

1.2. Combining Functions; Shifting and Scaling Graphs

Let's look at the main ways functions are combined or transformed to form new functions.

- Like numbers, functions can be added, subtracted, multiplied, and divided (except where the denominator is zero) to produce new functions. If f and g are functions, then for every x that belongs to the domains of both f and g (that is, for $x \in D_f \cap D_g$), we define functions $f + g$, $f - g$ and $f \cdot g$ by the formulas:

$$(f + g)(x) = f(x) + g(x),$$

$$(f - g)(x) = f(x) - g(x),$$

$$(f \cdot g)(x) = f(x) \cdot g(x).$$

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- At any point of $D_f \cap D_g$ at which $g(x) \neq 0$, we can also define the function f/g by the formula $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$ where $g(x) \neq 0$.

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- Functions can also be multiplied by constants: if c is a real number, then the function cf is defined for all x in the domain of f by

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- **Example 9.** Let $f(x) = x$ and $g(x) = x - 1$, sketch the graphs of these functions and consider the graph of $f + g$.

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- **Definition** If f and g are functions, the **composite function** $f \circ g$ (" f composed with g ") is defined by

$$(f \circ g)(x) = f(g(x))$$

The domain of $f \circ g$ consists of the numbers x in the domain of g for which $g(x)$ lies in the domain of f .

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- **Example 10.** Let $f(x) = x^2$ and $g(x) = 1 - \sqrt{x}$, find $(f \circ g)(x)$, $(g \circ f)(x)$, $(f \circ f)(x)$ and $(g \circ g)(x)$.

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- **Vertical Shifts** $y = f(x) + k$ Shifts the graph of f up k units if $k > 0$, shifts the graph of f down $|k|$ units if $k < 0$.

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- **Horizontal Shifts** $y = f(x + h)$ Shifts the graph of f left h units if $h > 0$, shifts the graph of f right $|h|$ units if $h < 0$.

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- **Example 11.** If the real function f is given with the rule $f(x) = -x^2 + 4x - 3$ sketch the graph of the curve $y = f(x) + 3$.

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- **Example 11.** If the real function f is given with the rule $f(x) = -x^2 + 4x - 3$ sketch the graph of the curve $y = f(x) + 3$.
- **Example 12.** Sketch the graph of

$$g(x) = (x - 2)^3 \text{ and } h(x) = (x + 1)^3$$

by using the graph of $f(x) = x^3$.

1.2. Combining Functions; Shifting and Scaling Graphs

- **Scaling and Reflecting a Graph of a Function:** To scale the graph of a function $y = f(x)$ is to stretch or compress it, vertically and horizontally. This is accomplished by multiplying the function f , or the independent variable x , by an appropriate constant c . Reflections across the coordinate axes are special cases where $c = -1$.

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- **For $c > 1$ the graph is scaled:**
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- $y = f(cx) \rightarrow$ Compresses the graph of f horizontally by a factor of c .
- $y = f\left(\frac{x}{c}\right) \rightarrow$ Stretches the graph of f horizontally by a factor of c .

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- **For $c = -1$ the graph is reflected:**
- $y = -f(x) \longrightarrow$ Reflects the graph of f across the x -axis.
- $y = f(-x) \longrightarrow$ Reflects the graph of f across the y -axis.

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Example 13. Sketch the graph of $y = 2f(x)$, $y = \frac{1}{2}f(x)$, $y = f(2x)$, $y = f(\frac{1}{2}x)$, $y = -f(x)$ and $y = f(-x)$ where $f(x) = x^2 + 2x$.

Exercises related with 1.1 and 1.2

- **1.** Find the domain of the following functions;

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- (a) $f(x) = \sqrt{1 - |x|}$,
- (b) $g(x) = \sqrt{|x - 1|} - 2$,
- (c) $h(x) = \sqrt{|x| + 4}$

Exercises related with 1.1 and 1.2

- (d) $t(x) = \sqrt{1 - [x]}$

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- (e) $s(x) = \frac{x}{[x]}$
- (f) $r(x) = \sqrt{x - [x]}$

Exercises related with 1.1 and 1.2

- $(g) \ f(x) = \frac{2^x + x}{[x] [x + 2]}$

Exercises related with 1.1 and 1.2

- (g) $f(x) = \frac{2^x + x}{\lfloor x \rfloor \lfloor x + 2 \rfloor}$
- (h) $g(x) = \frac{1}{\lfloor x - 2 \rfloor} + \sqrt{4 - x^2}$

Exercises related with 1.1 and 1.2

- (g) $f(x) = \frac{2^x + x}{\lfloor x \rfloor \lfloor x + 2 \rfloor}$
- (h) $g(x) = \frac{1}{\lfloor x - 2 \rfloor} + \sqrt{4 - x^2}$
- (i) $h(x) = \frac{\sqrt{9 - |2x + 1|}}{\lfloor \frac{x}{3} - 1 \rfloor}$

