

# Thermodynamik Formelsammlung

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$$\frac{d}{dt} \left\{ U + m \left( \frac{c^2}{2} + gz \right) \right\} = \sum_j \left[ \dot{m}_j \left( h + \frac{c^2}{2} + gz \right) \right]_j + \sum_l (\dot{Q}_t)_l + \sum_i (\dot{W}_t)_i - p \frac{dV}{dt}$$

## 1 Nomenklatur

<b>A<sub>n</sub></b> = Anergie[J]	<b>V</b> = Volumen[m <sup>3</sup> ]
<b>c<sub>s</sub></b> = Schallgeschwindigkeit[m/s]	<b>v</b> = Spezifisches Volumen[m <sup>3</sup> /kg]
<b>c<sub>p</sub></b> = Spezifische Wärmekapazität dp = 0 [J/kg*K]	<b>V<sub>m</sub></b> = Molares Volumen[m <sup>3</sup> /mol]
<b>c<sub>v</sub></b> = Spezifische Wärmekapazität dv = 0 [J/kg*K]	<b>W</b> = Arbeit[J]
<b>E</b> = Energie[J]	<b>w</b> = Spezifische Arbeit[J/kg]
<b>Ex</b> = - <b>W<sub>ex</sub></b> = Exergie[J]	<b>W<sub>v</sub></b> = Volumenänderungsarbeit[J]
<b>F</b> = Kraft[N]	<b>W<sub>el</sub></b> = Elektrische Arbeit[J]
<b>F</b> = <b>U</b> - <b>TS</b> = Freie Energie[J]	<b>W<sub>w</sub></b> = Wellenarbeit[J]
<b>f</b> = <b>u</b> - <b>Ts</b> = Spezifische freie Energie[J/kg]	<b>W<sub>diss</sub></b> = Dissipationsarbeit[J]
<b>f</b> = Fugazität[Pa]	<b>W<sub>t</sub></b> = Technische Arbeit[J]
<b>G</b> = <b>H</b> - <b>TS</b> = Freie Enthalpie[J]	<b>W<sub>virrev</sub></b> = Arbeitsverlust durch Irreversibilität[J]
<b>g</b> = <b>h</b> - <b>Ts</b> = Spezifische freie Enthalpie[J/kg]	<b>x</b> = $\frac{m''}{m' + m''}$ = Dampfanteil[-]
<b>g</b> = Erdbeschleunigung[m/s <sup>2</sup> ]	<b>x</b> = $\frac{m_{H_2O}}{m_L}$ = Wassergehalt
<b>H</b> = <b>U</b> + <b>pV</b> = Enthalpie[J]	<b>Z</b> = Allgemeine extensive Zustandsgrößen[Z]
<b>h</b> = <b>u</b> + <b>pv</b> = Spezifische Enthalpie[J/kg]	<b>z</b> = Allgemeine
<b>■H<sub>g</sub></b> = Molare Reaktionsenthalpie	<b>β</b> = Isobarer Ausdehnungskoeffizient[1/K]
<b>K</b> = Konstante des Massenwirkungsgesetzes[-]	<b>γ</b> = Isochorer Spannungskoeffizient[1/K]
<b>M</b> = Molmasse[kg/mol]	<b>δ<sub>T</sub></b> = Isothermer Drosselkoeffizient[m <sup>3</sup> /kg]
<b>ṁ</b> = Massestrom[kg/s]	<b>δ<sub>h</sub></b> = Isenthalper Drosselkoeffizient[Ks <sup>2</sup> m/kg]
<b>m'</b> = Masse in der flüssigen Phase[kg]	<b>ε</b> = Leistungsziffer[-]
<b>m''</b> = Masse in der gasförmigen Phase[kg]	<b>ε</b> = Verdichtungsverhältnis[-]
<b>Ma</b> = <b>c</b> / <b>c<sub>s</sub></b> = Machzahl[-]	<b>η<sub>th</sub></b> = Thermischer Wirkungsgrad[-]
<b>n</b> = <b>m</b> / <b>M</b> = Molzahl[mol]	<b>η<sub>mech</sub></b> = Mechanischer Wirkungsgrad[-]
<b>n</b> = Polytropenexponent[-]	<b>κ</b> = Adiabaten- oder Isentropenexponent[-]
<b>P<sub>t</sub></b> = technische Leistung[W]	<b>λ</b> = Reaktionslaufzahl[-]
<b>Q</b> = Wärme[J]	<b>μ<sub>i</sub></b> = Chemisches Potential[J/mol]
<b>Q̇</b> = Wärmestrom[W]	<b>v<sub>i</sub></b> = Stöchiometrische Koeffizienten[-]
<b>q</b> = Spezifische Wärme[J/kg]	<b>ξ<sub>i</sub></b> = Masseanteil[-]
<b>r</b> = Spezifische Verdampfungsenthalpie[J/kg]	<b>π</b> = Druckverhältnis[-]
<b>R</b> = Gaskonstante[J/(kg K)]	<b>ρ</b> = Dichte[kg/m <sup>3</sup> ]
<b>R<sub>m</sub></b> = Universelle Gaskonstante[J/(mol K)]	<b>τ</b> = Temperaturverhältnis[-]
<b>S</b> = Entropie[J/K]	<b>φ</b> = Relative Feuchte[-]
<b>s</b> = Spezifische Entropie[J/(kg K)]	<b>φ</b> = Einspritzverhältnis[-]
<b>T</b> = Temperatur[K]	<b>ξ</b> = Isothermer Kompressibilitätskoeffizient[m <sup>2</sup> /N]
<b>t</b> = Zeit[s]	<b>■</b> = Dissipationsenergie[J]
<b>t</b> = Temperatur[°C]	<b>ψ</b> = Spezifische Dissipationsenergie[J]
<b>T</b> = Sättigungstemperatur[K]	<b>ψ</b> = Drucksteigerungsverhältnis[-]
<b>U</b> = Innere Energie[J]	<b>ψ<sub>i</sub></b> = Molanteil[-]
<b>u</b> = Spezifische innere Energie [J/kg]	

## 2 Grundbegriffe

### Systeme

- Abgeschlossenes System - kein Stoff oder Energietransport
- Geschlossenes System - kein Stofftransport
- Adiabates System - kein  $\Delta q$ , aber Masse und Arbeit.
- Offenes System - Stoff und Energietransport
- Stationäres System  $\rightarrow \Delta U = 0$

### Messgrößen

- Prozessgrößen sind Wegabhängig (eg. Arbeit, Wärme)
- Zustandsgrößen sind Wegunabhängig (eg. Volumen, Druck)
- Extensive Zustandsgrößen sind abhängig von der Masse des Systems (V, m, H, S, F, G, E)
- Intensive Zustandsgrößen sind unabhängig von der Masse des Systems (T, p)

### Zustandsgleichungen

- Thermisch  $\rightarrow f(p, V, T) = 0$
- Kalorisch  $\rightarrow f(U, V, T) = 0, \quad U = U(V, T), \quad u = u(v, T)$

### Hauptsätze

- 0: Temperatur existiert, ihre Gleichheit ist notwendige Voraussetzung für das thermische Gleichgewicht.
- 1: Energie existiert, sie ist für abgeschlossene Systeme konstant.
- 2: Entropie existiert, sie wird bei allen irreversiblen Prozessen erzeugt.  $dS = \frac{\delta Q_{rev}}{T}$
- 3: 0K existiert, bei dieser Temperatur ist die Entropie = 0

## 3 Basisformeln

$$\begin{aligned}
 H &= U + pV \\
 dS &= \frac{\delta Q_{rev}}{T} \\
 F &= U - TS \\
 G &= H - ST \\
 W &= - \int p dV
 \end{aligned}
 \quad
 \begin{aligned}
 dS &= \frac{Q_{rev}}{T} + S_{prod} \\
 \Psi &= \int_1^2 T dS_{prod} \\
 W_{ir} &= \frac{T_u}{T} \Psi \\
 p_1 &= p_a + \frac{\varphi_1 - \varphi_a}{\varphi_b - \varphi_a} (p_b - p_a)
 \end{aligned}$$

## 4 Gibbs

$$\begin{aligned}
 dU &= Tds - pdV + \sum_{k=1}^K \mu_k dn_k \\
 dG &= -SdT + Vdp + \sum_{k=1}^K \mu_k dn_k \\
 dH &= TdS + Vdp + \sum_{k=1}^K \mu_k dn_k \\
 dF &= -SdT - pdV + \sum_{k=1}^K \mu_k dn_k \\
 dU &= \left( \frac{\partial U}{\partial S} \right)_V dS + \left( \frac{\partial U}{\partial V} \right)_S dV + \sum_{k=1}^K \left( \frac{\partial U}{\partial n_k} \right)_S dn_k
 \end{aligned}$$

## 5 Thermodynamische Beziehungen

$$\begin{aligned}
 T &= \left( \frac{\partial U}{\partial S} \right)_V = T(S, V) & -S &= \left( \frac{\partial F}{\partial T} \right)_V = S(T, V) \\
 T &= \left( \frac{\partial H}{\partial S} \right)_p = T(S, p) & -S &= \left( \frac{\partial G}{\partial T} \right)_p = S(T, p) \\
 p &= - \left( \frac{\partial U}{\partial V} \right)_S = p(V, S) & V &= \left( \frac{\partial G}{\partial p} \right)_T = V(p, T) \\
 -p &= \left( \frac{\partial F}{\partial V} \right)_T = p(T, V) & \mu &= \left( \frac{\partial U}{\partial n} \right)_{S, V} = \mu(S, V, n)
 \end{aligned}$$

## 6 Guggenheim

$$\begin{array}{ccccc}
 -S & U & V & U & = U(S, V) \\
 H & & F & H & = H(S, p) \\
 -p & G & T & F & = F(T, V) \\
 & & & G & = G(T, p)
 \end{array}$$

$$\overbrace{\frac{d}{dt} \left\{ U + m \left( \frac{c^2}{2} + gz \right) \right\}}^{\text{Stationäres System} \rightarrow 0} = \sum_j \overbrace{\left[ \dot{m}_j \left( h + \frac{c^2}{2} + gz \right) \right]_j}_{\text{Geschlossenes System} \rightarrow 0} + \overbrace{\sum_l (\dot{Q}_t)_l}_{\text{Kein Wärmestrom} \rightarrow 0} + \overbrace{\sum_i (\dot{W}_t)_i}_{\text{Keine Leistung} \rightarrow 0} - \overbrace{p \frac{dV}{dt}}^{\text{Keine Volumenänderung} \rightarrow 0}$$

## 7 Maxwell

$$\left(\frac{\partial T}{\partial p}\right)_{S,n_j} = \left(\frac{\partial V}{\partial S}\right)_{p,n_j}$$

$$\left(\frac{\partial S}{\partial V}\right)_{T,n_j} = \left(\frac{\partial p}{\partial T}\right)_{V,n_j}$$

$$\left(\frac{\partial S}{\partial p}\right)_{T,n_j} = -\left(\frac{\partial V}{\partial T}\right)_{p,n_j}$$

$$\left(\frac{\partial \mu_i}{\partial T}\right)_{p,n_j} = -\left(\frac{\partial S}{\partial n_i}\right)_{T,p,n_j \neq n_i}$$

$$\left(\frac{\partial \mu_i}{\partial p}\right)_{T,n_j} = \left(\frac{\partial V}{\partial n_i}\right)_{T,p,n_j \neq n_i}$$

## 8 Ideales Gas

$$pV = mRT$$

$$pv = RT$$

$$pV = nR_m T$$

$$\beta = \frac{1}{T}$$

$$\gamma = \frac{1}{T}$$

$$\chi = \frac{1}{p}$$

$$\beta = p\gamma\chi$$

$$R_m = 8,3143 \left[ \frac{kJ}{kmolK} \right]$$

$$R = c_p - c_v$$

$$R = \frac{R_m}{M}$$

$$U - U_0 = mc_v(T - T_0)$$

$$H - H_0 = mc_p(T - T_0) \quad \leftarrow \text{Für } c_p \text{ und } c_v \text{ const.}$$

$$s - s_0 = R \ln \left( \frac{v}{v_0} \right) + c_v \ln \left( \frac{T}{T_0} \right)$$

$$= c_v \ln \left( \frac{p}{p_0} \right) + c_p \ln \left( \frac{v}{v_0} \right)$$

$$= c_p \ln \left( \frac{T}{T_0} \right) - R \ln \left( \frac{p}{p_0} \right)$$

$$\beta = \frac{1}{T} = \frac{1}{V} \left( \frac{\partial V}{\partial T} \right)_p = \frac{1}{v} \left( \frac{\partial v}{\partial T} \right)_p = -\frac{1}{p} \left( \frac{\partial p}{\partial T} \right)_p$$

$$\gamma = \frac{1}{T} = \frac{1}{p} \left( \frac{\partial p}{\partial T} \right)_v$$

$$\chi = \frac{1}{p} = -\frac{1}{V} \left( \frac{\partial V}{\partial p} \right)_T = -\frac{1}{v} \left( \frac{\partial v}{\partial p} \right)_T$$

$$u_2 - u_1 = \int_{T_1}^{T_2} c_v(T) dT$$

## 9 Van-der-Waals

$$\left( p + \frac{a}{v^2} \right) (v - b) = RT$$

$$\left( \bar{p} + \frac{3}{\bar{v}^2} \right) (3\bar{v} - 1) = 8\bar{T}$$

$$\bar{p} = \frac{p}{p_K}, \quad \bar{v} = \frac{v}{v_K}, \quad \bar{T} = \frac{T}{T_K}$$

$$p_K = \frac{a}{27b^2}, \quad T_K = \frac{8}{27} \frac{a}{b} \frac{1}{R}, \quad \frac{p_K v_K}{RT_K} = \frac{3}{8}$$

$$a = 3p_K v_K^2, \quad b = \frac{v_K}{3}, \quad \frac{p_K v_K}{RT_K} = \frac{3}{8}$$

$$\beta = \frac{(v - b)Rv^2}{RTv^3 - 2a(v - b)^2}$$

$$\gamma = \frac{Rv^2}{RTv^2 - a(v - b)}$$

$$\chi = \frac{(v - b)^2 v^2}{RTv^3 - 2a(v - b)^2}$$

$$du = \frac{a}{v^2} dv + c_v(T) dT$$

$$u - u_0 = \left( \frac{a}{v_0} - \frac{a}{v} \right) + \int_{T_0}^T c_v(\tilde{T}) d\tilde{T}$$

$$u - u_0 = \left( \frac{a}{v_0} - \frac{a}{v} \right) + c_v(T - T_0) \quad \leftarrow \text{für } c_v = \text{const.}$$

$$c_p - c_v = \frac{Tv\beta^2}{\chi}$$

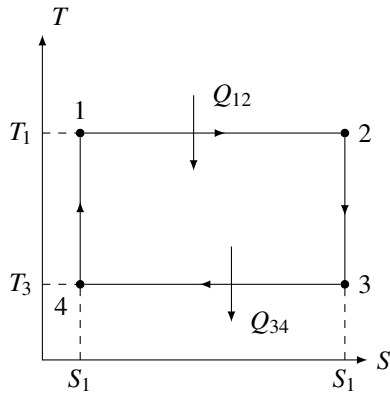
$$s - s_0 = c_v \ln \left( \frac{T}{T_0} \right) + R \ln \left( \frac{v - b}{v_0 - b} \right)$$

## 10 Carnot

$$\eta_{th} = 1 - \frac{-Q_{34}}{Q_{12}} = 1 - \frac{T_3(S_3 - S_4)}{T_1(S_2 - S_1)} = 1 - \frac{T_3}{T_1}$$

$$\frac{Q_{12}}{T_1} + \frac{Q_{34}}{T_3} = 0$$

$$\Delta S_{ges} = -Q_{34} \left( \frac{1}{T_{KK}} - \frac{T_1}{T_3} \frac{1}{T_{HK}} \right)$$



Adiabate Drosselung (ideal):

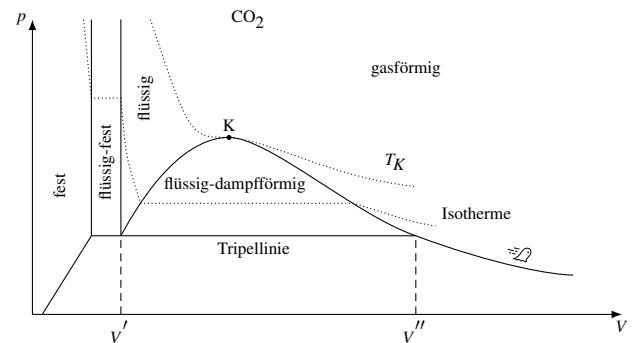
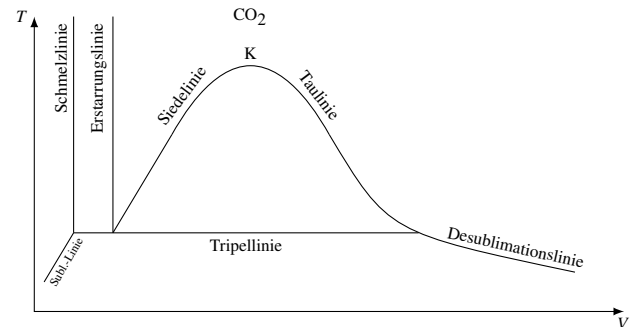
$$h + \frac{c^2}{2} + gz = \text{const.}$$

$$dh = 0$$

Adiabete Drosselung (real):

$$\delta_h = \left( \frac{\partial T}{\partial p} \right)_h = -\frac{v}{c_p} (1 - \beta T)$$

## 12 Nassdampf



## 11 Gemische Idealer Gase

$$\xi_i = \frac{m_i}{m}, \quad \psi_i = \frac{n_i}{n}, \quad p_i = \psi_i p$$

$$\xi_i = \frac{M_i n_i}{\sum_{k=1}^K M_k n_k} = \frac{M_i}{M_G} \psi$$

$$p_i V = m_i R_i T, \quad p_i V = n_i R_m T, \quad p V = m R_G T$$

$$\sum_{k=1}^K p_k = p$$

$$R_G = \frac{1}{m} \sum_{k=1}^K m_k R_k = \sum_{k=1}^K \xi_k R_k$$

$$U_G = \sum_{k=1}^K U_k = \sum_{k=1}^K m_k u_k = \sum_{k=1}^K c_{vk} m_k T \leftarrow c_v = \text{const}$$

$$H_G = \sum_{k=1}^K H_k = \sum_{k=1}^K m_k h_k = \sum_{k=1}^K c_{pk} m_k T \leftarrow c_p = \text{const.}$$

$$c_{vG} = \sum_{k=1}^K c_{vk} \xi_k, \quad c_{pG} = \sum_{k=1}^K c_{pk} \xi_k$$

$$S_2 - S_1 = R_m \left( n \ln n - \sum_{k=1}^K n_k \ln n_k \right)$$

$$v = (1-x)v' + xv''$$

$$v = v' + (v'' - v')x$$

$$u = (1-x)u' + xu''$$

$$u = u' + (u'' - u')x$$

$$h = (1-x)h' + xh''$$

$$h = h' + (h'' - h')x$$

$$s = (1-x)s' + xs''$$

$$s = s' + (s'' - s')x$$

$$r = h'' - h' = T(s'' - s')$$

$$T' = T''$$

$$p' = p''$$

$$g' = g''$$

$$dg' = v' dp' - s' dT'$$

$$dg'' = v'' dp'' - s'' dT''$$

$$dg' = dg''$$

$$\frac{dp}{dT} = \frac{s'' - s'}{v'' - v'}$$

$$\frac{dp}{dT} = \frac{1}{T} \frac{h'' - h'}{v'' - v'}$$

$$\frac{dp}{dT} = \frac{1}{T} \frac{r}{v'' - v'}$$

## 13 Realer Stoff im Nassdampfgebiet

Isobare Zustandsänderung

$$q_{12} = T(s_2 - s_1)$$

$$= T(s'' - s')(x_2 - x_1)$$

$$w_{V,12} = - \int_1^2 p dv$$

$$= -p(v_2 - v_1) = -p(v'' - v')(x_2 - x_1)$$

Isochore Zustandsänderung

$$q_{12} = u_2 - u_1 = u'_2 + x_2(u''_2 - u'_2) - u'_1 - x_1(u''_1 - u'_1)$$

Adiabate Zustandsänderung

$$w_{V,12} = u_2 - u_1 = u'_2 + x_2(u''_2 - u'_2) - u'_1 - x_1(u''_1 - u'_1)$$

Entropieänderung während des Mischvorgangs

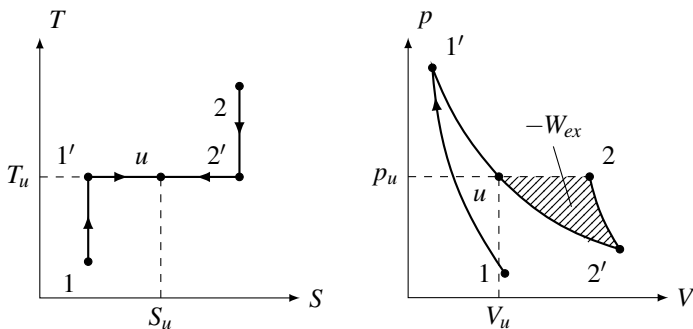
$$S_2 - S_1 = R_m \left( n \ln n - \sum_i n_i \ln n_i \right)$$

## 14 Maximale Arbeit und Exergie

Maximale nutzbare Arbeit → isentrop, reibungsfrei

1 → 1' : isentrop auf  $T_u$

1' → u : isotherm auf  $u$



$$-\dot{W}_{ex} = -(\dot{W}_t)_{rev} = -\frac{d}{dt} \left( U + m \left( \frac{c^2}{2} + gz \right) + p_u V - T_u S \right) + \sum_{j=1}^K \left( \dot{m}_j \left( h + \frac{c^2}{2} + gz - T_s \right) \right) + \sum_{l=1}^K \left( 1 - \frac{T_u}{T} \right) \dot{Q}_l$$

Die Exergie der Enthalpie (offenes, stationäres System)

$$-\dot{W}_{ex,1u} = \dot{m}(h_1 - h_u - T_u(s_1 - s_u))$$

Die Exergie der inneren Energie (geschlossenes, instationäres System)

$$-\dot{W}_{ex} = -\frac{d}{dt} (U + p_u V - T_u S)$$

$$-\dot{W}_{ex,1u} = U_1 - U_u - p_u(V_1 - V_u) - T_u(S_1 - S_u)$$

$$-\dot{W}_{ex,1u} = H_1 - (p_1 - p_u)V_1 - H_u - T_u(S_1 - S_u)$$

Für Ideales Gas

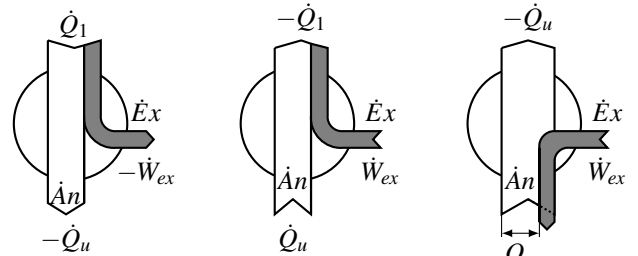
$$-W_{ex} = mc_v(T_1 - T_u) + p_u(V_1 - V_u) - T_u m \left( c_p \ln \left( \frac{T_1}{T_u} \right) - R_i \ln \left( \frac{p_1}{p_u} \right) \right)$$

Dampf/Luftdruckkammer

$$-W_{ex,1u} = m_1[u_1 - u_u + p_u(v_1 - v_u) - T_u(s_1 - s_u)]$$

Die Exergie der Wärme (geschlossenes, stationäres System)

$$-\dot{W}_{ex} = \left( 1 - \frac{T_u}{T_1} \right) \dot{Q}_1 = \eta_{th,C} \dot{Q}_1$$

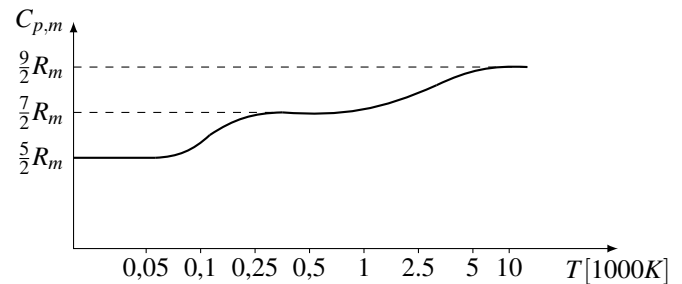


WärmeKraftprozess

Wärmepumpenprozess

Kälteprozess

## 15 Wärmekapazität



$$C_{v,m} = \frac{1}{\kappa - 1} R_m, \quad C_{p,m} = \frac{\kappa}{\kappa - 1} R_m$$

$$c_v = \frac{1}{\kappa - 1} R_j, \quad c_p = \frac{\kappa}{\kappa - 1} R_j$$

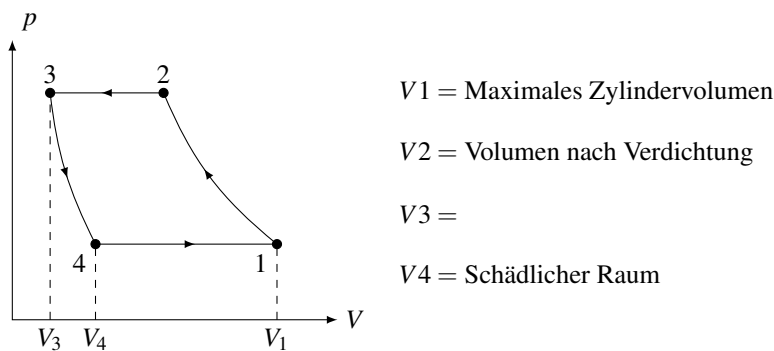
$$\kappa = \frac{c_p}{c_v}, \quad c_p - c_v = R$$

$$C_{v,m} = \underbrace{3 + \frac{R_m}{2}}_{\text{Translatorisch}} + \underbrace{\frac{n_{\text{rot}} R_m}{2}}_{\text{Rotatorisch}} + \underbrace{R_M(3n_{\text{Atome}} - 3 - n_{\text{rot}})}_{\text{Vibratorisch}} + \underbrace{C_{v,m, \text{Elektronenanregung}}}_{\text{Relevant ab: } T \approx 10^4 K}$$

