RESISTANCE COEFFICIENTS FOR SPHERES ON A PLANE BOUNDARY

by

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Signature of Author:

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Certified by: ______________________

Thesis Supervisor

ABSTRACT

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This thesis represents an investigation into the variation, with Reynolds number, of the resistance coefficients of spheres rolling on a plane boundary. The work was performed in the Hydrodynamics Laboratory of the Massachusetts Institute of Technology.

The procedure was to allow spheres of different size and specific gravity to roll down a smooth incline, under the action of gravity, submerged in fluids of different viscosity. The steady state translational velocity of the sphere was measured in each case and the Reynolds number and resistance coefficient computed.

A range of Reynolds numbers, $2x10^{-2} < N_R < 9x10^3$ was obtained with assurance of good accuracy. A few points of higher Reynolds number were obtained, but their accuracy is questionable. It was impossible to increase the upper limit of the Reynolds number range with the available equipment.

The conclusion reached is that the resistance coefficient - Reynolds number curve for the case of spheres rolling on a plane smooth boundary has the same general shape as that for spheres in an infinite body of fluid, with the drag coefficient at constant Reynolds number

being considerably larger in the former case.

The drag coefficient in the region of "creeping flow" (i.e. $N_R < 20$) appears to be given by $C_D = \frac{215}{N_R^{0.957}}$ in contrast with the Stokian value of $C_D = \frac{24}{N_R}$ for the spheres in an infinite fluid.

Cambridge, Massachusetts

May 20, 1957

Leicester F. Hamilton, Secretary of the Faculty

Massachusetts Institute of Technology

Cambridge, Massachusetts

Dear Sir:

In partial fulfillment of the requirements for the degree of Bachelor of Science in Civil Engineering from the Massachusetts Institute of Technology, I submit this thesis entitled "Resistance Coefficients for Spheres on a Plane Boundary."

Respectfully submitted,

Joseph James Carty, Jr.

ACKNOWLEDGEMENTS

The author is indebted to Dr. Peter S. Eagleson who initiated this study and whose interest, encouragement, and supervision stimulated me throughout the investigation.

An expression of appreciation is extended to Prof. Ronald E. Nece who provided a great deal of equipment, suggestions, and aid. Thanks also to Mr. T. A. Marlow and R. G. Dean for their advice and aid.

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LIST OF NOTATIONS

A	= projected area, square feet.
C_D	= coefficient of drag.
d	= diameter of cylinder, feet.
D	= diameter of sphere, feet.
F_D	= force due to drag, pounds.
L	= lift force, pounds.
N	= angular velocity, revolutions per minute.
N_R	= Reynolds number, as defined.
S.G.	= specific gravity of sphere.
$(S.G.)_f$	= specific gravity of fluid.
V_o	= velocity, feet per second.
W	= true weight, pounds.
W_B	= buoyant weight, pounds.
α (alpha)	= angle of elevation, degrees.
γ (gamma)	= density, pounds per cubic foot.
Γ (gamma)	= circulation, ft.^{2}/sec.
ν (nu)	= kinematic viscosity, ft.^{2}/sec.
ρ (rho)	= mass density, slugs per cubic foot.

SECTION I

INTRODUCTION

INTRODUCTION

The problem of finding coefficients of drag for spheres on and near a plane boundary arose from research done on sediment transportation and movement under the action of waves. There was a good curve available for values of drag coefficient as a function of Reynolds number in an infinite fluid, but nothing available for spheres on or near a plane boundary. No work has actually been done on this specific problem, but some has been carried out which touches on it. (See Survey of Literature.)

This report deals with experimentation for spheres rolling on a plane boundary. Coefficients were obtained for Reynolds numbers varying from a low of 2×10^{-2} to a maximum value of 5×10^{4}. The values up to 9×10^{3} were quite thoroughly covered and good accuracy obtained, but the higher values of Reynolds numbers are not as dependable.

From the results obtained, an accurate graph is plotted for the coefficient of drag as a function of Reynolds number.

SECTION II

SURVEY OF LITERATURE

SURVEY OF LITERATURE

There has been extensive research in the area of resistance coefficients for bodies of different shapes; however, most of this work has been done in the fluids of effectively infinite extent. The literature on drag coefficients for bodies subject to the effects of boundary proximity is not too extensive. The writer could find none concerning spheres in the vicinity of a plane boundary.

The coefficients of drag for common shapes with one or more axes of symmetry immersed in an "infinite" fluid have been determined by many investigators, therefore a dependable empirical relationship exists between coefficients of drag and Reynolds number, N_R.

At very low values of N_R, Stokes found theoretically that $C_D = \frac{24}{N_R}$ where

$$C_D = \frac{F_D}{\rho \frac{A}{2} Vo^2}$$

and:

F_D = drag force on body, lbs.

C_D = coefficient of drag, dimensionless

ρ = mass density of fluid, slugs/ cu. ft.

A = projected area of body = $\frac{\pi D^2}{4}$ for spheres, sq. ft.

Vo = velocity of body, ft./sec.

D = sphere diameter, ft.

This relationship has been found empirically to hold up to Reynolds numbers of about 0.1. This C_D - N_R relationship is commonly presented in graphical form over a Reynolds number range of about 10^8. It has been produced in many places and is readily available.

Substantial work on the subject of boundary effects on the resistance of spheres has been done by McNown (1) (2). The boundary conditions used were somewhat different than those employed in the study being reported here, but it is interesting to note some similarities. Experiments were conducted by dropping spheres through a cylinder rather than close to a plane boundary. In these experiments the sphere diameter is D and the cylinder diameter d. In reference (2) curves of C_D versus N_R were obtained for values of $\frac{D}{d}$ ranging from 0.2 to 0.98 and Reynolds numbers from 10^{-1} to 10^5. In reference (1) the same type of study covered values of $\frac{D}{d}$ from 0.2 to 0.99 and N_R from 10^{-2} to 10^3. In the latter paper emphasis was placed on very low Reynolds numbers. Results from reference (1) are reproduced in Figure 1.

The initial points of appreciable departure from the straight lines in Fig. 1 are indicated by crosses. A feature of the trend is the increase in the limiting value of N_R with increasing D/d. This is explained by the change of flow pattern with D/d so that the Reynolds number for the sphere alone is no longer a criterion for similarity.

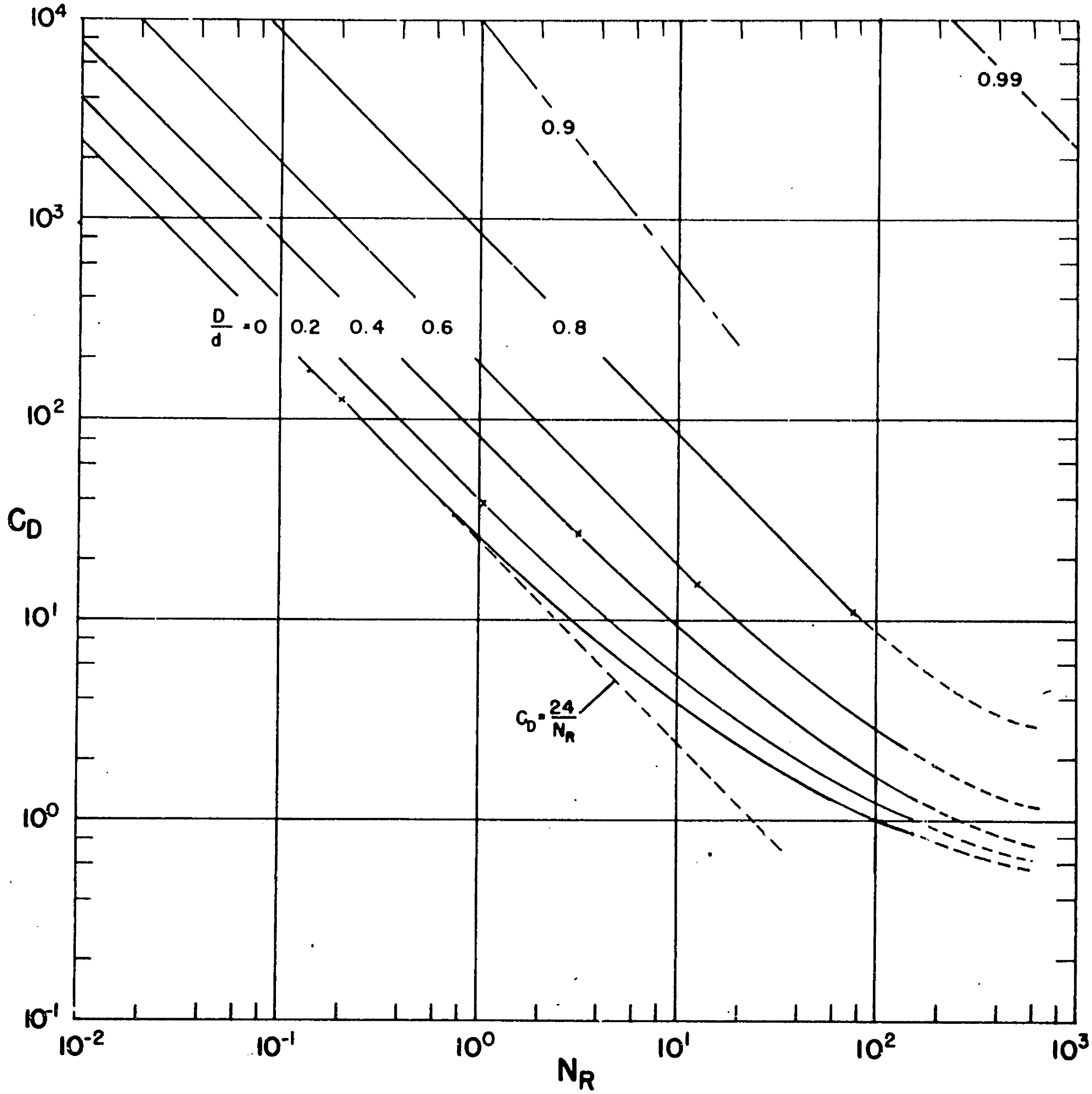


Figure 1: Variation of C_D with $\frac{D}{d}$ and R_N

The increase in viscous effects resulting from the increased velocity gradients more than offsets the increase in inertial effects.

An interesting question arises as to the effect of rolling on the drag coefficient for the spheres. Some work has been done on this problem. Davis (3) describes the effect of spin on the lift and drag forces acting on a golf ball. These, of course, are not smooth as are the spheres used in this work; however, some tests were done on smooth spheres for comparison. The general procedure was to drop a ball in a wind tunnel, first with topspin and then underspin. Equations were derived for drag and lift forces, and by measuring the horizontal movement for each case during its vertical fall, the forces could be computed. By comparing these cases with those for no spin, the effect of the spin is determined.

The values for the lift and drag for the standard balls show the effects of dimple and mesh markings to be similar. The values for the lift falls close to the curve represented by the equation.

$L = 0.064 \left[1 - \exp(-0.00026N)\right]$, where N is the rotational speed in r.p.m. and L is the lift in pounds. At the higher rotational speeds (above 5000 r.p.m.) the lift was about half the weight of the ball, 1.62 ounces. The drag increased nearly linearly from about 0.06 lb. at zero spin to just about the weight of the ball at 8,000 r.p.m.

For the smooth ball there was a surprising result for the lift. Up to about 5000 r.p.m., or an equatorial surface speed of 37 feet per second, the lift was just opposite in direction to that expected from theory. Above that speed it assumes the normal direction. The lift is much smaller than for the standard balls but there is an indication that at still higher rotational speeds it will be at least nearly as great. The drag of the smooth ball changes very little with change in rotational speed; above 4000 r.p.m. it is less than that of the standard balls.

Wayland and White (4) describe the work carried on at the U. S. Naval Ordnance Test Station concerning the boundary layer effects on spinning spheres. In this case, the spheres, up to a bowling ball in size, were dropped into water with various degrees of spin. Most of the results in this case are visual, as photographs were taken of the sphere as it fell through the water. These photographs are very enlightening, showing plainly that the spin has a marked effect on the boundary layer around the sphere.

The spheres were dropped into water with a sufficiently high velocity to cause a cavity of air to follow for some distance below the surface. Beyond a certain spin velocity a marked effect was seen on the character of the cavity wall and the separation angle. Since the drag of a body in cavity motion is closely correlated with the maximum diameter of the cavity, a marked difference occurs in the drag for low and high spin velocities. The transition from laminar to turbulent boundary layer is well defined in the photographs. At this point there is a marked reduction in the size of the wake, accompanied by a reduction in the drag force.

A concept which is of relevance and interest in this sort of experimentation is the phenomena of "virtual mass." In brief this concerns the added force resisting the acceleration of solid bodies due to the finite mass of fluid which they, in turn, must accelerate. This effect is beyond the scope of this thesis. Stelson and Mavis (5) give a good description of the effect on a body accelerating in an incompressible fluid of infinite extent.

SECTION III

THEORETICAL CONSIDERATIONS

THEORETICAL CONSIDERATIONS

The flow of a fluid around bodies of different shapes has been the object of much investigation. Most of this study has been done for bodies in an infinite fluid and a good deal is known about this case; however, the proximity of boundaries brings about interesting variations. This variation of flow pattern can best be attacked by describing it on a cylinder and realizing that a sphere results in a rather similar, but three dimensional, case.

For a cylinder falling at constant velocity through a fluid with its axis parallel to the direction of motion, there are only two forces acting, a net gravity force and a drag force exerted by the fluid. These two forces will be denoted by W_B (buoyant weight) and F_D respectively. (See Fig. 2) For this case, the flow pattern around the cylinder would be as shown in Fig. 3.

Fig. 2: Forces acting on a falling cylinder.

Fig. 2

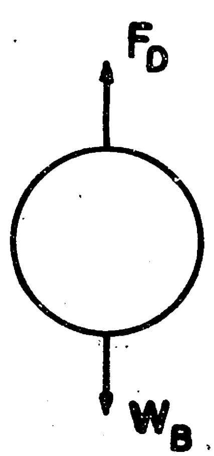


Fig. 3

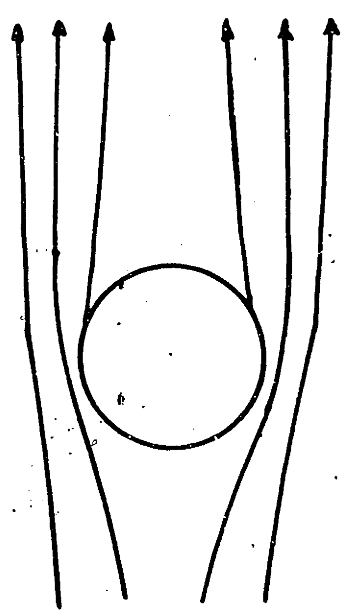


Fig. 3: Flow of a fluid around a freely falling cylinder.

For this case, there is a stagnation point, with attendant high pressure intensity, at the nose. In the wake (separation zone) on the downstream side the pressure intensity is near the low value associated with the point of separation. As long as there is no boundary near enough to affect this flow pattern around the body, there is no effect on the drag coefficient at this particular Reynolds number. However, as a boundary is brought nearer the body, a point is eventually reached where the flow begins to be "squeezed" and there is an effect on the drag force.

This investigation concerns spheres rolling on an inclined plane under the force of gravity. With the simplifying assumptions of a frictionless surface and no rotation of the sphere, the sphere would be acted on by a vertical gravity force and a resultant hydrodynamic drag force parallel to the plane as shown in Fig. 4.

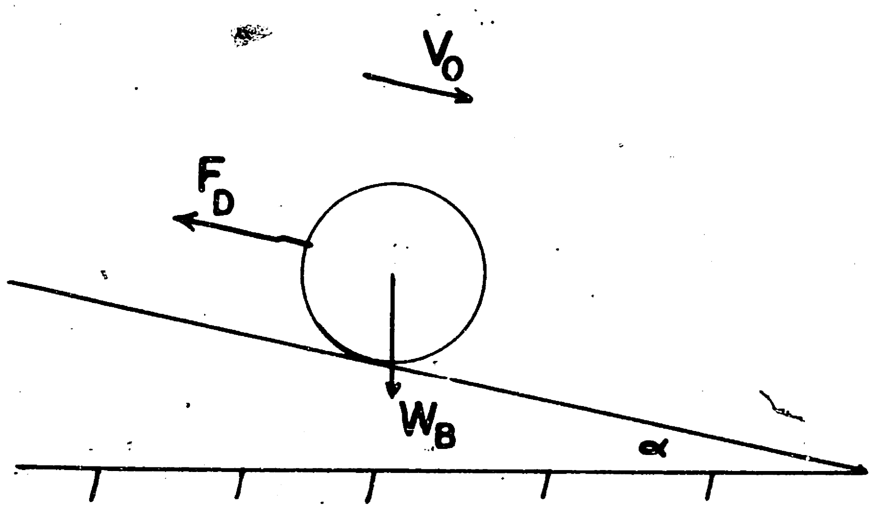


Figure 4: Forces on a body moving on a plane in a fluid, neglecting friction and rotation.

Starting from rest the sphere will accelerate under the action of gravity until, with increasing velocity, the forces build up to be equal and opposite. Under these assumptions at this point, $F_D = W_B \sin\alpha$, and the sphere ceases to accelerate and moves on at a constant velocity, Vo. As previously stated, the drag force is a function of the fluid density, projected area of the sphere, and velocity, and is given by:

$$F_D = C_D \rho \frac{A}{2} V^2 \qquad 1$$

therefore:

$$W_B \sin\alpha = C_D \rho \frac{A}{2} V^2 \qquad 2$$

which gives:

$$C_D = \frac{2 W_B \sin\alpha}{\rho A V^2} \qquad 3$$

but for a sphere:

$$A = \frac{\pi}{4} D^2 \qquad 4$$

therefore:

$$C_D = \frac{8 W_B \sin\alpha}{\rho \pi D^2 V^2} \qquad 5$$

In equation (5) all the terms are directly measurable, and C_D can be easily computed. As in the case of an infinite fluid, C_D can be plotted versus the Reynolds number which is also a function of velocity as well as diameter and kinematic viscosity, as follows:

$$N_R = \frac{VD}{\nu} \qquad 6$$

Here again V and D can be measured, and tables of ν for various fluids as a function of temperature are available. Therefore with these assumptions made, the coefficient of drag can be measured for any body for which the diameter, velocity, and weight are known; as well as the density of the fluid. If it is preferred to use the "true" weight, W, of the body rather than the buoyant weight equation (5) becomes:

$$C_D = \frac{8W \sin\alpha}{\rho \pi D^2 V^2} \times \left(\frac{S.G. - (S.G.)_f}{S.G.} \right) \qquad 7$$

where S.G. is the specific gravity of the body, and $S.G._f$ the specific gravity of the fluid.

If the mechanical friction force is not neglected in this computation, there must be another term in equation (2) which becomes:

$$W_B \sin\alpha = C_D \rho \frac{A}{2} V^2 + \mu W_B \cos \qquad 8$$

where μ is the coefficient of mechanical (rolling) friction. These coefficients are given in some tables, and for this work the effect will be discussed in Section VI. Applying this force to the sphere, the force picture becomes as shown in Fig. 5.

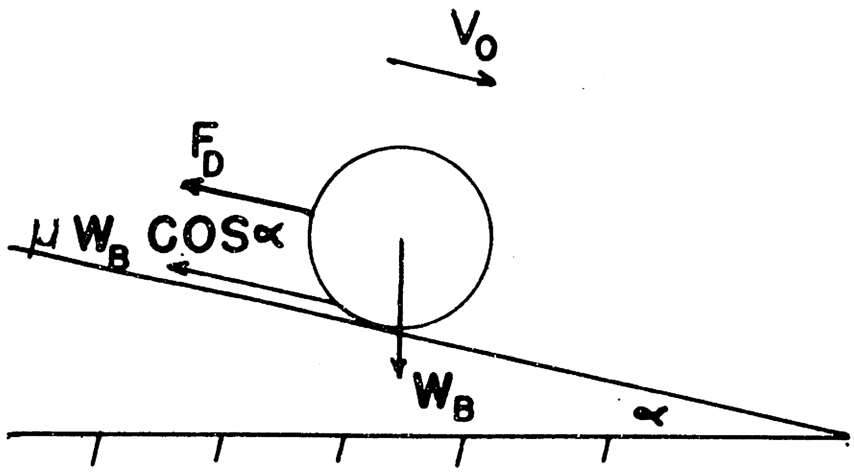


Figure 5: Forces on a body moving on a plane in a fluid with friction but neglecting rotation.

The rotation of the sphere will have an effect on the forces exerted on it. This can be seen by illustrating the flow patterns around a sphere with purely translational motion (Fig. 3) and one rotating in place (Fig. 6).

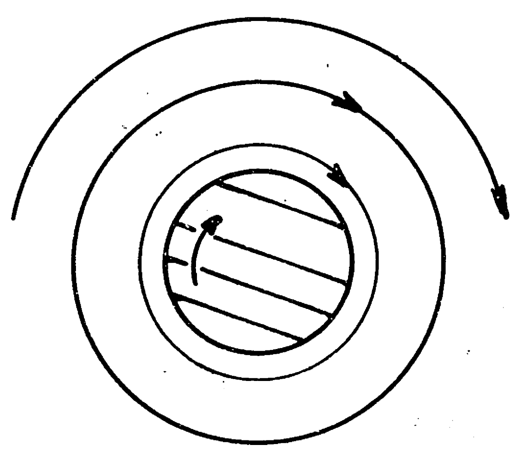


Figure 6: Flow pattern due to sphere rotation.

If these two patterns are superimposed the case of a rotating-translating sphere may be approximated. The flow pattern, as shown in Fig. 7, is now unsymmetrical with respect to axes perpendicular and parallel to the motion.

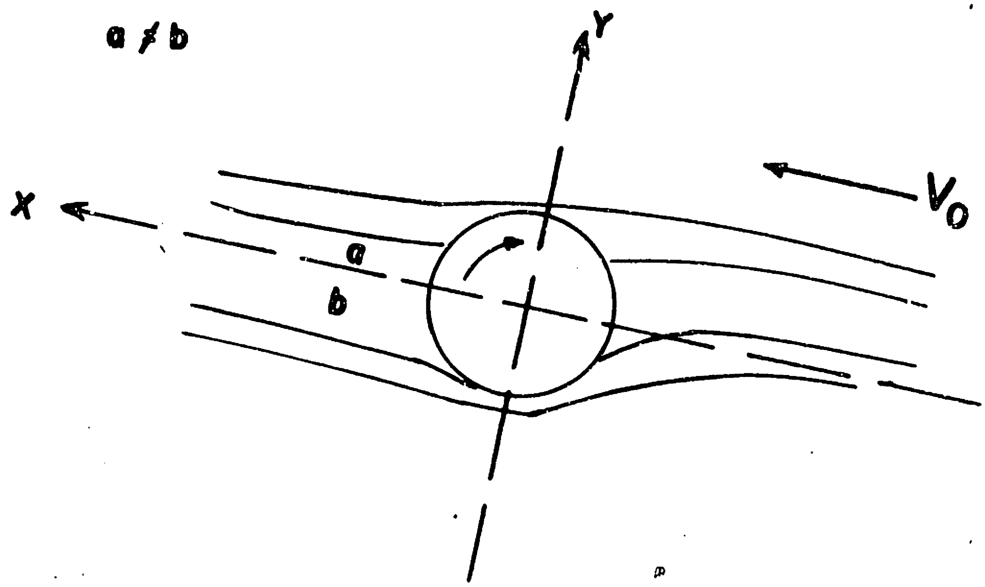


Figure 7: Flow around a rotating-translating sphere.

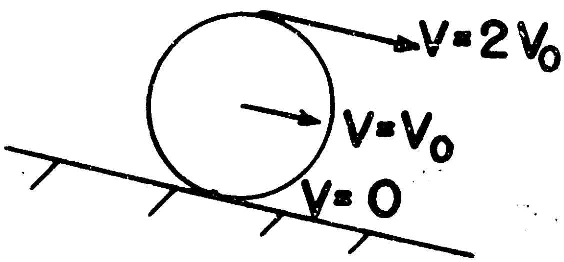
Because of the shifting of the stagnation point a flow such as this exerts a lifting force on the body in a direction normal to that of the free stream velocity, Vo.

With this basic flow picture for the rotating-translating sphere in an infinite fluid, the effects of proximity of a plane boundary parallel to the direction of motion may be hypothesized.

Due to rolling, the velocity of the sphere will be as shown in Fig. 8A. This is the equivalent in a perfect fluid, of a steady fluid velocity of Vo to the left, and an effective circulation,

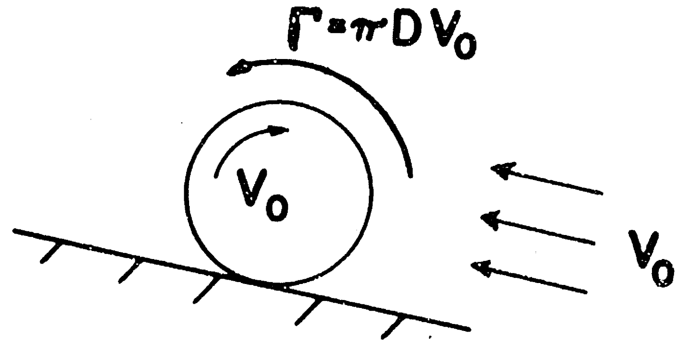
$\Gamma = \pi DVo$ as shown in Fig. 8B.

Figure 8 A



Velocity on a sphere due to rolling.

Figure 8 B



Effective circulation on a rolling sphere.

In a perfect fluid this combination of flows would produce coincidence of the fore and aft stagnation points at the bottom of the body. In a real fluid the circulation would be clockwise since the fluid is dragged by the clockwise rotation of the sphere and hence the stagnation points would be coincident at the top of the body. However, due to boundary layer formation on the sphere, separation should occur at approximately " 7 o'clock" and the stagnation point will adjust itself commensurate with the new limiting streamlines. This will yield a flow pattern similar to that of Figure 7.

The sphere rolling on a plane is therefore acted on by forces as shown in Figure 9.

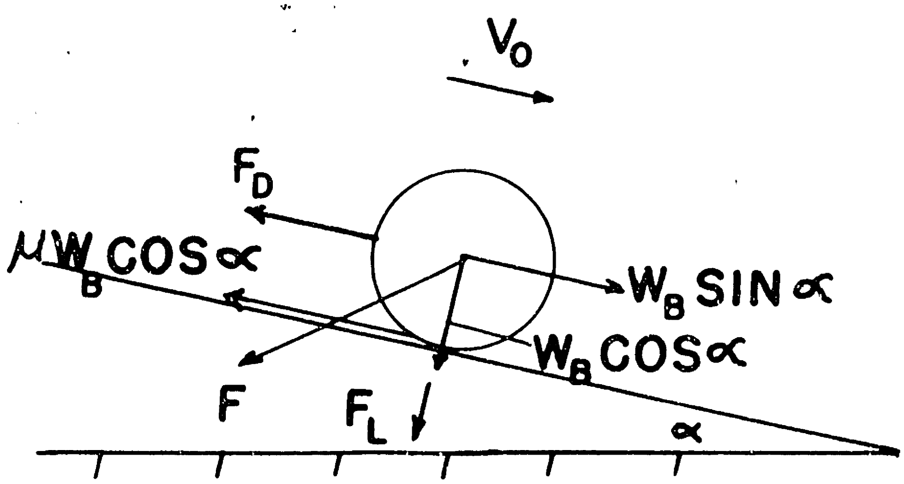


Figure 9: All forces acting on a body rolling on a plane in a fluid.

In the low Reynolds number region where surface resistance plays the major role, the drag coefficient should be higher with the presence of a boundary due to increased velocity gradients and hence increased boundary shear stresses on the bottom surface of the sphere.

As separation occurs, form drag plays a larger and larger part. The effect of the increased velocities on the underside may be to delay the separation point thereby reducing the drag coefficient over that for the infinite fluid.

The effect of lift force on C_D would vary with the weight and speed of the particle, and would yield a decrease in the defined C_D at constant N_R. The effect of rotation and friction force upon the determination will be discussed in Section VI.

SECTION IV

EXPERIMENTAL EQUIPMENT

EXPERIMENTAL EQUIPMENT

The spheres used in this test were lucite, glass, steel, and cellulose acetate (Figures 10A & 10B); and varied greatly in diameter and specific gravity. Their properties are listed in Table 1.

The fluids used were water, oil and air, the properties of which are presented in Appendix A.

The water testing was done on an inclined plane set up in a glass-walled channel $2\frac{1}{2}$ ft. wide and 3 ft. deep. This plane consisted of a piece of 3/4 inch marine plywood approximately $1\frac{1}{2}$ ft. wide and 6 ft. long, which was covered with several coats of marine spar varnish and gridded at 6 inch intervals. The plane was hinged at its lower end to a board secured to the bottom of the channel, and the slope varied by upright rods at the higher end. (See Fig. 11). A section of $\frac{1}{2}$ inch plate glass $1\frac{1}{2}$ ft. wide and approximately 4 ft. long was then placed on the plywood. This glass served a dual purpose as it formed a surface of low and uniform frictional properties and also reduced the effect of the dead weight sag in the board. A rubber tube approximately 2 inches in inner diameter was used as an entrance for the larger spheres. This tube was attached to the top rail of the channel and then butted against the top edge of the glass on the slope. By dropping the spheres into this tube and allowing them to roll out of it onto the plane, the entrance effects into the water were minimized and steady-state conditions reached at an early stage.

A second set-up was used for making runs in oil. This consisted of a small tank approximately 18 inches square and 4 inches deep, in which was placed a sheet of window glass 18 inches long to form the slope. The glass was marked off at 6 in. intervals and the slope easily computed by measuring the height of the supports under the upper end. (See Fig. 12). The oil used (Cities Service Co. No. 200) has a kinematic viscosity approximately 100 times that of water. Calibration curves for density and kinematic viscosity as a function of temperature are included in Appendix A.

For the few tests made with air as the fluid, an existing beach at one end of the channel was used. This beach has a fixed slope of 1 to 22.4 and is gridded every 6 inches. Its surface is made of varnished transite, but its frictional properties are far from uniform. Its effect will be discussed later.

A "Standard" timer which read directly to 0.10 seconds was used for measuring the velocities. The time was easily estimated to the nearest 0.05 seconds.

Table I Sphere Characteristics

Identification	Material	Diameter Ft. x 10^2	Weight lbs. x 10^2	S.G.
A	Lucite	3.57	0.175	1.17
C	Lucite	5.09	0.507	1.18
D	Lucite	6.13	0.886	1.18
E	Lucite	7.21	1.432	1.17
F	Lucite	8.23	2.151	1.18
G	Lucite	9.28	3.066	1.17
H	Lucite	10.37	4.290	1.18
M	Glass	6.00	1.750	2.48
N	Glass	6.10	1.834	2.47
R	Steel	2.08	0.229	7.82
S	Steel	8.87	17.690	7.80
Alex	Glass	1.64	.0325	2.26
Baker	Glass	1.18	.1210	2.28
Charlie	Glass	1.04	.0091	2.49
Dog	Glass	1.15	.0120	2.43
Easy	Glass	0.77	.0037	2.53
Franklyn	Acetate	1.60	.0170	1.26
Groucho	Acetate	0.93	.0033	1.26
I	Acetate	1.04	.0042	1.29
II	Glass	0.98	.0080	2.60
III	Acetate	0.66	.0012	1.29
IV	Acetate	0.98	.0038	1.23

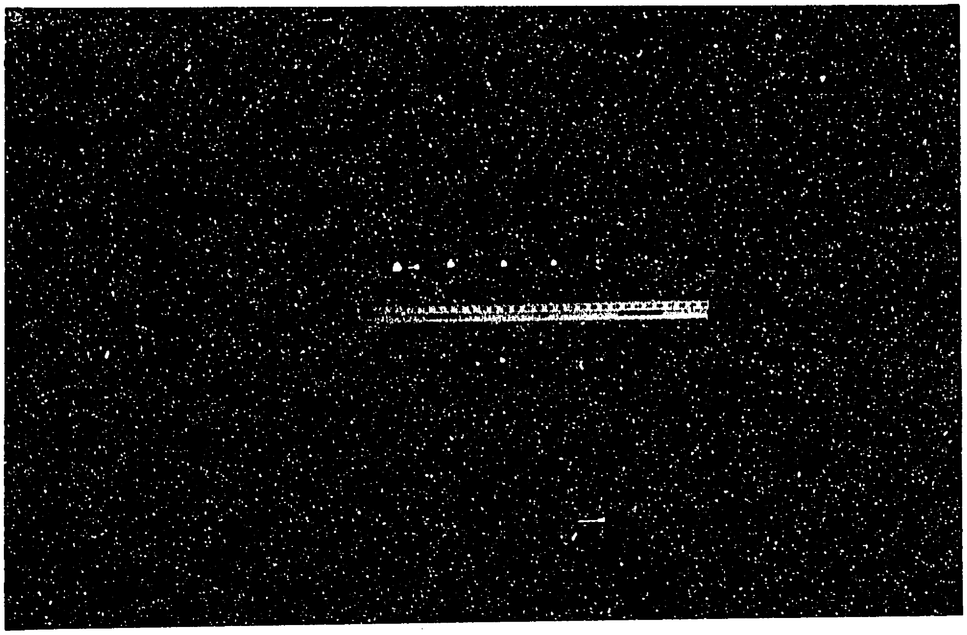


Figure 10 A.

Top: 7 lucite spheres used in testing.

Bottom: 2 steel spheres

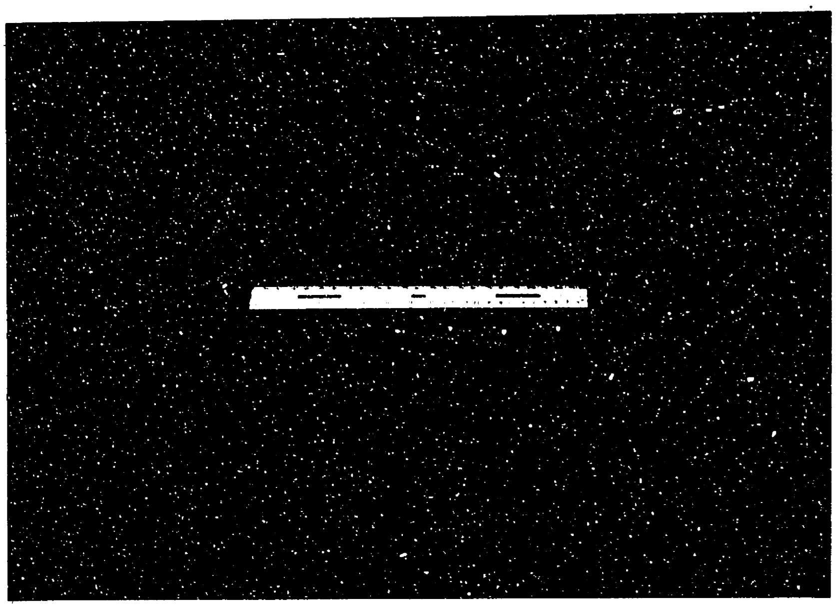
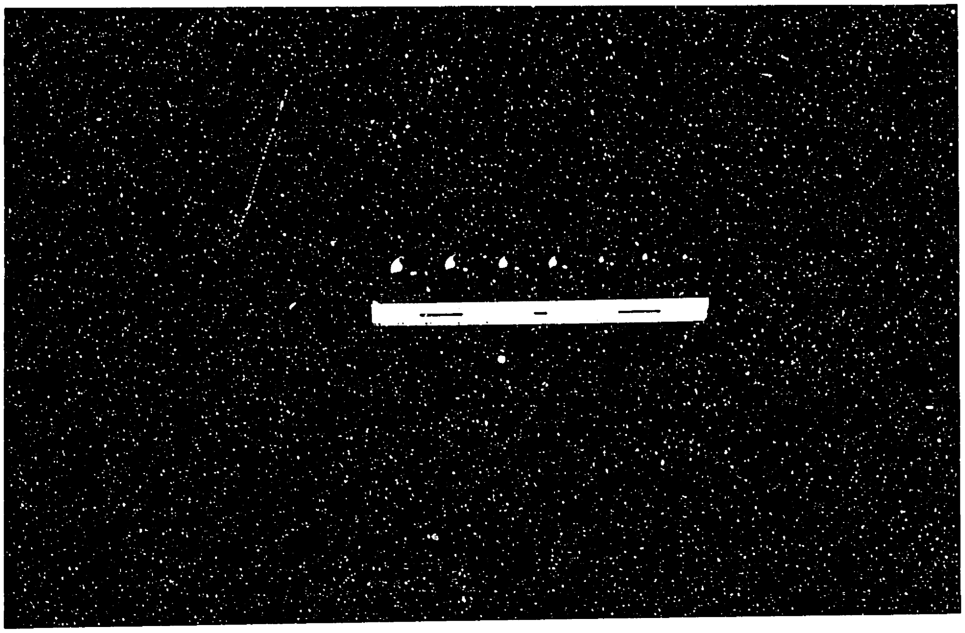


Figure 10 B.

Top: 5 cellulose acetate spheres.

Bottom: 8 glass spheres.



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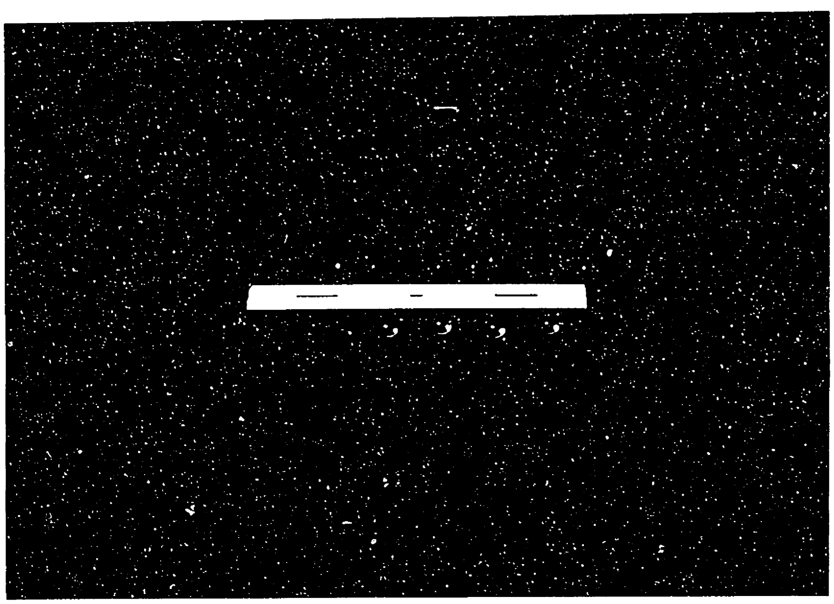

Figure 10 B.

Top: 5 cellulose acetate spheres.

