



**Western Cape  
Government**  
Education

**Western Cape Education Department**

**Telematics  
Learning Resource 2020**

**MATHEMATICS  
Grade 12**

## Dear Grade 12 Learner

In 2020 there will be 2 Telematics sessions on grade 12 content, 2 Telematics sessions on grade 11 content and 1 Telematics session on grade 10 content. In grade 12 in the June, September and end of year examination the grade 10 and 11 content will be assessed. It is thus important that you compile a study timetable which will consider the revision of the grade 10 and 11 content. The program in this book reflects the dates and times for all grade 10 - 12 Telematics sessions. It is highly recommended that you attend all Telematics sessions, this will support you with the revision of grade 10 and 11 content. This workbook however only have the material for the grade 12 Telematics sessions. The grade 10 and 11 material you will be able to download from the Telematics website. Please make sure that you bring this workbook along to each of the grade 12 Telematics sessions.

In the grade 12 examination,

- Trigonometry Graphs will be  $\pm 15$  marks of the 150 marks of Paper 2 and
- Probability will be  $\pm 15$  marks of the 150 marks of Paper 1



<http://bit.ly/2t3iicg>

There are altogether 6 proofs of theorems you must know because it could be examined. These theorems are clearly indicated in "MATHEMATICS EXAMINATION GUIDELINE Grade 12": Four are grade 11 theorems and two are grade 12 theorems. In the same document there are the acceptable shortened versions for each of the theorems. Ensure that you use these shortened versions as reasons when solving Geometry questions.

At school you should receive a book called "Grade 12 Tips for Success". In it you will have a breakdown of the weighting of the various Topics in Mathematics. Ensure that you download a QR reader, this will enable you to scan the various QR codes.

At the start of each lesson, the presenters will provide you with a summary of the important concepts and together with you will work through the activities. You are encouraged to come prepared, have a pen and enough paper (ideally a hard cover exercise book) and your scientific calculator with you.

You are also encouraged to participate fully in each lesson by asking questions and working out the exercises, and where you are asked to do so, sms or e-mail your answers to the studio.

Remember: "Success is not an event, it is the result of regular and consistent hard work".

GOODLUCK, Wishing you all the success you deserve!

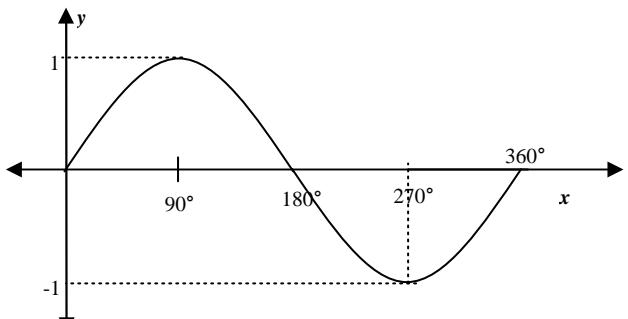
## 2020 Mathematics Telematics Program

Day	Date	Time		Mathematics	Topic
<b>Term 1: 15 Jan – 20 March</b>					
Tuesday	10 March	16:00 – 17:00	12	Mathematics	Trigonometry Graphs
Wednesday	11 March	16:00 – 17:00	12	Wiskunde	Trigonometrie Grafieke
<b>Term 2: 31 March to 12 June</b>					
Tuesday	21 April	15:00 – 16:00	11	Mathematics	Inequalities
Tuesday	21 April	16:00 – 17:00	11	Wiskunde	Ongelykhede
Wednesday	22 April	16:00 – 17:00	10	Mathematics	Functional Notation, Domain & Range
Thursday	23 April	16:00 – 17:00	10	Wiskunde	Funksionele Notasie, Omvang & Terrein
<b>Term 3: 7 July – 18 September</b>					
Wednesday	29 July	16:00 – 17:00	12	Mathematics	Probability
Thursday	30 July	16:00 – 17:00	12	Wiskunde	Waarskynlikheid
<b>Term 4: 29 September – 2 Desember</b>					
Tuesday	27 Oktober	15:00 – 16:00	11	Mathematics	Statistics
	27 Oktober	16:00 – 17:00	11	Wiskunde	Statistiek

# Trigonometric Graphs

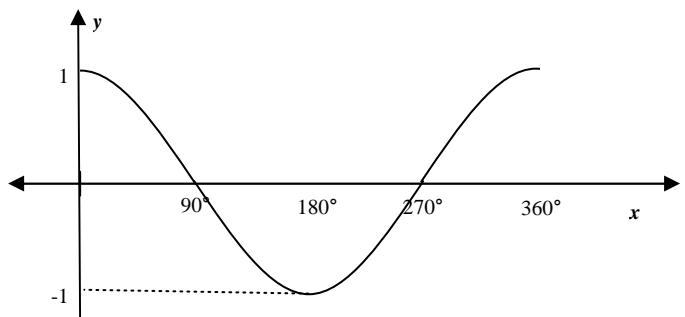
- There are three basic trigonometric graphs which you should be able to sketch. Below are the three sketches on the interval  $x \in [0^\circ ; 360^\circ]$ . These three trigonometric graphs you should be able to sketch on the interval  $x \in [-360^\circ ; 360^\circ]$ .

$$y = \sin x$$



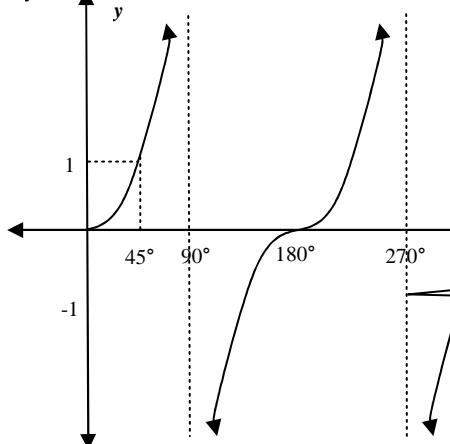
Maximum Value = 1  
Minimum Value = -1  
Domain =  $[0; 360^\circ]$   
Range =  $[-1; 1]$   
Amplitude = 1  
Period =  $360^\circ$

$$y = \cos x$$



Maximum Value = 1  
Minimum Value = -1  
Domain =  $[0^\circ; 360^\circ]$   
Range =  $[-1; 1]$   
Amplitude = 1  
Period =  $360^\circ$

$$y = \tan x$$



This graph does not have a Maximum or Minimum Value  
Domain =  $[0; 360^\circ]$   
Range = "real line"  
Period =  $180^\circ$   
Asymptotes at  $90^\circ$  And  $270^\circ$

Tan graph is undefined at the asymptotes

Sketching Trig Graphs



<http://bit.ly/2QL08oA>

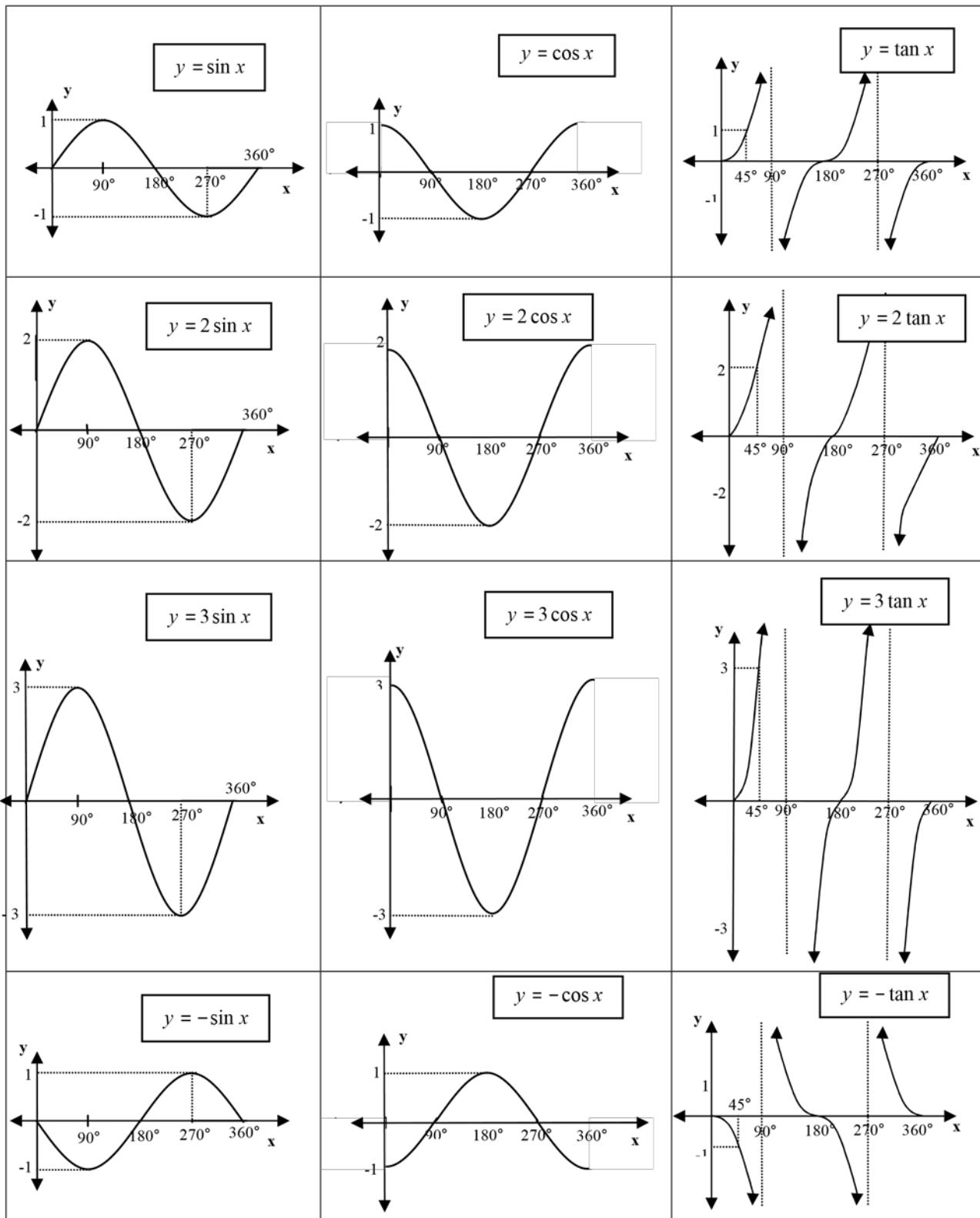
CAST Diagram: Trig Graphs



<http://bit.ly/2QQnJVb>

## Effect of "a" on trigometric graphs:

- In the table below are the graphs of  $y=a \sin x$ ,  $y=a \cos x$  and  $y=a \tan x$  for different values of "a".

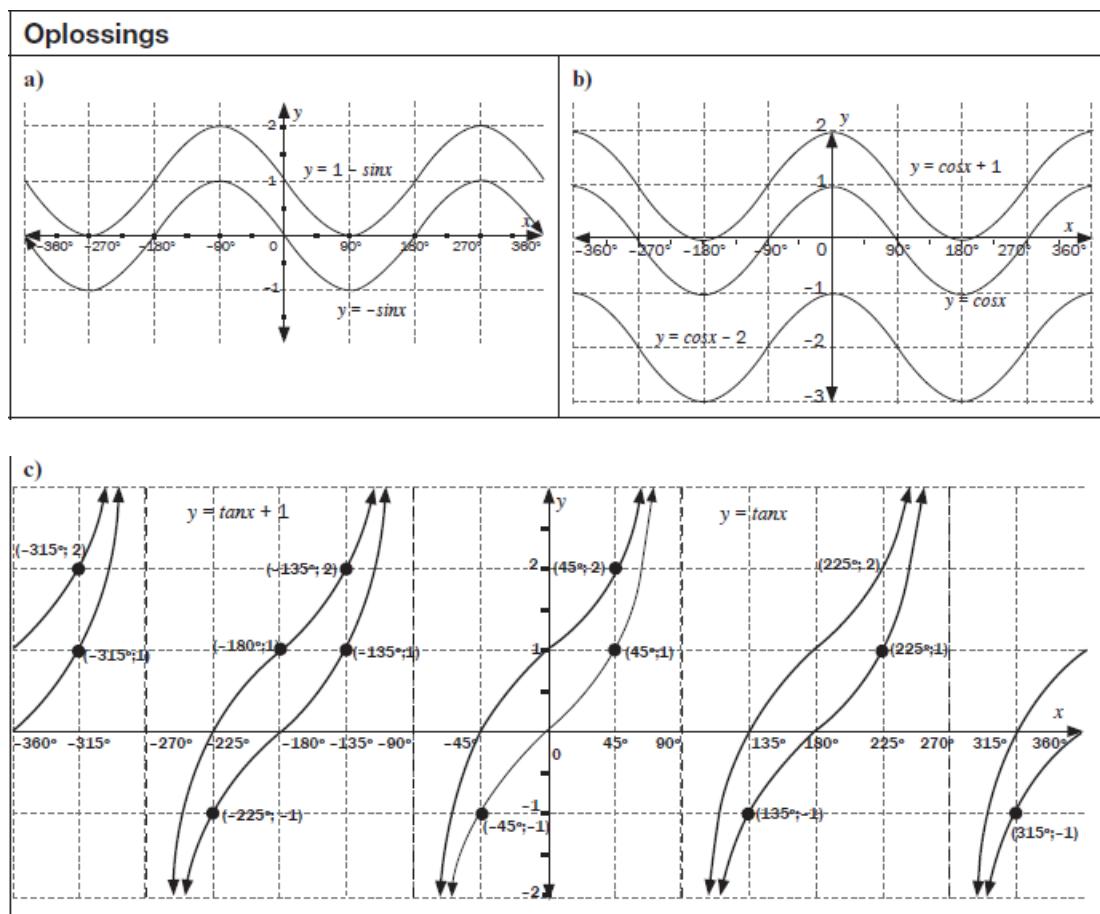


## The effect of “ $q$ ” on trigonometric graphs:

Examine the graphs:  $y = \sin x + q$ ,  $y = \cos x + q$  and  $y = \tan x + q$ .

Sketch the following graphs on the same system of axes with domain  $[-360^\circ; 360^\circ]$ :

- a)  $y = -\sin x$  en  $y = -\sin x + 1$
- b)  $y = \cos x$ ,  $y = \cos x + 1$ ,  $y = \cos x - 2$
- c)  $y = \tan x$  en  $y = \tan x + 1$

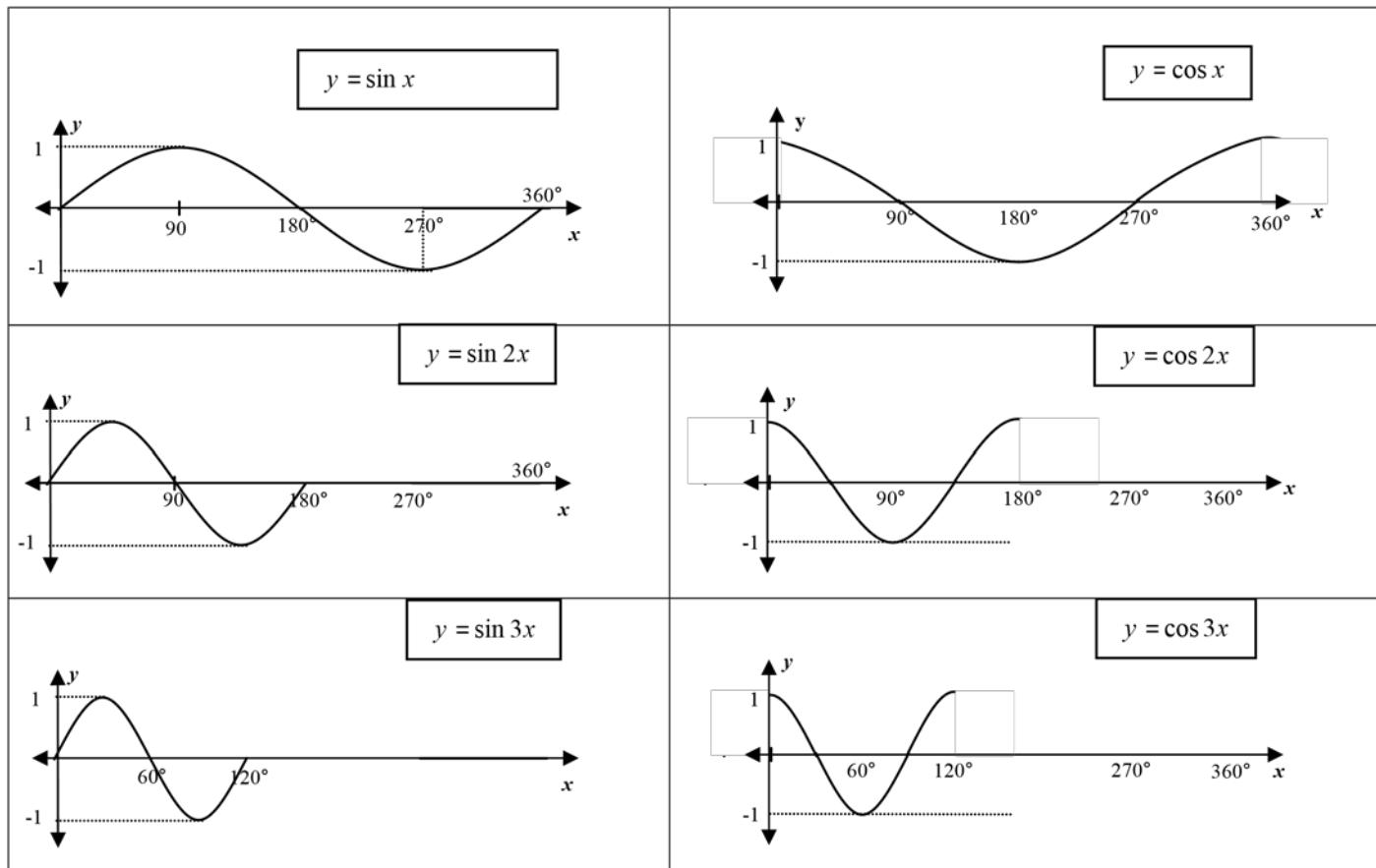


### Conclusion:

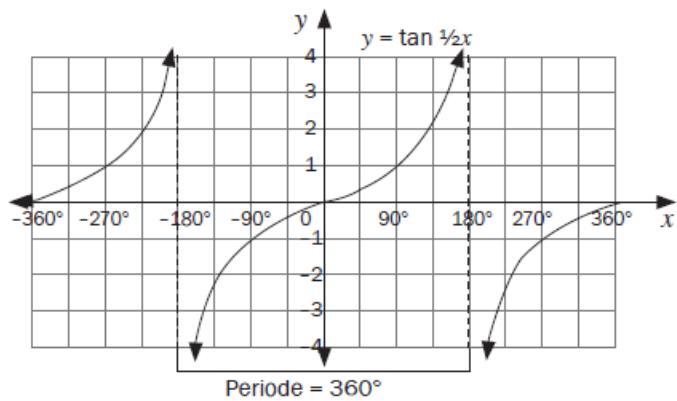
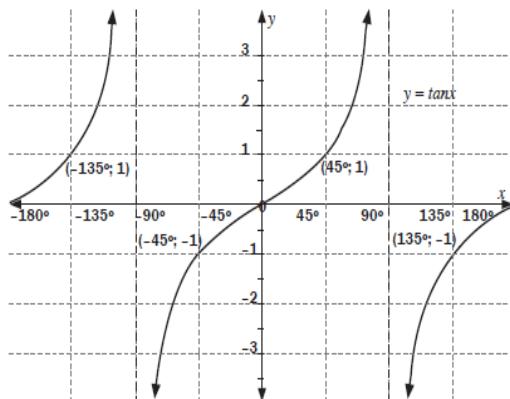
The parameter  $q$  shifts the graph  $q$  units up or down.

## The effect of “ $k$ ” on trigonometric graphs:

- In the table below are the graphs of:  $y = \sin kx$  and  $y = \cos kx$  for different values of “ $k$ ”.



- Below are the graphs of,  $y = \tan kx$ , for two different values of “ $k$ ”.



## The effect of “ $p$ ” on trigonometric graphs:

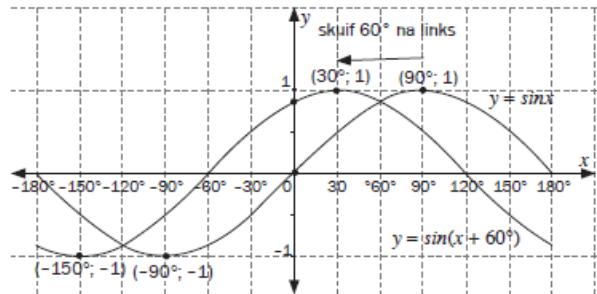
Examine the graphs:  $y = \sin(x + p)$ ,  $y = \cos(x + p)$  and  $y = \tan(x + p)$ .

1. Sketch each of the following on the same system of axes for  $x \in [-180^\circ, 180^\circ]$ :

- a)  $y = \sin x$  and  $y = \sin(x + 60^\circ)$
- b)  $y = \cos x$  and  $y = \cos(x - 45^\circ)$
- c)  $y = \tan x$  and  $y = \tan(x + 45^\circ)$

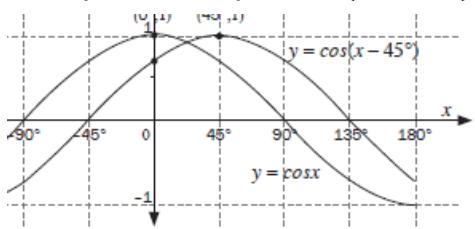
Solutions:

1. a)  $y = \sin x$ ;  $y = \sin(x + 60^\circ)$



The graph  $y = \sin(x + 60^\circ)$ , is the same as the graphs of  $y = \sin x$  shifted  $60^\circ$  to the left.

