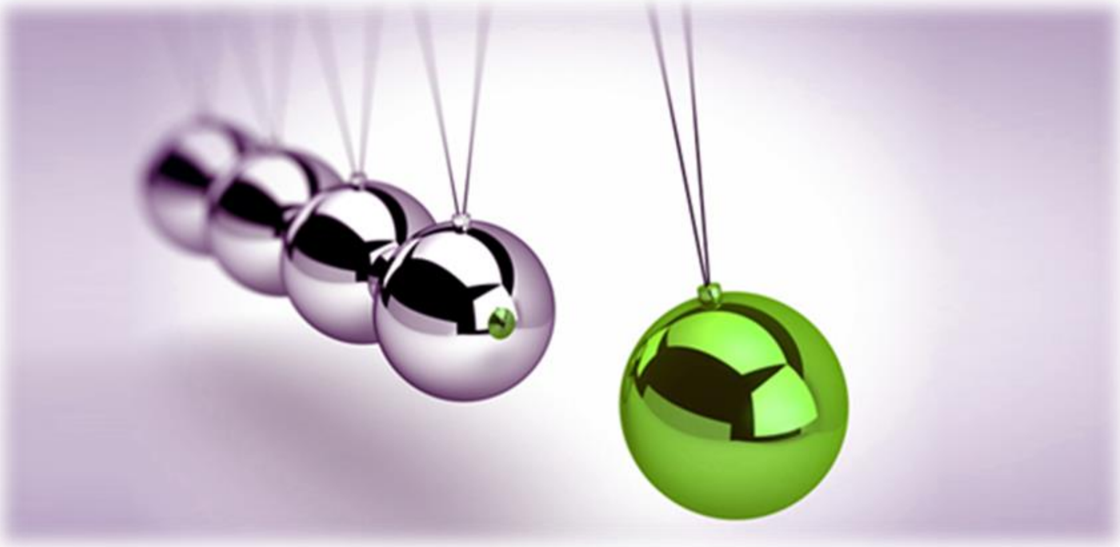
PHYSICS I - MECHANICS

SOME APPLICATIONS of NEWTON's LAW



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CHAPTER 6. Some Applications of Newton's Law

Learning Objectives

6.1 Using Newton’s First Law: Particles in Equilibrium

6.2 Using Newton’s Second Law: Dynamics of Particles

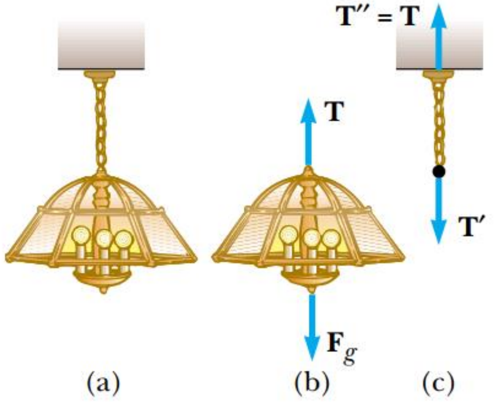
6.3 Frictional Forces

Some Applications of Newton's Law

When we apply Newton's laws to an object, we are interested only in external forces that act on the object.

When a rope attached to an object is pulling on the object, the rope exerts a force T on the object, and the magnitude T of that force is called the tension in the rope.

Objects in Equilibrium



If the acceleration of an object that can be modeled as a particle is zero, the particle is in **equilibrium**.

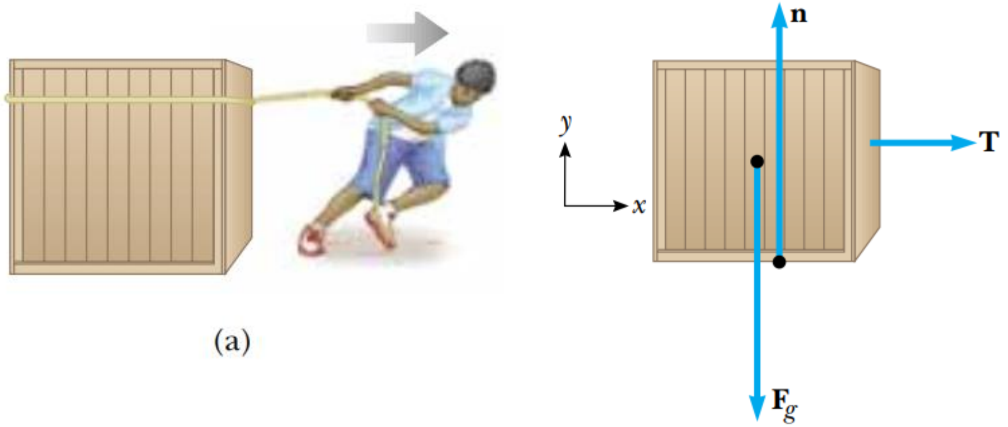
$$\sum F_y = T - F_g = 0 \qquad \text{or} \qquad T = F_g$$

Objects Experiencing a Net Force

Consider a crate being pulled to the right on a frictionless, horizontal surface and the magnitude of T is equal to the tension in the rope.

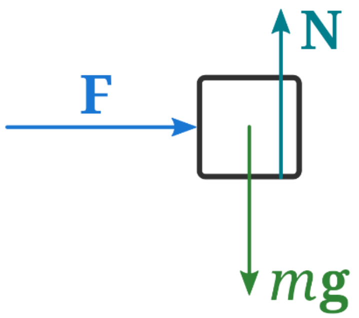
$$\sum F_x = T = ma_x \qquad \text{or} \qquad a_x = \frac{T}{m}$$

$$n + (-F_g) = 0 \qquad \text{or} \qquad n = F_g$$



Some Applications of Newton's Law

When dealing with the problem the following procedure is recommended:



- ✓ Draw the free body diagram of each object individually.
- ✓ Then determine the force exerting on the object.
- ✓ Find the components of each force along each of the body’s coordinate axes.
- ✓ If any acceleration component is zero, the particle is in equilibrium in this direction and $\Sigma F=0$.
- ✓ If not, the particle is undergoing an acceleration, the problem is one of nonequilibrium in this direction, and $\Sigma F=ma$.
- ✓ Set the sum of all x-components of force equal to zero or *ma*. In a separate equation, set the sum of all y-components equal to zero or *ma*.
- ✓ If there are two or more bodies, repeat all of the above steps for each body.

➢ *When we apply the Newton’s Law on a system, we usually neglect the mass of any ropes, strings, or cables and the friction as long as it is not given in the system.*

6.1 Using Newton's First Law: Particles in Equilibrium

In this section we consider only equilibrium of a body that can be modeled as a particle.

