DESIGN OF PID

Goal → To design a PID controller with active feedback
•9 control loop for angular movement of a dc motor

•10 Problem → No specs given

•11 Assumptions → small general purpose dc motor and typical values.

•12

Armature resistance $(R) = 1\Omega$
•1 Armature inductance $(L) = 0.5H$
Torque constant $(K_i) = 0.01\ Nm/A$
•2 Back EMF constant $(K_e) = 0.01\ Vs/rad$
Rotor inertia $(J) = 0.01\ kg\,m^2$
•3 Viscous friction coeff. $(B) = 0.1\ Nms/rad$

•4 We are trying to model how the motor responds to voltage input & $V(t)$ and produces angular position as output $\theta(t)$

•5

$\boxed{\text{Input } V(t)} \xrightarrow{i(t)} \boxed{\text{Armature}} \rightarrow \boxed{\text{Torque}} \rightarrow \boxed{\text{Inertia + Friction}}$
•6 ↳ Angular position

DC Motor has two interacting systems:
- Electrical current (through armature coil)
- mechanical rotation of the shaft.

ELECTRICAL DYNAMICS [Behaves as RL circuit with Back EMF] (KVL)

$$V(t) = L\frac{d\,i(t)}{dt} + R\,i(t) + e_b(t)$$

$$\left\{ e_b(t) = K_e \frac{d\theta(t)}{dt} \right\}$$

$$P(s) = \frac{\theta(s)}{V(s)} \leftrightarrow \frac{O/P}{I/P}$$

MECHANICAL DYNAMICS:- Torque = Inertia × Ang. Acc.

$$T(t) = J\frac{d^2\theta(t)}{dt^2} + B\frac{d\theta(t)}{dt}$$

$$\boxed{T(t) = k_i\, i(t)}$$

Combining the two equations by eliminating $i(t)$

Laplace transform (IC = 0)

$$V(s) = LsI(s) + RI(s) + k_b\,\omega(s)$$

$$k_t I(s) = Js\,\omega(s) + B\,\omega(s)$$

$$I(s) = \frac{Js+B}{k_t}\,\omega(s)$$

$$V(s) = [Ls+R]\,\frac{Js+B}{k_t}\,\omega(s) + k_b\,\omega(s)$$

$$\boxed{\frac{\omega(s)}{s} = \theta(s)}$$

$$\frac{\omega(s)}{V(s)} = \frac{k_t}{(Ls+R)(Js+B)+k_b k_t}$$

$$\frac{\theta(s)}{V(s)} = \frac{k_t}{s[(Ls+R)(Js+B)+k_b k_t]}$$

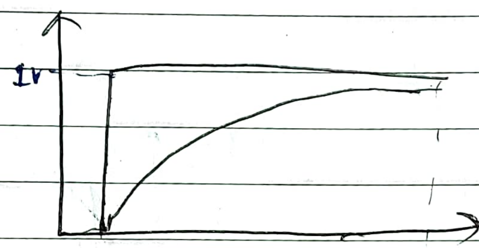
$$= \frac{0.01}{s(0.005s^2+0.06s+0.100))}$$

M	T	W	T	F	S	S	
				1	2	3	N
4	5	6	7	8	9	10	O
11	12	13	14	15	16	17	V
18	19	20	21	22	23	24	
25	26	27	28	29	30		'19

In Simulink :-

•9 • DC motor model (plant)
• PID block
•10 • Setpoint (refrence input)
• Feedback loop
•11 • Scope to Visualise Results.

•12 Sim I



T = 50 s

•1 Obs : overdamped

•2 Probable causes :-

$k_p = 1$
$k_i = 1$
$kd = 0$

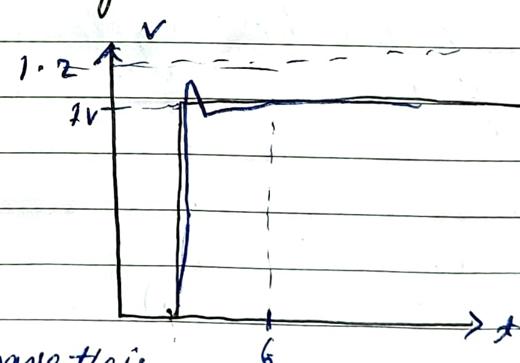
• k_p too low
•3 • k_d high or zero
• k_i not contributing enough

•4

SII # After 1 hour of mindnumbing trial and error

•5

$Kp = 200$
•6 $Ki = 100$
$Kd = \cdot 8$



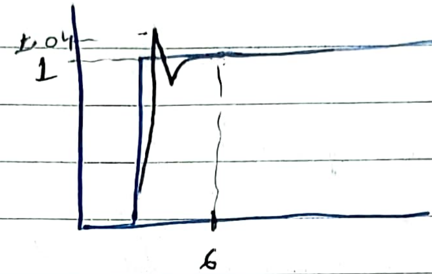
But to such high value have their own cost and tend to be impractical

STIII

$K_p = 100$

$k_i = 100$

•9 $k_d = 4$



•10 Peak 1.04 v

Settling time = 6s

•11

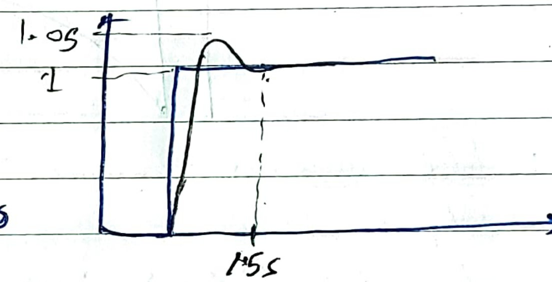
•12 STIV Finally!!