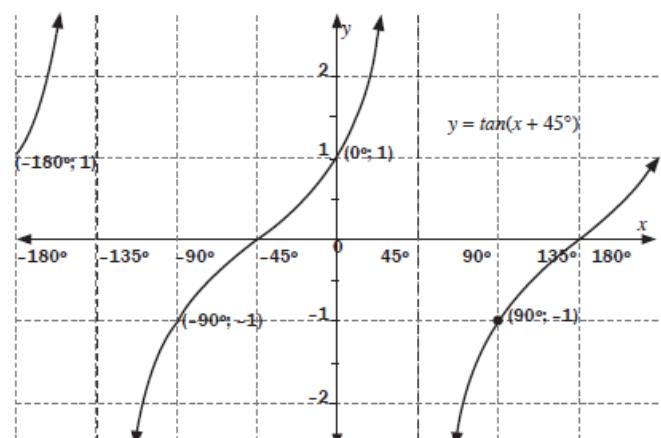
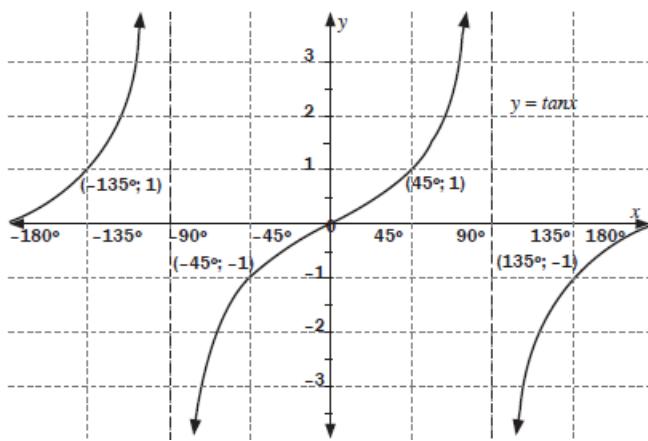
- b)  $y = \cos x$ ,  $y = \cos(x - 45^\circ)$



The graph of  $y = \cos(x - 45^\circ)$ , is the same as the graph of  $y = \cos x$  shifted  $45^\circ$  to the right.

The graph of  $y = \tan(x + 45^\circ)$ , is the same as the graph of  $y = \tan x$ , shifted  $45^\circ$  to the left,

- c)  $y = \tan x$  en  $y = \tan(x + 45^\circ)$



### Conclusion:

The effect of  $p$  on the shape of trigonometric graphs: If  $p$  is positive then the graph is shifted “ $p$ ” degrees to the left, and if “ $p$ ” is negative then the graph is shifted “ $p$ ” degrees to the right.

Sketch the graph of  $y = \cos(45^\circ - x)$  for  $x \in [-180^\circ, 180^\circ]$ .

## TRIGONOMETRIC GRAPHS

	Sine Function $y = a \sin k(x + p) + q$	Cosine Function $y = a \cos k(x + p) + q$	Tangent Function $y = a \tan k(x + p) + q$
<b>Function</b>			
<b><math>a &gt; 0</math></b>			
<b><math>a &lt; 0</math></b>			
<b>Amplitude</b>	$\frac{a}{ a }$	$\frac{a}{ a }$	
<b>Period</b>	$\frac{360^\circ}{ k }$	$\frac{360^\circ}{ k }$	$\frac{180^\circ}{ k }$

## What you need to be able to do

- Recognise the basic shapes of the graphs associated with their equations.
- Sketch functions and show the effect of different parameters  $a$ ,  $q$ ,  $k$  and  $p$ .
- Draw each graph using the critical points: intercepts with the axes and turning points, where applicable
- Show any asymptotes and include any other points you might need.
- Determine the features of graphs including
  - domain and range of functions
  - turning points
  - asymptotes
  - intercepts with axes
- Find the equation from the graph. Find the equation of any of the transformations of the graphs.
- Sketch trig functions, any shifts and changes in amplitude and period.
- The solution of trigonometric equations involving double and composite angles is important in this question. This is required when determining the coordinates of the points of intersections of two graphs, where the coordinates can not be read off.

Negative Angles : graphically



<http://bit.ly/37WwH91>

Trig equations: Method 1



<http://bit.ly/2Ti8Lcb>

Trig equations: Method 2



<http://bit.ly/2QQ3F5n>

General Solution:  $\tan x = -1$

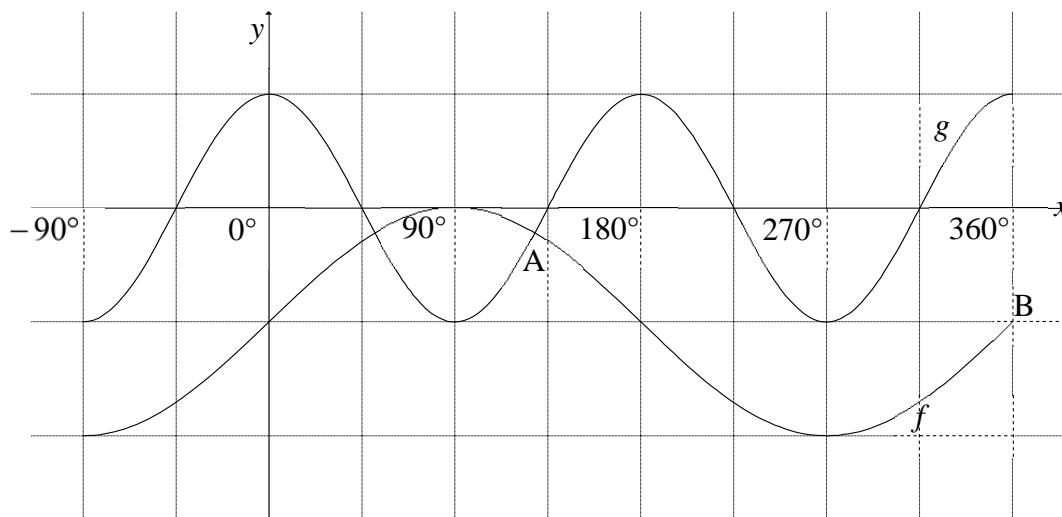


<http://bit.ly/36Q7pcC>

**A. QUESTION 6**

DBE/November 2019

In the diagram, the graphs of  $f(x) = \sin x - 1$  and  $g(x) = \cos 2x$  are drawn for the interval  $x \in [-90^\circ; 360^\circ]$ . Graphs  $f$  and  $g$  intersect at A. B( $360^\circ$ ; -1) is a point on  $f$ .



- 6.1 Write down the range of  $f$ . (2)
- 6.2 Write down the values of  $x$  in the interval  $x \in [-90^\circ; 360^\circ]$  for which graph  $f$  is decreasing. (2)
- 6.3 P and Q are points on graphs  $g$  and  $f$  respectively such that PQ is parallel to the y-axis. If PQ lies between A and B, determine the value(s) of  $x$  for which PQ will be a maximum. (6)  
[10]

**B. QUESTION 6**

DBE/November 2018

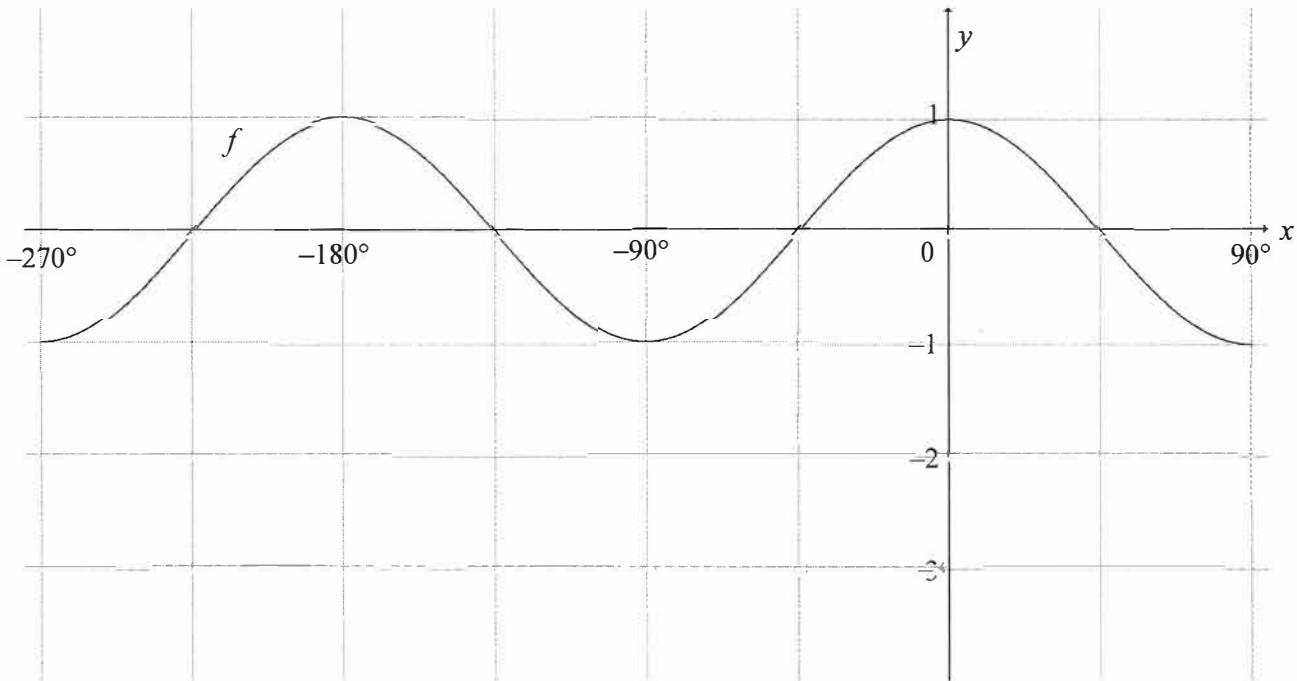
Consider:  $f(x) = -2 \tan \frac{3}{2}x$

- 6.1 Write down the period of  $f$ . (1)
- 6.2 The point A( $t$ ; 2) lies on the graph. Determine the general solution of  $t$ . (3)
- 6.3 On the grid provided in the ANSWER BOOK, draw the graph of  $f$  for the interval  $x \in [-120^\circ; 180^\circ]$ . Clearly show ALL asymptotes, intercepts with the axes and endpoint(s) of the graph. (4)
- 6.4 Use the graph to determine for which value(s) of  $x$  will  $f(x) \geq 2$  for  $x \in [-120^\circ; 180^\circ]$ . (3)
- 6.5 Describe the transformation of graph  $f$  to form the graph of  $g(x) = -2 \tan\left(\frac{3}{2}x + 60^\circ\right)$ . (2)  
[13]

**C. QUESTION 6**

DBE/November 2017

In the diagram, the graph of  $f(x) = \cos 2x$  is drawn for the interval  $x \in [-270^\circ; 90^\circ]$ .