Newton's First Law:

An object at rest will stay at rest, and an object in motion will stay in motion at constant velocity, unless acted upon by an unbalanced force.

- ✓ In simpler terms, we can say that when no force acts on an object, the acceleration of the object is zero.
- ✓ If nothing acts to change the object's motion, then its velocity does not change.

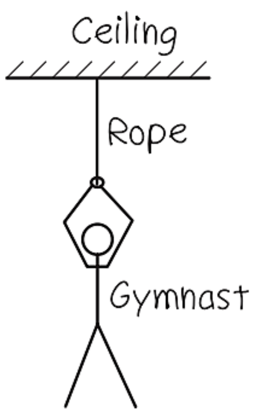
$$\sum \vec{F} = 0 \qquad \text{(body in equilibrium)}$$

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \text{(body in equilibrium)}$$

6.1 Using Newton's First Law: Particles in Equilibrium

Example: One-dimensional equilibrium: Tension in a massless rope

(a) The situation



A gymnast with mass m_G=50.0 kg suspends herself from the lower end of a hanging rope of negligible mass. The upper end of the rope is attached to ceiling.

a) What is the gymnast's weight?
b) What force (magnitude and direction) does the rope exert on her?
c) What is the tension at the top of the rope?

$$w_G = m_G g = (50.0\ \text{kg})(9.80\ \text{m/s}^2) = 490\ \text{N}$$

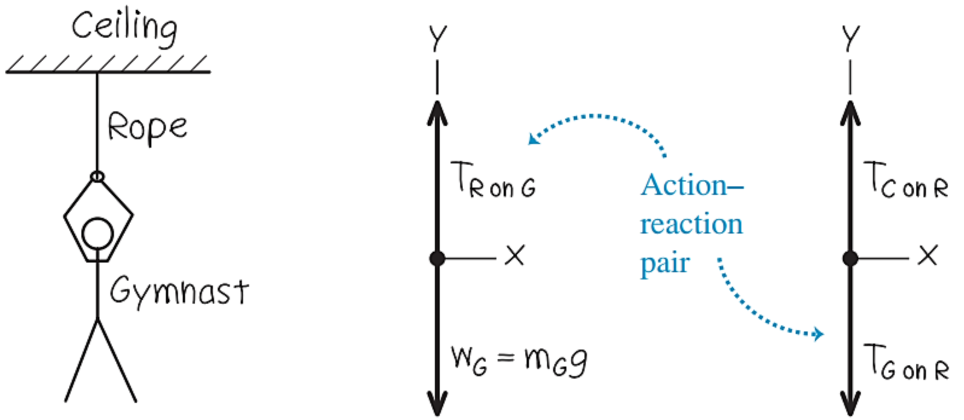
Gymnast: $\sum F_y = T_{\text{R on G}} + (-w_G) = 0$ so

$$T_{\text{R on G}} = w_G = 490\ \text{N}$$

Rope: $\sum F_y = T_{\text{C on R}} + (-T_{\text{G on R}}) = 0$ so

$$T_{\text{C on R}} = T_{\text{G on R}} = 490\ \text{N}$$

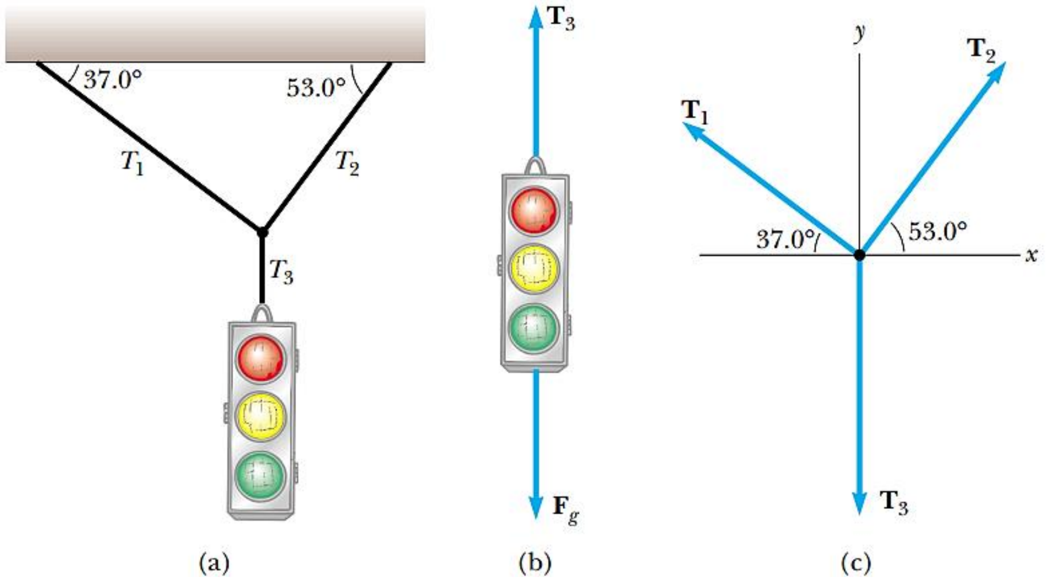
(a) The situation (b) Free-body diagram for gymnast (c) Free-body diagram for rope



6.1 Using Newton's First Law: Particles in Equilibrium

Example: A Traffic Light at Rest

A traffic light weighing 122 N hangs from a cable tied to two other cables fastened to a support, as in figure. The upper cables make angles of 37.0° and 53.0° with the horizontal. Find the tension for three cables?



Force	x Component	y Component
$\mathbf{T}_1$	$-T_1 \cos 37.0°$	$T_1 \sin 37.0°$
$\mathbf{T}_2$	$T_2 \cos 53.0°$	$T_2 \sin 53.0°$
$\mathbf{T}_3$	0	-122 N