Exercises related with 1.1 and 1.2

- (j) $f(x) = \operatorname{sgn} \left(\frac{x-2}{\sqrt{[x]^2 - 9}} \right)$

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- (j) $f(x) = \operatorname{sgn}\left(\frac{x-2}{\sqrt{[x]^2-9}}\right)$
- (k) $s(x) = \operatorname{sgn}\left(\frac{x-2}{x+2}\right)$

Exercises related with 1.1 and 1.2

- (j) $f(x) = \operatorname{sgn} \left(\frac{x-2}{\sqrt{\lfloor x \rfloor^2 - 9}} \right)$
- (k) $s(x) = \operatorname{sgn} \left(\frac{x-2}{x+2} \right)$
- (l) $r(x) = \frac{|x| \operatorname{sgn} x}{\lfloor x \rfloor}$

Exercises related with 1.1 and 1.2

- $(m) \ g(x) = \sqrt{\operatorname{sgn}(x+2)}$

Exercises related with 1.1 and 1.2

- (m) $g(x) = \sqrt{\operatorname{sgn}(x+2)}$
- (n) $h(x) = \sqrt{x|x|}$

Exercises related with 1.1 and 1.2

- (m) $g(x) = \sqrt{\operatorname{sgn}(x+2)}$
- (n) $h(x) = \sqrt{x|x|}$
- (o) $t(x) = \frac{\sqrt{\operatorname{sgn}(x+1)}}{\lfloor x-2 \rfloor}$

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- **2.** Sketch the graph of the following functions;

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- (a) $f(x) = 2|x - 1| + 3|x + 1|$

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- (a) $f(x) = 2|x - 1| + 3|x + 1|$
- (b) $g(x) = |x + 2| + 1$

Exercises related with 1.1 and 1.2

- **2.** Sketch the graph of the following functions;
- (a) $f(x) = 2|x - 1| + 3|x + 1|$
- (b) $g(x) = |x + 2| + 1$
- (c) $h(x) = \left\lfloor \frac{x}{2} \right\rfloor$, over $[-2, 2]$

Exercises related with 1.1 and 1.2

- (d) $s(x) = x^2 + 6x + 5$

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- (d) $s(x) = x^2 + 6x + 5$
- (e) $y = |s(x)|$

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- (d) $s(x) = x^2 + 6x + 5$
- (e) $y = |s(x)|$
- (f) $y = s(|x|)$

Exercises related with 1.1 and 1.2

- (d) $s(x) = x^2 + 6x + 5$
- (e) $y = |s(x)|$
- (f) $y = s(|x|)$
- (g) $y = |s(|x|)|$

Exercises related with 1.1 and 1.2

- $(h) \ f(x) = \operatorname{sgn}(x^2 + x - 6)$

Exercises related with 1.1 and 1.2

- (h) $f(x) = \operatorname{sgn}(x^2 + x - 6)$
- (i) $g(x) = \lfloor |x| \rfloor$, over $[-2, 2]$

Exercises related with 1.1 and 1.2

- (j) $f(x) = x + |x| + \operatorname{sgn}(|x|)$

Exercises related with 1.1 and 1.2

- (j) $f(x) = x + |x| + \operatorname{sgn}(|x|)$
- (k) $s(x) = |x + 1| - x \operatorname{sgn}(x - 2)$

Exercises related with 1.1 and 1.2

- (I) $r(x) = \lfloor |x| \rfloor$, over $[-2, 2]$

Exercises related with 1.1 and 1.2

- (l) $r(x) = \lfloor |x| \rfloor$, over $[-2, 2]$
- (m) $g(x) = x^2 + \operatorname{sgn}(x - 1)$

Exercises related with 1.1 and 1.2

- (n) $h(x) = |x^2 - 1| + \operatorname{sgn}(x^2 - 1)$

Exercises related with 1.1 and 1.2

- (n) $h(x) = |x^2 - 1| + \operatorname{sgn}(x^2 - 1)$
- (o) $t(x) = \frac{\lfloor x \operatorname{sgn} x \rfloor}{\lfloor x - 2 \rfloor}$, over $[-2, 2]$

Exercises related with 1.1 and 1.2

- **3.** Use the graph of the function $f(x) = x^2 - 2x - 3$ and sketch the graph of the following;

$$\begin{array}{llll} y = 2f(x), & y = f(x-1), & y = f(x+1), & y = -f(x) \\ y = -2f(x), & y = f(x) + 3, & y = f(x) - 1 & y = f(x-1) + 1 \end{array}$$