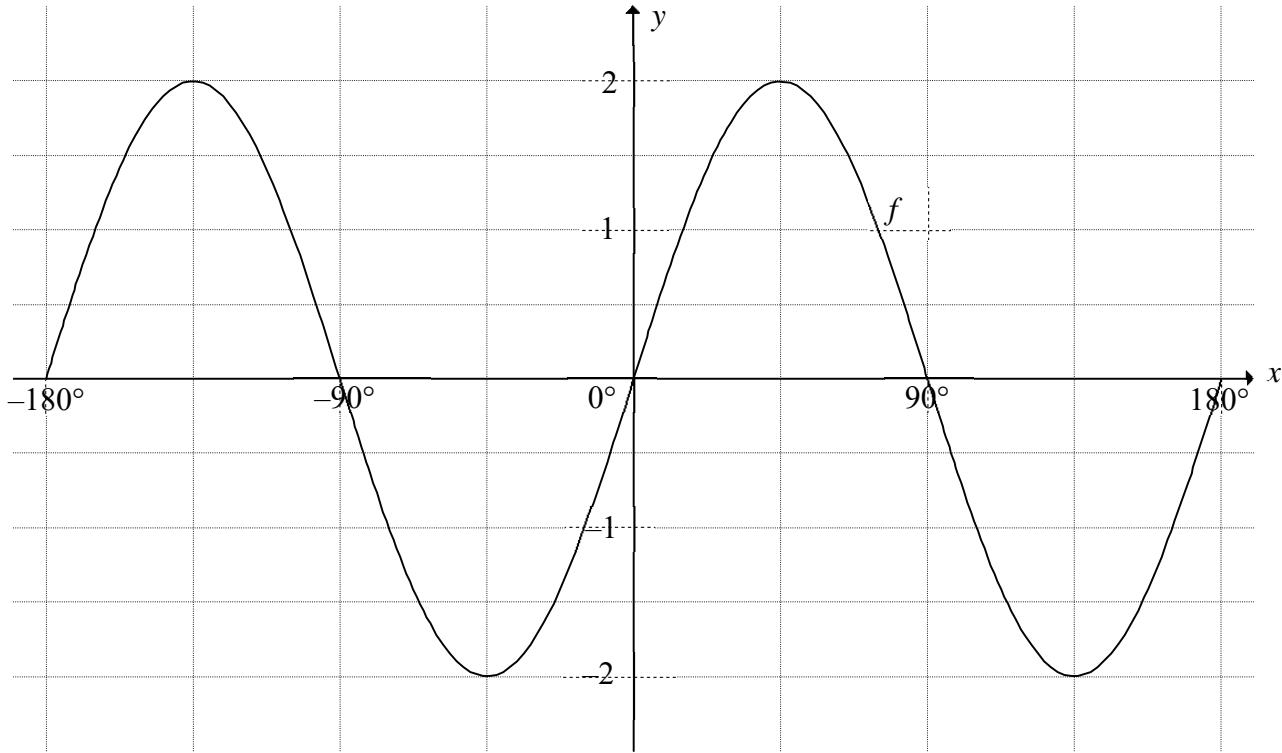


- 6.1 Draw the graph of  $g(x) = 2\sin x - 1$  for the interval  $x \in [-270^\circ; 90^\circ]$  on the grid given in your ANSWER BOOK. Show ALL the intercepts with the axes, as well as the turning points. (4)
- 6.2 Let A be a point of intersection of the graphs of  $f$  and  $g$ . Show that the  $x$ -coordinate of A satisfies the equation  $\sin x = \frac{-1 + \sqrt{5}}{2}$ . (4)
- 6.3 Hence, calculate the coordinates of the points of intersection of graphs of  $f$  and  $g$  for the interval  $x \in [-270^\circ; 90^\circ]$ . (4)  
[12]

**D. QUESTION 6**

DBE/November 2016

In the diagram the graph of  $f(x) = 2 \sin 2x$  is drawn for the interval  $x \in [-180^\circ ; 180^\circ]$ .

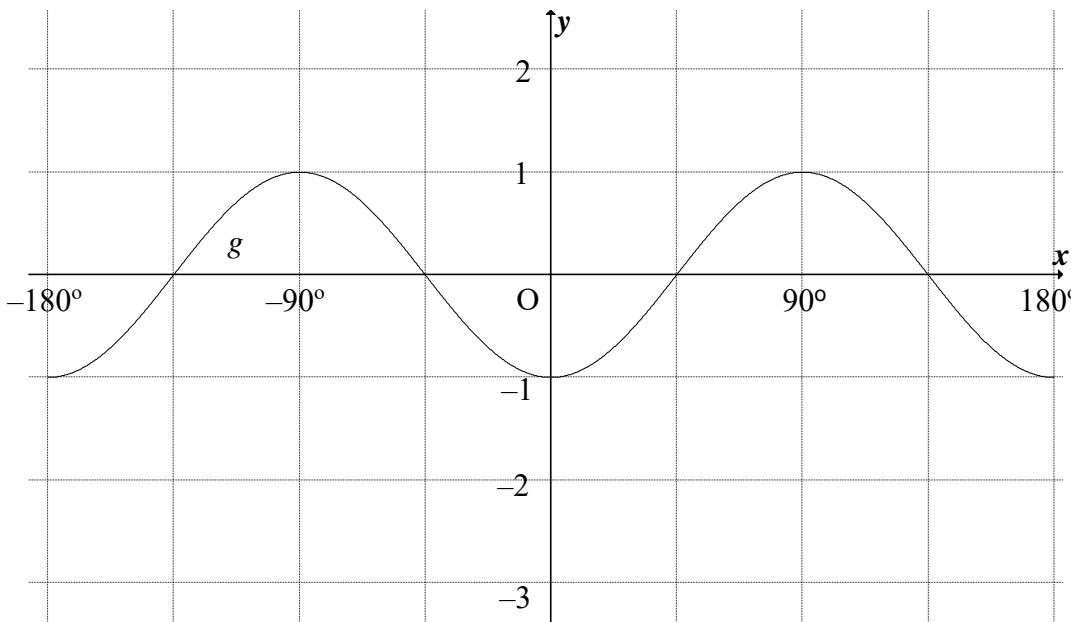


- 6.1 On the system of axes on which  $f$  is drawn in the ANSWER BOOK, draw the graph of  $g(x) = -\cos 2x$  for  $x \in [-180^\circ ; 180^\circ]$ . Clearly show all intercepts with the axes, the coordinates of the turning points and end points of the graph. (3)
- 6.2 Write down the maximum value of  $f(x) - 3$ . (2)
- 6.3 Determine the general solution of  $f(x) = g(x)$ . (4)
- 6.4 Hence, determine the values of  $x$  for which  $f(x) < g(x)$  in the interval  $x \in [-180^\circ ; 0^\circ]$ . (3)  
[12]

**E. QUESTION 6**

6.1 Determine the general solution of  $4\sin x + 2\cos 2x = 2$  (6)

6.2 The graph of  $g(x) = -\cos 2x$  for  $x \in [-180^\circ ; 180^\circ]$  is drawn below.



6.2.1 Draw the graph of  $f(x) = 2\sin x - 1$  for  $x \in [-180^\circ ; 180^\circ]$  on the set of axes provided in the ANSWER BOOK. (3)

6.2.2 Write down the values of  $x$  for which  $g$  is strictly decreasing in the interval  $x \in [-180^\circ ; 0^\circ]$  (2)

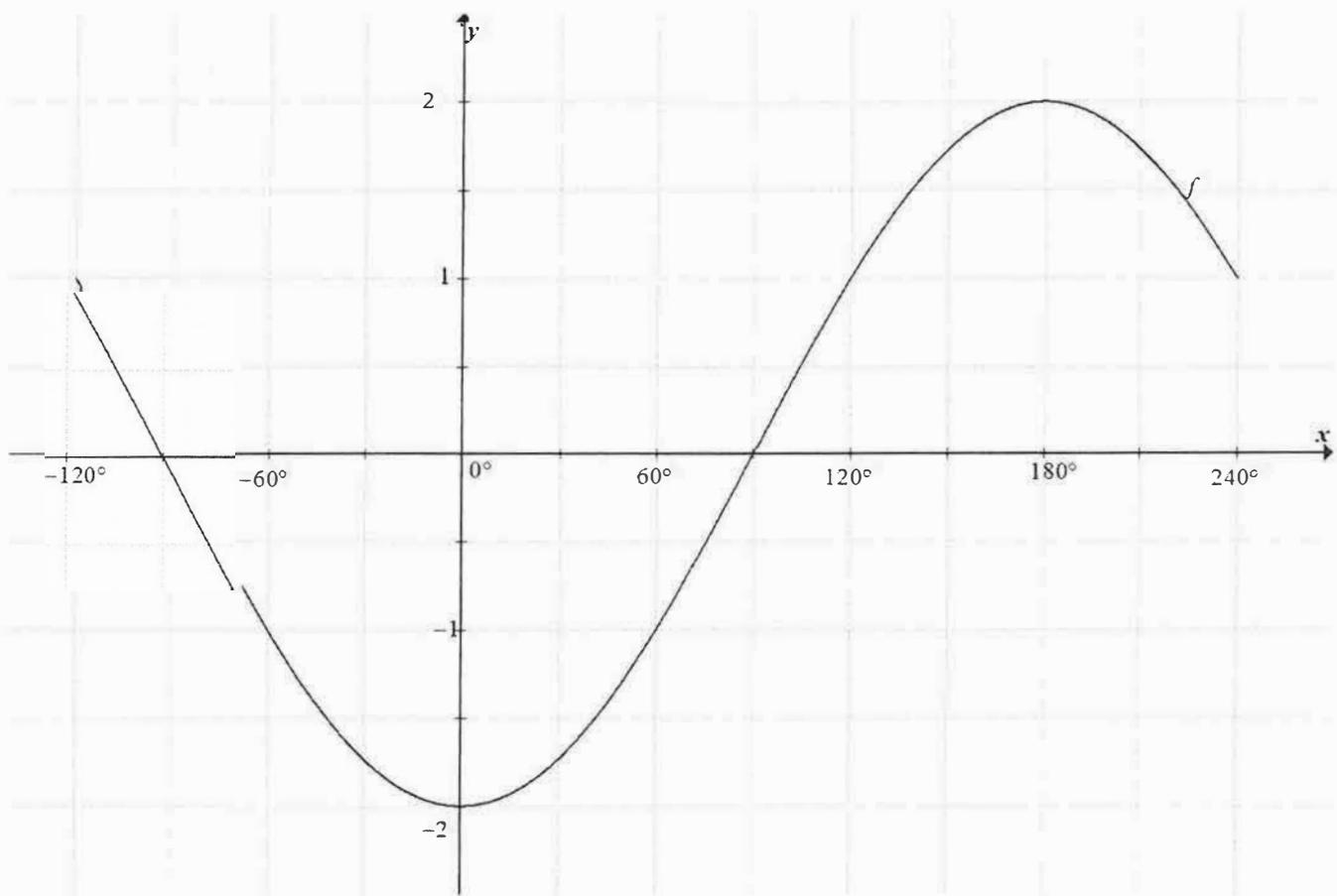
6.2.3 Write down the value(s) of  $x$  for which  $f(x + 30^\circ) - g(x + 30^\circ) = 0$  for  $x \in [-180^\circ ; 180^\circ]$  (2)  
[13]

