

Type Systems

Question 1. Consider the the following language definition based on WHILE:

```
(statements) s ::= x := e | s1; s2
(expression) e ::= c | x | e1 + e2 | e1 * e2
(integer constant) c
(integer variable) x
(parity) p ::= even | odd
```

Develop a type system for tracking the parity (even vs. odd) of integer expressions in this language. Assume that environment A is a map from each variable to even, odd, or top. Add the eight missing rules, each of which has judgments of the form $A \vdash e : p$, meaning that “under environment A , expression e has parity p .” You may assume the following statements:

$\frac{[c \text{ is an odd constant}]}{A \vdash c : \text{odd}}$	$\frac{[c \text{ is an even constant}]}{A \vdash c : \text{even}}$	$\frac{}{A \vdash x : A(x)}$
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In your answer, write each rule in the following form, where each [] is filled in with an appropriate statement.

$$\frac{[\quad] \quad [\quad]}{[\quad]}$$

a. Four rules for ‘ $e1 + e2$ ’:

b. Four rules for ‘ $e1 * e2$ ’: