

4.1 The Position, Velocity, and Acceleration Vectors

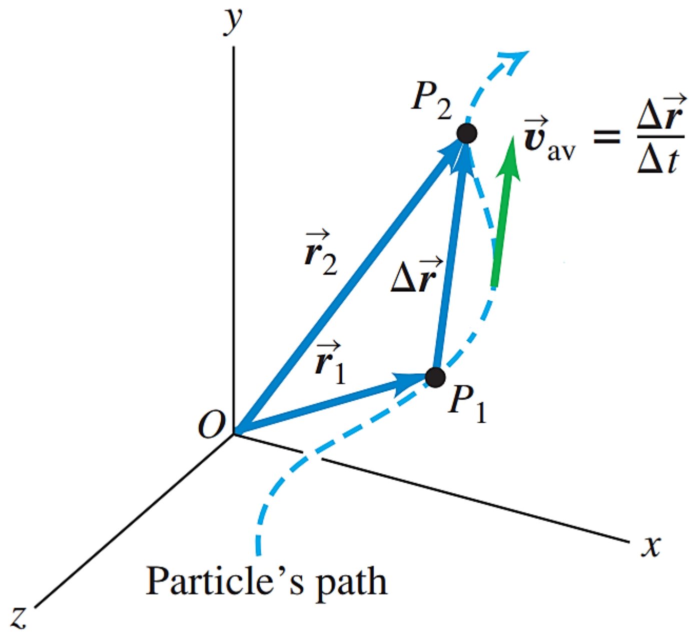
Average Velocity

- The average velocity is the ratio of the displacement to the time interval for the displacement:

$$Average\ velocity = \frac{displacement}{time\ interval}$$

$$\overline{\boldsymbol{v}} = \boldsymbol{v}_{avg} = \frac{\Delta \boldsymbol{r}}{\Delta t}$$

$$\overline{\boldsymbol{v}} = \boldsymbol{v}_{avg} = \frac{\Delta x}{\Delta t}\hat{\boldsymbol{\imath}} + \frac{\Delta y}{\Delta t}\hat{\boldsymbol{\jmath}} + \frac{\Delta z}{\Delta t}\hat{\boldsymbol{k}}$$



- The direction of the average velocity is the direction of the displacement vector
- The average velocity between points is *independent of the path* taken
  - This is because it is dependent on the displacement, also independent of the path

4.1 The Position, Velocity, and Acceleration Vectors

Instantaneous Velocity

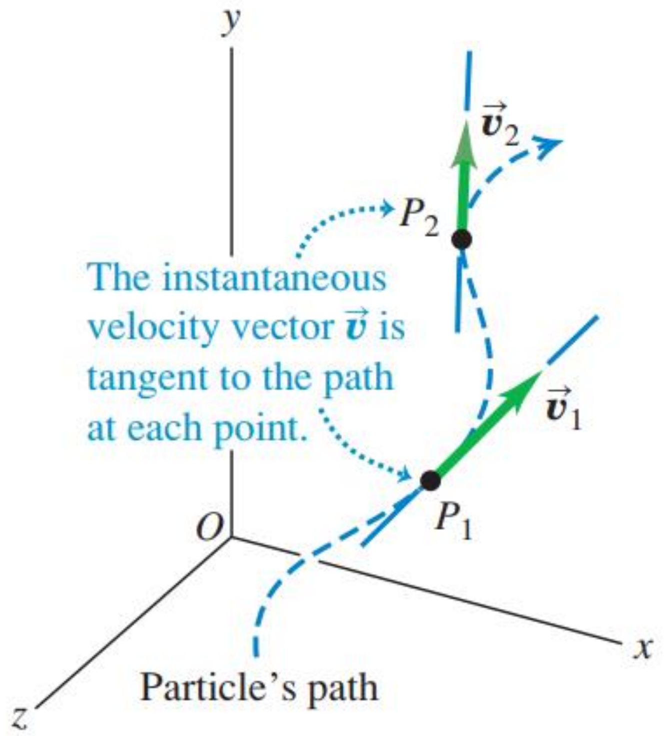
The instantaneous velocity is the limit of the average velocity as Δt approaches zero.

As the time interval becomes smaller, the direction of the displacement approaches that of the line tangent to the curve.

$$\boldsymbol{v} = \lim_{\Delta t \to 0} \frac{\Delta \boldsymbol{r}}{\Delta t} = \frac{d\boldsymbol{r}}{dt}$$

$$\boldsymbol{v} = \frac{d\boldsymbol{r}}{dt} = \frac{dx}{dt}\hat{\boldsymbol{\imath}} + \frac{dy}{dt}\hat{\boldsymbol{\jmath}} + \frac{dz}{dt}\widehat{\boldsymbol{k}}$$

$$\boldsymbol{v} = v_x\hat{\boldsymbol{\imath}} + v_y\hat{\boldsymbol{\jmath}} + v_z\widehat{\boldsymbol{k}}$$



- The magnitude of the instantaneous velocity vector is the speed

$$\boldsymbol{v} = |v| = \sqrt{\boldsymbol{v}_x^2 + \boldsymbol{v}_y^2 + \boldsymbol{v}_z^2}$$

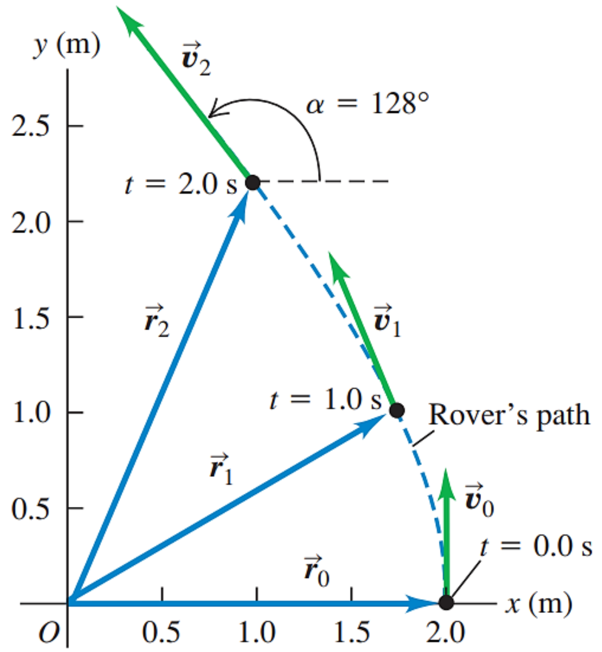
4.1 The Position, Velocity, and Acceleration Vectors

Example:

The rover, which we represent as a point, has *x*- and *y*-coordinates that vary with time:

$$x = 2.0\ \text{m} - (0.25\ \text{m/s}^2)t^2$$
$$y = (1.0\ \text{m/s})t + (0.025\ \text{m/s}^3)t^3$$

(a) Find the rover's coordinates and distance from the lander at t=2.0 s.



Solution:

At the rover's coordinates are t=2.0 s:

$$x = 2.0\ \text{m} - (0.25\ \text{m/s}^2)(2.0\ \text{s})^2 = 1.0\ \text{m}$$
$$y = (1.0\ \text{m/s})(2.0\ \text{s}) + (0.025\ \text{m/s}^3)(2.0\ \text{s})^3 = 2.2\ \text{m}$$

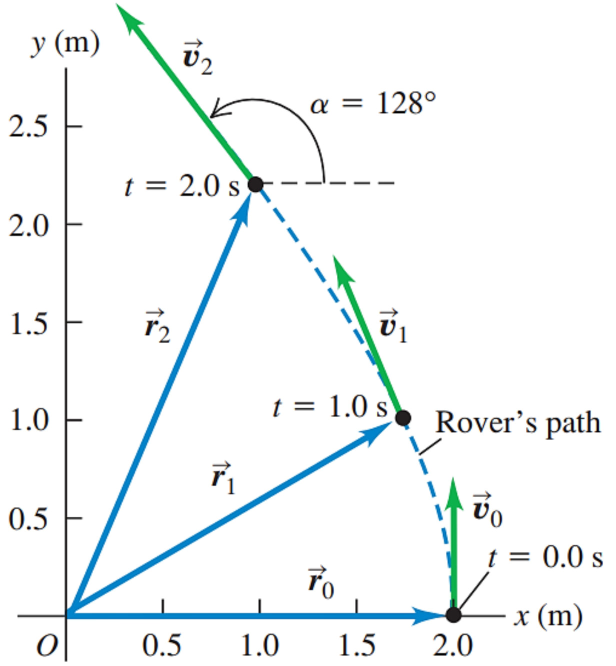
The rover's distance from the origin at this time is

$$\boxed{r = \sqrt{x^2 + y^2} = \sqrt{(1.0\ \text{m})^2 + (2.2\ \text{m})^2} = 2.4\ \text{m}}$$

4.1 The Position, Velocity, and Acceleration Vectors

Example (cont.):

(b) Find the rover's displacement and average velocity vectors for the interval t=0.0 s to t=2.0 s.



Solution:

To find the displacement and average velocity over the given time interval, we first express the position vector ***r*** as a function of time.

$$\begin{aligned}\vec{r} &= x\hat{\imath} + y\hat{\jmath} \\ &= [2.0\ \text{m} - (0.25\ \text{m/s}^2)t^2]\hat{\imath} \\ &\quad + [(1.0\ \text{m/s})t + (0.025\ \text{m/s}^3)t^3]\hat{\jmath}\end{aligned}$$

At t=0.0 s the position vector $\boldsymbol{r_0}$ is $\vec{r}_0 = (2.0\ \text{m})\hat{\imath} + (0.0\ \text{m})\hat{\jmath}$

From part (a), the position vector $\boldsymbol{r_2}$ at t=2.0 s is $\vec{r}_2 = (1.0\ \text{m})\hat{\imath} + (2.2\ \text{m})\hat{\jmath}$

The displacement from t=0.0 s to t=2.0 s is therefore

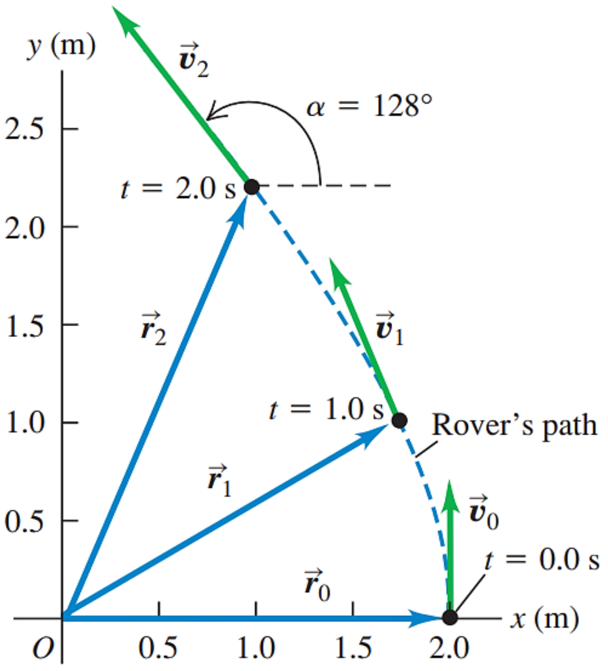
$$\boxed{\begin{aligned}\Delta\vec{r} &= \vec{r}_2 - \vec{r}_0 = (1.0\ \text{m})\hat{\imath} + (2.2\ \text{m})\hat{\jmath} - (2.0\ \text{m})\hat{\imath} \\ &= (-1.0\ \text{m})\hat{\imath} + (2.2\ \text{m})\hat{\jmath}\end{aligned}}$$

$$\boxed{\begin{aligned}\vec{v}_{\text{av}} &= \frac{\Delta\vec{r}}{\Delta t} = \frac{(-1.0\ \text{m})\hat{\imath} + (2.2\ \text{m})\hat{\jmath}}{2.0\ \text{s} - 0.0\ \text{s}} \\ &= (-0.50\ \text{m/s})\hat{\imath} + (1.1\ \text{m/s})\hat{\jmath}\end{aligned}}$$

4.1 The Position, Velocity, and Acceleration Vectors

Example (cont.):

(c) Find a general expression for the rover's instantaneous velocity vector ***v*.** Express ***v*** at t=2.0 s in component form and in terms of magnitude and direction.



Solution:

The components of instantaneous velocity are the time derivatives of the coordinates:

$$v_x = \frac{dx}{dt} = (-0.25\ \text{m/s}^2)(2t)$$

$$v_y = \frac{dy}{dt} = 1.0\ \text{m/s} + (0.025\ \text{m/s}^3)(3t^2)$$

Hence the instantaneous velocity vector is

$$\vec{v} = v_x\hat{\imath} + v_y\hat{\jmath} = (-0.50\ \text{m/s}^2)t\hat{\imath} + [1.0\ \text{m/s} + (0.075\ \text{m/s}^3)t^2]\hat{\jmath}$$

At t=2.0 s the velocity vector $\boldsymbol{v_2}$ has components

$$v_{2x} = (-0.50\ \text{m/s}^2)(2.0\ \text{s}) = -1.0\ \text{m/s}$$

$$v_{2y} = 1.0\ \text{m/s} + (0.075\ \text{m/s}^3)(2.0\ \text{s})^2 = 1.3\ \text{m/s}$$

The magnitude of the instantaneous velocity t=2.0 s

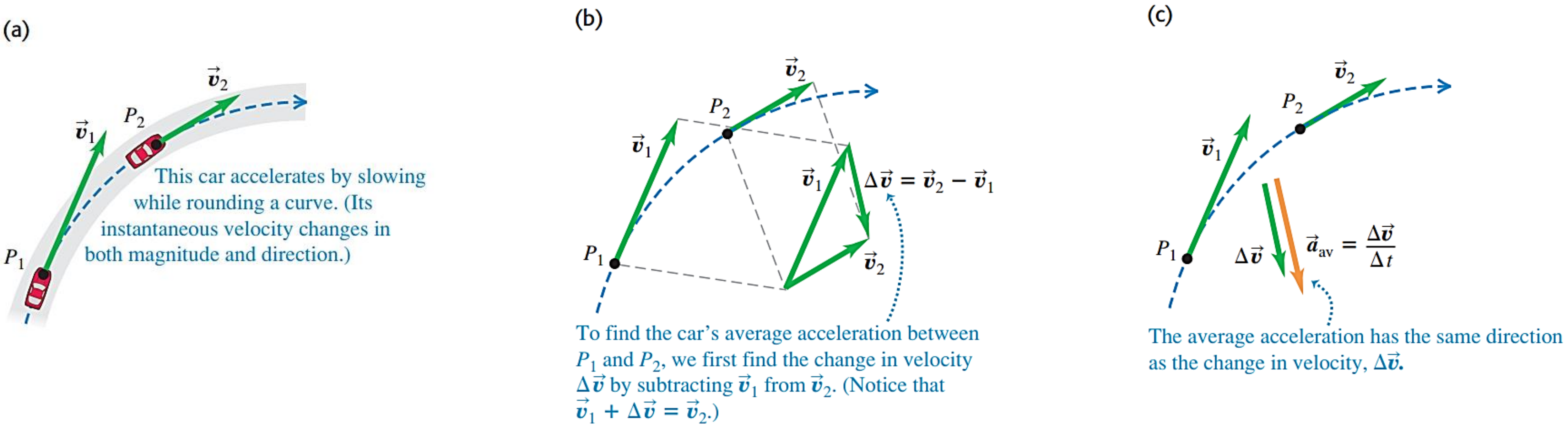
$$\boxed{v_2 = \sqrt{v_{2x}^2 + v_{2y}^2} = \sqrt{(-1.0\ \text{m/s})^2 + (1.3\ \text{m/s})^2} = 1.6\ \text{m/s}}$$

$$\arctan\frac{v_y}{v_x} = \arctan\frac{1.3\ \text{m/s}}{-1.0\ \text{m/s}} = -52°$$

$$\boxed{\alpha = 180° - 52° = 128°}$$

4.1 The Position, Velocity, and Acceleration Vectors

Average Acceleration



The average accelaration is the ratio of the velocity to the time interval for the velocity:

$$\textit{Average acceleration} = \frac{\textit{change in velocity}}{\textit{time interval}}$$

$$\bar{\boldsymbol{a}} = \boldsymbol{a}_{avg} = \frac{\boldsymbol{v}_2 - \boldsymbol{v}_1}{\Delta t} = \frac{\Delta \boldsymbol{v}}{\Delta t}$$

Average acceleration is a *vector* quantity in the same direction as the vector $\Delta\boldsymbol{v}$.

4.1 The Position, Velocity, and Acceleration Vectors

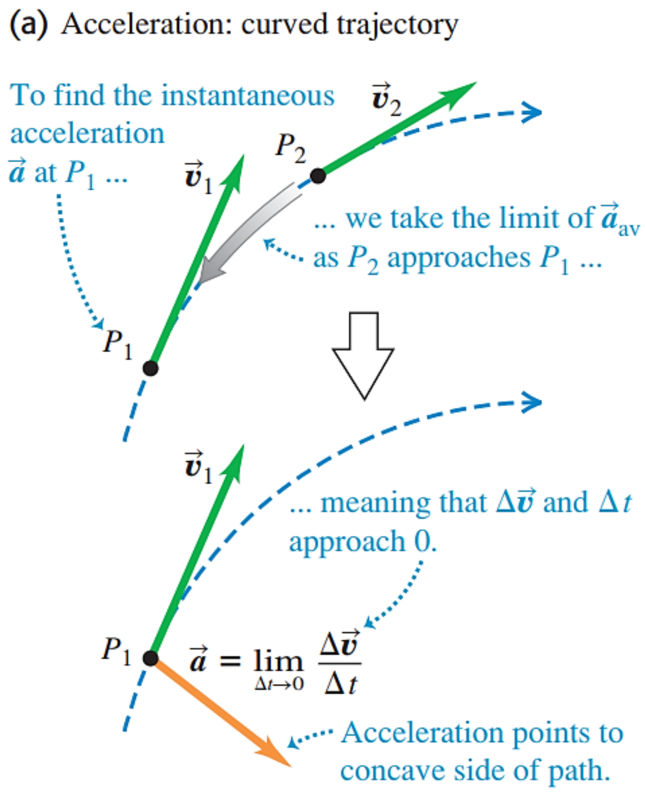
Instantaneous Accelaration

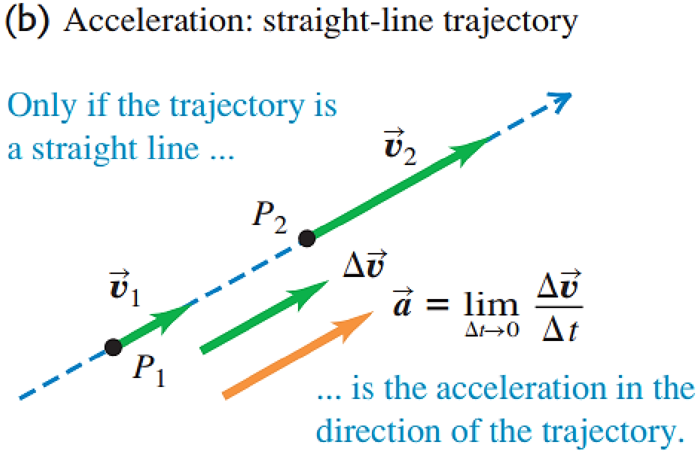
- The instantaneous acceleration is the limit of the average accelaration as Δt approaches zero.

$$\boldsymbol{a} = \lim_{\Delta t \to 0} \frac{\Delta \boldsymbol{v}}{\Delta t} = \frac{d\boldsymbol{v}}{dt}$$

$$\boldsymbol{a} = \frac{d\boldsymbol{v}}{dt} = \frac{dv_x}{dt}\hat{\boldsymbol{\imath}} + \frac{dv_y}{dt}\hat{\boldsymbol{\jmath}} + \frac{dv_z}{dt}\hat{\boldsymbol{k}}$$

$$\boldsymbol{a} = a_x\hat{\boldsymbol{\imath}} + a_y\hat{\boldsymbol{\jmath}} + a_z\hat{\boldsymbol{k}}$$





The acceleration is tangent to the path only if the particle moves in a straight line.