**F. QUESTION 6**

DBE/Feb.–Mar. 2016

Given the equation:  $\sin(x + 60^\circ) + 2\cos x = 0$

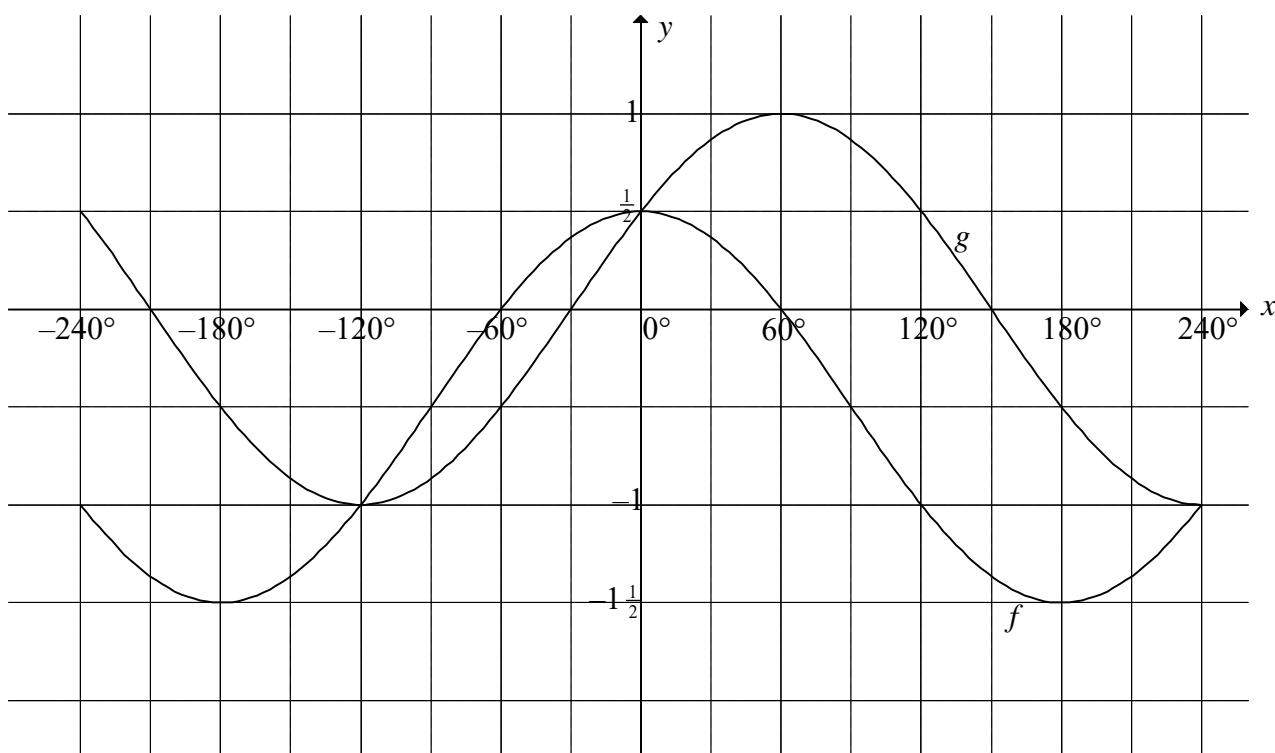
- 6.1 Show that the equation can be rewritten as  $\tan x = -4 - \sqrt{3}$ . (4)
- 6.2 Determine the solutions of the equation  $\sin(x + 60^\circ) + 2\cos x = 0$  in the interval  $-180^\circ \leq x \leq 180^\circ$ . (3)
- 6.3 In the diagram below, the graph of  $f(x) = -2 \cos x$  is drawn for  $-120^\circ \leq x \leq 240^\circ$ .



- 6.3.1 Draw the graph of  $g(x) = \sin(x + 60^\circ)$  for  $-120^\circ \leq x \leq 240^\circ$  on the grid provided in the ANSWER BOOK. (3)
- 6.3.2 Determine the values of  $x$  in the interval  $-120^\circ \leq x \leq 240^\circ$  for which  $\sin(x + 60^\circ) + 2\cos x > 0$ . (3)
- [13]

## G. QUESTION 6

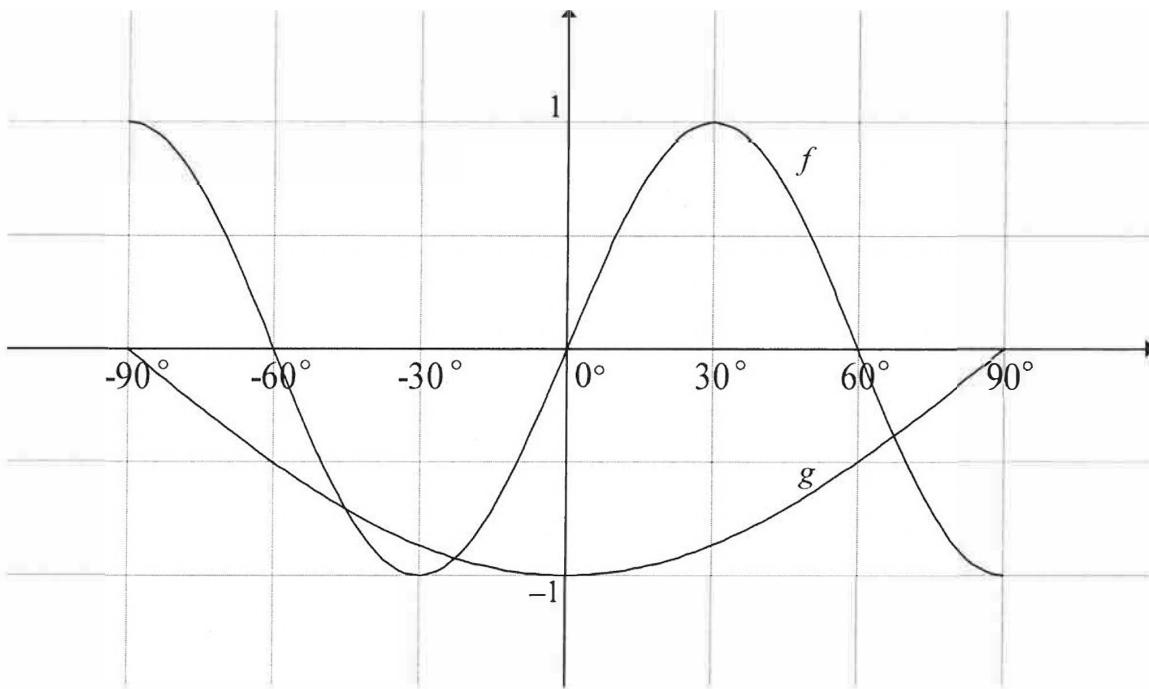
In the diagram below, the graphs of  $f(x) = \cos x + q$  and  $g(x) = \sin(x + p)$  are drawn on the same system of axes for  $-240^\circ \leq x \leq 240^\circ$ . The graphs intersect at  $\left(0^\circ; \frac{1}{2}\right)$ ,  $(-120^\circ; -1)$  and  $(240^\circ; -1)$ .



- 6.1 Determine the values of  $p$  and  $q$ . (4)
- 6.2 Determine the values of  $x$  in the interval  $-240^\circ \leq x \leq 240^\circ$  for which  $f(x) > g(x)$ . (2)
- 6.3 Describe a transformation that the graph of  $g$  has to undergo to form the graph of  $h$ , where  $h(x) = -\cos x$ . (2)  
[8]

**H. QUESTION 6**

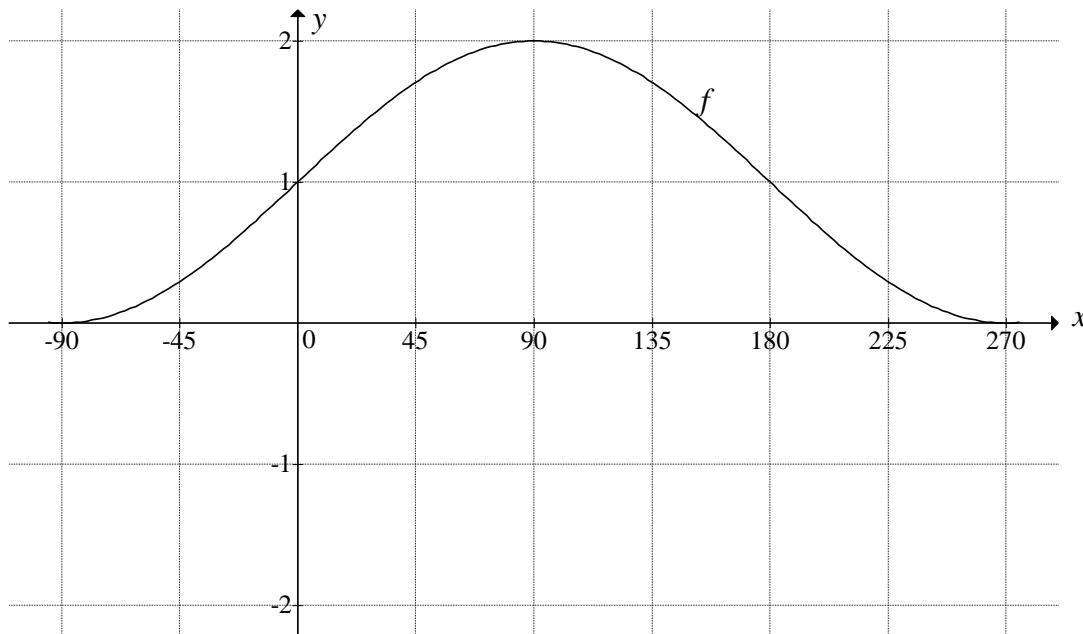
In the diagram below the graphs of  $f(x) = \sin bx$  and  $g(x) = -\cos x$  are drawn for  $-90^\circ \leq x \leq 90^\circ$ . Use the diagram to answer the following questions.



- 6.1 Write down the period of  $f$ . (1)
- 6.2 Determine the value of  $b$ . (1)
- 6.3 The general solutions of the equation  $\sin bx = -\cos x$  are  $x = 67,5^\circ + k.90^\circ$  or  $x = 135^\circ + k.180^\circ$  where  $k \in \mathbb{Z}$ . Determine the  $x$ -values of the points of intersection of  $f$  and  $g$  for the given domain. (3)
- 6.4 Write down the values of  $x$  for which  $\sin bx + \cos x < 0$  for the given domain. (4)  
[9]

**I. QUESTION 7**

In the diagram below, the graph of  $f(x) = \sin x + 1$  is drawn for  $-90^\circ \leq x \leq 270^\circ$ .



- 7.1 Write down the range of  $f$ . (2)
- 7.2 Show that  $\sin x + 1 = \cos 2x$  can be rewritten as  $(2\sin x + 1)\sin x = 0$ . (2)
- 7.3 Hence, or otherwise, determine the general solution of  $\sin x + 1 = \cos 2x$ . (4)
- 7.4 Use the grid on DIAGRAM SHEET 2 to draw the graph of  $g(x) = \cos 2x$  for  $-90^\circ \leq x \leq 270^\circ$ . (3)
- 7.5 Determine the value(s) of  $x$  for which  $f(x + 30^\circ) = g(x + 30^\circ)$  in the interval  $-90^\circ \leq x \leq 270^\circ$ . (3)
- 7.6 Consider the following geometric series:
- $$1 + 2 \cos 2x + 4 \cos^2 2x + \dots$$
- Use the graph of  $g$  to determine the value(s) of  $x$  in the interval  $0^\circ \leq x \leq 90^\circ$  for which this series will converge. (5)
- [19]**

# Probability

- We can work out the probability using the formula:

$$\text{probability} = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$$

- This ratio can be expressed as a common fraction, a decimal fraction or a percentage.  
So, a probability of 5 out of 8 can be written as  $\frac{5}{8}$  or as 0,625 or as 62,5%.
- Probability always lies between 0 and 1, measured as a fraction or as a decimal.
- If probability is shown as a percentage, then it lies between 0% and 100%.

## Empirical/Experimental probability and Theoretical probability

**Eksperimentele waarskynlikheid:** If you perform an experiment where you toss a coin say, 10 times and you obtain heads 8 of the 10 times. Then 8 is the relative frequency.

$$\text{Experiment Probability} = \frac{8}{10} = \frac{4}{5} = 0,5$$



### Theoretical Probability:

If a coin is tossed:

- The possible outcomes are H (heads) or T (tails).
- There are two possible outcomes. Each has a 50% chance of happening.
- We say that there is a theoretical probability of  $\frac{1}{2}$  for each outcome.

Probability Explained



[http://bit.ly/prob\\_1](http://bit.ly/prob_1)

### Revise:

Consider two events A and B:

$$P(\text{A or B}) = P(\text{A}) + P(\text{B}) - P(\text{A and B})$$

If A and B are **mutually exclusive** then,  $P(\text{A and B}) = 0$ ;

$$\therefore P(\text{A or B}) = P(\text{A}) + P(\text{B})$$

If A and B are complementary then:  $P(\text{B}) = P(\text{not A}) = 1 - P(\text{A})$

Product rule for independent events:

$$P(\text{A and B}) = P(\text{A}) \times P(\text{B})$$

Events are independent if the probability of one event happening is not influenced by another event happening.

# Counting Principal

## The basic counting principle:

The number of ways of making several decisions in succession (call them  $m_1$ ;  $m_2$  and  $m_3$  etc...) is determined by multiplying the numbers of choices that can be made in each decision.  $m_1 \times m_2 \times m_3 \dots$

## Permutations

- The number of permutations of  $m$  different items is  $m!$
- The number of permutations of  $m$  items of which:  $a$  are alike,  $b$  are alike,  $c$  are alike, is:  $\frac{m!}{a! \times b! \times c!}$
- The number of permutations of  $m$  items taken  $n$  at a time, when each of the items may be repeated any number of times, is:  $m \times m \times m \times m \times \dots$  to  $n$  factors =  $m^n$  times.
- The number of ways that  $m$  items taken  $n$  at a time can be arranged, is  ${}_m P_n = \frac{m!}{(m-n)!}$

## What you need to be able to do:

- Revise the addition rule for mutually exclusive events:  
 $P(A \text{ or } B) = P(A) + P(B)$
- Revise the complementary rule:  $P(A') = 1 - P(A)$
- Revise the identity  $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$  for all possible events.
- Identify dependent and independent events and use the product rule
- Use Venn diagrams to solve problems for up to three events. Introduce an  $x$  for an event to solve problems.
- Use tree diagrams and contingency tables for probability of consecutive events or simultaneous events which are not necessarily independent.
- Understand and use Counting principles in probability.

Venn Diagram



<http://bit.ly/35HvPn6>

Tree Diagrams



<http://bit.ly/2NeZcqz>

Mutually Exclusive &amp; Independent Events



<http://bit.ly/2R9bGRM>



<http://bit.ly/2t2mrj>

Example:

1. How many different 3 digit numbers greater than 220 can be formed using the numbers 0, 2, and 3.  
List all of them

**A. QUESTION 10**

DBE/November 2019

The school library is open from Monday to Thursday. Anna and Ben both studied in the school library one day this week. If the chance of studying any day in the week is equally likely, calculate the probability that Anna and Ben studied on:

- 10.1 The same day (2)
- 10.2 Consecutive days (3)  
[5]

**QUESTION 11**

- B.** 11.1 Events **A** and **B** are independent.  $P(A) = 0,4$  and  $P(B) = 0,25$ .

- 11.1.1 Represent the given information on a Venn diagram. Indicate on the Venn diagram the probabilities associated with each region. (3)
- 11.1.2 Determine  $P(A \text{ or NOT } B)$ . (2)

- 11.2 Motors Incorporated manufacture cars with 5 different body styles, 4 different interior colours and 6 different exterior colours, as indicated in the table below.

BODY STYLES	INTERIOR COLOURS	EXTERIOR COLOURS
Five body styles	Blue	Silver
	Grey	Blue
	Black	White
		Green
	Red	Red
		Gold

The interior colour of the car must NOT be the same as the exterior colour.

Motors Incorporated wants to display one of each possible variation of its car in their showroom. The showroom has a floor space of  $500 \text{ m}^2$  and each car requires a floor space of  $5 \text{ m}^2$ .

Is this display possible? Justify your answer with the necessary calculations. (6)

**C. QUESTION 11**

DBE/November 2018

Given the digits: 3 ; 4 ; 5 ; 6 ; 7 ; 8 and 9

- 11.1 Calculate how many unique 5-digit codes can be formed using the digits above, if:
- 11.1.1 The digits may be repeated (2)
- 11.1.2 The digits may not be repeated (2)
- 11.2 How many unique 3-digit codes can be formed using the above digits, if:
- Digits may be repeated
  - The code is greater than 400 but less than 600 (3)
  - The code is divisible by 5 [7]

**D. QUESTION 12**

DBE/November 2018

- 12.1 Given:  $P(A) = 0,45$ ;  $P(B) = y$  and  $P(A \text{ or } B) = 0,74$

Determine the value(s) of  $y$  if A and B are mutually exclusive. (3)

- 12.2 An organisation decided to distribute gift bags of sweets to a Grade R class at a certain school. There is a mystery gift in exactly  $\frac{1}{4}$  of the total number of bags.

Each learner in the class may randomly select two gift bags of sweets, one after the other. The probability that a learner selects two bags of sweets with a mystery gift is  $\frac{7}{118}$ . Calculate the number of gift bags of sweets with a mystery gift inside. (6)

[9]

**E. QUESTION 10**

DBE/November 2017

A survey was conducted among 100 Grade 12 learners about their use of Instagram (I), Twitter (T) and WhatsApp (W) on their cell phones. The survey revealed the following:

- 8 use all three.
- 12 use Instagram and Twitter.
- 5 use Twitter and WhatsApp, but not Instagram.
- $x$  use Instagram and WhatsApp, but not Twitter.
- 61 use Instagram.
- 19 use Twitter.
- 73 use WhatsApp.
- 14 use none of these applications.

- 10.1 Draw a Venn diagram to illustrate the information above. (4)

- 10.2 Calculate the value of  $x$ . (2)

- 10.3 Calculate the probability that a learner, chosen randomly, uses only ONE of these applications. (2)

[8]

**F. QUESTION 11**

DBE/November 2017

A company uses a coding system to identify its clients. Each code is made up of two letters and a sequence of digits, for example AD108 or RR 45789.

The letters are chosen from A; D; R; S and U. Letters may be repeated in the code.

The digits 0 to 9 are used, but NO digit may be repeated in the code.

- 11.1 How many different clients can be identified with a coding system that is made up of TWO letters and TWO digits? (3)
- 11.2 Determine the least number of digits that is required for a company to uniquely identify 700 000 clients using their coding system. (3) [6]

**G. QUESTION 11**

DBE/November 2016

A survey was conducted among 100 boys and 60 girls to determine how many of them watched TV in the period during which examinations were written. Their responses are shown in the partially completed table below.

	WATCHED TV DURING EXAMINATIONS	DID NOT WATCH TV DURING EXAMINATIONS	TOTALS
Male	80	$a$	
Female	48	12	
Totals	$b$	32	160

- 11.1 Calculate the values of  $a$  and  $b$ . (2)
- 11.2 Are the events 'being a male' and 'did not watch TV during examinations' mutually exclusive? Give a reason for your answer. (2)
- 11.3 If a learner who participated in this survey is chosen at random, what is the probability that the learner:
- 11.3.1 Watched TV in the period during which the examinations were written? (2)
  - 11.3.2 Is not a male and did not watch TV in the period during which examinations were written? (2) [8]

**H. QUESTION 12**

The digits 1 to 7 are used to create a four-digit code to enter a locked room. How many different codes are possible if the digits may not be repeated and the code must be an even number bigger than 5 000? [5]

**I. QUESTION 10**

- 10.1 A tournament organiser conducted a survey among 150 members at a local sports club to find out whether they play tennis or not. The results are shown in the table below.

	PLAYING TENNIS	NOT PLAYING TENNIS
Male	50	30
Female	20	50

- 10.1.1 What is the probability that a member selected at random is:
- (a) Female (2)
  - (b) Female and plays tennis (1)
- 10.1.2 Is playing tennis independent of gender? Motivate your answer with the necessary calculations. (3)
- 10.2 The probability of events A and B occurring are denoted by  $P(A)$  and  $P(B)$  respectively.

For any two events A and B it is given that:

- $P(B') = 0,28$
- $P(B) = 3P(A)$
- $P(A \text{ or } B) = 0,96$

Are events A and B mutually exclusive? Justify your answer. (4)

[10]

**J. QUESTION 11**

Five boys and four girls go to the movies. They are all seated next to each other in the same row.

- 11.1 One boy and girl are a couple and want to sit next to each other at any end of the row of friends. In how many different ways can the entire group be seated? (3)
- 11.2 If all the friends are seated randomly, calculate the probability that all the girls are seated next to each other. (3)
- [6]

**K. QUESTION 10**

DBE/Feb.–Mrt. 2016

- 10.1 Each passenger on a certain Banana Airways flight chose exactly one beverage from tea, coffee or fruit juice. The results are shown in the table below.

	<b>MALE</b>	<b>FEMALE</b>	<b>TOTAL</b>
Tea	20	40	60
Coffee	$b$	$c$	80
Fruit juice	$d$	$e$	20
<b>TOTAL</b>	60	100	$a$

- 10.1.1 Write down the value of  $a$ . (1)
- 10.1.2 What is the probability that a randomly selected passenger is male? (2)
- 10.1.3 Given that the event of a passenger choosing coffee is independent of being a male, calculate the value of  $b$ . (4)
- 10.2 A Banana Airways aeroplane has 6 seats in each row.
- 10.2.1 How many possible arrangements are there for 6 people to sit in a row of 6 seats? (2)
- 10.2.2 Xoliswa, Anees and 4 other passengers sit in a certain row on a Banana Airways flight. In how many different ways can these 6 passengers be seated if Xoliswa and Anees must sit next to each other? (2)
- 10.2.3 Mary and 5 other passengers are to be seated in a certain row. If seats are allocated at random, what is the probability that Mary will sit at the end of the row? (4)

**L. QUESTION 11**

November 2015

- 11.1 For two events, A and B, it is given that:

$$\begin{aligned} P(A) &= 0,2 \\ P(B) &= 0,63 \\ P(A \text{ and } B) &= 0,126 \end{aligned}$$

Are the events, A and B, independent? Justify your answer with appropriate calculations. (3)