(1) $$\sum F_x = -T_1 \cos 37.0° + T_2 \cos 53.0° = 0$$

(2) $$\sum F_y = T_1 \sin 37.0° + T_2 \sin 53.0° + (-122 \text{ N}) = 0$$

(3) $$T_2 = T_1\left(\frac{\cos 37.0°}{\cos 53.0°}\right) = 1.33 T_1$$

$$T_1 \sin 37.0° + (1.33 T_1)(\sin 53.0°) - 122 \text{ N} = 0$$

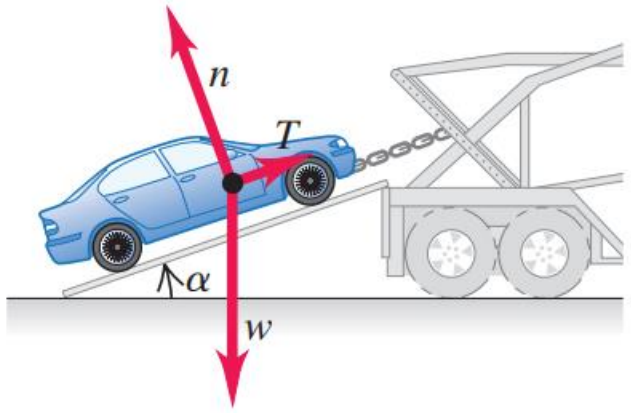
$$T_1 = 73.4 \text{ N}$$

$$T_2 = 1.33 T_1 = 97.4 \text{ N}$$

6.1 Using Newton’s First Law: Particles in Equilibrium

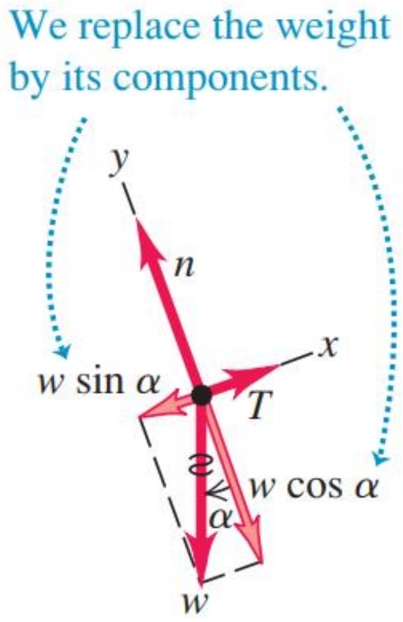
Example: An Inclined Plane

A car of weight w rests on a slanted ramp attached to a trailer. Only a cable running from the trailer to the car prevents the car from rolling off the ramp. (The car’s brakes are off and its transmission is in neutral.) Find the **tension in the cable** and **the force that the ramp exerts on the car’s tires**.



$$\sum F_x = T + (-w \sin \alpha) = 0$$
$$\sum F_y = n + (-w \cos \alpha) = 0$$

$$T = w \sin \alpha$$
$$n = w \cos \alpha$$

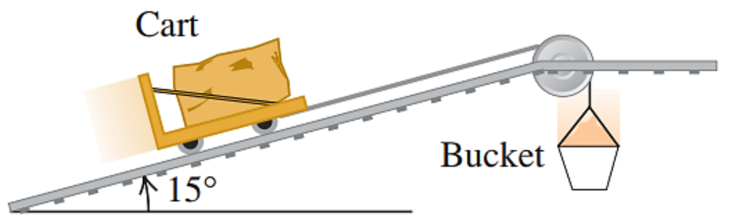


6.1 Using Newton's First Law: Particles in Equilibrium

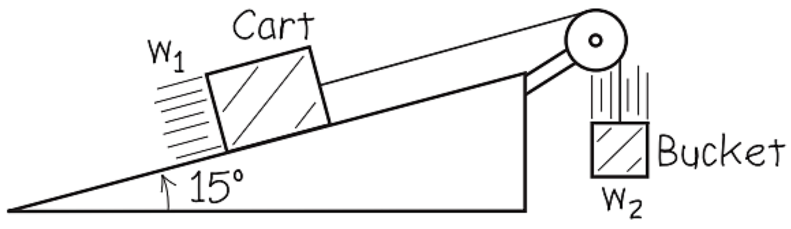
Example: Equilibrium of bodies connected by cable and pulley

Blocks of granite are to be hauled up a 15° slope out of a quarry, and dirt is to be dumped into the quarry to fill up old holes. To simplify the process, you design a system in which a granite block on a cart with steel wheels (weight w_1 including both block and cart) is pulled uphill on steel rails by a dirt-filled bucket (weight w_2 including both dirt and bucket) that descends vertically into the quarry. How must the weights w_1 and w_2 be related in order for the system to move with constant speed? Ignore friction in the pulley and wheels, and ignore the weight of the cable.

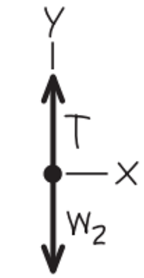
(a) Dirt-filled bucket pulls cart with granite block



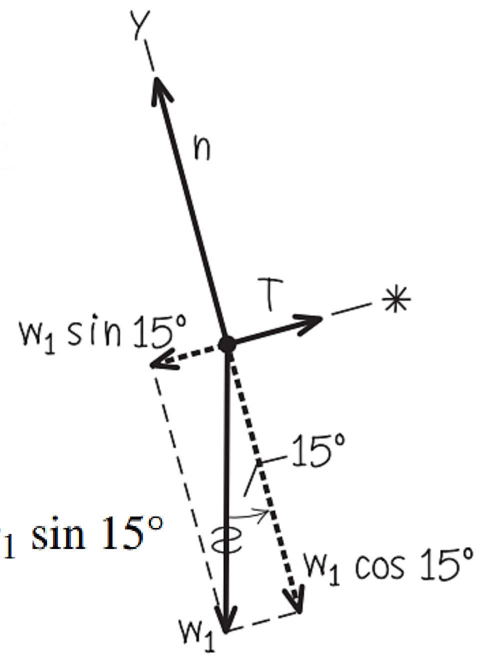
(b) Idealized model of the system



(c) Free-body diagram for bucket



(d) Free-body diagram for cart



$$\sum F_y = 0$$

$$\sum F_x = 0$$

$$\sum F_y = T + (-w_2) = 0 \quad \text{so} \quad T = w_2$$

$$\sum F_x = T + (-w_1 \sin 15°) = 0 \quad \text{so} \quad T = w_1 \sin 15°$$

$$w_2 = w_1 \sin 15° = 0.26 w_1$$

6.2 Using Newton's Second Law: Dynamics of Particles

We are now ready to discuss dynamics problems. In these problems, we apply Newton's second law to bodies on which the net force is not zero. These bodies are not in equilibrium and hence are accelerating.