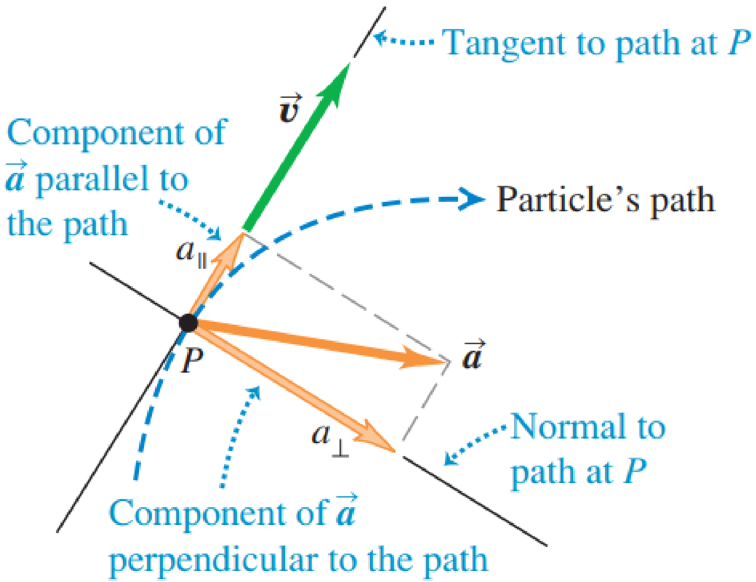
$$\vec{a} = \frac{d^2x}{dt^2}\hat{\imath} + \frac{d^2y}{dt^2}\hat{\jmath} + \frac{d^2z}{dt^2}\hat{k}$$

4.1 The Position, Velocity, and Acceleration Vectors

Parallel and Perpendicular Components of Acceleration

The particle's instantaneous acceleration vector ***a*** has component along x, y and z-axes.

$$\boldsymbol{a_x} = \frac{dv_x}{dt} \qquad \boldsymbol{a_y} = \frac{dv_y}{dt} \qquad \boldsymbol{a_z} = \frac{dv_z}{dt}$$



component *parallel* to the particle's path—that is, parallel to the velocity $a_{//}$

component *perpendicular* to the path—that is perpendicular to the velocity $a_{\perp}$

$a_{//}$ → changes in the particle's *speed*

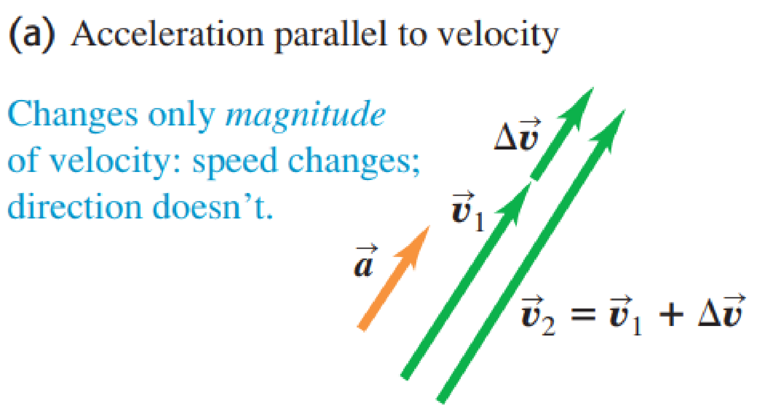
$a_{\perp}$ → changes in the particle's *direction of motion*

4.1 The Position, Velocity, and Acceleration Vectors

Parallel and Perpendicular Components of Acceleration

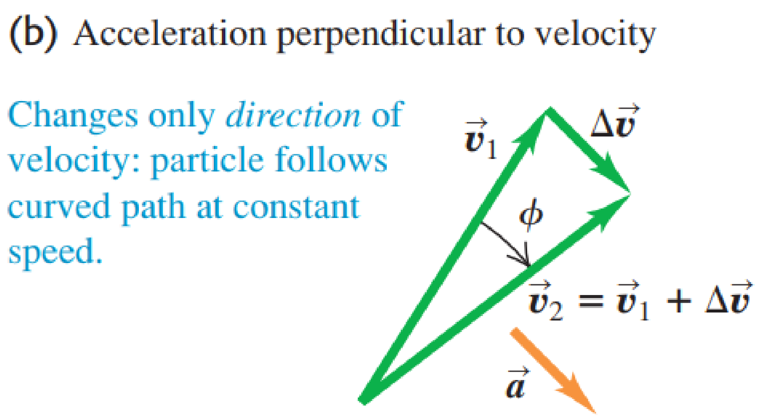
The acceleration vector is in the same direction as the velocity $\boldsymbol{v}_1$ so $\boldsymbol{a}$ has only a parallel component $a_{//}$ (that is $a_{\perp}$=0).

The velocity $\boldsymbol{v}_2$ at the end of Δt is in the same direction as $\boldsymbol{v}_1$ but has greater magnitude. The particle in moved in a straight line with increasing speed.



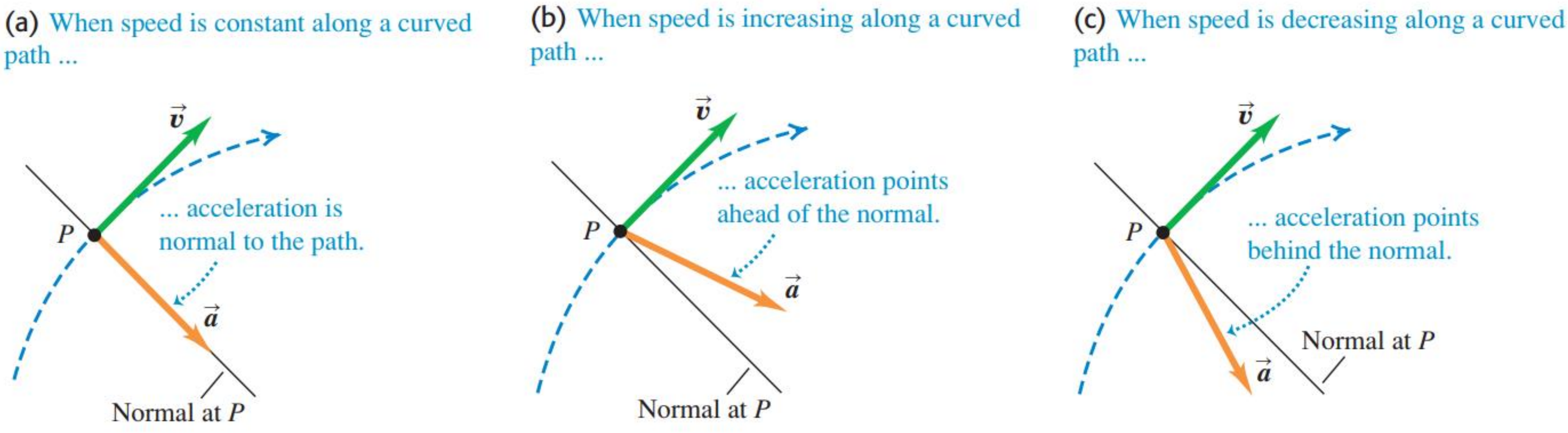
The acceleration is *perpendicular* to the velocity, so $\boldsymbol{a}$ has only a perpendicular component $a_{\perp}$ (that is, $a_{//}$ =0).

The speed of the particle stays the same, but the direction of motion changes and the path of the particle curves.



4.1 The Position, Velocity, and Acceleration Vectors

Parallel and Perpendicular Components of Acceleration



If the speed is constant, ***a*** is perpendicular, or *normal,* to the path and to ***v***

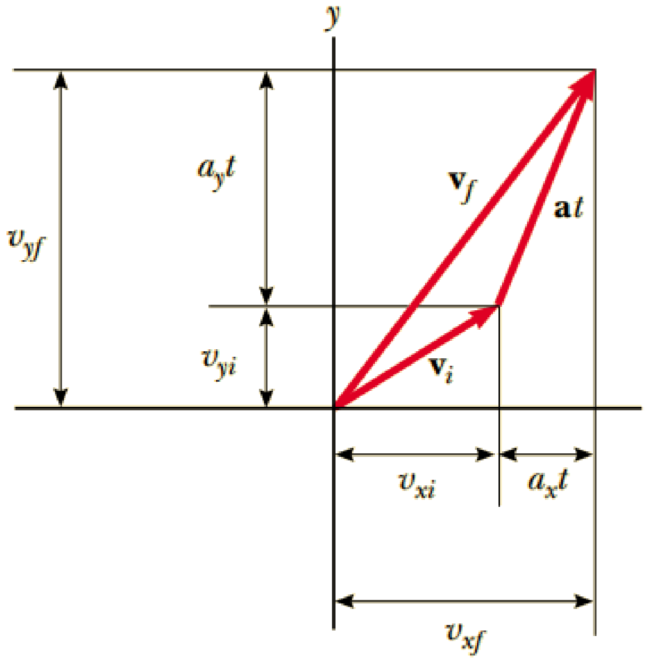
If the speed is increasing, the final velocity is greater and ***a*** points ahead of the normal to the path

If the speed is decreasing, the final velocity is lower and ***a*** points behind the normal to the path

4.2 Two-Dimensional Motion with Constant Acceleration

Position vector for a particle moving in the xy plane $\boldsymbol{r} = x\hat{\mathbf{\imath}} + y\hat{\boldsymbol{\jmath}}$

$$\boldsymbol{v} = \frac{d\boldsymbol{r}}{dt} = \frac{dx}{dt}\hat{\boldsymbol{\imath}} + \frac{dy}{dt}\hat{\boldsymbol{\jmath}} = v_x\hat{\boldsymbol{\imath}} + v_y\hat{\boldsymbol{\jmath}}$$



Since acceleration is constant, we can also find an expression for the velocity as a function of time:

$$\vec{\mathbf{v}}_f = \vec{\mathbf{v}}_i + \vec{\mathbf{a}}t$$

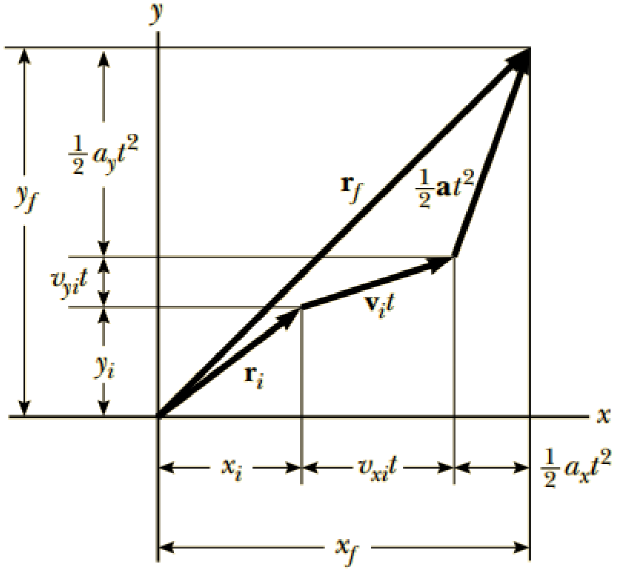
$$\begin{aligned}\mathbf{v}_f &= (v_{xi} + a_x t)\hat{\mathbf{i}} + (v_{yi} + a_y t)\hat{\mathbf{j}} \\ &= (v_{xi}\hat{\mathbf{i}} + v_{yi}\hat{\mathbf{j}}) + (a_x\hat{\mathbf{i}} + a_y\hat{\mathbf{j}})t\end{aligned}$$