November 2015

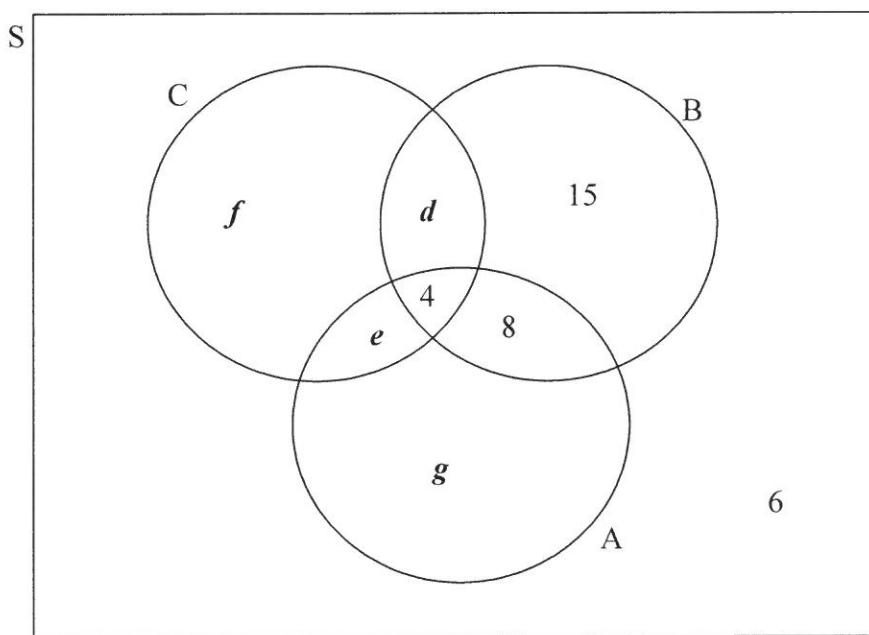
- L. 11.2** The letters of the word DECIMAL are randomly arranged into a new 'word', also consisting of seven letters. How many different arrangements are possible if:
- 11.2.1 Letters may be repeated (2)
- 11.2.2 Letters may not be repeated (2)
- 11.2.3 The arrangements must start with a vowel and end in a consonant and no repetition of letters is allowed (4)
- 11.3** There are  $t$  orange balls and 2 yellow balls in a bag. Craig randomly selects one ball from the bag, records his choice and returns the ball to the bag. He then randomly selects a second ball from the bag, records his choice and returns it to bag. It is known that the probability that Craig will select two balls of the same colour from the bag is 52%.
- Calculate how many orange balls are in the bag. (6)
- DBE/ 2015
- M. 11.1** Zebra High School offers only two sporting activities, namely rugby and hockey.
- The following information is given:
- There are 600 learners in the school.
  - 372 learners play hockey.
  - 288 learners play rugby.
  - 56 of the learners play NO sport.
  - The number of learners that play both hockey and rugby is  $x$ .
- 11.1.1 Represent the given information in a Venn diagram, in terms of  $x$ . (3)
- 11.1.2 Calculate the value of  $x$ . (2)
- 11.1.3 Are the events playing rugby and playing hockey mutually exclusive? Justify your answer. (2)
- 11.2** Another school, Tulani High, has a sports awards ceremony. Tulani High has a basketball team consisting of 5 players and a volleyball team consisting of 6 players.
- 11.2.1 All the basketball players sit in a single row at the ceremony. There are no restrictions on who sits in which position. In how many different ways can they be seated? (1)
- 11.2.2 The decision is taken that the captain must sit in the first seat of the row. The two vice-captains have to be seated next to each other in any of the remaining seats. In how many different ways can the basketball players be seated now? (3)
- 11.2.3 After the interval, the basketball team and the volleyball team sit in the same row at the ceremony. Calculate the probability that the basketball players will sit together and the volleyball players will sit together. Assume that seating positions are allocated randomly. Give your answer as a simplified fraction. (3)

**N. QUESTION 10**

- 10.1 Research was conducted about driving under the influence of alcohol. Information obtained from traffic authorities in 54 countries on the methods that are used to measure alcohol levels in a person, are summarised below:

- 4 countries use all three methods (A, B and C).
- 12 countries use the alcohol content of breath (A) and blood-alcohol concentration (B).
- 9 countries use blood-alcohol concentration (B) and certificates issued by doctors (C).
- 8 countries use the alcohol content of breath (A) and certificates issued by doctors (C).
- 21 countries use the alcohol content of breath (A).
- 32 countries use blood-alcohol concentration (B).
- 20 countries use certificates issued by doctors (C).
- 6 countries use none of these methods.

Below is a partially completed Venn diagram representing the above information.



- 10.1.1 Use the given information and the Venn diagram to determine the values of  $d$ ,  $e$ ,  $f$  and  $g$ . (4)
- 10.1.2 For a randomly selected country, calculate:
- $P(A \text{ and } B \text{ and } C)$  (1)
  - $P(A \text{ or } B \text{ or } C)$  (1)
  - $P(\text{only } C)$  (1)
  - $P(\text{that a country uses exactly two methods})$  (1)

**N. 10.2** Nametso may choose DVDs from three categories as listed in the table below:

Drama	Romance	Comedy
<ul style="list-style-type: none"> <li>• <i>Last Hero</i></li> <li>• <i>Midnight</i></li> <li>• <i>Stranger Calls</i></li> <li>• <i>Missing in Action</i></li> <li>• <i>Only 40 Seconds Left</i></li> </ul>	<ul style="list-style-type: none"> <li>• <i>One Heart</i></li> <li>• <i>You and Me</i></li> <li>• <i>Love Song</i></li> <li>• <i>Bird's First Nest</i></li> </ul>	<ul style="list-style-type: none"> <li>• <i>Laughing Dragon</i></li> <li>• <i>Falling Down</i></li> <li>• <i>Sitting on the Stairs</i></li> </ul>

- 10.2.1 Nametso must choose ONE DVD from the Drama category. What is the probability that she will choose *Midnight*? (2)
- 10.2.2 How many different selections are possible if her selection must include ONE drama, ONE romance and ONE comedy? (2)
- 10.2.3 Calculate the probability that she will have *Last Hero* and *Laughing Dragon* as part of her selection in QUESTION 10.2.2. (2)

## O. QUESTION 11

DBE/November 2014

A survey concerning their holiday preferences was done with 180 staff members. The options they could choose from were to:

- Go to the coast
- Visit a game park
- Stay at home

The results were recorded in the table below:

	Coast	Game Park	Home	Total
<b>Male</b>	46	24	13	83
<b>Female</b>	52	38	7	97
<b>Total</b>	98	62	20	180

- 11.1 Determine the probability that a randomly selected staff member:
- 11.1.1 Is male (1)
- 11.1.2 Does not prefer visiting a game park (2)
- 11.2 Are the events 'being a male' and 'staying at home' independent events. Motivate your answer with relevant calculations. (4)
- [7]

**P. QUESTION 12**

- 12.1 A password consists of five different letters of the English alphabet. Each letter may be used only once. How many passwords can be formed if:
- 12.1.1 All the letters of the alphabet can be used (2)
  - 12.1.2 The password must start with a 'D' and end with an 'L' (2)
- 12.2 Seven cars of different manufacturers, of which 3 are silver, are to be parked in a straight line.
- 12.2.1 In how many different ways can ALL the cars be parked? (2)
  - 12.2.2 If the three silver cars must be parked next to each other, determine in how many different ways the cars can be parked. (3)
- [9]**

**Q. QUESTION 11**

NSC – Grade 12 Exemplar

DBE/2014

- 11.1 Events A and B are mutually exclusive. It is given that:
- $P(B) = 2P(A)$
  - $P(A \text{ or } B) = 0,57$
- Calculate  $P(B)$ . (3)
- 11.2 Two identical bags are filled with balls. Bag A contains 3 pink and 2 yellow balls. Bag B contains 5 pink and 4 yellow balls. It is equally likely that Bag A or Bag B is chosen. Each ball has an equal chance of being chosen from the bag. A bag is chosen at random and a ball is then chosen at random from the bag.
- 11.2.1 Represent the information by means of a tree diagram. Clearly indicate the probability associated with each branch of the tree diagram and write down all the outcomes. (4)
  - 11.2.2 What is the probability that a yellow ball will be chosen from Bag A? (1)
  - 11.2.3 What is the probability that a pink ball will be chosen? (3)
- [11]**

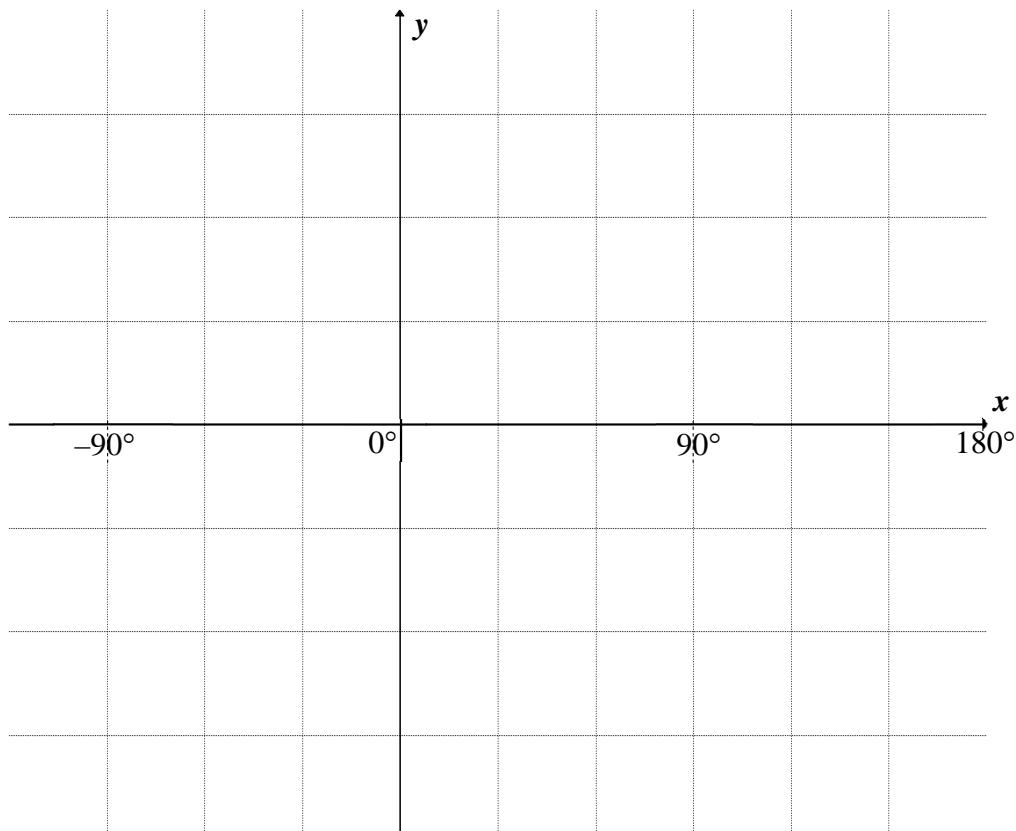
**R. QUESTION 12**

Consider the word M A T H S.

- 12.1 How many different 5-letter arrangements can be made using all the above letters? (2)
- 12.2 Determine the probability that the letters S and T will always be the first two letters of the arrangements in QUESTION 12.1. (3)
- [5]**

DBE/November 2018

6.3

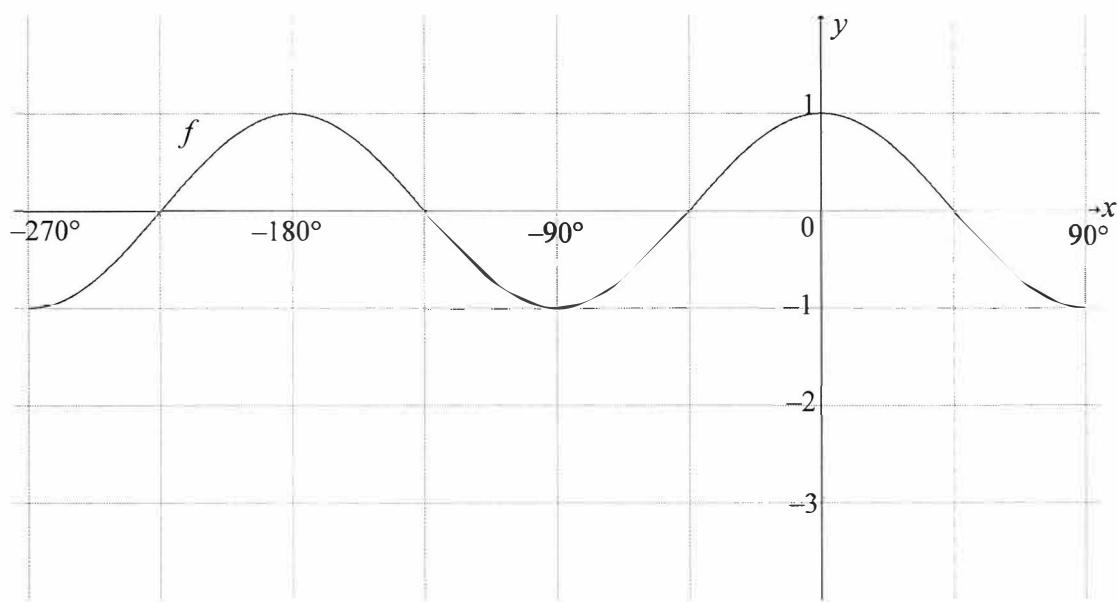


(4)