***Newton's second law*:**

If a net external force acts on a body, the body accelerates. The direction of acceleration is the same as the direction of the net force. The mass of the body times the acceleration of the body equals the net force Vector.

$$\sum \vec{\boldsymbol{F}} = m\vec{\boldsymbol{a}} \qquad \text{(Newton's second law of motion)}$$

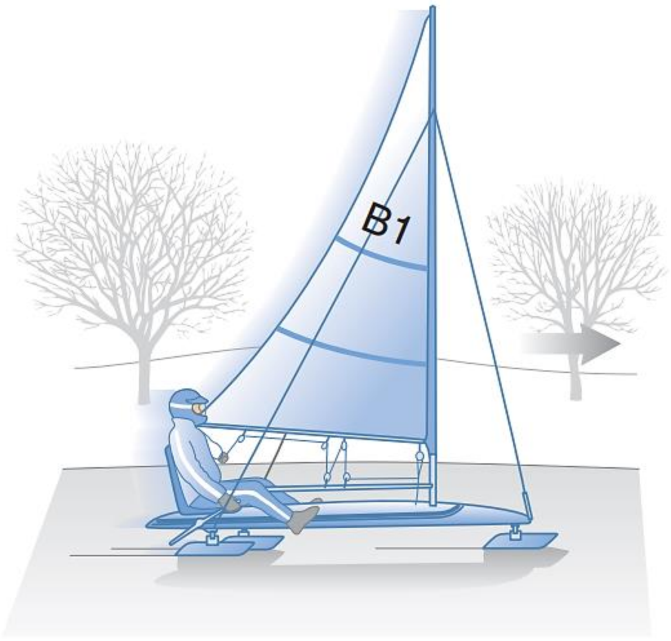
$$\sum F_x = ma_x \qquad \sum F_y = ma_y \qquad \text{(Newton's second law, component form)}$$

6.2 Using Newton's Second Law: Dynamics of Particles

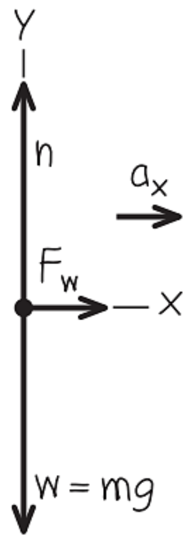
Example: Straight-line motion with a constant force

An iceboat is at rest on a frictionless horizontal surface. A wind is blowing along the direction of the runners so that 4.0 s after the iceboat is released, it is moving at 6 m/s (about 22 km/h or 13 mi/h). What constant horizontal force F_W does the wind exert on the iceboat? The combined mass of iceboat and rider is 200 kg.

(a) Iceboat and rider on frictionless ice



(b) Free-body diagram for iceboat and rider



$$\sum F_x = F_W = ma_x$$
$$\sum F_y = n + (-mg) = 0 \quad \text{so} \quad n = mg$$

$$v_x = v_{0x} + a_x t$$

$$a_x = \frac{v_x - v_{0x}}{t} = \frac{6.0\text{ m/s} - 0\text{ m/s}}{4.0\text{ s}} = 1.5\text{ m/s}^2$$

$$F_W = ma_x = (200\text{ kg})(1.5\text{ m/s}^2) = 300\text{ kg}\cdot\text{m/s}^2$$

Since $1\text{ kg}\cdot\text{m/s}^2 = 1\text{ N}$, the final answer is

$$F_W = 300\text{ N}$$

6.2 Using Newton's Second Law: Dynamics of Particles

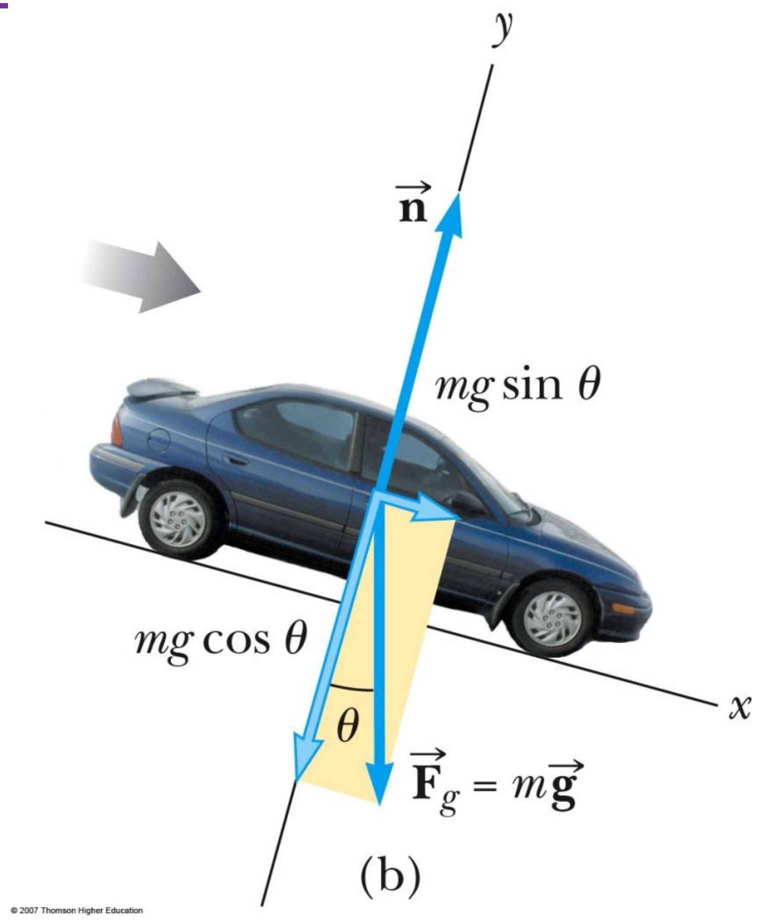
Example: The Runaway Car

A car of mass m is on an icy driveway inclined at an angle θ, as in figure.
a) Find the acceleration of the car, assuming that the driveway is frictionless.

$$\sum F_x = mg \sin\theta = ma_x$$

$$\sum F_y = n - mg\cos\theta = 0$$

$$a_x = \boxed{g \sin\theta}$$



b) Suppose the car is released from rest at the top of the incline, and the distance from the car's front bumper to the bottom of the incline is *d*. How long does it take the front bumper to reach the bottom, and what is the car's speed as it arrives there?

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2 \qquad x_i = 0 \qquad x_f = d \qquad v_{xi} = 0$$

$$v_{xf}^{\,2} = 2a_x d$$

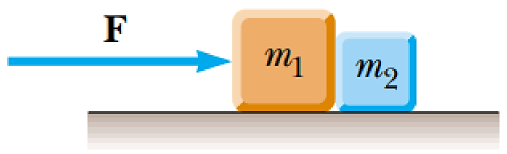
$$t = \sqrt{\frac{2d}{a_x}} = \boxed{\sqrt{\frac{2d}{g\sin\theta}}}$$

$$v_{xf} = \sqrt{2a_x d} = \boxed{\sqrt{2gd\sin\theta}}$$

6.2 Using Newton's Second Law: Dynamics of Particles

Example: One Block Pushes Another

Two blocks of masses m_1 and m_2, with $m_1 > m_2$, are placed in contact with each other on a frictionless, horizontal surface, as in figure. A constant horizontal force **F** is applied to m_1 as shown.