The position vector can also be expressed as a function of time:

$$x_f = x_i + v_{xi}t + \tfrac{1}{2}a_x t^2 \qquad y_f = y_i + v_{yi}t + \tfrac{1}{2}a_y t^2$$

$$\begin{aligned}\mathbf{r}_f &= (x_i + v_{xi}t + \tfrac{1}{2}a_x t^2)\hat{\mathbf{i}} + (y_i + v_{yi}t + \tfrac{1}{2}a_y t^2)\hat{\mathbf{j}} \\ &= (x_i\hat{\mathbf{i}} + y_i\hat{\mathbf{j}}) + (v_{xi}\hat{\mathbf{i}} + v_{yi}\hat{\mathbf{j}})t + \tfrac{1}{2}(a_x\hat{\mathbf{i}} + a_y\hat{\mathbf{j}})t^2\end{aligned}$$

$$\mathbf{r}_f = \mathbf{r}_i + \mathbf{v}_i t + \tfrac{1}{2}\mathbf{a}t^2$$



4.2 Two-Dimensional Motion with Constant Acceleration

Example:

A particle initially located at the origin has an acceleration of **a** = $3.00\hat{\mathbf{j}}$m/s^2 and an initial velocity of $\mathbf{v}_i$ = $500\hat{\mathbf{i}}$ m/s. Find

(a) the vector position and velocity at any time *t* and
(b) the coordinates and speed of the particle at *t* = 2.00 s.

$$\mathbf{a} = 3.00\hat{\mathbf{j}}\ \mathrm{m/s^2}\,;\ \mathbf{v}_i = 5.00\hat{\mathbf{i}}\ \mathrm{m/s}\,;\ \mathbf{r}_i = 0\hat{\mathbf{i}} + 0\hat{\mathbf{j}}$$

(a)

$$\mathbf{r}_f = \mathbf{r}_i + \mathbf{v}_i t + \frac{1}{2}\mathbf{a}t^2 = \boxed{\left[5.00t\hat{\mathbf{i}} + \frac{1}{2}3.00t^2\hat{\mathbf{j}}\right]\ \mathrm{m}}$$

$$\mathbf{v}_f = \mathbf{v}_i + \mathbf{a}t = \boxed{\left(5.00\hat{\mathbf{i}} + 3.00t\hat{\mathbf{j}}\right)\ \mathrm{m/s}}$$

(b)

$$t = 2.00\ \mathrm{s}\,,\ \mathbf{r}_f = 5.00(2.00)\hat{\mathbf{i}} + \frac{1}{2}(3.00)(2.00)^2\hat{\mathbf{j}} = \left(10.0\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}}\right)\ \mathrm{m}$$

$$\text{so } x_f = \boxed{10.0\ \mathrm{m}}\,,\ y_f = \boxed{6.00\ \mathrm{m}}$$

$$\mathbf{v}_f = 5.00\hat{\mathbf{i}} + 3.00(2.00)\hat{\mathbf{j}} = \left(5.00\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}}\right)\ \mathrm{m/s}$$

$$v_f = \left|\mathbf{v}_f\right| = \sqrt{v_{xf}^2 + v_{yf}^2} = \sqrt{(5.00)^2 + (6.00)^2} = \boxed{7.81\ \mathrm{m/s}}$$

4.3 Projectile Motion

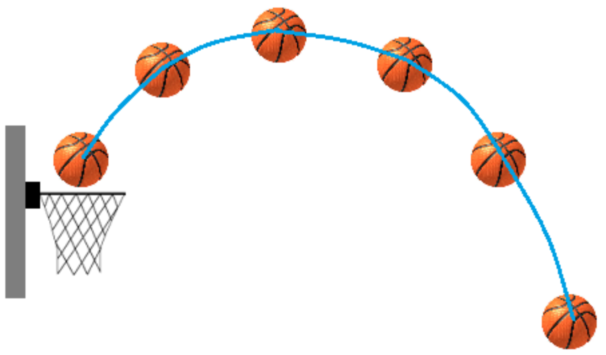
An object may move in both the *x* and *y* directions simultaneously.

The form of two-dimensional motion we will deal with is called **projectile motion.**

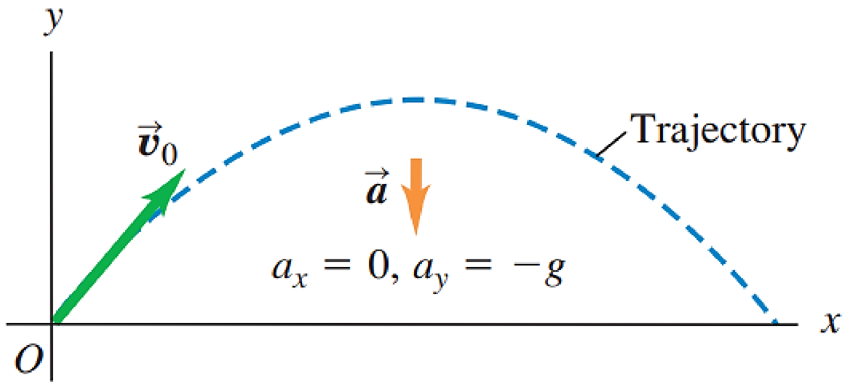
Assumptions of Projectile Motion

- The free-fall acceleration is constant over the range of motion
  - It is directed downward
- $a_y = -g$
- Constant-velocity motion in the horizontal direction $a_x = 0$
- The effect of air friction is negligible
- With these assumptions, an object in projectile motion will follow a parabolic path
  - This path is called the ***trajectory***

Projectile motion

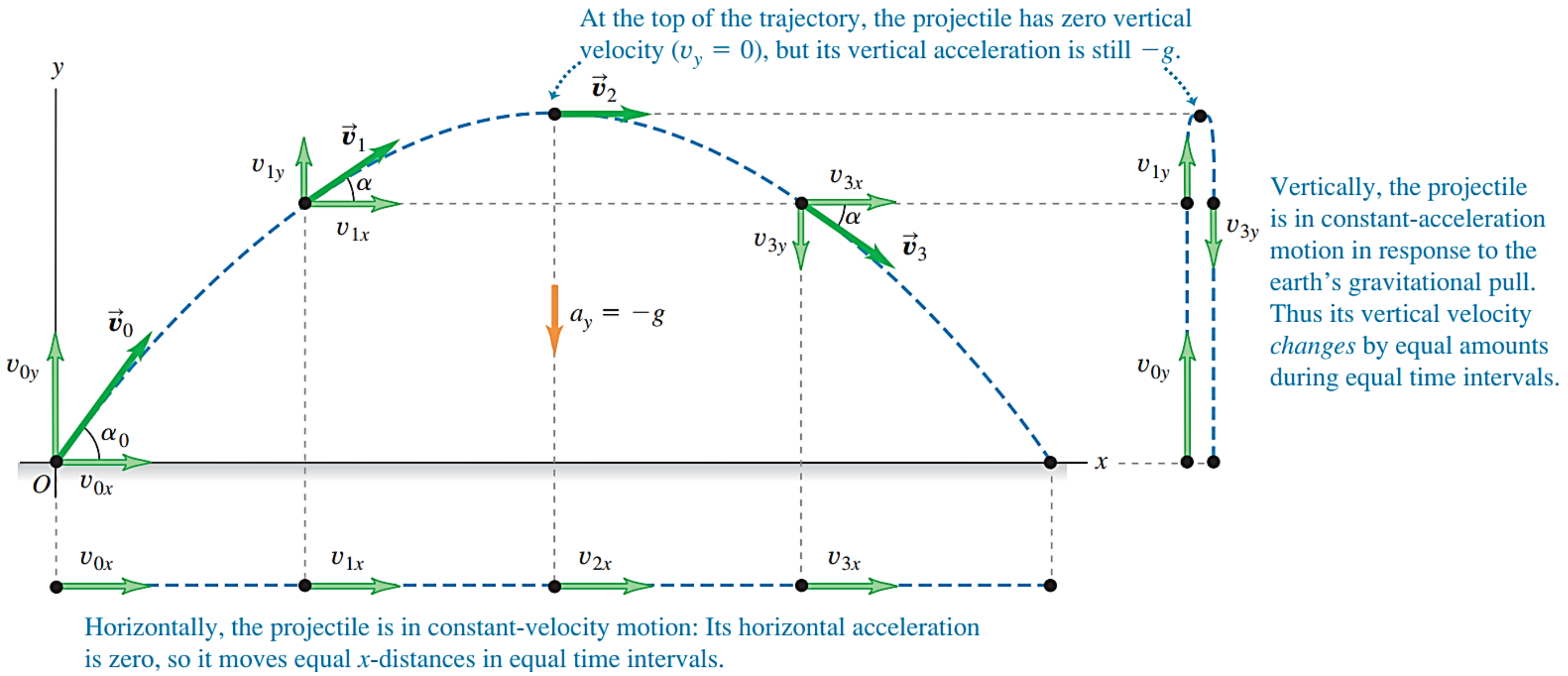


- A projectile moves in a vertical plane that contains the initial velocity vector $\vec{v}_0$.
- Its trajectory depends only on $\vec{v}_0$ and on the downward acceleration due to gravity.



