Ball collision experiments

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Abstract
Experiments are described on collisions between two billiard balls and between a bat and a ball. The experiments are designed to extend a student's understanding of collision events and could be used either as a classroom demonstration or for a student project.

S Online supplementary data available from stacks.iop.org/PED/50/010052

1. Introduction

The head-on collision between two balls is a standard high school or undergraduate physics problem that can be solved theoretically by applying conservation of momentum and energy. In fact, kinetic energy is never conserved in collisions of this type, and a more generally useful and even simpler approach is to solve the problem in terms of the coefficient of restitution (COR) for the collision. For example, consider the collision shown in figure 1 where a mass m_1 collides head-on with a mass m_2 that is initially at rest.

Conservation of momentum indicates that

$$m_1 v_1 = m_1 v_2 + m_2 V. \tag{1}$$

The COR, e, is a measure of energy loss during the collision and is defined as the ratio of the relative speed after the collision to that before the collision, so

$$e = \frac{V - v_2}{v_1}. \tag{2}$$

These two equations are easily solved to give

$$v_2 = \frac{(m_1 - e m_2)\, v_1}{m_1 + m_2} \tag{3}$$

and hence v_2 will be positive, zero or negative depending on whether m_1/m_2 is greater than, equal to or less than e. In general, a light ball will therefore bounce backwards off a heavy ball, while the incident ball will come to a dead stop if $m_1 = e m_2$.

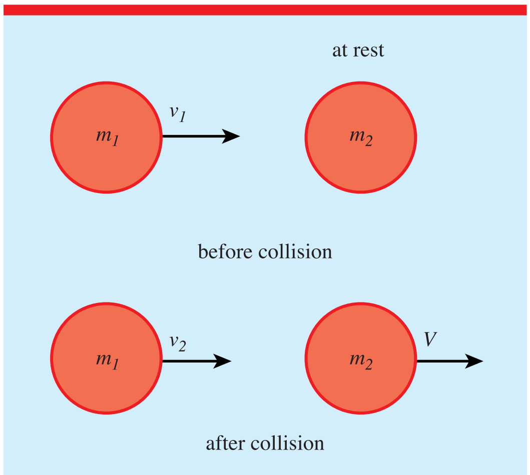

Figure 1. A head-on collision between two balls when one is initially at rest.

2. Billiard ball collisions

The collision between two billiard balls is an interesting case, since the COR is close to unity, meaning that very little energy is lost during the collision. One might expect that billiard ball collision experiments should also be standard in physics courses, but they are not. Part of the reason is that air track experiments are also relatively free of energy losses and are even simpler to set up and analyze. In addition, billiard balls don't always behave in an ideal manner.

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It has been the author's experience that simple experiments are never as simple as they first appear, and that interesting new physics can be learnt from almost any experiment, no matter how simple. Billiard ball collisions provide an interesting case study. Most of the published articles on billiard ball collisions are concerned with the fact that billiard balls don't behave as expected [1, 2]. For example, if a billiard ball collides head-on with an identical ball at rest, then the expected result is that the incident ball comes to a dead stop and transfers all of its momentum and energy to the stationary ball. Anyone who has played billiards, pool or snooker knows that the end result depends on the amount of topspin or backspin applied to the incident ball. Even if the incident ball is not deliberately given any spin, it will quickly acquire topspin as it rolls on the table. The ball will stop for a brief instant following the collision with a stationary ball, as expected, but there is almost no change in spin since billiard balls are almost frictionless. The ball therefore starts rolling forward again after a momentary pause [3], unless it is given sufficient backspin to counteract the tendency to roll forward.

The author recently conducted several billiard ball experiments to see what other surprises might lie in store. The biggest surprise occurred when the stationary ball was held at rest by hand to stop it moving. A billiard ball was projected along a table to collide head-on with the stationary ball, as indicated in figure 2. The incident ball did not bounce backward, as expected, but it stopped dead as if the added weight of my hand was zero. Close inspection showed that the stationary ball did not remain perfectly stationary but it moved forward by about 0.5 mm as a result of the collision, compressing the soft tissue of my hand. The soft tissue then expanded back to its original position, and returned the ball to where it was before the collision. In other words, there were two collisions. The first was the very short duration collision between the two balls, bringing the incident ball to rest, and the second was a longer duration collision between the struck ball and my hand. By using other balls of varying stiffness, I was then able to measure, at least in a qualitative fashion, the stiffness of the soft tissue in my hand. A ball with similar stiffness to soft tissue (e.g. a tennis ball) compresses on the same time scale, in which case the two separate collisions merge into just one collision and the incident ball then bounces back off the hand-held ball.

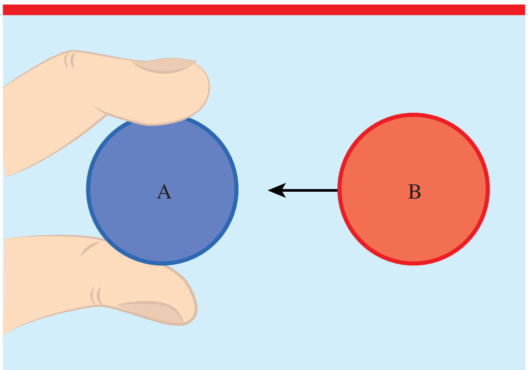

Figure 2. If ball A is held at rest and is struck by an identical ball B, then ball B can either come to a dead stop or bounce backwards, depending on the stiffness of the two balls. See movies 1 and 2 in the supplementary material (stacks.iop.org/PED/50/010052).

A similar result was obtained when adding plasticine or Blu-Tack to a billiard ball in order to increase its mass. In that case, the effect of the additional mass depended on whether the weighted ball was initially at rest or whether it struck an unmodified ball at rest. The two situations are shown in figure 3. The two balls were mounted to collide as pendulum bobs. If the unmodified ball was incident on the weighted ball at rest, as shown in figure 3(*a*), then the incident ball came almost to a dead stop. The ball bounced back by only about 1 mm. If the unmodified ball was initially at rest and was struck by the weighted ball, as in figure 3(*b*), then the weighted ball did not come to a dead stop. Instead, the weighted ball continued to move forward after the collision, as one would normally expect when a heavy ball strikes a light ball.

The latter results can also be understood in terms of two separate collisions. In both cases, there was a short duration collision of the two equal mass billiard balls, where the incident ball transferred essentially all of its momentum to the other ball. That event was then followed by a longer duration collision between the added mass and the ball to which it was attached. When the heavier ball was incident on the lighter ball, the heavier ball itself came to a dead stop for an

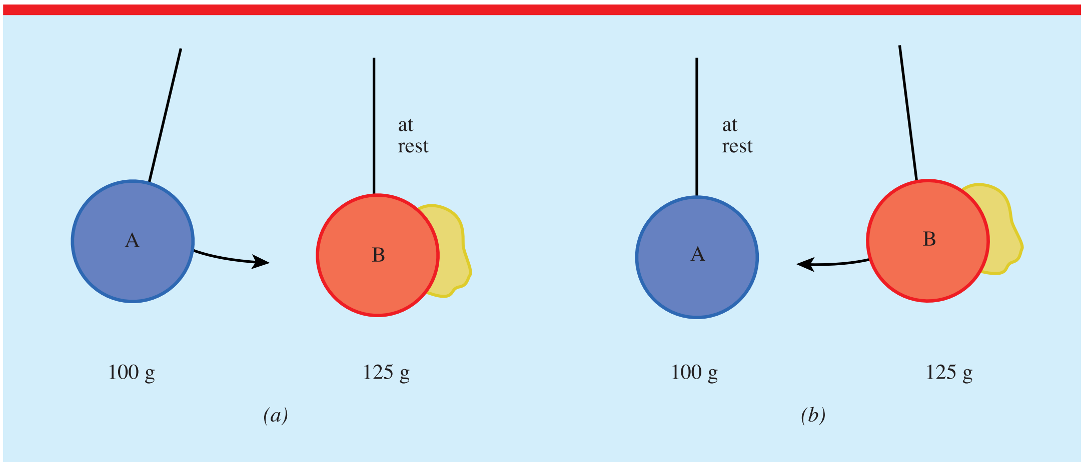

Figure 3. Balls A and B are identical billiard balls. An additional 25 g of plasticene was added to ball B. (*a*) If A collided with B then A came to a dead stop. (*b*) If B collided with A, then B continued to move forward after the collision, in the expected manner. See movies 3–5 in the supplementary material (stacks.iop.org/PED/50/010052).

instant, but it was then projected forwards as a result of the continued forward motion of the added mass attached to the rear of the ball.

Differences between the two collisions in figure 3 can be analyzed if we assume that in both cases the incident ball is incident at 1 m s^{-1}. In figure 3(*a*), ball A comes to rest and ball B exits at 0.8 m s^{-1} to conserve momentum. The energy loss is then 0.01 J. In figure 3(*b*), ball A exits at 1 m s^{-1}, and ball B comes to a complete stop after the first collision. The initial momentum carried by the plasticene is then shared with ball B, so ball B moves forward at 0.2 m s^{-1}. The energy loss is then 0.01 J, which is the same as in figure 3(*a*) despite the fact that the initial energy in figure 3(*b*) is greater than in figure 3(*a*). In both cases, the COR is 0.8, which is less than unity due to energy loss in the plasticene. In fact, both collisions are essentially the same collision viewed in two different reference frames. An insect sitting on ball A in figure 3(*a*) would see the collision shown in figure 3(*b*) where A is at rest as B approaches the insect at 1 m s^{-1}. A change in reference frame changes the speed of each ball, and hence changes its kinetic energy, but there is no change in the energy loss.

There are many other types of collision like those in figure 3, for example where one football player collides with another or where a relatively solid ball contains a soft core or a soft cover. Momentum will be conserved in such collisions, but there will be an increase in energy loss if a solid part of the ball collides with a softer part. The second collision may be totally inelastic, as in figure 3, or the two parts of the ball may continue to vibrate for a long time after the initial collision, in which case the vibrational energy will eventually be dissipated in the ball. Even a solid ball can vibrate after a collision, although the energy loss in a solid spherical ball due to ball vibrations is typically very small. That is partly why billiard balls and steel balls have such a high coefficient of restitution.

3. Bat and ball collisions

Friction between a billiard ball and a billiard table adds to the complexity of real billiard ball collisions. One way to avoid the problem is to suspend two balls as pendulum bobs and allow them to collide in mid air. That is how Sir Isaac Newton and his contemporaries discovered experimentally that momentum is conserved in head-on collisions. The same type of experiment can be done by colliding any two objects, say a bat and a ball or any two balls, with the added advantage that ‘real’ objects are involved and the energy loss or the coefficient of restitution can be quantified.

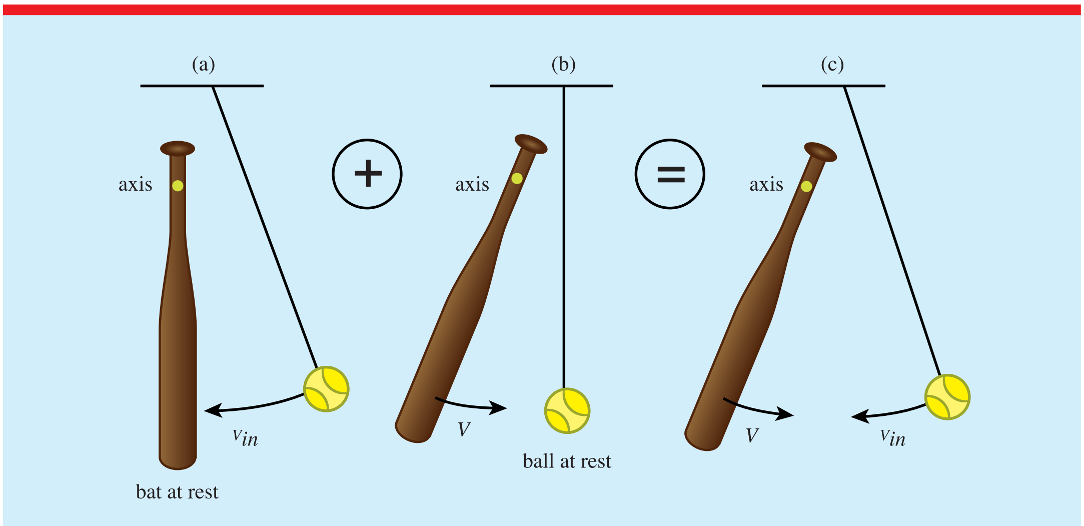

Figure 4. The outgoing speed of a batted ball in (c) can be calculated by adding (a) the measured outgoing speed when the bat is initially at rest to (b) the measured outgoing speed when the ball is initially at rest. The first measured speed represents the 'inbuilt power' of the bat and the second measured speed depends on the power of the batter and the moment of inertia of the bat. See movies 6 and 7 in the supplementary material (stacks.iop.org/PED/50/010052).

Figure 4 shows an arrangement used by the author to investigate the physics of the collision between a baseball and a baseball bat [4]. The same physics applies to any striking implement impacting a ball. The ball was suspended as a pendulum bob and the bat was allowed to swing as a physical pendulum by pivoting it about an axis near the end of the handle. The bat could be swung at a stationary ball, or the ball could be swung at a stationary bat, or both bat and ball could be swung towards each other. In addition, impacts could be studied at various impact points along the barrel by raising or lowering the ball. This arrangement is slightly more complicated than colliding two identical balls, but the essential physics is the same and the experimental procedure is the same. Bat and ball speeds are easily measured using a video camera. A project such as this is not beyond any good physics student and would provide a valuable and entertaining lesson in the physics of colliding objects of practical interest.

Each impact point along the bat can be treated as an isolated ball, with an effective mass less than the total bat mass and depending on (*a*) the moment of inertia of the bat about an axis through the swing axis, and (*b*) the actual impact point. A simple formula is derived below. Measurements of the bat and ball collision speeds can then be used to work out the COR at each impact point and to find the points along the bat with (*a*) the largest COR and (*b*) the largest outgoing ball speed. The largest COR point is the point along the bat that vibrates the least, commonly known as the 'sweet spot'. However, it is not necessarily the point that generates the greatest outgoing ball speed, since the velocity of the bat itself varies from a minimum at the handle end to a maximum at the tip.

The effective mass, M_e, of any given impact point on the bat can be calculated from figure 5. If a force, F, is exerted on the bat at a distance R from the axis then the bat will rotate about the axis at angular velocity ω. The point at which F is applied recoils at speed $V = R\omega$. If we treat that point as a ball of mass M_e then $F = M_e\mathrm{d}V/\mathrm{d}t$, while the torque about the axis is $FR = I_A\mathrm{d}\omega/\mathrm{d}t$ where I_A is the moment of inertia of the bat about the axis. Consequently,

$$F = M_e R\frac{\mathrm{d}\omega}{\mathrm{d}t} = \frac{I_A}{R}\frac{\mathrm{d}\omega}{\mathrm{d}t} \tag{4}$$

and hence

$$M_e = \frac{I_A}{R^2} \tag{5}$$

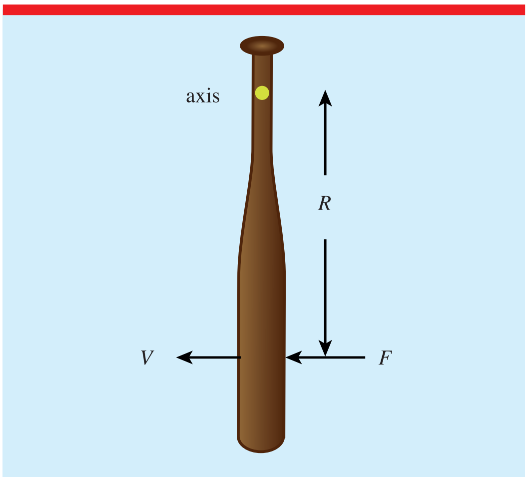

Figure 5. When a transverse force F is applied to a point on the bat, the bat recoils at speed V at that point.

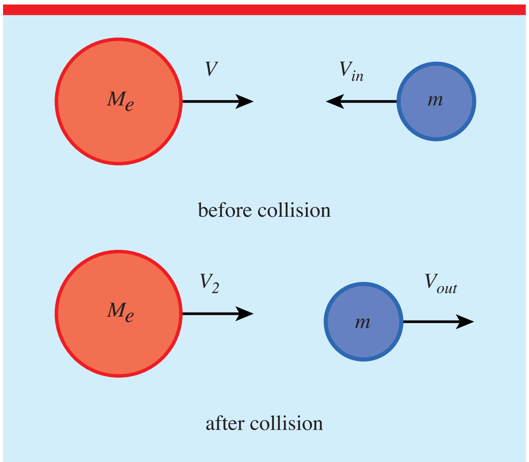

Figure 6. The collision between a bat and a ball of mass m can be described by replacing the bat with a ball of mass M_e.

The effective mass of the bat therefore decreases as R increases, and may even be as small as the mass of the ball, or even smaller, at the tip of the bat. In that case, a ball dropped onto the tip of a stationary, horizontal bat will not bounce off the bat at all and may come to a dead stop. The tip of a bat is the point with the least 'inbuilt power', but it partly makes up for its low effective mass by being the point that is swung the fastest.

To calculate the outgoing speed of the ball, v_{out}, we can treat the impact point as a ball of mass M_e as indicated in figure 6. Then

$$M_e V - m v_{\text{in}} = M_e V_2 + m v_{\text{out}} \tag{6}$$

where

$$e = \frac{v_{\text{out}} - V_2}{V + v_{\text{in}}} \tag{7}$$

and hence

$$v_{\text{out}} = e_A v_{\text{in}} + (1 + e_A)\, V \tag{8}$$

where

$$e_A = \frac{eM_e - m}{M_e + m} \tag{9}$$

is the apparent coefficient of restitution at the impact point. e_A depends on energy losses in the ball and the bat, and on M_e, but is easily measured using the arrangement in figure 4(a) where the bat is initially at rest with $V = 0$. In that case, $e_A = v_{\text{out}}/v_{\text{in}}$. Alternatively, the arrangement in figure 4(b) gives $(1 + e_A) = v_{\text{out}}/V$ when $v_{\text{in}} = 0$. The performance of any given bat (or racket or other striking implement) can be specified by plotting a graph of e_A versus position along the length of the barrel, or equivalently by plotting a graph of v_{out} versus V for an assumed v_{in} and swing speed ω, at different points along the barrel. Typical results, obtained with an Easton BK7 aluminium bat of mass 0.85 kg and length 0.84 m, are shown in figure 7. The COR was measured from the relative speeds of the bat and the ball at each impact point, not at the bat centre of mass. Results qualitatively

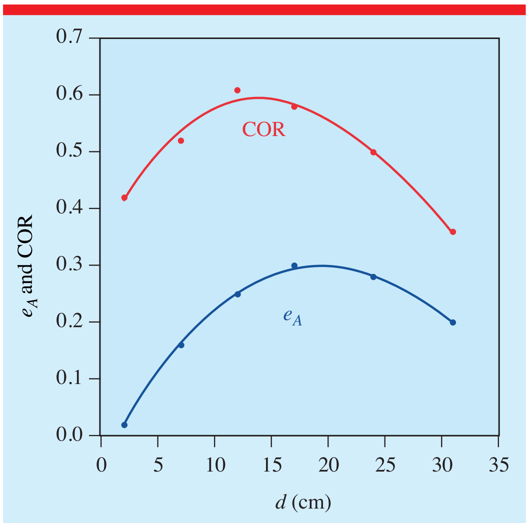

Figure 7. Experimental results obtained with a hollow aluminium baseball bat, with d being the distance from the impact point to the tip of the barrel.

similar to these are also found with cricket bats and tennis rackets.

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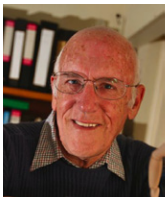

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