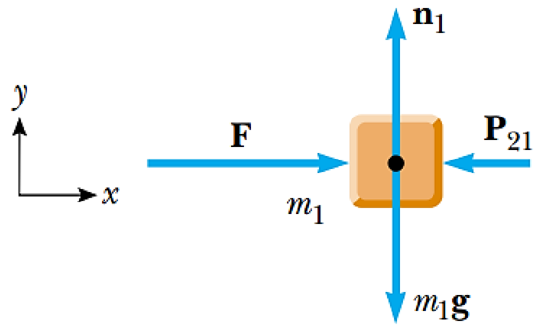
(a)

(A) Find the magnitude of the acceleration of the system.

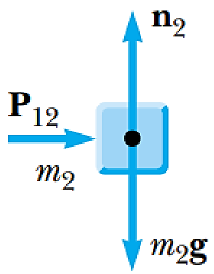
$$\sum F_x(\text{system}) = F = (m_1 + m_2)a_x$$

$$a_x = \boxed{\frac{F}{m_1 + m_2}}$$

(B) Determine the magnitude of the contact force between the two blocks.



(b)



(c)

$$\sum F_x = P_{12} = m_2 a_x$$

$$P_{12} = m_2 a_x = \boxed{\left(\frac{m_2}{m_1 + m_2}\right)F}$$

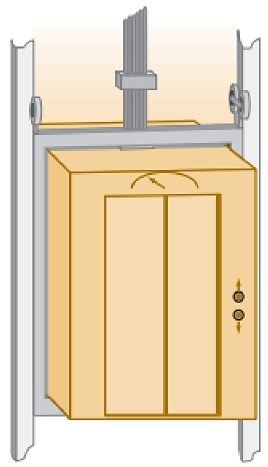
$$\sum F_x = F - P_{21} = F - P_{12} = m_1 a_x$$

$$P_{12} = F - m_1 a_x = F - m_1\left(\frac{F}{m_1 + m_2}\right) = \left(\frac{m_2}{m_1 + m_2}\right)F$$

6.2 Using Newton's Second Law: Dynamics of Particles

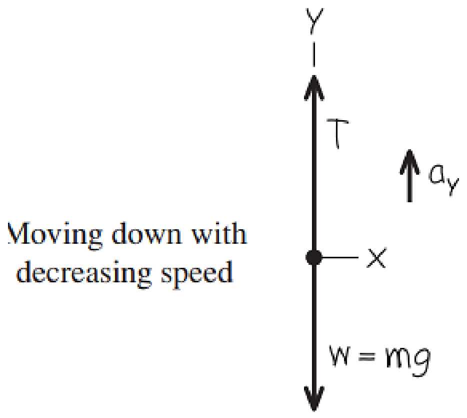
Example: Tension in an elevator cable

(a) Descending elevator



An elevator and its load have a combined mass of 800 kg. The elevator is initially moving downward at 10.0 m/s; it slows to a stop with constant acceleration in a distance of 25.0 m. What is the tension T in the supporting cable while the elevator is being brought to rest?

(b) Free-body diagram for elevator



The elevator is moving downward with decreasing speed, so its acceleration is upward; we chose the positive y-axis to be upward

$$v_{0y} = -10.0\ \text{m/s} \qquad y - y_0 = -25.0\ \text{m}$$

$$v_y = 0$$

$$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$$

$$a_y = \frac{v_y^2 - v_{0y}^2}{2(y - y_0)} = \frac{(0)^2 - (-10.0\ \text{m/s})^2}{2(-25.0\ \text{m})} = +2.00\ \text{m/s}^2$$

$$\sum F_y = T + (-w) = ma_y$$

$$w = mg = (800\ \text{kg})(9.80\ \text{m/s}^2) = 7840\ \text{N}$$

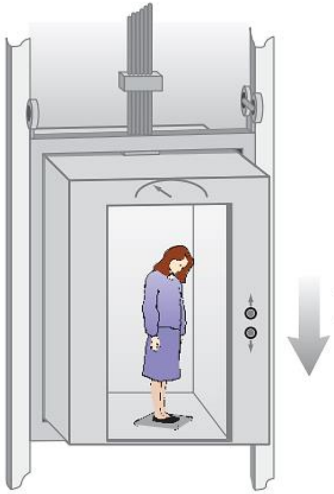
$$T = m(g + a_y) = (800\ \text{kg})(9.80\ \text{m/s}^2 + 2.00\ \text{m/s}^2)$$
$$= 9440\ \text{N}$$

6.2 Using Newton's Second Law: Dynamics of Particles

Example: Apparent weight in an accelerating elevator

A 50.0-kg woman stands on a bathroom scale while riding in the elevator in the previous example. What is the reading on the scale?

(a) Woman in a descending elevator



$$w = mg = (50.0\ \text{kg})(9.80\ \text{m/s}^2) = 490\ \text{N}$$

$$\begin{aligned} \sum F_y &= n + (-mg) = ma_y \\ n &= mg + ma_y = m(g + a_y) \\ &= (50.0\ \text{kg})(9.80\ \text{m/s}^2 + 2.00\ \text{m/s}^2) = 590\ \text{N} \end{aligned}$$

The scale reads 590 N, which is 100 N more than her actual weight. The scale reading is called the passenger's **apparent weight**.

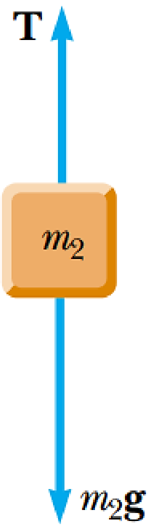
What would the woman feel if the elevator were accelerating *downward,* $a_y = -2.00\ \text{m/s}^2$

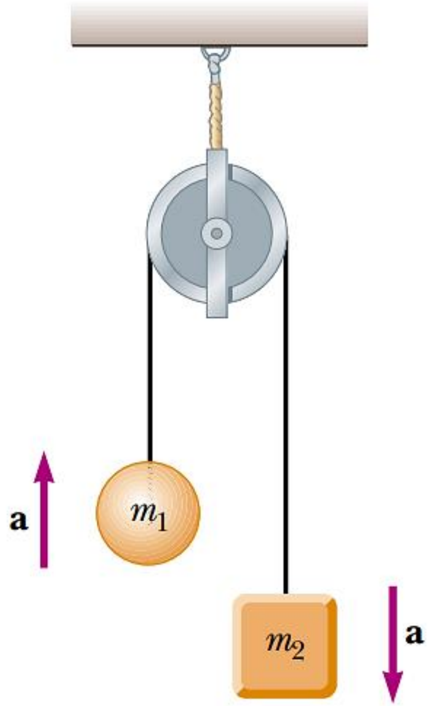
$$\begin{aligned} n &= m(g + a_y) = (50.0\ \text{kg})[9.80\ \text{m/s}^2 + (-2.00\ \text{m/s}^2)] \\ &= 390\ \text{N} \end{aligned}$$