## 16 Technische Anwendung

adiabat ( $c_p = \text{const.}$ )	$W_{t,12} = mc_p(T_2 - T_1) = \frac{\kappa}{\kappa - 1}(p_2 V_2 - p_1 V_1)$	$Q_{12} = 0$
reversibel adiabat $\kappa = \text{const.}$	$W_{t,12} = \frac{\kappa}{\kappa - 1}(p_1 V_1) \left[ \left( \frac{p_2}{p_1} \right)^{\frac{\kappa-1}{\kappa}} - 1 \right]$	$Q_{12} = 0$
irreversibel adiabat als Polytrope $n > \kappa; n, \kappa = \text{const.}$	$W_{t,12} = \frac{\kappa}{\kappa - 1}(p_1 V_1) \left[ \left( \frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right]$	$Q_{12} = 0$
reversibel polytrop $n, \kappa = \text{const.}$	$W_{t,12} = \frac{n}{n-1}(p_2 V_2 - p_1 V_1)$ $= \frac{n}{n-1}mR(T_2 - T_1)$ $= \frac{n}{n-1}(p_1 V_1) \left[ \left( \frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right]$	$Q_{12} = mc_n(T_2 - T_1)$ $= \frac{n-\kappa}{(n-1)(\kappa-1)}(p_1 V_1) \left[ \left( \frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right]$ $c_n = \frac{n-\kappa}{n-1}c_v$
isotherm	$W_{t,12} = (p_1 V_1) \ln \left( \frac{p_2}{p_1} \right)$	$Q_{12} = -W_{t,12}$

### 16.1 Kolbenverdichter



$$\begin{aligned}
 \mu &= \frac{V_1 - V_4}{V_1 - V_3}, & \varepsilon_s &= \frac{V_3}{V_1 - V_3} \\
 \mu &= 1 - \varepsilon_s \left[ \left( \frac{p_2}{p_1} \right)^{\frac{1}{n}} - 1 \right] \\
 W_{t,12} &= \int_1^2 V dp \\
 &= \underbrace{p_2 V_2}_{\text{Ausschiebearbeit}} - \underbrace{p_1 V_1}_{\text{Einschiebearbeit}} - \int_1^2 p dV \\
 &= \frac{n}{n-1} p_1 (V_1 - V_4) \left[ \left( \frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right]
 \end{aligned}$$

Verdichter Wirkungsgrad

$$\eta_{sV} = \frac{w_{t,12,rev}}{w_{t,12}} = \frac{h_{2,rev} - h_1}{h_2 - h_1}$$

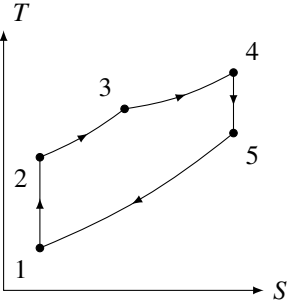
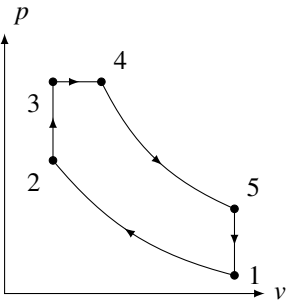
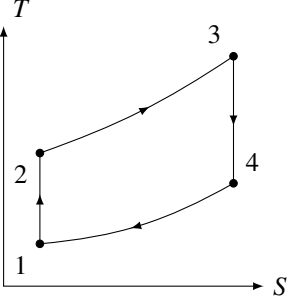
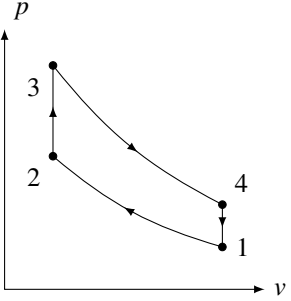
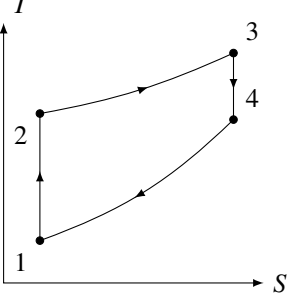
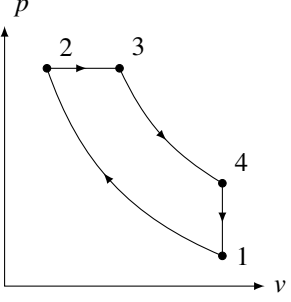
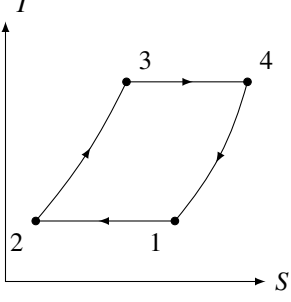
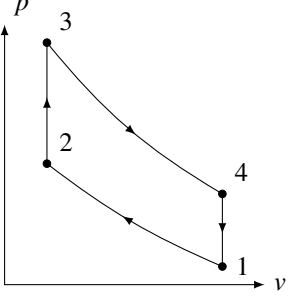
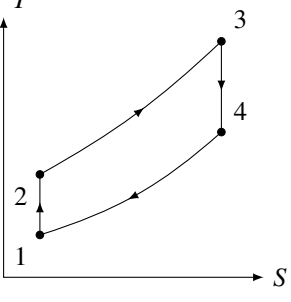
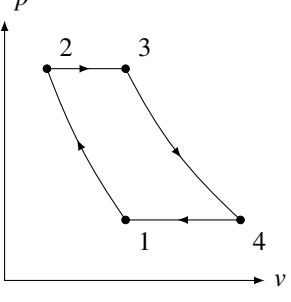
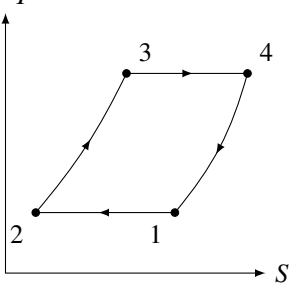
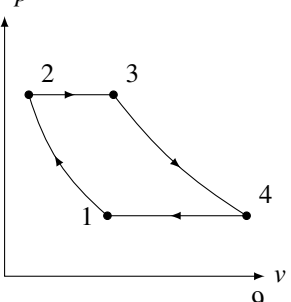
Verdichter wirkungsgrad (Ideales Gas,  $c_p = \text{const.}$ )

$$\eta_{sV} = \frac{T_{2,rev} - T_1}{T_2 - T_1}$$

Technische Verlustarbeit

$$\begin{aligned}
 w_{t,Verl,12} &= w_{t,12} - w_{t,12,rev} = h_2 - h_{2,rev} \\
 &= \int_{2,rev}^2 T|_{p_2=\text{const.}} ds
 \end{aligned}$$



		<p>Seiliger Prozess</p> $\eta_{th} = 1 - \frac{\phi^{\kappa} \psi - 1}{\varepsilon^{\kappa-1} [\psi - 1 + \kappa \psi (\phi - 1)]}$ $\varepsilon = \frac{v_1}{v_2} \quad \psi = \frac{p_3}{p_2} \quad \phi = \frac{v_4}{v}$
		<p>Otto Prozess</p> $\eta_{th} = 1 - \frac{1}{\varepsilon^{\kappa-1}}$ $\varepsilon = \frac{v_1}{v_2}$
		<p>Diesel Prozess</p> $\eta_{th} = 1 - \frac{\phi^{\kappa} - 1}{\varepsilon^{\kappa-1} \kappa (\phi - 1)}$ $\varepsilon = \frac{v_1}{v_2} \quad \phi = \frac{v_4}{v}$
		<p>Stirling Prozess</p> $\eta_{th} 1 - \frac{ q_{12} }{q_{34}} = \frac{RT_1 \ln \left( \frac{v_1}{v_2} \right)}{RT_3 \ln \left( \frac{v_4}{v_3} \right)} = 1 - \frac{T_1}{T_3}$
		<p>Joule Prozess</p> $\eta_{th} = 1 - \frac{T_1}{T_2} = 1 - \left( \frac{p_1}{p_2} \right)^{\frac{\kappa-1}{\kappa}} = 1 - \left( \frac{1}{\pi} \right)^{\frac{\kappa-1}{\kappa}}$ $\pi = \frac{p_2}{p_1}$
		<p>Ericsson Prozess</p> $\eta_{th} 1 - \frac{ q_{12} }{q_{34}} = \frac{RT_1 \ln \left( \frac{p_1}{p_2} \right)}{RT_3 \ln \left( \frac{p_4}{p_3} \right)} = 1 - \frac{T_1}{T_3}$

# Ideales Gas

	Isothermo	Isobare	Isochore	Isentrop	Polytrope
konstant:	T	p	v	$\delta q = 0$	$p v^n$
	-	-	-	$p_1 v_1^\kappa = p_2 v_2^\kappa$	$v_1^n = p_2 v_2^n$
	$p_1 p_2 = p_2 v_2$	$\frac{v_1}{v_2} = \frac{T_1}{T_2}$	$\frac{p_1}{T_1} = \frac{p_2}{T_2}$	$T_1 v_1^{\kappa-1} = T_2 v_2^{\kappa-1}$	$T_1 v_1^{n-1} = T_2 v_2^{n-1}$
	-	-	$\neq \mathcal{Q}$	$\frac{T_1^{\frac{\kappa-1}{\kappa}}}{p_1} = \frac{T_2^{\frac{\kappa-1}{\kappa}}}{p_2}$	$\frac{T_1^{\frac{n-1}{n}}}{p_1} = \frac{T_2^{\frac{n-1}{n}}}{p_2}$
$p, v$	$p = \frac{p_1 v_1}{v}$	$p = p_1$	$v = v_1$	$p = \frac{p_1 v_1^\kappa}{v^\kappa}$	$p = \frac{p_1 v_1^n}{v^n}$
$p, T$	$p = \frac{p_1 v_1}{v}$	$p = p_1$	$p = \frac{p_1}{T_1} T$	$p = \frac{p_1^{\frac{\kappa}{\kappa-1}} T^{\frac{\kappa}{\kappa-1}}}{T_1^{\frac{\kappa}{\kappa-1}}}$	$p = \frac{p_1^{\frac{n}{n-1}} T^{\frac{n}{n-1}}}{T_1^{\frac{n}{n-1}}}$
$v, T$	$T = T_1$	$v = \frac{v_1}{T_1} T$	$v = v_1$	$T = \frac{T_1 v_1^{\kappa-1}}{v^{\kappa-1}}$	$T = \frac{T_1 v_1^{n-1}}{v^{n-1}}$
$q_{12}$	$= p_1 v_1 \ln \frac{p_1}{p_2}$	$= c_p (T_2 - T_1)$	$= c_v (T_2 - T_1)$	$= 0$	$= c_v \frac{n-\kappa}{n-1} (T_2 - T_1)$
$w_{V,12}$	$= -q_{12}$	$= -p_1 (v_2 - v_1)$	$= 0$	$= \frac{p_1 v_1}{k-1} \left[ \left( \frac{v_1}{v_1} \right)^{k-1} - 1 \right]$	$= \frac{p_1 v_1}{n-1} \left[ \left( \frac{v_1}{v_2} \right)^{n-1} - 1 \right]$
$s_2 - s_1$	$s_2 - s_1 = R \ln \left( \frac{p_1}{p_2} \right)$	$= c_p \ln \left( \frac{T_2}{T_1} \right)$	$= c_v \ln \left( \frac{T_2}{T_1} \right)$	$= 0$	$= c_v \frac{n-\kappa}{n-1} \ln \left( \frac{T_2}{T_1} \right)$