•1 $K_p = 100$

$K_i = 180$

•2 $K_d = 5$

N (filter coefficient) = 100



•3

Settling time = 1.5s

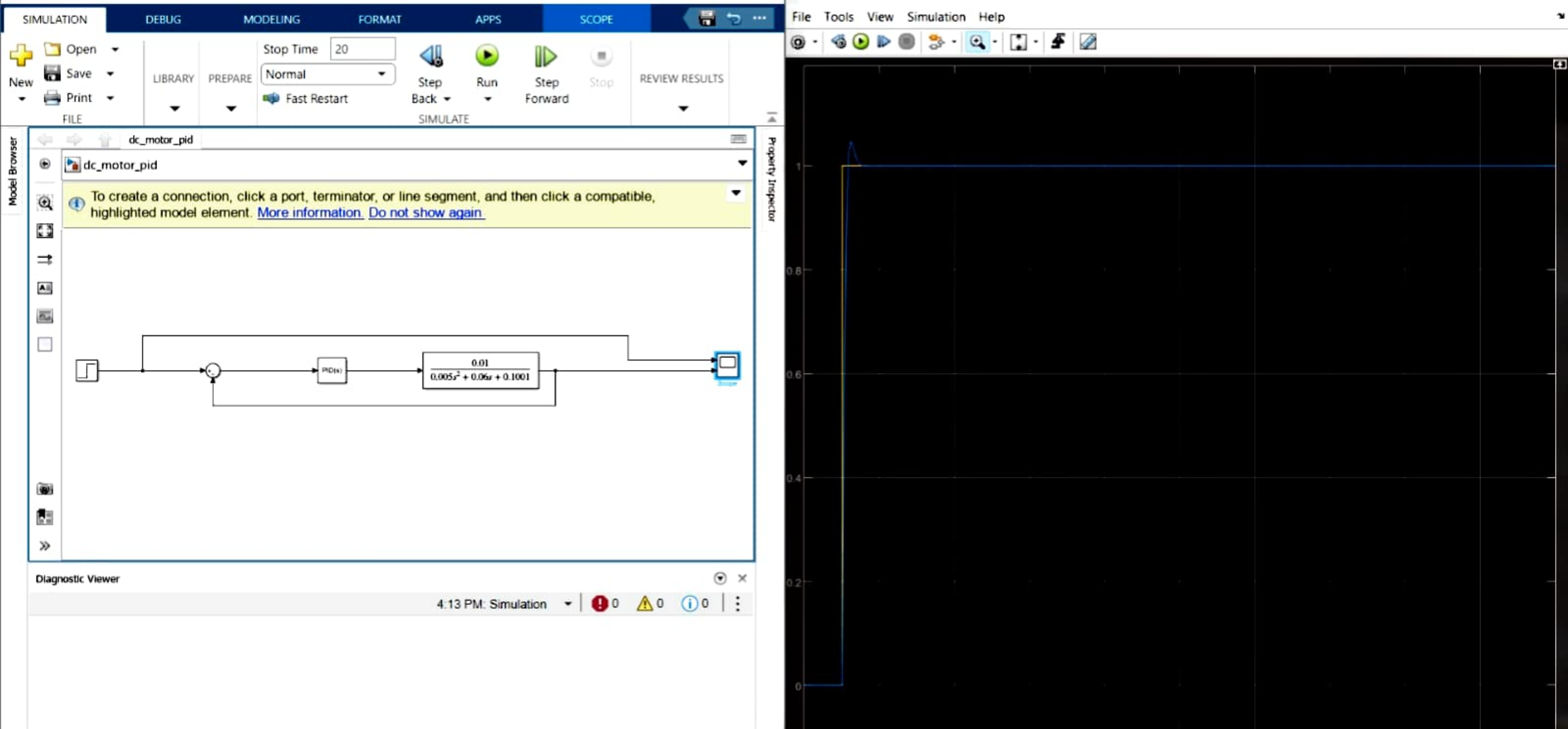
•4 Peak overshoot = 1.05V

•5 One of the best system designed by me but there are tradeoffs.

•6

PROS:

- Fast Settling without being erratic or aggressive
- Minimal overshoot (≈ 5%)
- Doesnot rely solely on massive brute force (High K values)


SIMULATION
DEBUG
MODELING
FORMAT
APPS
SCOPE
New
Open
Save
Print
FILE
LIBRARY
PREPARE
Stop Time
20
Normal
Fast Restart
Step Back
Run
Step Forward
Stop
SIMULATE
REVIEW RESULTS
dc_motor_pid
To create a connection, click a port, terminator, or line segment, and then click a compatible, highlighted model element. More information. Do not show again
Model Browser
Property Inspector
PID(s)
$\frac{0.01}{0.005s^2 + 0.06s + 0.1001}$
Diagnostic Viewer
4:13 PM: Simulation
0
0
0
File
Tools
View
Simulation
Help
1
0.8
0.6
0.4
0.2
0