6.2 Using Newton's Second Law: Dynamics of Particles

Example:The Atwood Machine

When two objects of unequal mass are hung vertically over a frictionless pulley of negligible mass, as in figure, the arrangement is called an Atwood machine. The device is sometimes used in the laboratory to measure the free-fall acceleration. Determine the magnitude of the acceleration of the two objects and the tension in the lightweight cord.





$$(1) \qquad \sum F_y = T - m_1 g = m_1 a_y$$

$$(2) \qquad \sum F_y = m_2 g - T = m_2 a_y$$

$$-m_1 g + m_2 g = m_1 a_y + m_2 a_y$$

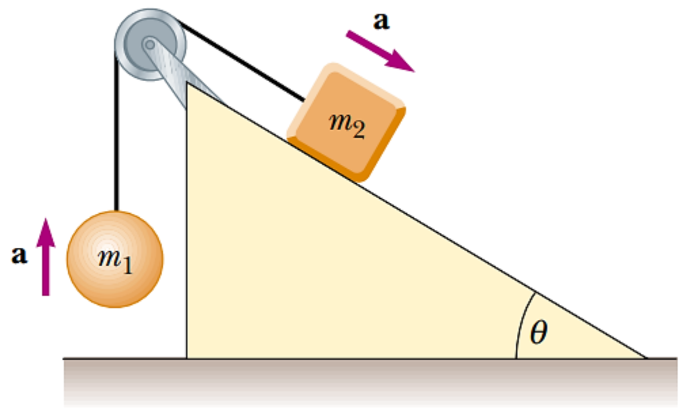
$$(3) \qquad a_y = \boxed{\left(\frac{m_2 - m_1}{m_1 + m_2}\right) g}$$

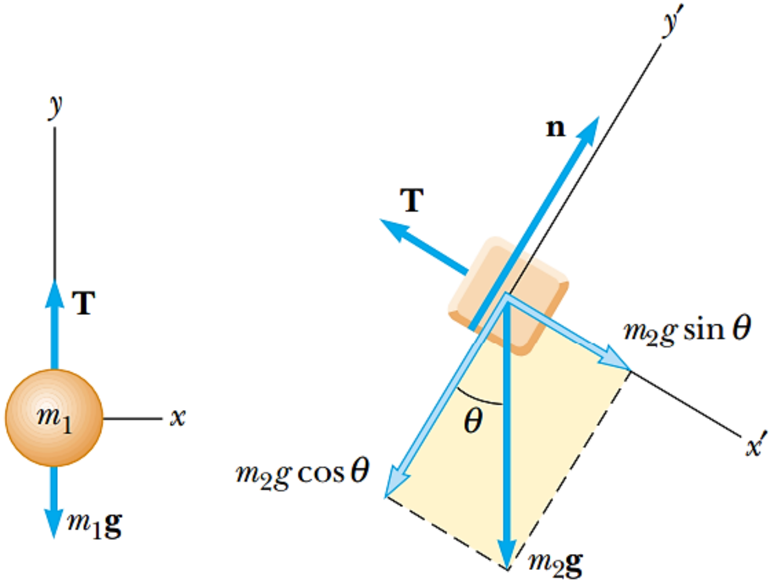
$$(4) \qquad T = \boxed{\left(\frac{2 m_1 m_2}{m_1 + m_2}\right) g}$$

6.2 Using Newton's Second Law: Dynamics of Particles

Example: Acceleration of Two Objects Connected by a Cord

A ball of mass m_1 and a block of mass m_2 are attached by a lightweight cord that passes over a frictionless pulley of negligible mass, as in figure. The block lies on a frictionless incline of angle θ. Find the magnitude of the acceleration of the two objects and the tension in the cord.





$$(1) \qquad \sum F_x = 0$$

$$(2) \qquad \sum F_y = T - m_1 g = m_1 a_y = m_1 a$$

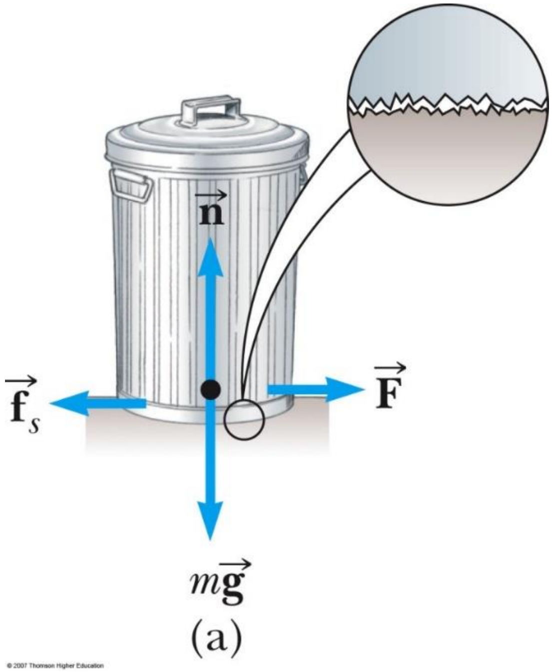
$$(3) \qquad \sum F_{x'} = m_2 g \sin\theta - T = m_2 a_{x'} = m_2 a$$

$$(4) \qquad \sum F_{y'} = n - m_2 g \cos\theta = 0$$

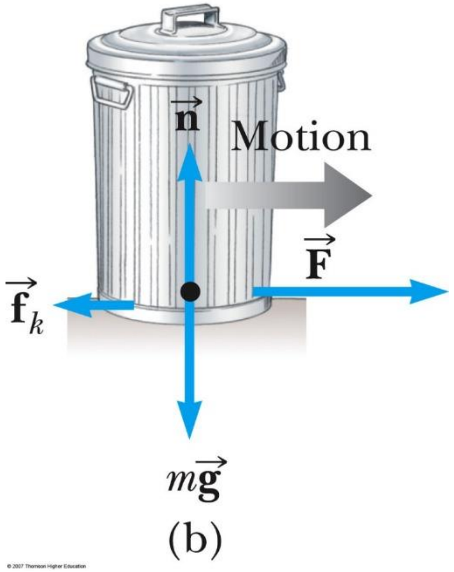
$$(5) \qquad a = \boxed{\frac{m_2 g \sin\theta - m_1 g}{m_1 + m_2}} \qquad\qquad (6) \qquad T = \boxed{\frac{m_1 m_2 g(\sin\theta + 1)}{m_1 + m_2}}$$