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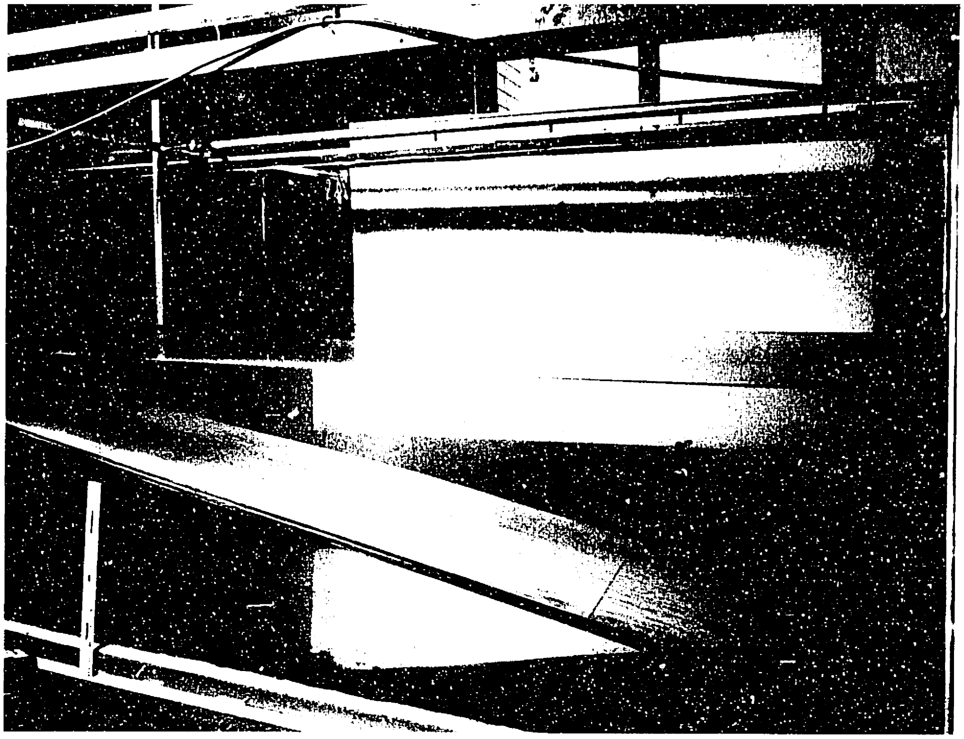

Figure 11.

Elevation view of slope in glass-walled channel.

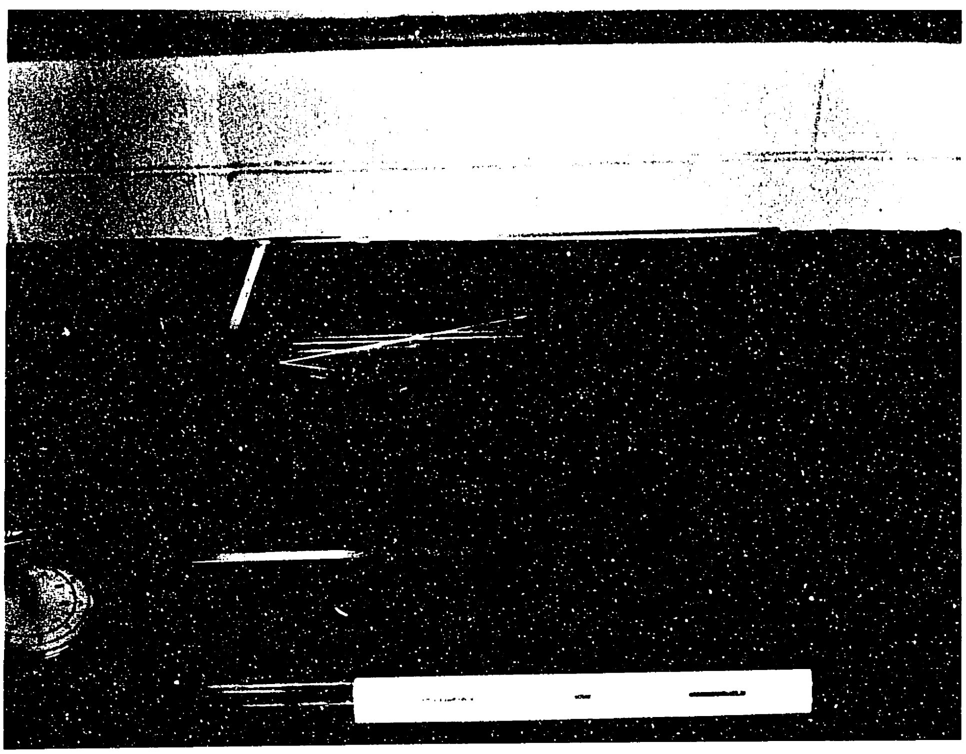

Figure 12.

Tank and slope for testing in oil.

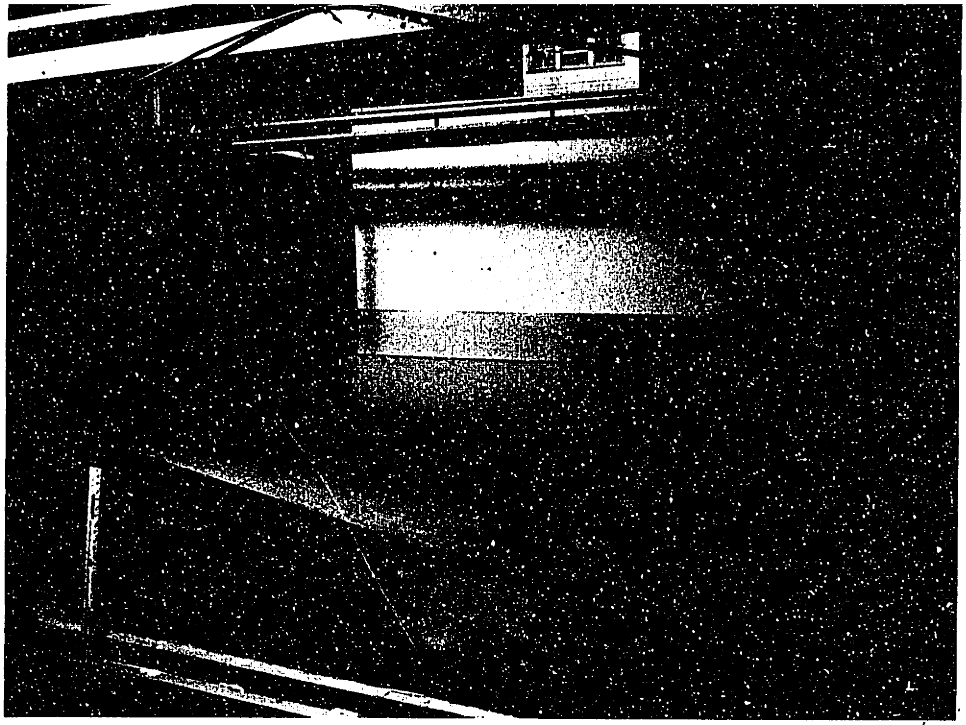

Figure 11.

Elevation view of slope in glass-walled channel.

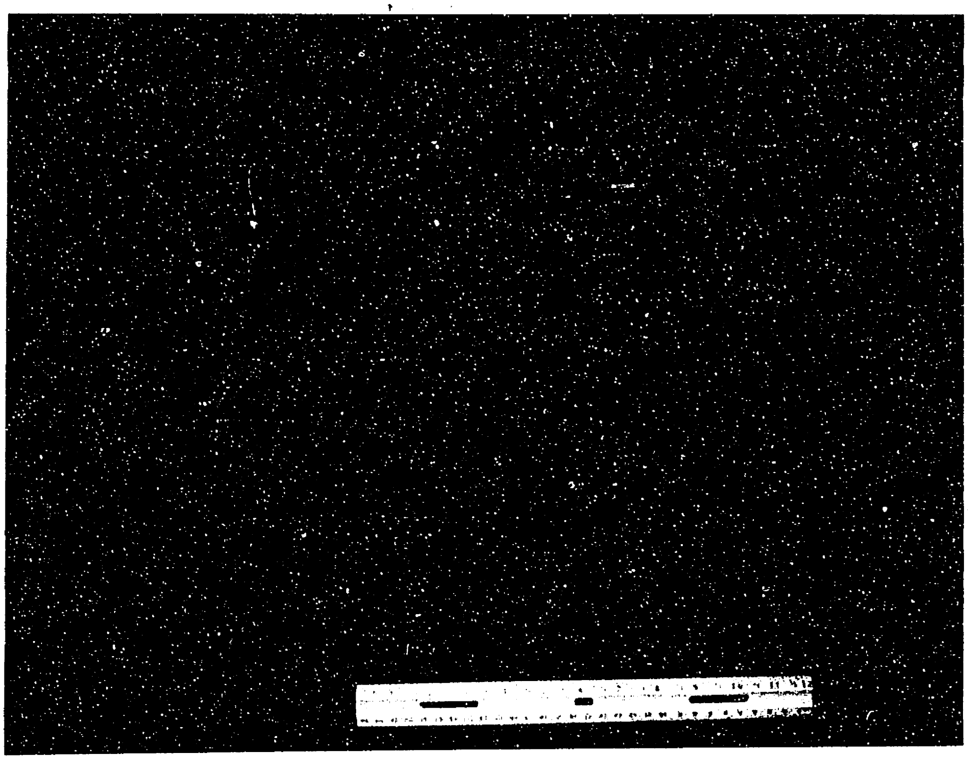

Figure 12.

Tank and slope for testing in oil.

SECTION V

EXPERIMENTAL PROCEDURE

EXPERIMENTAL PROCEDURE

A tabulation of the properties of all the spheres to be used was made. This included diameter, weight, and specific gravity; which were the necessary factors for determining C_D and N_R. These properties were known from previous investigations for some of the spheres. The diameters for the other spheres to be used were measured with a micrometer, and they were weighed to the nearest .0001 gram on balances in the Physical Chemistry Laboratory at M. I. T. From the known diameter, the specific gravity was easily computed.

The spheres used in the experimentation were coded and kept in separate containers for easy identification. Some were marked by letters, some by name, and a few were kept in containers which were marked with their properties. This latter method was used on the smallest glass and the acetate spheres, which had previously been measured and used in the research which suggested this investigation. (Reference 6)

This method of identification, linked with the table of properties, made the task of recording data and computing results relatively direct and efficient.

For the runs in water with the lucite, glass, and steel spheres, the entrance tube was used to eliminate any splashing and to have the sphere moving in the desired direction at approximately its terminal velocity upon reaching the plane. The spheres were allowed to roll several diameters before time was taken so as to be sure a steady velocity had been reached. By using this system it was possible to get a run of two to four feet and therefore a good time reading.

For the small acetate and glass spheres in water, a slightly diferent procedure was used. Since many of these spheres were too light to roll through the tube, they had to be held under the surface with the fingers and then dropped. While held under the surface, the spheres were rolled in the fingers to remove any air bubbles, and then released. The small size of these spheres caused a considerable time lag between release and their reaching the plane, and therefore all surface disturbance had died out before any velocity measurements were made. Their size also necessitated only a very short rolling distance before terminal velocity was reached.

For the experimentation with oil as the fluid; glass, lucite, and acetate spheres were used. In this case the spheres were simply placed on the glass plane in the small tank and time taken between marked off intervals as their weight carried them down the slope.

There were a few runs made on the beach in air. Two of the lucite spheres were used, and one additional run was made with a basketball as the sphere. This was done with the hope of increasing the Reynolds number, but proved unsuccessful for various reasons.

For the tests in water and air there were ten runs made at each slope per sphere, and the average velocity for these ten was used in the computations. In the oil, five runs were made and the average velocity used. This was due to the smaller velocity, longer time required for each run, and the higher accuracy time determination for these longer periods.

The temperature of the fluid was taken and recorded whenever data was taken so as to obtain an accurate value for kinematic viscosity. This is necessary for there is a large enough change with temperature to have considerable effect on the results.

The reduction of the data was carried out in the following manner. The Reynolds number was easily computed from the relation,

$$N_R = \frac{VD}{\nu}$$

where the average value of V for 10 runs was taken. The coefficient of drag was computed from equation (7).

$$C_D = \frac{8W \sin \alpha}{\rho \pi D^2 V^2} \times \left(\frac{S.G. - S.G._f}{S.G.} \right)$$

In this relation π is a constant, ρ a function of temperature, α is set, and W, D, V, and the specific gravities are all measured. A tabulation of the data is presented in Appendix A.

SECTION VI

PRESENTATION AND DISCUSSION OF RESULTS

PRESENTATION AND DISCUSSION OF RESULTS

When this work was undertaken, it was expected that the resistance coefficients for spheres on the plane would be greater than those for comparable Reynolds numbers in an infinite fluid. This assumption was made due to the effect of the boundary on the flow profile around the sphere as discussed in Section III.

The results of the tests may be seen in the curve of Figure 13, with a tabulation of the data recorded in Table 2. It is readily seen that these results are in general accord with the expectations.

At low values of Reynolds number, the curve becomes linear on logarithmic paper. This corresponds to the Stokian region for the infinite fluid which extends to a maximum Reynolds number of 0.1. For this case, the linear region extends to a N_R as high as approximately 20, and is defined by the equation

$$C_D = \frac{215}{N_R^{\ 0.957}} \qquad 9$$

This extension of the zone of purely viscous effects to higher Reynolds numbers when in the presence of a boundary was mentioned by McNown (1).

As in the case of an infinite fluid, this drag coefficient could probably be reached by theoretical considerations, however, such calculations are beyond the scope of this thesis. The effects of rotation and friction are certainly

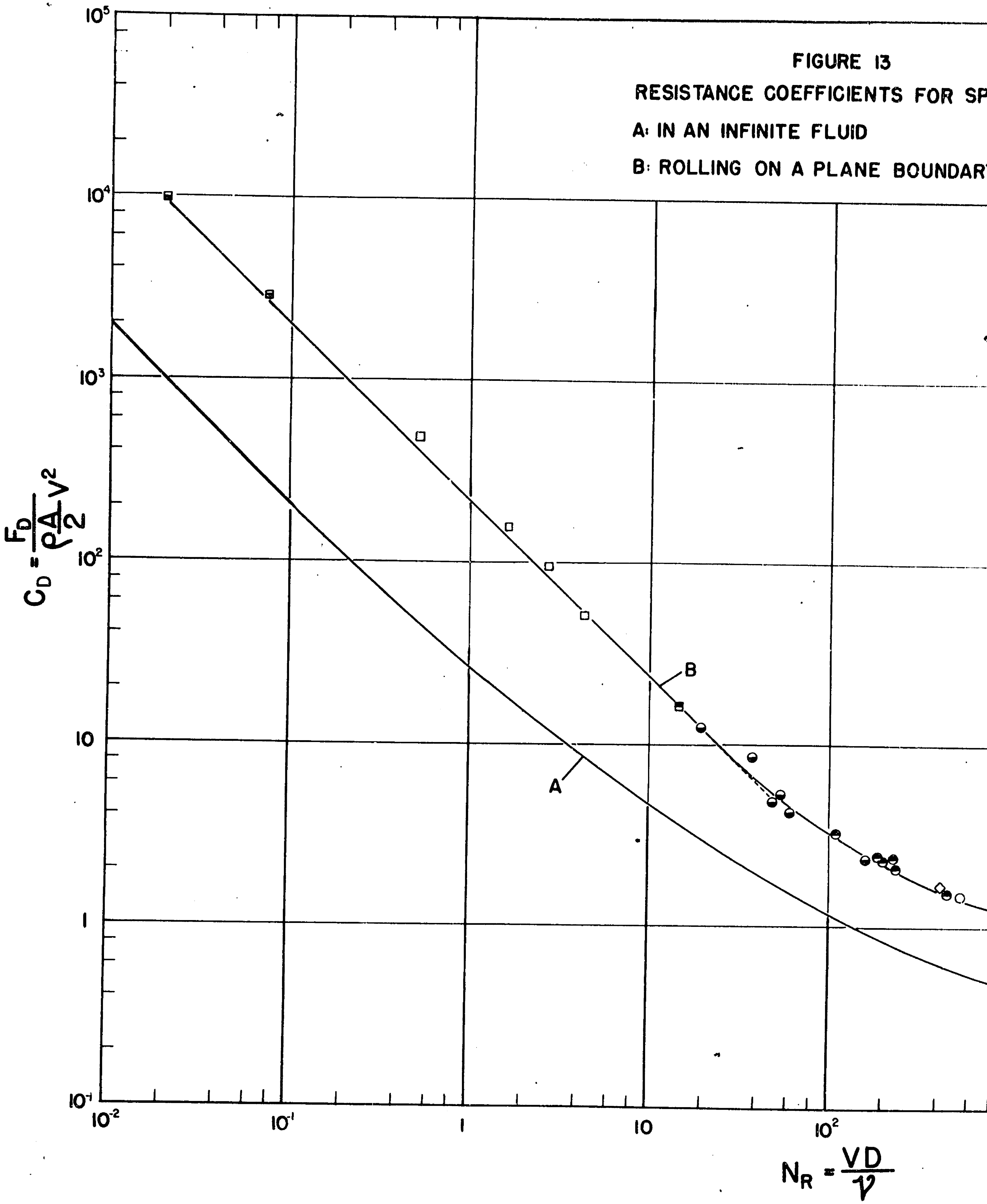

FIGURE 13
RESISTANCE COEFFICIENTS FOR SP
A: IN AN INFINITE FLUID
B: ROLLING ON A PLANE BOUNDAR
$C_D = \frac{F_D}{\rho \frac{A}{2} V^2}$
$N_R = \frac{VD}{\nu}$
A
B
10^5
10^4
10^3
10^2
10
1
10^{-1}
10^{-2}
10^{-1}
1
10
10^2

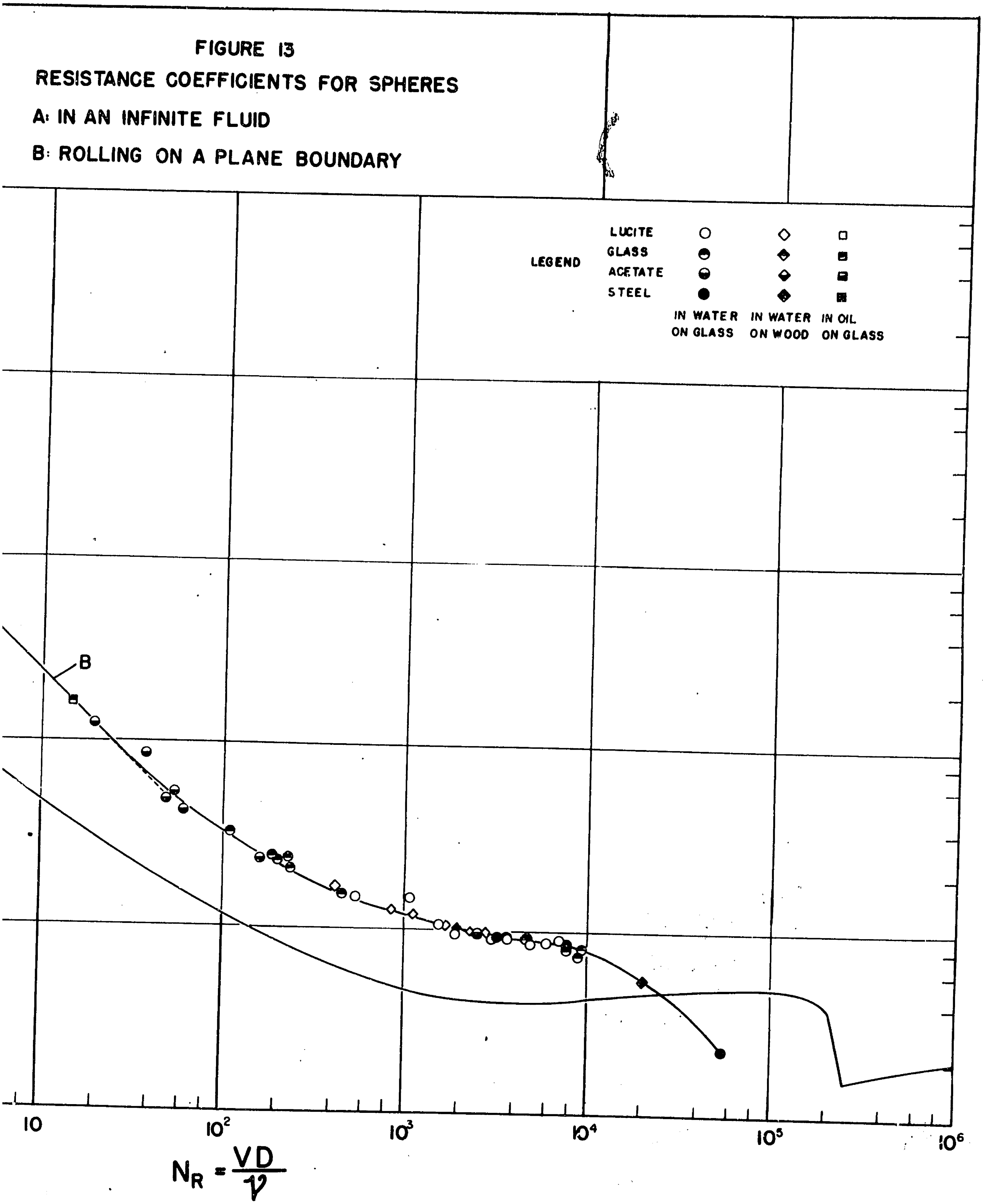

FIGURE 13

RESISTANCE COEFFICIENTS FOR SPHERES

A: IN AN INFINITE FLUID

B: ROLLING ON A PLANE BOUNDARY

of interest with regard to these results.

Rotation effects:

As seen earlier, the rotation of the sphere undoubtedly has some effect on the drag coefficient. For a sphere rotating in an infinite fluid, a value of $C_D = 0.5^-$ was given for a Reynolds number of 10^5. (7). This is essentially the same value obtained at this Reynolds number without rotation. Since rotation has been shown to change the wake geometry, the pressure intensity within the wake must change, at least at this Reynolds number, in such a manner as to keep C_D constant.

Friction effects:

Effort was made to minimize the effect of friction. In water, the spheres were rolled on a scratch free, clean, glass plate which should be as nearly frictionless as possible and therefore produce accurate results. A small, clean piece of glass was used in the oil runs and these results should also have minimum friction effects. The tests in air were made on a relatively rough beach, and the results for the drag coefficient varied considerably from all the others obtained. This variation would indicate that a good degree of consistency was maintained for frictional effects on the other spheres, since the effect of beach roughness was so marked.

The runs taken with lucite spheres were done both on the varnished plywood and the glass surface. It is seen that the points for both cases fall on the curve, showing that the frictional effects in each case are the same. Since there is a considerable difference between the surface character of these planes, it appears that frictional effects are negligible. It appears that the only serious effect of friction occurred in those test performed on the beach, and that in this case the friction was of such importance that it masked the effects of viscosity. This is highly probable, for the lucite spheres used were quite small and any roughness on the beach would have a relatively large effect on their motion. The one large sphere used was a rubber basketball, with a friction producing surface.

From these results, it seems reasonable to assume that friction was of no consequence in the drag force exerted on the sphere. As expected, the drag coefficients for spheres seem to be increased by the presence of a boundary near the sphere. Since friction and rotation seem to be somewhat eliminated as a factor in this work, the rise in C_D appears to be caused primarily by the presence of the boundary.

Only two points were obtained for Reynolds numbers greater than 10^4; and because these are not enough to define the curve accurately, it is less dependable in this area. There is a possibility that the curve swings off sharply and the final point is on the upward sloping line corresponding to that portion at $N_R > 2.5 \times 10^5$ on the infinite fluid curve. The curve as drawn seems more likely, which follows from the slightly downward turn at values of N_R slightly below 10^4. It is easily possible that this gradual fall occurs rather than the sharper one, due to the presence of the boundary and the rotation.

SECTION VII

CONCLUSIONS

CONCLUSIONS

From the results obtained, it appears that:

1. The curve for coefficient of drag for spheres rolling on a plane boundary is accurate and a good representation of actual values for $10^{-2} < N_R < 10^4$.

2. The curve is similar to that for spheres in an infinite fluid.

3. The zone of viscous influence extends to $N_R = 20$ and is described by $C_D = \dfrac{215}{N_R^{\,0.957}}$.

4. A point of boundary layer transition is apparently reached at a Reynolds number of 10^4.

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1. The curve for coefficient of drag for spheres rolling on a plane boundary is accurate and a good representation of actual values for $10^{-2} < N_R < 10^4$.

2. The curve is similar to that for spheres in an infinite fluid.

3. The zone of viscous influence extends to $N_R = 20$ and is described by $C_D = \frac{215}{N_R^{\ 0.957}}$.

4. A point of boundary layer transition is apparently reached at a Reynolds number of 10^4.

SECTION VIII

RECOMMENDATIONS FOR FUTURE WORK

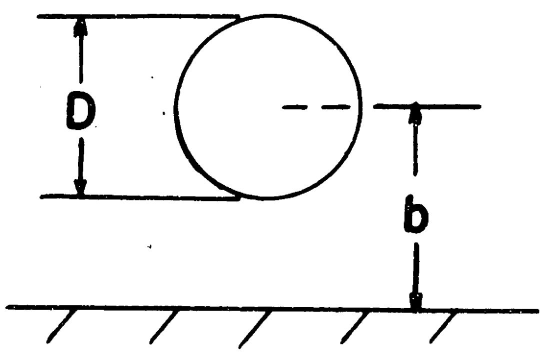
RECOMMENDATIONS FOR FUTURE WORK

There are two general areas of interest for future work:

1. Extension of this case of rolling spheres to higher Reynolds numbers in order to better define the zone of boundary layer transitions.

2. Compilation of resistance data for spheres without rolling and with various ratios of diameter to separation from the plane. (See Figure 14).

Figure 14: Definition Sketch



D = diameter of sphere.

b = separation of center from plane.

SECTION IX

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REFERENCES

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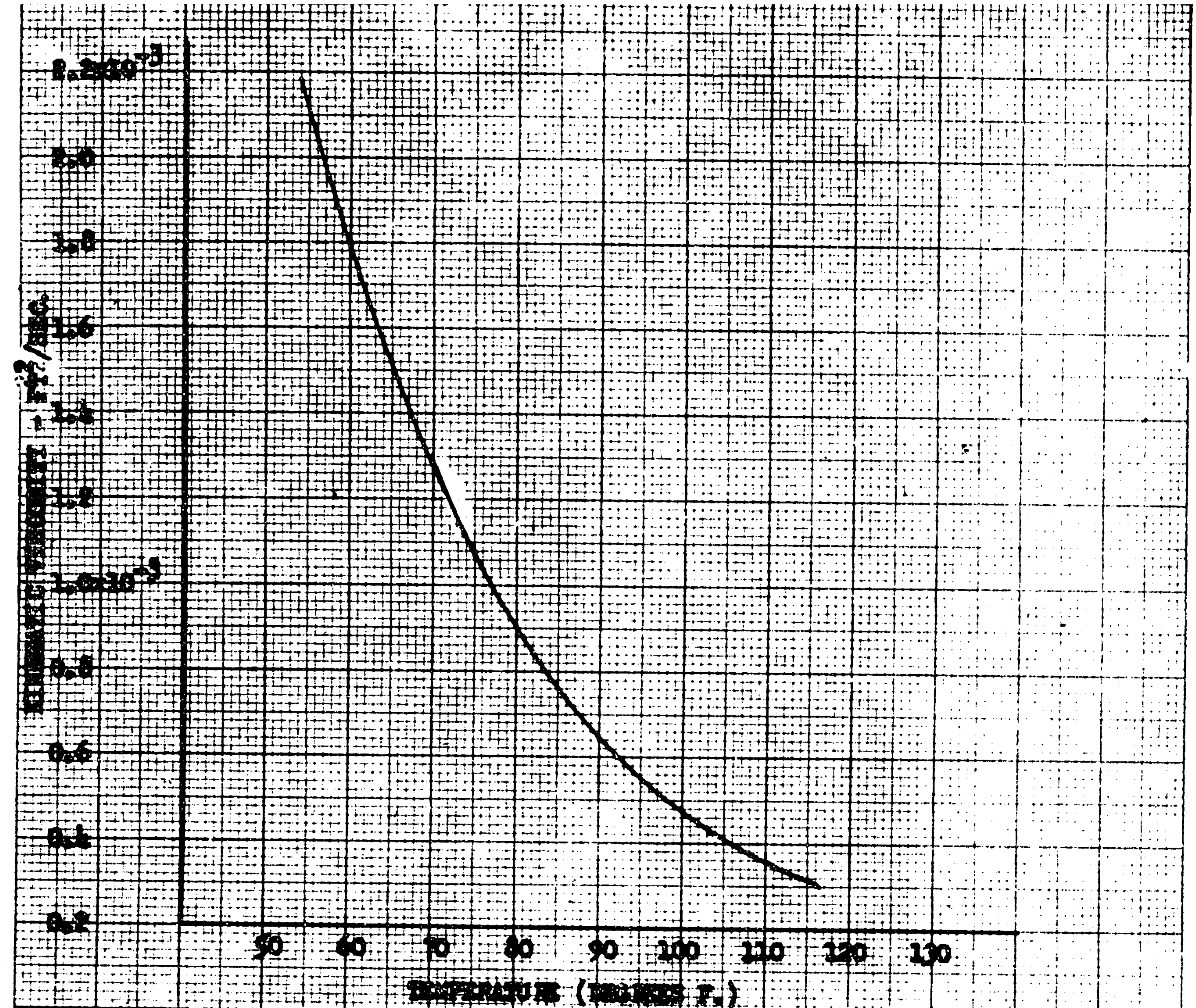
APPENDIX A

Mechanical Properties of Air at Atmospheric Pressure

Temperature T	Density ρ	Kinematic Viscosity ν
Degrees F	Slug/Ft.^{3}	Ft.^{2}/sec.
0	0.00268	1.26×10^{-4}
20	0.00257	1.36
40	0.00247	1.46
60	0.00237	1.58
80	0.00228	1.69×10^{-4}
100	0.00220	1.80
120	0.00215	1.89
150	0.00204	2.07
200	0.00187	2.40

Properties of Fresh Water

T	ρ	$\nu \times 10^5$
32	1.940	1.931
35	1.940	1.823
39.2	1.941	1.687
40	1.940	1.664
45	1.940	1.528
50	1.940	1.410
55	1.939	1.307
60	1.938	1.217
65	1.937	1 134
70	1.936	1.059
75	1.935	0.991
80	1.934	0.930
85	1.932	0.876
90	1.931	0.826
95	1.929	0.780
100	1.927	0.739
110	1.923	0.667
120	1.918	0.609
130	1.913	0.558
140	1.908	0.514
150	1.902	0.476
160	1.896	0.442
170	1.890	0.413
180	1.883	0.385
190	1.876	0.362
200	1.868	0.341
212	1.860	0.319


2.2x10^-5
2.0
1.8
1.6
1.4
1.2
1.0x10^-5
0.8
0.6
0.4
0.2
50
60
70
80
90
100
110
120
130
TEMPERATURE (DEGREES F.)

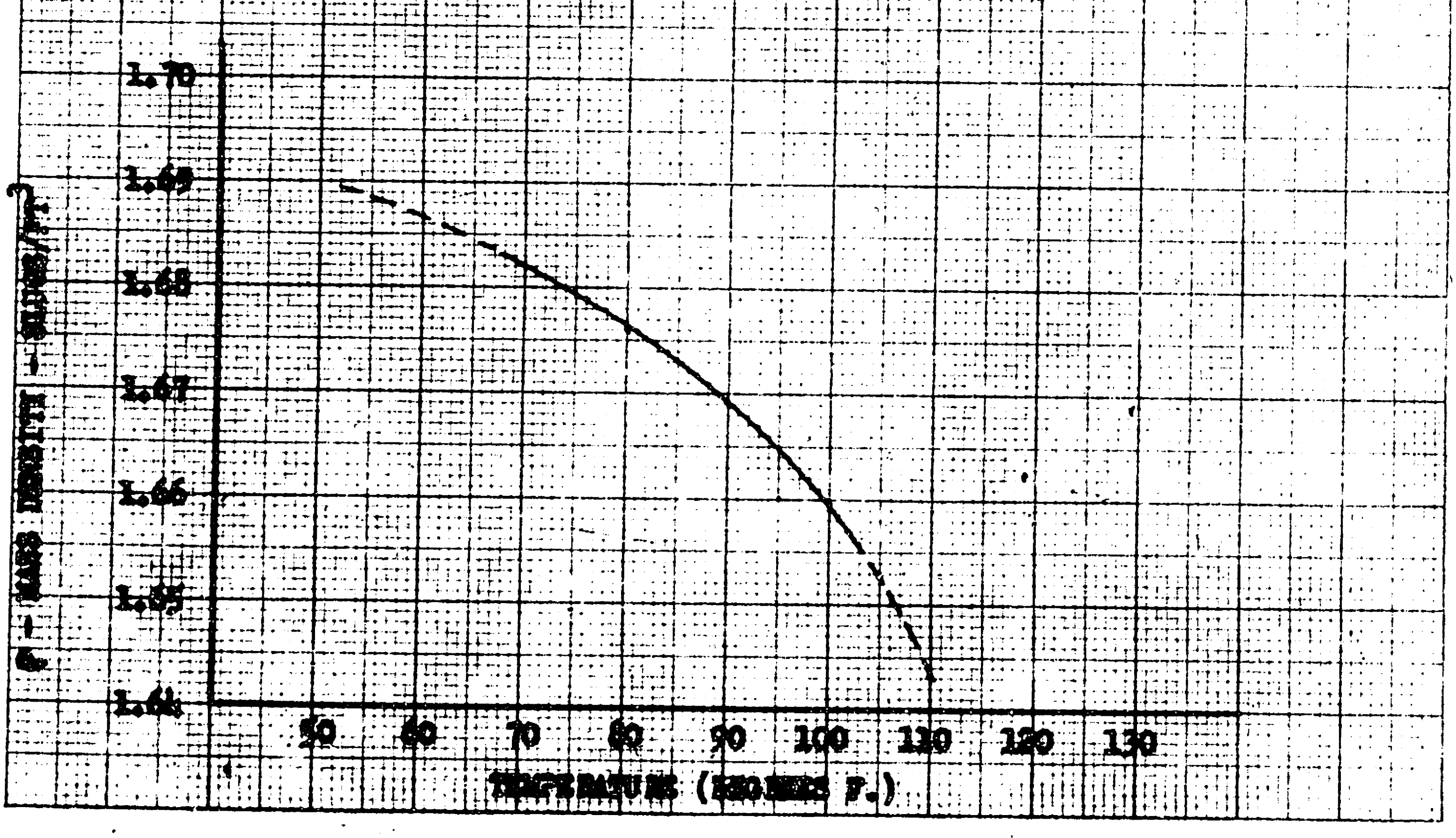

1.70
1.69
1.68
1.67
1.66
1.65
1.64
50
60
70
80
90
100
110
120
130
TEMPERATURE (DEGREES F.)

Table 2: Results

Sphere	N_R	C_D	$\nu \times 10^5$
A	0.532	482.5	94.0
A	415.0	1.680	1.41
A	535.0	1.484	1.167
C	1.630	152.3	94.0
C	837.0	1.258	1.41
C	1073.	1.220	1.119
D	2.720	95.40	94.0
D	1126.	1.214	1.41
D	1517.	1.058	1.119
E	4.680	50.90	94.0
E	1503.	1.053	1.41
E	1984.	0.961	1.119
F	1908.	1.031	1.41
F	2479.	0.971	1.119
G	2277.	0.982	1.41
G	2994.	0.905	1.119
G	4820.	0.853	1.005
H	2773.	0.973	1.41
H	3633.	0.902	1.119
H	5789.	0.868	1.005
H	6882.	0.897	1.005
M	3583.	0.932	1.41
M	4595.	0.900	1.119
M	7642.	0.794	1.005

Table 2: (Contd.)

Sphere	N_R	C_D	$\nu \times 10^5$
M	8776.	0.730	1.005
M	9182.	0.804	1.005
N	14.73	16.40	94.0
N	4672.	0.908	1.119
N	7587.	0.842	1.005
R	1660.	1.050	1.253
R	3104.	0.920	1.005
S	20,224.	0.547	1.253
S	54,305.	0.232	1.005
Alex	450.	1.54	1.151
Baker	236.	2.10	1.151
Charlie	201.	2.33	1.151
Dog	227.	2.38	1.151
Easy	108.	3.28	1.151
Franklyn	.0747	2837.	94.0
Franklyn	160.	2.35	1.151
Groucho	49.0	4.94	1.151
I	.0204	9983.	94.0
I	54.0	5.30	1.253
I	61.0	4.23	1.167
II	186.	2.44	1.151
III	19.44	12.32	1.151
IV	37.7	8.56	1.151

APPENDIX B

HYDRODYNAMICS LABORATORY
DEPARTMENT OF CIVIL AND SANITARY ENGINEERING
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
CAMBRIDGE 39, MASS.

PAGE 1
DATE 3-25-57
RUN NO:
PROJ. NO:
PARTY: Carty

PROJECT TITLE: Thesis

REMARKS: Water Temp. = 50° F. $\nu = 1.41 \times 10^{-5}$
Slope = 10"
On board

1	2	3	4	5	6	7	8	9	10
Run	Dist.	Time	Vel.		W	D	IR	C_D	S. G.
sphere H	(ft.)	(sec.)	(ft./sec.)		(lbs.)	(ft.)			
1	4	10.7			.0428	.1037	2773.	.973	1.18
2	"	10.5							
3	"	10.6							
4	"	10.5							
5	"	10.7							
6	"	10.7							
7	"	10.6							
8	"	10.5							
9	"	10.5							
10	"	10.7							
Avg.	4'	10.6	0.377						
Sphere G									
1	4'	11.60			.0306	.0928	2277.	.982	1.17
2	"	11.55							
3	"	11.50							
4	"	11.50							
5	"	11.55							
6	"	11.40							
7	"	11.50							
8	"	11.60							
9	"	11.70							
10	"	11.40							
Avg.	4'	11.55	0.346						

HYDRODYNAMICS LABORATORY

DEPARTMENT OF CIVIL AND SANITARY ENGINEERING

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

CAMBRIDGE 39, MASS.

PAGE 2

DATE 3-25-52

RUN NO.

PROJECT TITLE: Thesis

PROJ. NO.

PARTY: Canty

REMARKS: Water = 50°F.
Slope = 10°
On board

1	2	3	4	5	6	7	8	9	10
Run	Dist.	Time	Vel.		W	D	R	C_D	S.G.
Sphere A.	(ft.)	(sec)	(ft/sec)		(lbs.)	(ft.)			
1	4'	24.20			.00174	.0357	415.2	1.68	1.17
2	"	24.40							
3	"	24.20							
4	"	24.30							
5	"	24.30							
6	"	23.80							
7	"	24.40							
8	"	24.40							
9	"	24.20							
10	"	24.10							
Avg.	4'	24.35	.164						
Sphere C					.00505	.0509	837.5	1.258	1.18
1	4	17.50							
2	"	17.10							
3	"	17.40							
4	"	17.40							
5	"	17.15							
6	"	17.40							
7	"	17.20							
8	"	17.20							
9	"	17.00							
10	"	17.25							
Avg.	4	17.25	.232						

HYDRODYNAMICS LABORATORY

DEPARTMENT OF CIVIL AND SANITARY ENGINEERING

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

CAMBRIDGE 39, MASS.

PAGE 3

DATE: 3-25-57

RUN NO:

PROJECT TITLE: Thesis

PROJ. NO:

PARTY: Carty

REMARKS: Water = 50°F.
Slope = 10° on board

1	2	3	4	5	6	7	8	9	10
Run	Dist.	Time	Vel.		W	D	R	C_D	S.G.
Sphere D	(ft.)	(sec)			(lbs.)	(ft.)			
1	4	15.60			.00885	.0613	1126.	1.214	1.18
2	"	15.40							
3	"	15.90							
4	"	15.45							
5	"	15.40							
6	"	15.20							
7	"	15.60							
8	"	15.20							
9	"	15.60							
10	"	15.20							
Avg.	4	15.49	.259						
Sphere E					.0143	.0721	1503.	1.653	1.17
1	4	13.85							
2	"	14.10							
3	"	13.50							
4	"	13.30							
5	"	13.50							
6	"	13.75							
7	"	13.35							
8	"	13.40							
9	"	13.50							
10	"	13.65							
Avg.	4	13.60	.294						

HYDRODYNAMICS LABORATORY

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MASSACHUSETTS INSTITUTE OF TECHNOLOGY

CAMBRIDGE 39, MASS.

PAGE 4

DATE [illegible]

RUN NO.

PROJECT TITLE: Thesis

PROJ. NO.

PARTY: Canty

REMARKS: Water 50° F
Slope 10°
On board

Run	Dist.	Time	Vel.		W	D	R	C_D	S. G.
Sphere F	(ft.)	(sec.)	(ft/sec)		(lbs.)	(ft.)			
1	4	12.40			.0215	.0823	1903.7	1.031	1.18
2	"	12.20							
3	"	12.30							
4	"	12.55							
5	"	12.20							
6	"	12.15							
7	"	12.10							
8	"	12.25							
9	"	12.20							
10	"	12.20							
Avg.	4	12.25	.327						
Sphere M					.0600	.0175	3583	0.932	2.48
1	4	4.70							
2	"	4.90							
3	"	4.70							
4	"	4.75							
5	"	4.75							
6	"	4.75							
7	"	4.70							
8	"	4.80							
9	"	4.65							
10	"	4.80							
Avg.	4	4.75	.842						

HYDRODYNAMICS LABORATORY

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MASSACHUSETTS INSTITUTE OF TECHNOLOGY
CAMBRIDGE 39, MASS.

PAGE:

DATE: [illegible] 57

RUN NO:

PROJECT TITLE: Thesis

PROJ. NO:

PARTY: Carty

REMARKS: Water = 58° F
Slope = 10° On board

1	2	3	4	5	6	7	8	9	10
Run	Dist.	Time	Vel.		W	D	R	C_D	S.G.
Sphere R	(ft.)	(sec.)	(ft/sec.)		(lbs.)	(ft.)			
1	4	4.20			.00229	.0208	1660.	1.05	7.82
2	"	4.10							
3	"	4.00							
4	"	3.75							
5	"	4.00							
6	"	3.95							
7	"	4.00							
8	"	4.00							
9	"	4.00							
10	"	4.00							
Avg.	4	4.00	1.00						
Sphere	Glass plate on plane					3.17 mm 1/8"			
1	2	30.50			47.41×10^{6}	.0104'	54.0	5.30	1.29
2	2	30.90							
3	2	30.30							
4	2	30.20							
5	2	31.10							
6	2	31.20							
7	2	30.10							
8	2	30.80							
9	2	30.40							
10	2	31.50							
Avg.	2'	30.70	.0651						

HYDRODYNAMICS LABORATORY

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MASSACHUSETTS INSTITUTE OF TECHNOLOGY

CAMBRIDGE 39, MASS.

PAGE 2

DATE: 3-26-57

RUN NO:

PROJECT TITLE: Thesis

PROJ. NO:

PARTY: Co. Ty.

REMARKS:
Water = 58°F
Slope = 10° On board

1	2	3	4	5	6	7	8	9	10
Run	Dist.	Time	Vel.		W	D	R	C_D	S. G.
Sphere S	(ft.)	(sec)	(ft/sec)		(lbs.)	(ft.)			
1	4	1.50			.1769	.0837	20,224.	.547	7.11
2	"	1.35							
3	"	1.50							
4	"	1.40							
5	"	1.35							
6	"	1.30							
7	"	1.40							
8	"	1.40							
9	"	1.40							
10	"	1.35							
Avg.	4	1.40	2.857						
Sphere K									
1	4	4.95			.0123	.0525	3318.	1.00	2.60
2	"	5.05							
3	"	5.05							
4	"	5.05							
5	"	5.00							
6	"	4.95							
7	"	5.10							
8	"	5.05							
9	"	5.10							
10	"	5.10							
Avg.	4	5.05	0.792						

No Good

Wrong Sphere

HYDRODYNAMICS LABORATORY
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MASSACHUSETTS INSTITUTE OF TECHNOLOGY
CAMBRIDGE 39, MASS.

PAGE 1
DATE: 3-27-57
RUN NO:
PROJECT TITLE: Thesis
PROJ. NO:
PARTY: Conty

REMARKS: Water 63° $\nu = 1.167 \times 10^{-5}$
Slope 10°
Glass Bottom

1	2	3	4	5	6	7	8	9	10
Run	Dist.	Time	Vel.		W	D	R	C_D	S. G.
sphere A	(ft.)	(sec.)	(ft/sec.)		(lbs.)	(ft.)			
1	1.50	8.55			.00175	.0357	535.3	1.484	1.17
2	1.50	8.35							
3	"	8.50							
4	"	8.45							
5	"	8.60							
6	"	8.85							
7	"	8.35							
8	"	8.80							
9	"	8.70							
10	"	8.50							
Avg.	1.50	8.55	0.175						
						3.17mm.			
sphere						1/8"			
					$.42 \times 10^{-4}$	.0104'	61.13	4.23	1.29
1	2	28.85							
2	"	30.20							
3	"	30.30							
4	"	29.15							
5	"	29.20							
6	"	29.60							
7	"	28.30							
8	"	28.40							
9	"	28.30							
10	"	29.20							
Avg.	2.0	29.15	.0686						

HYDRODYNAMICS LABORATORY

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MASSACHUSETTS INSTITUTE OF TECHNOLOGY
CAMBRIDGE 39, MASS.

PAGE 2

DATE: 3-27-57

RUN NO:

PROJECT TITLE: Thesis

PROJ. NO:

PARTY: Carty

REMARKS: Water 64° F. $\nu = 1.151 \times 10^{-5}$
Slope 10°
Glass Bottom

1	2	3	4	5	6	7	8	9	10
Run	Dist.	Time	Vel.		W	D	$\mathbb{R}$	C_D	S.G.
sphere	(ft.)	(sec.)	(ft/sec.)		(lbs.)	(ft.)			
1	1.50	6.70			$.8 \times 10^{-4}$	3 mm.	186.5	2.44	2.60
2	"	6.80							
3	"	6.90				.0098'			
4	"	6.70							
5	"	7.00							
6	"	7.05							
7	"	6.95							
8	"	6.80							
9	"	7.00							
10	"	6.80							
Avg.	1.50	6.85	.219						
Sphere									
1	2	45.35			$.379 \times 10^{-4}$	3 mm.	37.70	8.57	1.23
2	"	45.70							
3	"	43.60				.0098'			
4	"	47.05							
5	"	44.50							
6	"	44.30							
7	"	45.20							
8	"	45.85							
9	"	44.20							
10	"	45.60							
Avg.	2.0	45.15	.0443						

HYDRODYNAMICS LABORATORY

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RUN NO:

PROJECT TITLE: Thesis

PROJ. NO:

PARTY: Carty

REMARKS: Water 64° F.
Slope 10°
glass bottom

1	2	3	4	5	6	7	8	9	10
Run	Dist.	Time	Vel.		W	D	R	C_D	S.G.
sphere	(ft.)	(sec)	(ft/sec)		(lbs.)	(ft.)			
1	1.0	31.10			$.12 \times 10^{-4}$	2 mm.	19.41	12-32	1.29
2	"	29.20							
3	0.50	15.90				.0066'			
4	"	14.50							
5	"	14.70							
6	"	14.40							
7	"	13.90							
8	"	14.80							
9	"	15.00							
10	"	14.10							
Avg.	0.50	14.75	.0339						
Sphere alex					3.25×10^{-4}	.0164	450.2	1.54	2.26
1	1.50	4.75							
2	"	4.80							
3	"	4.75							
4	"	4.70							
5	"	4.75							
6	"	4.75							
7	"	4.70							
8	"	4.85							
9	"	4.70							
10	"	4.80							
Avg.	1.50	4.75	.316						

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PROJ. NO:
PARTY: Carty

PROJECT TITLE: Thesis

REMARKS: Water 64°
Slope 10°
Glass Bottom

1	2	3	4	5	6	7	8	9	10
Run	Dist.	Time	Vel.		W	D	IR	C_D	S.G.
sphere Baker	(ft.)	(sec.)	(ft/sec.)		(lbs.)	(ft.)			
1	1.50	6.70			1.21×10^{-4}	.01175	235.8	2.10	2.28
2	"	6.65							
3	"	6.50							
4	"	6.60							
5	"	6.55							
6	"	6.45							
7	"	6.50							
8	"	6.40							
9	"	6.40							
10	"	6.50							
Avg.	1.50	6.50	.231						
Sphere Charlie					$.912 \times 10^{-4}$	.0104	200.6	2.33	2.49
1	1.50	7.00							
2	"	6.85							
3	"	6.50							
4	"	6.70							
5	"	6.70							
6	"	6.75							
7	"	6.90							
8	"	6.80							
9	"	6.65							
10	"	6.80							
	"								
Avg.	1.50	6.75	.222						

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PROJ. NO:
PARTY: Carty

PROJECT TITLE: Thesis

REMARKS: Water 64° F.
Slope 10°
Glass Bottom

1	2	3	4	5	6	7	8	9	10
Run	Dist.	Time	Vel.		W	D	R	C_D	S. G.
Sphere Dog	(ft.)	(sec.)	(ft./sec.)		(lbs.)	(ft.)			
1	1.50	6.50			1.21×10^{-4}	.0115	226.8	2.38	2.43
2	1.50	6.70							
3	"	6.50							
4	"	6.75							
5	"	6.60							
6	"	6.60							
7	"	6.70							
8	"	6.40							
9	"	6.60							
10	"	6.60							
Avg.	1.50	6.60	.227						
Sphere EASY					$.372 \times 10^{-4}$	.00767	108.6	3.28	2.53
1	1.50	9.20							
2	"	9.15							
3	"	9.20							
4	"	9.00							
5	"	9.00							
6	"	9.20							
7	"	9.30							
8	"	9.20							
9	"	9.25							
10	"	9.30							
Avg.	1.50	9.20	.163						

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PROJECT TITLE: Thesis

PROJ. NO:

PARTY: Carty

REMARKS:
Water 64°F
Slope 10°
Glass Bottom

1	2	3	4	5	6	7	8	9	10
Run	Dist.	Time	Vel.		W	D	R	C_D	S.G.
Sphere Franklyn	(ft.)	(sec.)	(ft./sec.)		(lbs.)	(ft.)			
1	1.50	13.00			$.169 \times 10^{-4}$	.0160	160.0	2.35	1.26
2	"	12.90							
3	"	12.90							
4	"	12.90							
5	"	13.05							
6	"	13.40							
7	"	13.40							
8	"	13.20							
9	"	13.25							
10	"	13.00							
Avg.	1.50	13.10	.115						
Sphere Graucho					$.335 \times 10^{-4}$	.00933	49.0	4.94	1.26
1	1.50	24.90							
2	"	24.95							
3	"	24.70							
4	"	25.20							
5	"	24.30							
6	"	24.40							
7	"	24.60							
8	"	25.40							
9	"	24.40							
10	"	25.05							
Avg.	1.50	24.80	.0605						

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PROJ. NO:
PARTY: Carty

PROJECT TITLE: Thesis

REMARKS: Water 66° F. $\nu = 1.119 \times 10^{-5}$
~~Slope 10°~~
~~Glass~~

1	2	3	4	5	6	7	8	9	10
Run	Dist.	Time	Vel.		W	D	$\mathbb{R}$	C_D	S.G.
sphere C	(ft.)	(sec.)	(ft/sec.)		(lbs.)	(ft.)			
					.00507	.0509	1073.	1.22	1.18
1	3.50	14.90							
2	"	14.70							
3	"	14.75							
4	"	14.90							
5	"	15.10							
6	"	14.95							
7	"	14.80							
8	"	14.60							
9	"	14.55							
10	"	14.70							
Avg.	3.50	14.80	.236						
Sphere D					.00886	.0613	1517.	1.058	1.18
1	3.50	12.50							
2	"	12.50							
3	"	12.60							
4	"	12.60							
5	"	12.70							
6	"	12.60							
7	"	12.80							
8	"	12.70							
9	"	12.70							
10	"	12.80							
Avg.	3.50	12.65	.277						