4.3 Projectile Motion

We can analyze projectile motion as a combination of horizontal motion with constant velocity and vertical motion with constant acceleration.



$$a_x = 0$$

$$v_x = v_{0x}$$

$$x = x_0 + v_{0x}t$$

$$v_x = v_{0x} + a_x t$$

$$x = x_0 + v_{0x}t + \tfrac{1}{2}a_x t^2$$

$$a_y = -g$$

$$v_y = v_{0y} - gt$$

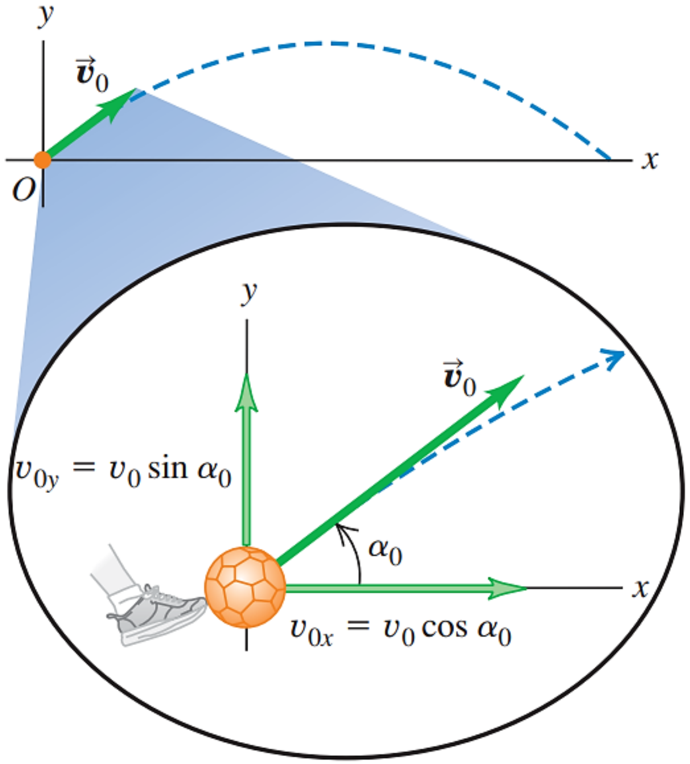
$$y = y_0 + v_{0y}t - \tfrac{1}{2}gt^2$$

4.3 Projectile Motion

It's usually simplest to take the initial position (at t=0) as the origin; then x_0=y_0=0

We can also represent the initial velocity $\boldsymbol{v_0}$ by its magnitude v_0 (the initial speed) and its angle $\boldsymbol{\alpha_0}$ with the positive *x*-axis.

In terms of these quantities, the components $\boldsymbol{v_{0x}}$ and $\boldsymbol{v_{0y}}$ of the initial velocity are



$$v_{0x} = v_0 \cos \alpha_0 \qquad v_{0y} = v_0 \sin \alpha_0$$

$$x = (v_0 \cos \alpha_0)t$$

$$y = (v_0 \sin \alpha_0)t - \tfrac{1}{2}gt^2$$

$$v_x = v_0 \cos \alpha_0$$

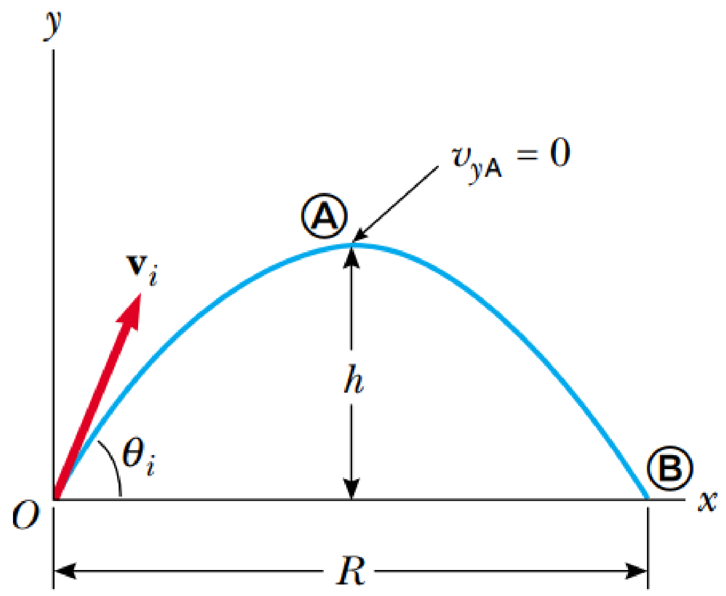
$$v_y = v_0 \sin \alpha_0 - gt$$

4.3 Projectile Motion

Maximum Height and Horizontal Range of a Projectile

The maximum height the projectile reaches is *h*

$$y_f = v_{yi}t + \tfrac{1}{2}a_y t^2 = (v_i \sin\theta_i)\,t - \tfrac{1}{2}g t^2$$



$$v_{yf} = v_{yi} + a_y t$$

The maximum height of the projectile can be found in terms of the initial velocity vector:

$$0 = v_i \sin\theta_i - g t_A$$

$$t_A = \frac{v_i \sin\theta_i}{g}$$

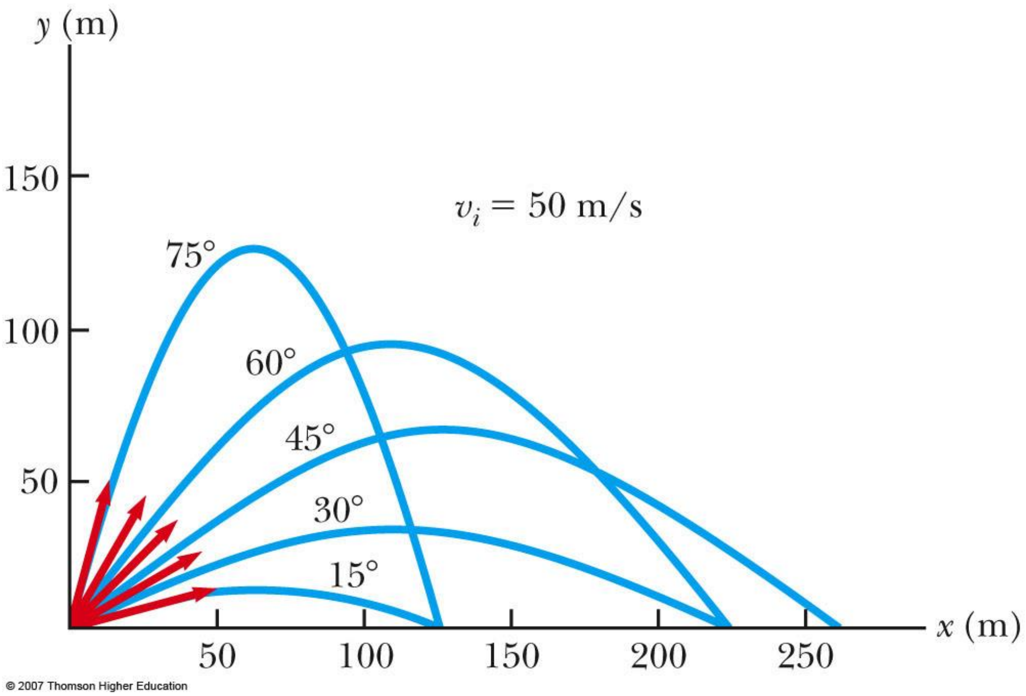
$$h = (v_i \sin\theta_i)\frac{v_i \sin\theta_i}{g} - \tfrac{1}{2}g\left(\frac{v_i \sin\theta_i}{g}\right)^2$$

$$\boxed{h = \frac{v_i^2 \sin^2\theta_i}{2g}}$$

This equation is valid only for symmetric motion!!!

4.3 Projectile Motion

Maximum Height and Horizontal Range of a Projectile



$$t_B = 2t_A$$

$$v_{xi} = v_{xB} = v_i \cos\theta_i$$

$$x_B = R \text{ at } t = 2t_A$$

$$R = v_{xi}t_B = (v_i \cos\theta_i)2t_A$$

$$= (v_i \cos\theta_i)\frac{2v_i \sin\theta_i}{g} = \frac{2v_i^2 \sin\theta_i \cos\theta_i}{g}$$

$$2\theta = 2\sin\theta\cos\theta$$

The range of a projectile can be expressed in terms of the initial velocity vector:

$$\boxed{R = \frac{v_i^2 \sin 2\theta_i}{g}}$$

4.3 Projectile Motion

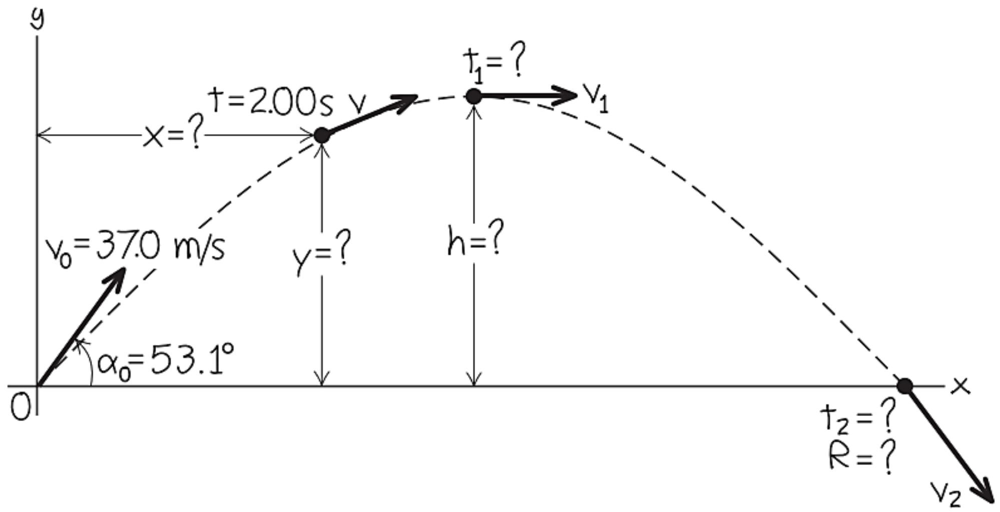
Example:

A baseball leaves the bat at speed $v_0 = 37.0\ m/s$ at an angle $\alpha_0 = 53.1^o$.

a) Find the position of the ball and its velocity (magnitude and direction) at t=2.00 s.

b) Find the time when the ball reaches the highest point of its flight, and its height *h* at this time.

c) Find the *horizontal range R*—that is, the horizontal distance from the starting point to where the ball hits the ground.