6.3 Frictional Forces

When an object is in motion on a surface or through a viscous medium, there will be a resistance to the motion. This resistance is called the ***force of friction.***



Static Friction



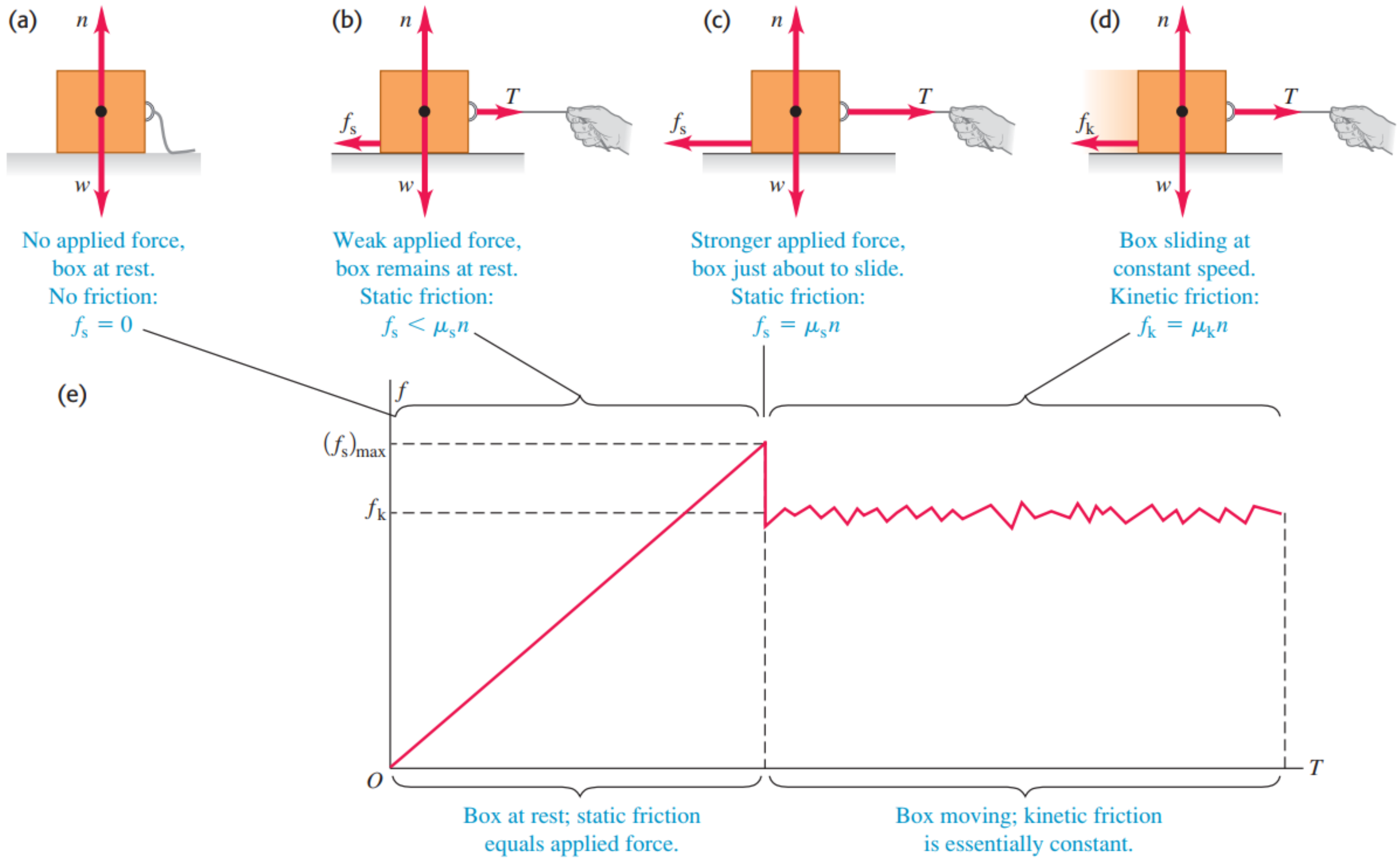
Kinetic Friction

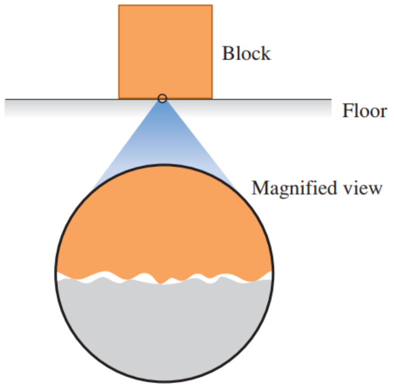
Static friction acts to keep the object from moving
If $\vec{F}$ increases, so does
If $\vec{F}$ decreases, so does
$f_s \leq \mu_s n$

The force of kinetic friction acts when the object is in motion
Although μ_k can vary with speed, we shall neglect any such variations
$f_k = \mu_k n$

μ is the **coefficient of friction**

6.3 Frictional Forces





On a microscopic level, even smooth surfaces are rough; they tend to catch and cling.

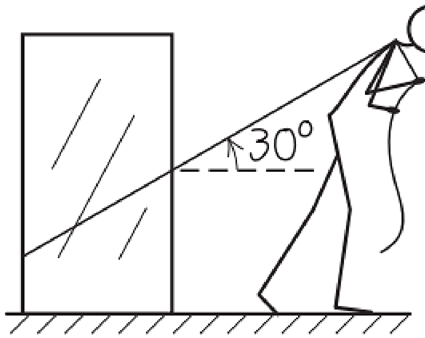
a), (b), (c) When there is no relative motion, the magnitude of the static friction force f_s is less than or equal to $\mu_s n$ (d) When there is relative motion, the magnitude of the kinetic friction force f_k equals $\mu_k n$ (e) A graph of the friction force magnitude f as a function of the magnitude T of the applied force. The kinetic friction force varies somewhat as intermolecular bonds form and break.