# Van-Der-Waals-Gas

	Isotherme	Isobare	Isochore	Isentrop
konst.	T	p	v	$\delta = 0$
	$(p_1 + \frac{a}{v_1^2})(v_1 - b)$ $= (p_2 + \frac{a}{v_2^2})(v_2 - b)$	$\frac{RT_1}{v_1 - b} - \frac{a}{v_1^2} = \frac{RT_2}{v_2 - b} - \frac{a}{v_2^2}$	$\frac{p_1 + \frac{a}{v_1^2}}{T_1} = \frac{p_2 + \frac{a}{v_2^2}}{T_2}$	$(p_1 + \frac{a}{v_1^2})(v_1 - b)^{\frac{c_v + R}{c_v}}$ $= (p_2 + \frac{a}{v_2^2})(v_2 - b)^{\frac{c_v + R}{c_v}}$ $T_1(v_1 - b)^{R/c_v} = T_2(v_2 - b)^{R/c_v}$
$p, v$	$p = (p + \frac{a}{v^2})\frac{v_u}{v - b} - \frac{a}{v^2}$	$p = p_1$	$v = v_1$	$p = -\frac{a}{v^2} + (p_1 + \frac{a}{v_1^2})\left(\frac{v_1 - b}{v_m}\right)^{\frac{v_v + R}{R}}$
$p, T$	$T = T_1$	$p = p_1$	$p = \frac{T}{T_1}(p_1 + \frac{a}{v_1^2}) - \frac{a}{v_1^2}$	$p = -\frac{a}{v^2} + (p_1 + \frac{a}{v_1^2})\left(\frac{T}{T_1}\right)^{\frac{c_v + R}{R}}$
$v, T$	$T = T_1$	$T = T_1 \frac{v - b}{v_1 - b} + \frac{a}{R}(v - b)\left(\frac{1}{v^2} - \frac{1}{v_1^2}\right)$	$v = v_1$	$T = T_1 \left(\frac{v_1 - b}{v - b}\right)^{\frac{R}{c_v}}$
$q_{12}$	$= RT_1 \ln \left(\frac{v_2 - b}{v_1 - b}\right)$	$= \frac{a}{v_1} - \frac{a}{v_2} + c_v(T_2 - T_1) + p_1(v_2 - v_1)$	$= c_v(T_2 - T_1)$	$= 0$
$w_{V,12}$	$= -RT_1 \ln \left(\frac{v_2 - b}{v_1 - b}\right) + \frac{a}{v_1} - \frac{a}{v_2}$	$= -p_1(v_2 - v_1)$	$= 0$	$= \frac{a}{v_1} - \frac{a}{v_2} + c_v(T_2 - T_1)$
$s_2 - s_1$	$= R \ln \left(\frac{v_2 - b}{v_1 - b}\right)$	$= c_v \ln \left(\frac{T_2}{T_1}\right) + R \ln \left(\frac{v_2 - b}{v_1 - b}\right)$	$= c_v \ln \left(\frac{T_2}{T_1}\right)$	$= 0$

p [bar]	$h'$ [kJ/kg]	$h''$ [kJ/kg]	$s'$ [kJ/(kgK)]	$s''$ [kJ/(kgK)]
0,01	29,3	2513,3	0,1058	8,9732
0,03	101,0	2544,7	0,3543	8,5754
0,06	151,4	2566,7	0,5207	8,3283
0,08	173,8	2576,3	0,5922	8,2266
0,10	191,7	2583,9	0,6489	8,1480
0,30	289,1	2624,4	0,9435	7,7657
0,50	340,4	2644,7	1,0906	7,5903
0,80	391,6	2664,3	1,2325	7,4300
1,00	417,4	2673,8	1,3022	7,3544
2,00	504,6	2704,6	1,5295	7,1212
3,00	561,3	2723,2	1,6711	6,9859
4,00	604,5	2736,5	1,7758	6,8902
6,00	670,2	2755,2	1,9301	6,7555
8,00	720,6	2768,0	2,0448	6,6594
10,00	762,2	2777,5	2,1372	6,5843