**QUESTION/VRAAG 6**

DBE/November 2017

6.1

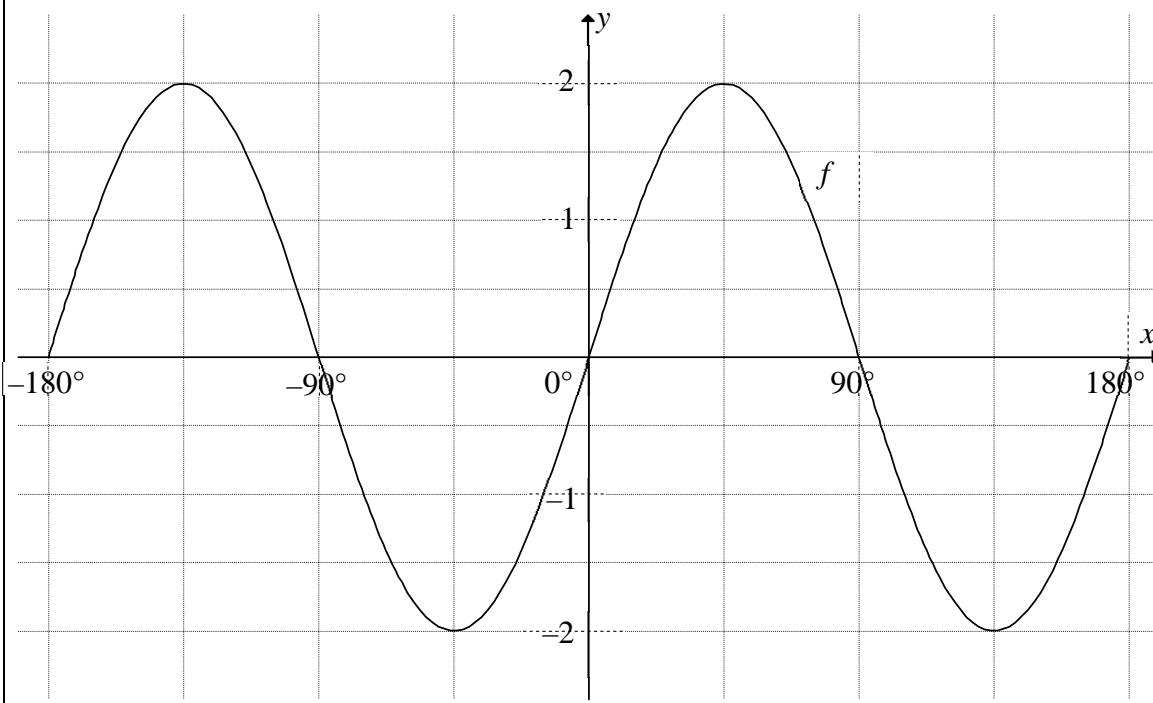


(4)

**QUESTION/VRAAG 6**

DBE/November 2016

6.1

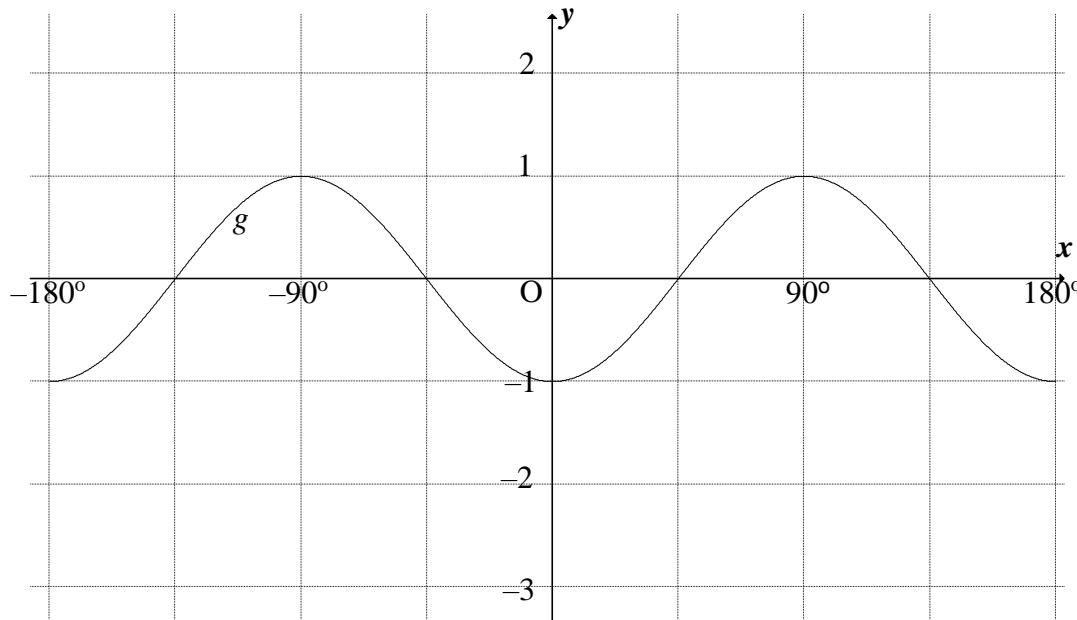


(3)

**QUESTION/VRAAG 6**

DBE/2016

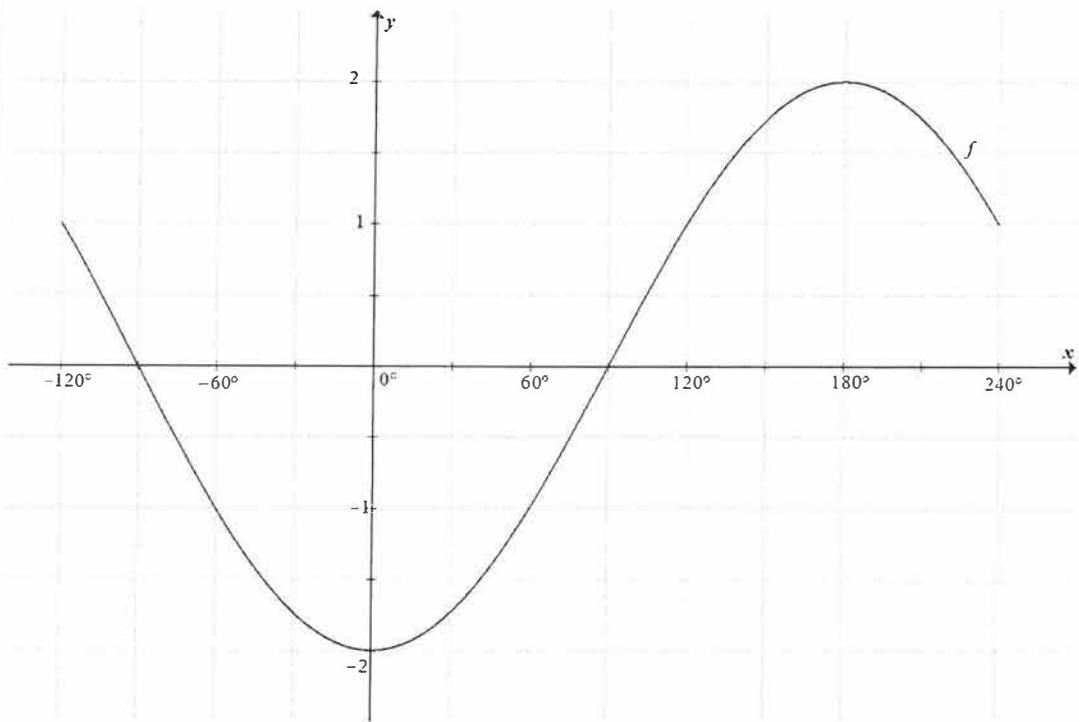
6.2.1



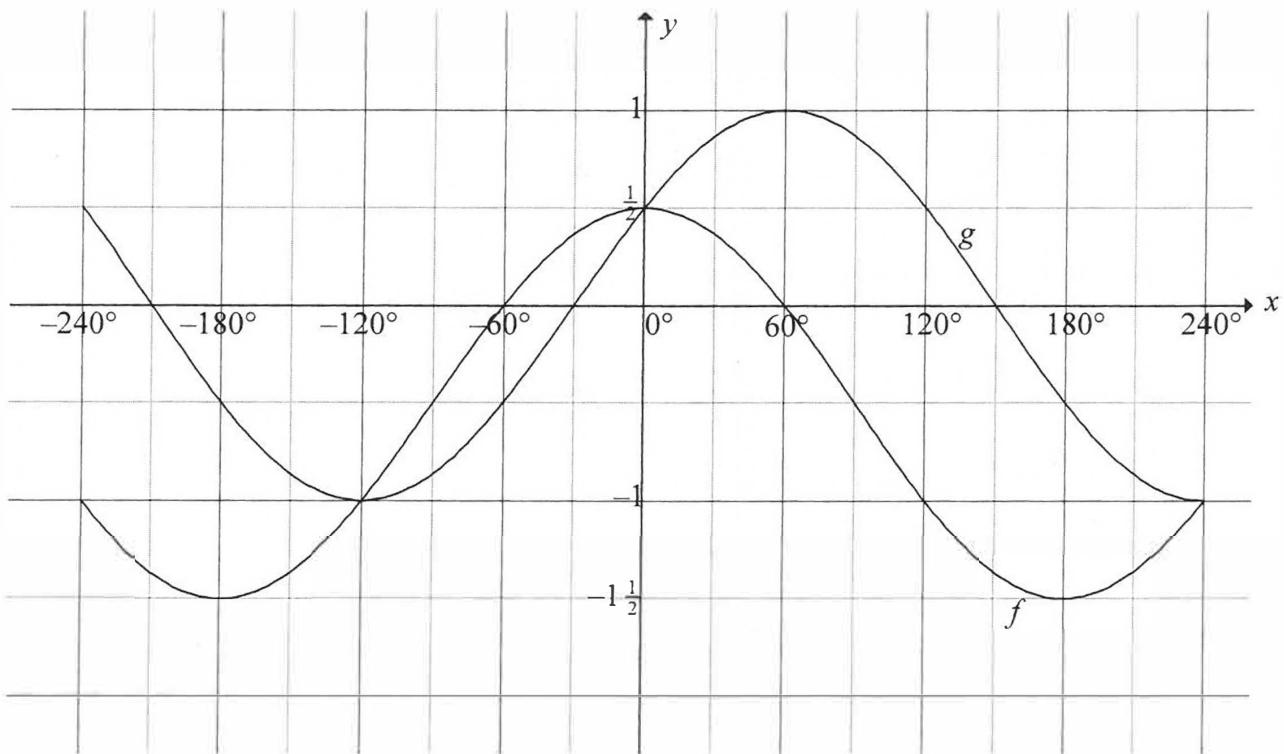
(3)

DBE/Feb.–Mar./Feb.–Mrt. 2016

6.3.1

**QUESTION/VRAAG 6**

DBE/November 2015



**DIAGRAM SHEET 2**  
**QUESTION 7.4**

DBE/November 2014