Solution:

a) The initial velocity of the ball has components

$$v_{0x} = v_0 \cos\alpha_0 = (37.0\ \text{m/s})\cos 53.1° = 22.2\ \text{m/s}$$
$$v_{0y} = v_0 \sin\alpha_0 = (37.0\ \text{m/s})\sin 53.1° = 29.6\ \text{m/s}$$

$$v_x = v_{0x} = 22.2\ \text{m/s}$$
$$v_y = v_{0y} - gt = 29.6\ \text{m/s} - (9.80\ \text{m/s}^2)(2.00\ \text{s}) = 10.0\ \text{m/s}$$

$$\boxed{\begin{aligned} v &= \sqrt{v_x^2 + v_y^2} = \sqrt{(22.2\ \text{m/s})^2 + (10.0\ \text{m/s})^2} \\ &= 24.4\ \text{m/s} \\ \alpha &= \arctan\left(\frac{10.0\ \text{m/s}}{22.2\ \text{m/s}}\right) = \arctan 0.450 = 24.2° \end{aligned}}$$

$$x = v_{0x}t = (22.2\ \text{m/s})(2.00\ \text{s}) = 44.4\ \text{m}$$

$$\begin{aligned} y &= v_{0y}t - \tfrac{1}{2}gt^2 \\ &= (29.6\ \text{m/s})(2.00\ \text{s}) - \tfrac{1}{2}(9.80\ \text{m/s}^2)(2.00\ \text{s})^2 \\ &= 39.6\ \text{m} \end{aligned}$$

4.3 Projectile Motion

Solution:

b) At the highest point, the vertical velocity v_y is zero. Call the time when this happens t_1 ; then

$$v_y = v_{0y} - gt_1 = 0$$
$$t_1 = \frac{v_{0y}}{g} = \frac{29.6\ \mathrm{m/s}}{9.80\ \mathrm{m/s^2}} = 3.02\ \mathrm{s}$$

The height h at the highest point is the value of y at time t_1 :

$$\begin{aligned} h &= v_{0y}t_1 - \tfrac{1}{2}gt_1^{\,2} \\ &= (29.6\ \mathrm{m/s})(3.02\ \mathrm{s}) - \tfrac{1}{2}(9.80\ \mathrm{m/s^2})(3.02\ \mathrm{s})^2 \\ &= 44.7\ \mathrm{m} \end{aligned}$$

c) At the highest point, the vertical velocity v_y is zero. Call the time when this happens t_1 ; then

$$y = 0 = v_{0y}t_2 - \tfrac{1}{2}gt_2^{\,2} = t_2\left(v_{0y} - \tfrac{1}{2}gt_2\right)$$

$$t_2 = 0 \qquad \text{and} \qquad t_2 = \frac{2v_{0y}}{g} = \frac{2(29.6\ \mathrm{m/s})}{9.80\ \mathrm{m/s^2}} = 6.04\ \mathrm{s}$$

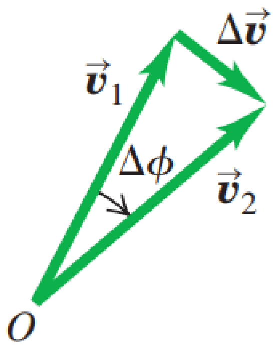
$$R = v_{0x}t_2 = (22.2\ \mathrm{m/s})(6.04\ \mathrm{s}) = 134\ \mathrm{m}$$

$$\begin{aligned} v_y &= v_{0y} - gt_2 = 29.6\ \mathrm{m/s} - (9.80\ \mathrm{m/s^2})(6.04\ \mathrm{s}) \\ &= -29.6\ \mathrm{m/s} \end{aligned}$$

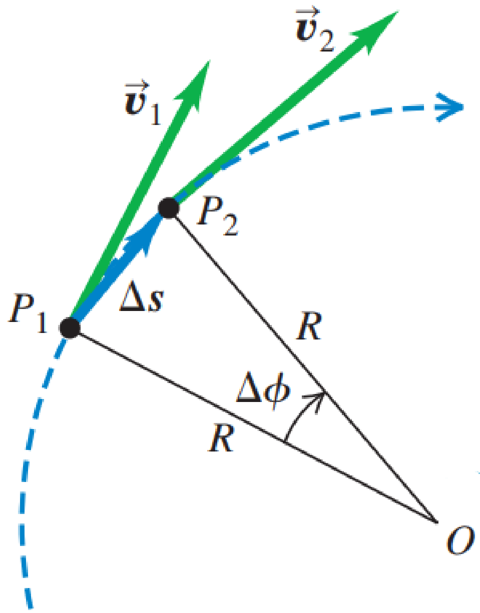
4.4 Motion in a Circle

Uniform Circular Motion

A particle moving with constant speed in a circular path of radius R with center at O.

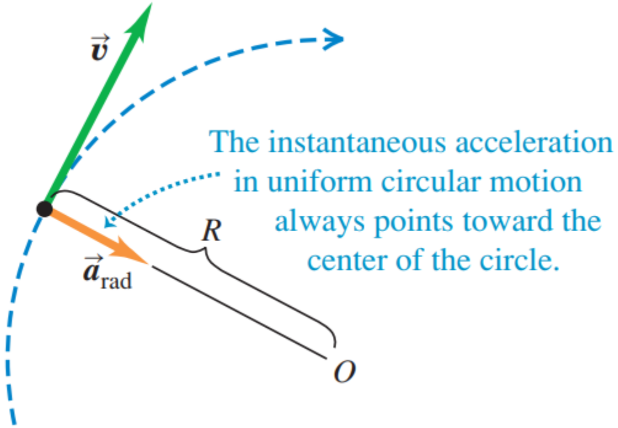


$$\frac{|\Delta\vec{v}|}{v_1} = \frac{\Delta s}{R} \qquad \text{or} \qquad |\Delta\vec{v}| = \frac{v_1}{R}\Delta s$$



The magnitude of the average acceleration a_{av} during is therefore

$$a_{av} = \frac{|\Delta\vec{v}|}{\Delta t} = \frac{v_1}{R}\frac{\Delta s}{\Delta t}$$



The magnitude a of the *instantaneous* acceleration

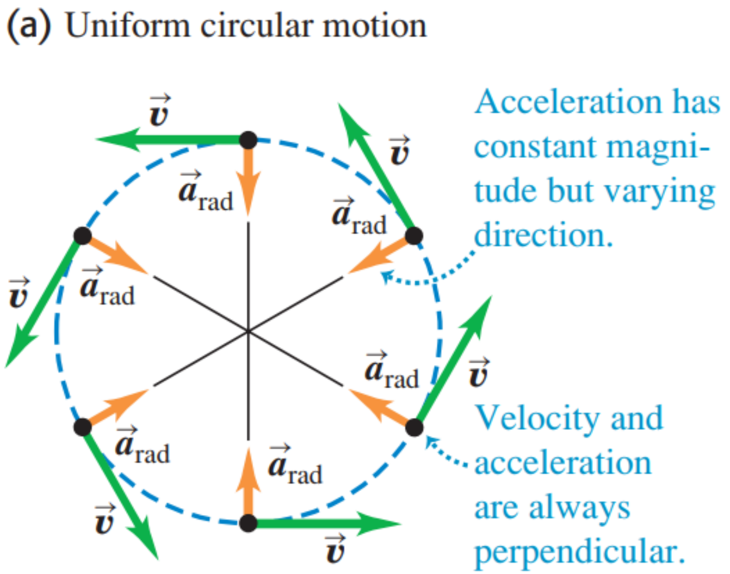
$$a = \lim_{\Delta t \to 0} \frac{v_1}{R}\frac{\Delta s}{\Delta t} = \frac{v_1}{R}\lim_{\Delta t \to 0}\frac{\Delta s}{\Delta t}$$

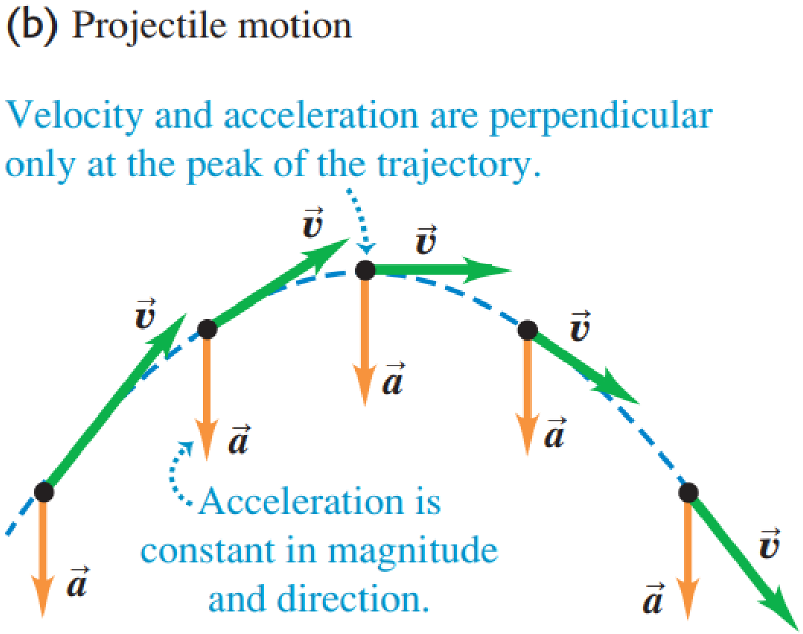
$$a_{rad} = \frac{v^2}{R} \qquad \text{(uniform circular motion)}$$

4.4 Motion in a Circle

Uniform Circular Motion

In uniform circular motion the direction of ***a*** changes continuously so that it always points toward the center of the circle. In projectile motion, by contrast, the direction of remains the same at all times.





In a time ***T*** (period) the particle travels a distance equal to the circumference of the circle, so its speed is:

$$v = \frac{2\pi R}{T} \quad \longrightarrow \quad a_{rad} = \frac{4\pi^2 R}{T^2} \quad \text{(uniform circular motion)}$$

4.4 Motion in a Circle

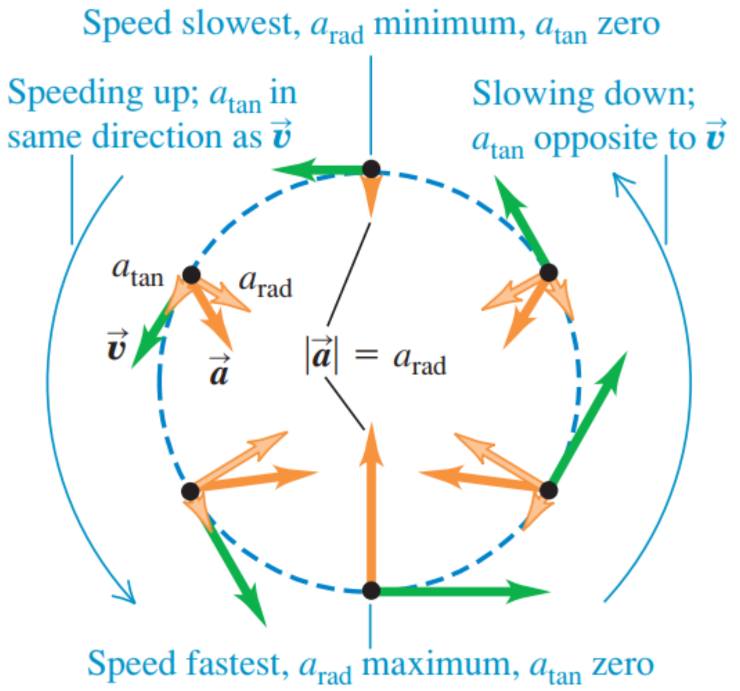
Nonuniform Circular Motion

In circular motion the magnitude of the velocity could also be changing. If the speed varies, we call the motion **nonuniform circular motion.**

In this case, the velocity vector is always tangent to the path.