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PARTY: Carty

PROJECT TITLE: Thesis

REMARKS: Water 66°
Slope 10°
Glass

1	2	3	4	5	6	7	8	9	10
Run	Dist.	Time	Vel.		W	D	R	C_D	S.G.
sphere E	(ft.)	(sec.)	(ft/sec.)		(lbs.)	(ft.)			
1	3.50	11.25			.01432	.0721	1984.	0.961	1.17
2	"	11.35							
3	"	11.35							
4	"	11.30							
5	"	11.40							
6	"	11.35							
7	"	11.40							
8	"	11.45							
9	"	11.40							
10	"	11.40							
Avg.	3.50	11.35	.308						
sphere F					.0215	.0823	2479.	0.971	1.18
1	3.50	10.30							
2	"	10.50							
3	"	10.40							
4	"	10.50							
5	"	10.40							
6	"	10.60							
7	"	10.35							
8	"	10.35							
9	"	10.30							
10	"	10.45							
Avg.	3.50	10.40	.337						

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PROJ. NO:
PARTY: Carty

PROJECT TITLE: Thesis

REMARKS: Water 66°
Slope 10°
Glass

Run	Dist.	Time	Vel.		W	D	R	C_D	S.G.
sphere G	(ft.)	(sec.)	(ft./sec.)		(lbs.)	(ft.)			
1	3.50	9.80			.0307	.0928	2994.	.905	1.17
2	"	9.80							
3	"	9.70							
4	"	9.70							
5	"	9.70							
6	"	9.70							
7	"	9.70							
8	"	9.75							
9	"	9.60							
10	"	9.60							
Avg.	3.50	9.70	.361						
sphere H					.0429	.1037	3633.	.902	1.18
1	3.00	7.70							
2	"	7.50							
3	"	7.60							
4	"	7.70							
5	"	7.60							
6	"	7.60							
7	"	7.65							
8	"	7.75							
9	"	7.75							
10	"	7.60							
Avg.	3.00	7.65	.392						

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PROJ. NO:
PARTY: Caity

PROJECT TITLE: Thesis

REMARKS: Water 66°
Slope 10°
Glass

1	2	3	4	5	6	7	8	9	10
Run	Dist	Time	Vel.		W	D	R	C_D	S. G.
sphere M	(ft.)	(sec.)	(ft./sec.)		(lbs.)	(ft.)			
1	3.00	3.45			.0175	.0600	4595.	.900	2.48
2	"	3.40							
3	"	3.50							
4	"	3.50							
5	"	3.60							
6	"	3.60							
7	"	3.60							
8	"	3.60							
9	"	3.45							
10	"	3.50							
Avg.	3.00	3.50	.857						
sphere N					.0183	.0610	4672.	.908	2.47
1	3.00	3.50							
2	"	3.55							
3	"	3.50							
4	"	3.65							
5	"	3.50							
6	"	3.50							
7	"	3.45							
8	"	3.45							
9	"	3.60							
10	"	3.50							
Avg.	3.00	3.50	.857						

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PROJECT TITLE: Thesis

RUN NO:

PROJ. NO:

PARTY: Carty

REMARKS: Oil "200" $\rho_{oil} = 1.677$
Temp. = 78° F. $\nu = 0.94 \times 10^{-3}$
Slope = 3" to 17"

Run	Dist. (ft.)	Time (sec.)	Vel. (ft/sec.)		W (lbs.)	D (ft.)	R	C_D	S.G. sphere
sphere A									
1	0.5'	34.6			.00175	.0357	0.532	482.5	1.17
2	"	36.8							
3	"	36.2							
4	"	34.6							
5	"	35.6							
Avg.	0.5'	35.6	.0140						
Sphere C					.00507	.0509	1.63	152.3	1.18
1	0.5'	16.6							
2	"	16.6							
3	"	16.8							
4	"	16.4							
5	"	16.6							
Avg.	0.5'	16.6	.0301						
Sphere D					.00886	.0613	2.72	95.4	1.18
1	0.5	12.6							
2	"	12.0							
3	"	12.0							
4	"	12.0							
5	"	11.6							
Avg.	0.5'	12.0	.0417						

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CAMBRIDGE 39, MASS.

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RUN NO:

PROJECT TITLE: Thesis

PROJ. NO:

PARTY: Caity

REMARKS: Oil "200" Temp. 78° F. Slope 3 to 17

$\rho_{oil} = 1.677$ $\nu = 0.94 \times 10^{-3}$

1	2	3	4	5	6	7	8	9	10
Run	Dist.	Time	Vel.		W	D	R	C_D	S.G.
Sphere	(ft.)	(sec.)	(ft./sec.)		(lbs)	(ft.)			Sphere
1	0.5	273.2			$.42\times10^{-4}$	.0104	.0204	9983.	1.29
2	"	266.8							
3	"	266.6							
4	"	275.0							
5	"	276.0							
Avg.	0.5	271.6	.00184						
Sphere E					.0143	.0721	4.68	50.9	1.17
1	0.5	8.0							
2	"	8.0							
3	"	8.2							
4	"	8.2							
5	"	8.4							
Avg.	0.5	8.2	.061						
Sphere Franklyn					1.69×10^{-4}	.0160	.0747	2837.	1.26
1	0.5	112.8							
2	"	116.2							
3	"	110.8							
4	"	116.0							
5	"	114.0							
Avg.	0.5	114.0	.00439						
Sphere * N					.0183	.0610	14.73	16.4	2.47
1	1.0	4.4							
2	"	4.4							
3	"	4.4							
4	"	4.8							
5	"	4.2							

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MASSACHUSETTS INSTITUTE OF TECHNOLOGY

CAMBRIDGE 39, MASS.

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DATE: 4-23-57

RUN NO:

PROJECT TITLE: Thesis

PROJ. NO:

PARTY: Carty

REMARKS: Water Temp = 73° $\nu = 1.005 \times 10^{-5}$
Slope = 20°
Glass Bottom

1	2	3	4	5	6	7	8	9	10
Run	Dist.	Time	Vel.		W	D	R	C_D	S.G.
sphere G	(ft.)	(sec.)	(ft./sec.)		(lbs.)	(ft.)			
1	3.0	5.75			.0307	.0928	4820.	.853	1.17
2	"	5.75							
3	"	5.70							
4	"	5.80							
5	"	5.90							
6	"	5.80							
7	"	5.80							
8	"	5.70							
9	"	5.75							
10	"	5.70							
Avg.	3.0	5.75	0.522						
Sphere H									
1	3.0	5.35			.0429	.1037	5789.	.868	1.18
2	"	5.40							
3	"	5.30							
4	"	5.30							
5	"	5.50							
6	"	5.20							
7	"	5.30							
8	"	5.35							
9	"	5.40							
10	"	5.35							
Avg.	3.0	5.35	0.561						

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PROJ. NO:

PARTY: Carty

REMARKS: Water 73° $\nu = 1.005 \times 10^{-5}$

Slope = 20°

Glass Bottom

1	2	3	4	5	6	7	8	9	10
Run	Dist.	Time	Vel.		W	D	R	C_D	S.G.
Sphere R	(ft.)	(Sec.)	(ft./sec.)		(lbs.)	(ft.)			
1	3.0	2.00			.00229	.0208	3104.	.920	7.82
2	"	2.00							
3	"	2.00							
4	"	1.95							
5	"	2.00							
6	"	2.00							
7	"	1.90							
8	"	2.10							
9	"	2.00							
10	"	2.00							
Avg.	3.00	2.00	1.50						
Sphere M					.0175	.0600	7642.	.774	2.48
1	3.00	2.40							
2	"	2.30							
3	"	2.30							
4	"	2.45							
5	"	2.40							
6	"	2.35							
7	"	2.35							
8	"	2.35							
9	"	2.40							
10	"	2.30							
Avg.	3.00	2.35	1.28						

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PARTY: Carty

PROJECT TITLE: Thesis

REMARKS: Water 73° $\nu = 1.005 \times 10^{-5}$
Slope = 20°
Glass Bottom

1	2	3	4	5	6	7	8	9	10
Run	Dist.	Time	Vel.		W	D	R	C_D	S.G.
Sphere N	(ft.)	(sec.)	(ft/sec.)		(lbs.)	(ft.)			
1	3.00	2.40			.01834	.0610	7587.	.842	2.47
2	"	2.40							
3	"	2.40							
4	"	2.40							
5	"	2.40							
6	"	2.35							
7	"	2.40							
8	"	2.40							
9	"	2.40							
10	"	2.40							
Avg.	3.00	2.40	1.25						
Sphere S					.1769	.0887	54,305	.232	7.80
1	4.00	0.50							
2	"	0.50							
3	"	0.60							
4	"	0.70							
5	"	0.75							
6	"	0.75							
7	"	0.65							
8	"	0.70							
9	"	0.75							
10	"	0.70							
Avg.	4.00	0.65	6.153						

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PARTY: Carty

PROJECT TITLE: Thesis

REMARKS:
Water 72° $\nu = 1.005 \times 10^{-5}$
Slope 24.5°
Glass

1	2	3	4	5	6	7	8	9	10
Run	Dist.	Time	Vel.		W	D	R	C_D	S. G.
sphere M	(ft.)	(sec.)	(ft/sec.)		(lbs.)	(ft.)			
1	2.50	1.60			.0175	.0600	8776	.730	2.48
2	"	1.75							
3	"	1.70							
4	"	1.70							
5	"	1.65							
6	"	1.70							
7	"	1.70							
8	"	1.75							
9	"	1.75							
10	"	1.70							
Avg.	2.50	1.70	1.47						
sphere									
1									
2									
3									
4									
5									
6									
7									
8									
9									
10									
Avg.									

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CAMBRIDGE 39, MASS.

DATE: 4-23-57

RUN NO:

PROJECT TITLE: Thesis

PROJ. NO:

PARTY: Carty

REMARKS: Water 73° $\nu = 1.005 \times 10^{-5}$
Slope 30°
Glass

1	2	3	4	5	6	7	8	9	10
Run	Dist.	Time	Vel.		W	D	R	C_D	S.G.
sphere M	(ft.)	(sec.)	(ft/sec.)		(lbs.)	(ft.)			
1	2.00	1.40			.0175	.0660	9182	.704	2.4[illegible]
2	"	1.30							
3	"	1.35							
4	"	1.30							
5	"	1.25							
6	"	1.35							
7	"	1.30							
8	"	1.30							
9	"	1.25							
10	"	1.35							
Avg.	2.00	1.30	1.538						
sphere H					.0429	.1037	6872	.897	1.18
1	2.00	2.95							
2	"	2.90							
3	"	3.10							
4	"	2.90							
5	"	3.10							
6	"	3.10							
7	"	2.90							
8	"	3.00							
9	"	3.00							
10	"	3.10							
Avg.	2.00	3.00	0.667						

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DATE: 4-25-57
RUN NO:
PROJ. NO:
PARTY: Canty

PROJECT TITLE: Thesis

REMARKS: Temp. = 72° air ν =
Slope = 1:22.4
Beach

1	2	3	4	5	6	7	8	9	10
Run	Dist.	Time	Vel.		W	D	R	C_D	S.G.
Sphere H	(ft.)	(sec.)	(ft/sec.)		(lbs.)	(ft.)			
1	5.00	1.20			.0429	.1037	2609.	12.1	1.18
2	"	1.25							
3	"	1.35							
4	"	1.20							
5	"	1.20							
6	"	1.25							
7	"	1.20							
8	"	1.25							
9	"	1.30							
10	"	1.15							
Avg. ✓	5.00	1.25	4.00						
Sphere D					.00886	.0613	917.5	20.2	1.179
1	5.00	2.20							
2	"	2.30							
3	"	2.50							
4	"	1.90							
5	"	2.20							
6	"	2.15							
7	"	2.00							
8	"	1.90							
9	"	1.80							
10	"	2.05							
Avg. ✓	5.00	2.10	2.38						

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MASSACHUSETTS INSTITUTE OF TECHNOLOGY

CAMBRIDGE 39, MASS.

DATE: 4-25-57

RUN NO:

PROJECT TITLE: Thesis

PROJ. NO:

PARTY: Carty

REMARKS: Beach
Slope = 1:22.4
Temp = 72° Air

1	2	3	4	5	6	7	8	9	10
Run	Dist.	Time	Vel.		W	D	$\mathbb{R}$	C_D	S.G.
Basket-ball	(ft.)	(sec.)	(ft./sec.)		(lbs.)	(ft.)			
1	8.00	1.65			1.40	.792	25,700	4.07	
2	"	1.50							
3	"	1.45							
4	"	1.75							
5	"	1.80							
6	"	1.55							
7	"	1.50							
8	"	1.60							
9	"	1.60							
10	"	1.25							
Avg.	8.00	1.55	5.16						