6.3 Frictional Forces

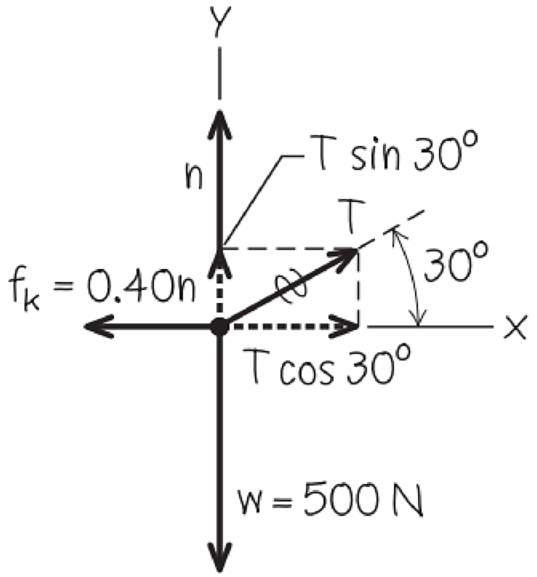
Example:

Suppose you move the crate by pulling upward on the rope at an angle of 30 ° above the horizontal. How hard must you pull to keep it moving with constant velocity? Assume that μ_k=0.40.

(a) Pulling a crate at an angle



$$\sum F_x = T\cos 30° + (-f_k) = 0 \quad \text{so} \quad T\cos 30° = \mu_k n$$
$$\sum F_y = T\sin 30° + n + (-w) = 0 \quad \text{so} \quad n = w - T\sin 30°$$



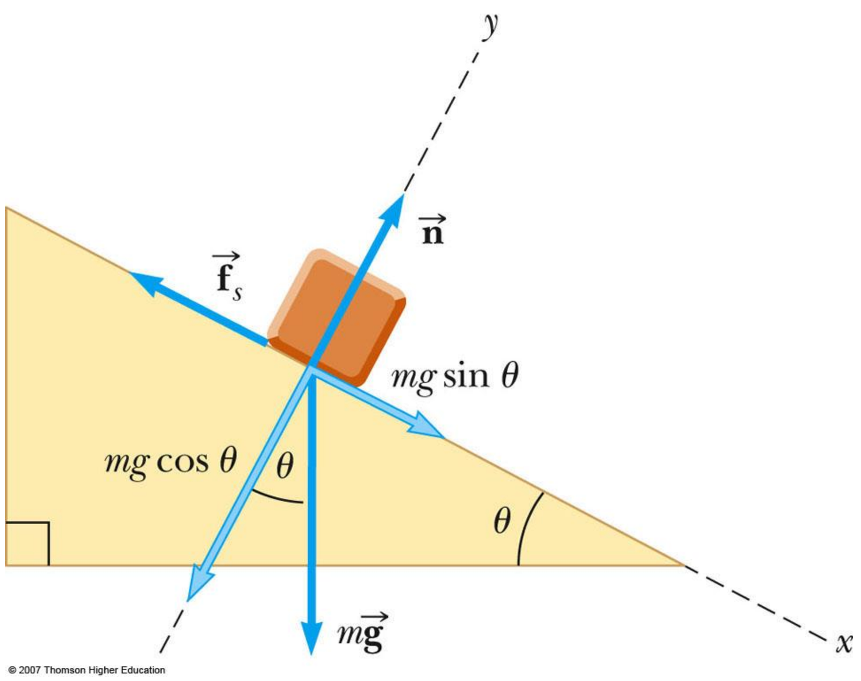
$$T\cos 30° = \mu_k(w - T\sin 30°)$$
$$T = \frac{\mu_k w}{\cos 30° + \mu_k \sin 30°} = 188\text{ N}$$

$$n = w - T\sin 30° = (500\text{ N}) - (188\text{ N})\sin 30° = 406\text{ N}$$

6.3 Frictional Forces

Example:

- The block is sliding down the plane, so friction acts up the plane
- This setup can be used to experimentally determine the coefficient of friction



$$\sum F_x = mg\sin\theta - f_s = ma_x = 0$$

$$\sum F_y = n - mg\cos\theta = ma_y = 0$$

$$mg = n/\cos\theta$$

$$f_s = mg\sin\theta = \left(\frac{n}{\cos\theta}\right)\sin\theta = n\tan\theta$$

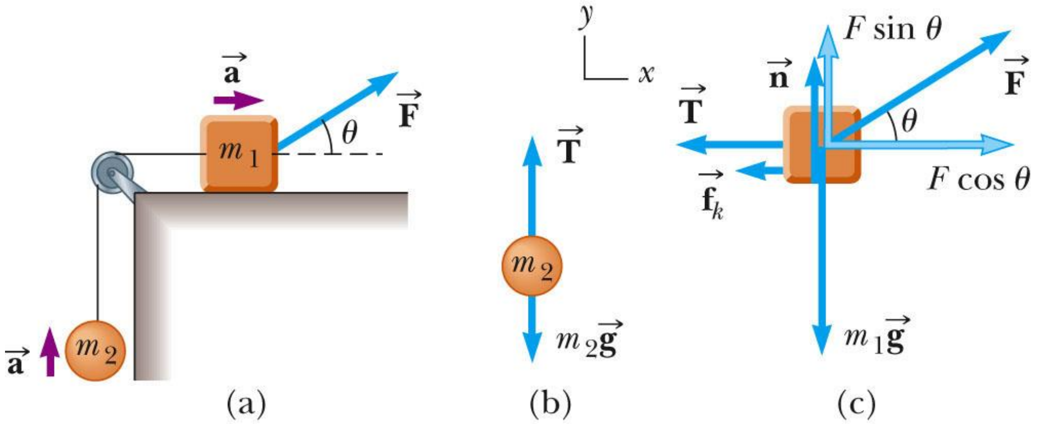
$$\mu_s n = n\tan\theta_c$$

the block starts to move at $\theta \geq \theta_c$ it accelerates down the incline

6.3 Frictional Forces

Example:

A force of magnitude F at an angle θ with the horizontal is applied to the block as shown. The coefficient of kinetic friction between the block and surface is μ_k. Determine the magnitude of the acceleration of the two objects.



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Motion of block: (1) $\sum F_x = F\cos\theta - f_k - T = m_1 a_x = m_1 a$

(2) $\sum F_y = n + F\sin\theta - m_1 g = m_1 a_y = 0$

Motion of ball: $\sum F_x = m_2 a_x = 0$

(3) $\sum F_y = T - m_2 g = m_2 a_y = m_2 a$

$$f_k = \mu_k(m_1 g - F\sin\theta)$$

$$a = \frac{F(\cos\theta + \mu_k \sin\theta) - g(m_2 + \mu_k m_1)}{m_1 + m_2}$$