The acceleration vector **a** has two components

–Along the Radius a_r
–Perpendicular to the radius a_t

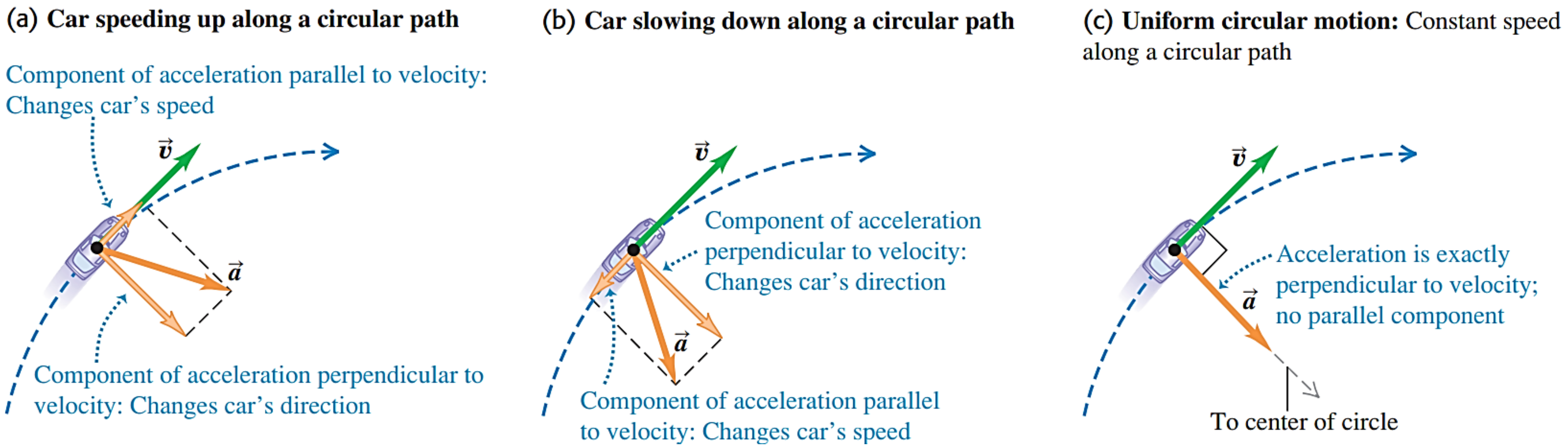


$$a_{rad} = \frac{v^2}{R} \quad \text{and} \quad a_{tan} = \frac{d|\vec{v}|}{dt} \quad \text{(nonuniform circular motion)}$$

4.4 Motion in a Circle

Nonuniform Circular Motion

The tangential component is in the same direction as the velocity if the particle is speeding up, and in the opposite direction if the particle is slowing down. If the particle's speed is constant, $a_{tan}=0$.



- ✓ The *tangential acceleration* causes the change in the speed of the particle.
- ✓ The *radial acceleration* comes from a change in the direction of the velocity vector.

4.5 Relative Velocity and Relative Acceleration

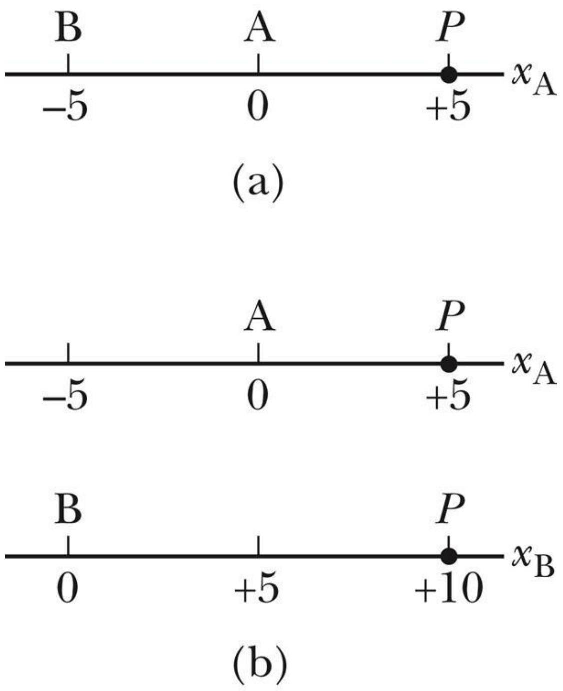
In general, when two observers measure the velocity of a moving body, they get different results if one observer is moving relative to the other.

The velocity seen by a particular observer is called the velocity *relative* to that observer, or simply **relative velocity.**

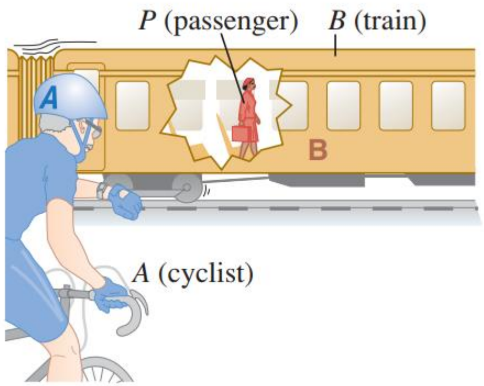
Observer A measures point P at +5 m from the origin

Observer B measures point P at +10 m from the origin

The difference is due to the different frames of reference being used



4.5 Relative Velocity and Relative Acceleration



Relative Motion in One Dimensions

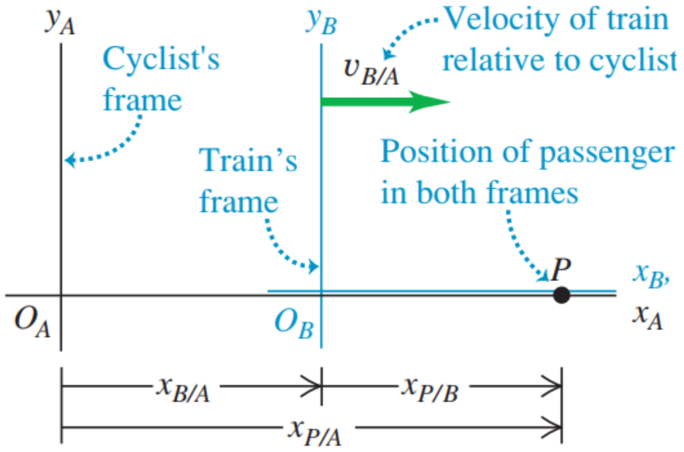
the symbol *A* for the observer frame of reference (at rest with respect to the ground)

the symbol *B* for the frame of reference of the moving train

point *P* represents the passenger

In straight-line motion the position of a point P relative to frame A is given by x_{PA}

$$x_{PA} = x_{PB} + x_{BA}$$



Taking the time derivative

$$\frac{d}{dt}(x_{PA}) = \frac{d}{dt}(x_{PB}) + \frac{d}{dt}(x_{BA}) \longrightarrow v_{PA} = v_{PB} + v_{BA}$$

$$\frac{d}{dt}(v_{PA}) = \frac{d}{dt}(v_{PB}) + \frac{d}{dt}(v_{BA})$$

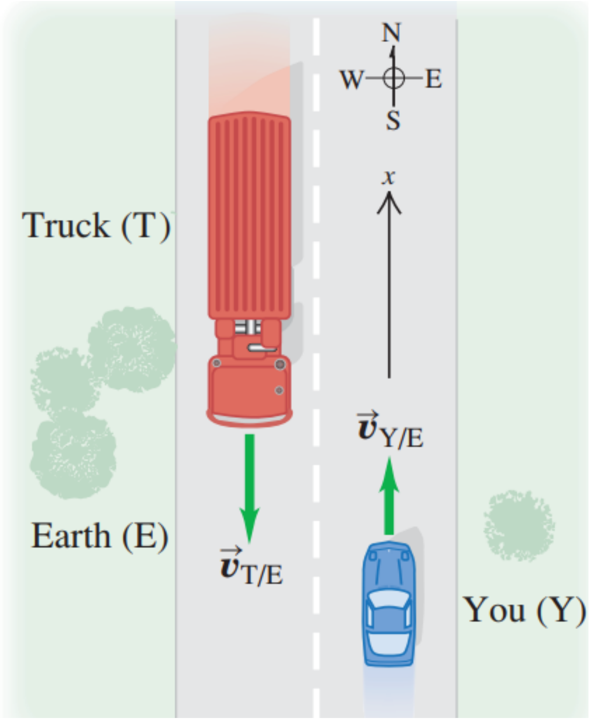
Because v_{BA} is constant, the last term is zero and we have: $a_{PA} = a_{PB}$

Observers on different frames of reference that move at constant velocity relative to each other will measure the same acceleration for a moving particle.

4.5 Relative Velocity and Relative Acceleration

Example:

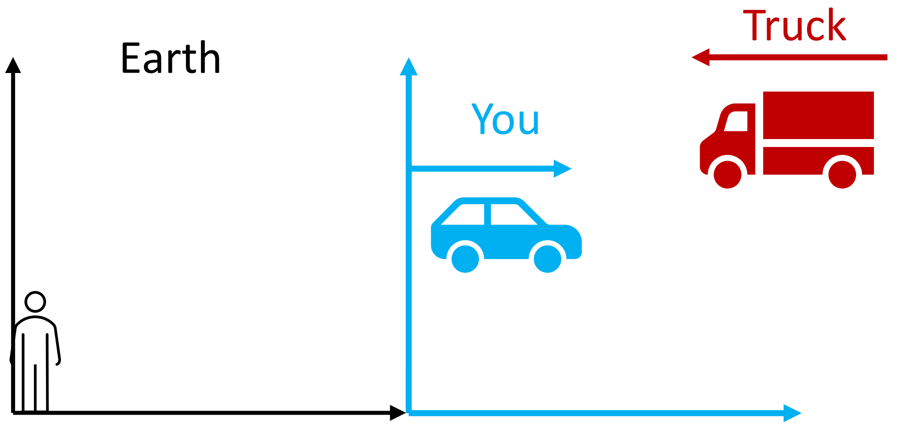
You drive north on a straight two-lane road at a constant 88 km/h. A truck in the other lane approaches you at a constant 104 km/h.
Find **(a)** the truck's velocity relative to you and **(b)** your velocity relative to the truck.



There are three reference frame: you (Y), truck (T) and Erath (E)

Your velocity relative to Earth $v_{YE} = +88\ km/h$

Truck's velocity relative to Earth $v_{TE} = -104\ km/h$



$$v_{TE} = v_{TY} + v_{YE} \Rightarrow -104\ km/h = v_{TY} + 88\ km/h$$

$$v_{TY} = -192\ km/h$$

$$v_{YT} = +192\ km/h$$

4.5 Relative Velocity and Relative Acceleration

Example:

Suppose that frame B's velocity relative to frame A is a constant v_{BA} =+3 m/s and passenger *P* is moving in the positive direction of the *x* axis with v_{PB} =+1 m/s

Passenger's velocity v_{PA} relative to observer in frame A is: $v_{PA} = v_{PB} + v_{BA} = +1.0\ m/s + 3.0\ m/s = 4.0\ m/s$

Example:

The train velocity (frame B) relative to observer frame A is a constant v_{BA} = 52 km/h. A car (*C)* is moving in the negative direction of the x axis.

(a) If frame A measures a constant v_{CA} =-78 km/h for car, what velocity v_{CB} will frame B measure?

$$-78\ km/h = v_{CB} + 52\ km/h \qquad v_{CB} = -130\ km/h$$

(b) If car (C) stops relative to A (and thus relative to the ground) in time *t* =10 s at constant acceleration, what is its acceleration a_{CA} relative to A?

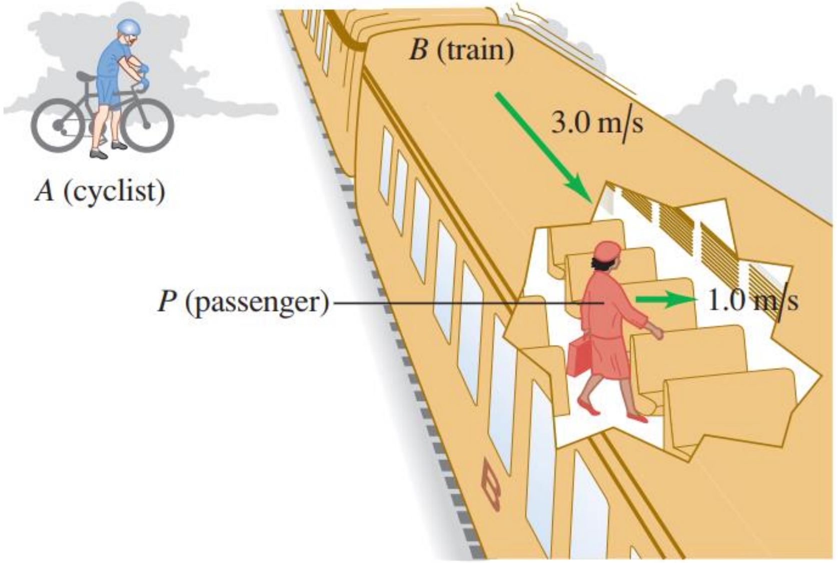
To calculate the acceleration of car relative to frame *A,* we must use the car velocities relative to frame *A*.

$v_{CA} = -78\ km/h$ and final velocity=0

$$a_{CA} = \frac{v - v_0}{t} = \frac{0 - (-78\ km/h)}{10\ s} \cdot \frac{1\ m/s}{3.6\ km/h} = 2.2\ m/s^2$$

4.5 Relative Velocity and Relative Acceleration

Relative Motion in Two Dimensions



At that instant, the position vector of the origin of *B* relative to the origin of *A* is $\boldsymbol{r}_{BA}$.

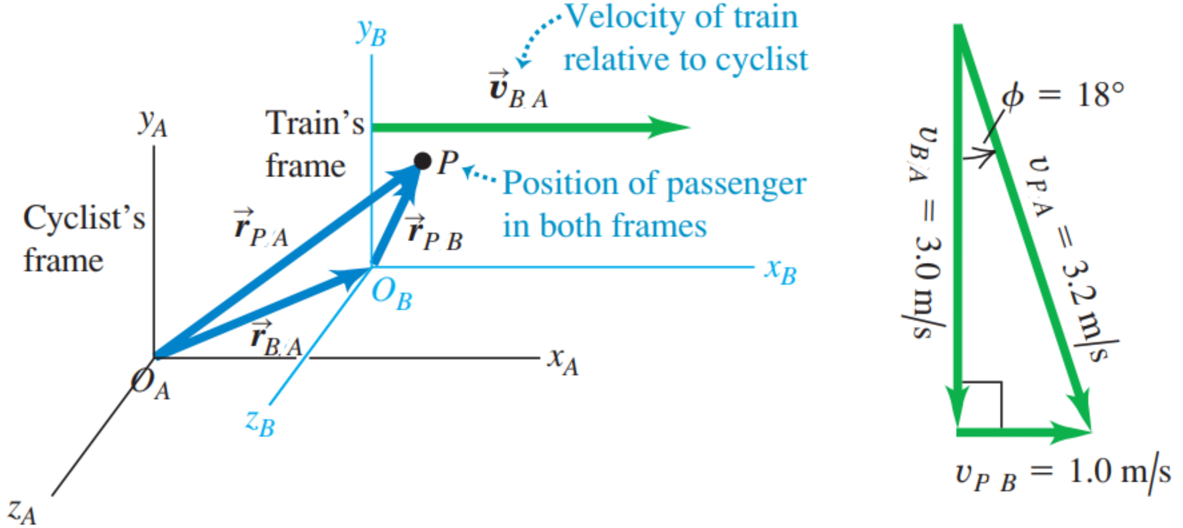
The position vectors of passenger P are $\boldsymbol{r}_{PA}$ relative to the origin of *A* and $\boldsymbol{r}_{PB}$ relative to the origin of *B*

$$\boldsymbol{r}_{PA} = \boldsymbol{r}_{PB} + \boldsymbol{r}_{BA}$$

By taking the time derivative:

$$\boldsymbol{v}_{PA} = \boldsymbol{v}_{PB} + \boldsymbol{v}_{BA}$$

$\boldsymbol{v}_{PA}$: is the velocity of the passenger P measured by observer A
$\boldsymbol{v}_{PB}$: is the velocity of the passenger P measured by observer B
$\boldsymbol{v}_{BA}$: is the velocity of the B measured by observer A



By taking the time derivative of this relation:

$$\boldsymbol{a}_{PA} = \boldsymbol{a}_{PB}$$

4.5 Relative Velocity and Relative Acceleration

Example:

An airplane's compass indicates that it is headed due north, and its airspeed indicator shows that it is moving through the air at 240 km/h. If there is a 100-km/h wind from west to east, what is the velocity of the airplane relative to the earth?

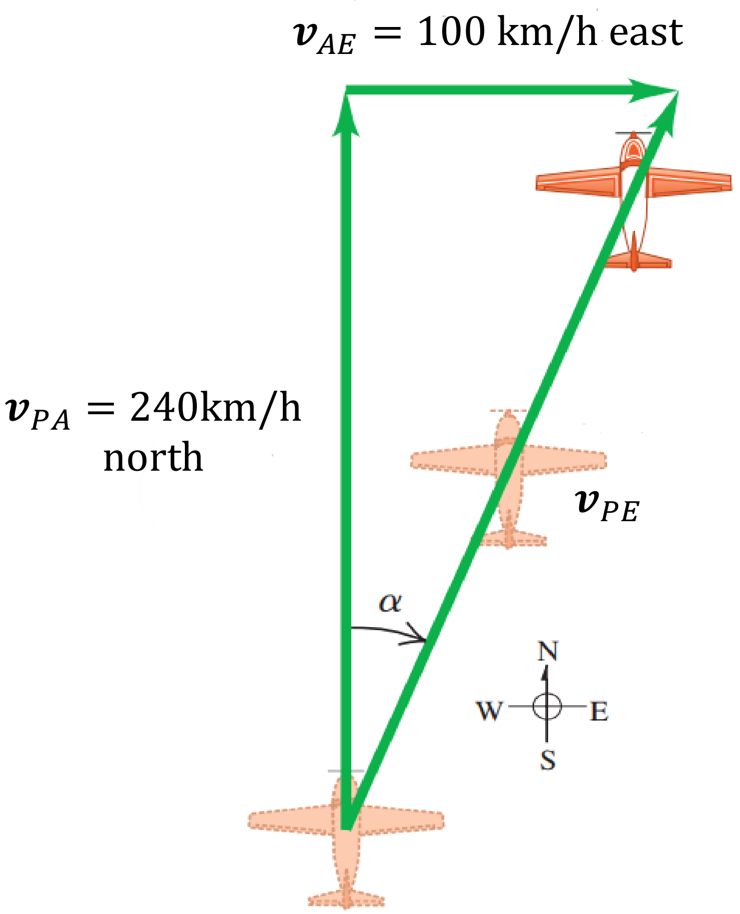
$$\boldsymbol{v}_{PA} = 240\ km/h \qquad \text{due north}$$

$$\boldsymbol{v}_{AE} = 100\ km/h \qquad \text{due east}$$

$$\boldsymbol{v}_{PE} = \boldsymbol{v}_{PA} + \boldsymbol{v}_{AE}$$

$$v_{PE} = \sqrt{(240\ km/h)^2 + (100\ km/h)^2} = 260\ km/h$$

$$\alpha = \arctan\left(\frac{100\ km/h}{40\ km/h}\right) = 23^0 east\ of\ north$$



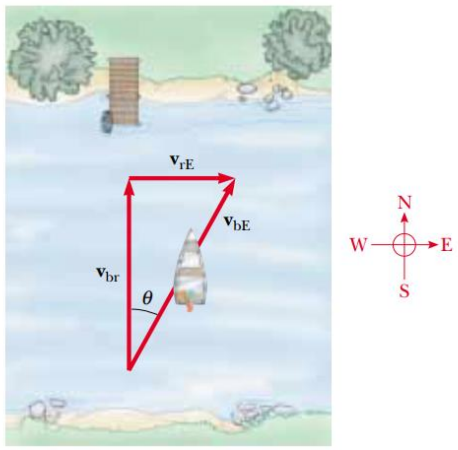


Figure 4.24 (Example 4.10) A boat aims directly across a river and ends up downstream.