

Thermodynamik Formelsammlung

Jonas Walkling *

January 20, 2020

*Maximilian Goldapp

Contents

1	Nomenklatur	3
2	Grundbegriffe	4
3	Basisformeln	4
4	Gibbs	4
5	Thermodynamische Beziehungen	4
6	Guggenheim	4
7	Maxwell	5
8	Ideales Gas	5
9	Van-der-Waals	5
10	Carnot	6
11	Gemische Idealer Gase	6
12	Nassdampf	6
13	Realer Stoff im Nassdampfgebiet	7
14	Maximale Arbeit und Exergie	7
15	Wärmekapazität	7
16	Technische Anwendung	8
17	Eindimensionale Strömungsvorgänge	10
18	Feuchte Luft	10
19	Chemische Reaktionen	10
20	Stoffwerte einiger Gase	14
21	Stoffdaten einiger Stoffe	14
22	Zahlenwerte feuchte Luft	14
23	Obskure Zusammenhänge	15
24	Dinge die man eigentlich wissen sollte	15

$$\frac{d}{dt} \left\{ U + m \left(\frac{c^2}{2} + gz \right) \right\} = \sum_j \left[\dot{m}_j \left(h + \frac{c^2}{2} + gz \right) \right]_j + \sum_l (\dot{Q}_t)_l + \sum_i (\dot{W}_t)_i - p \frac{dV}{dt}$$

1 Nomenklatur

A_n = Anergie[J]	V = Volumen[m ³]
c_s = Schallgeschwindigkeit[m/s]	v = Spezifisches Volumen[m ³ /kg]
c_p = Spezifische Wärmekapazität dp = 0 [J/kg*K]	V_m = Molares Volumen[m ³ /mol]
c_v = Spezifische Wärmekapazität dv = 0 [J/kg*K]	W = Arbeit[J]
E = Energie[J]	w = Spezifische Arbeit[J/kg]
Ex = - W_{ex} = Exergie[J]	W_v = Volumenänderungsarbeit[J]
F = Kraft[N]	W_{el} = Elektrische Arbeit[J]
F = U - TS = Freie Energie[J]	W_w = Wellenarbeit[J]
f = u - Ts = Spezifische freie Energie[J/kg]	W_{diss} = Dissipationsarbeit[J]
f = Fugazität[Pa]	W_t = Technische Arbeit[J]
G = H - TS = Freie Enthalpie[J]	W_{virrev} = Arbeitsverlust durch Irreversibilität[J]
g = h - Ts = Spezifische freie Enthalpie[J/kg]	$x = \frac{m''}{m' + m''}$ = Dampfanteil[-]
g = Erdbeschleunigung[m/s ²]	$x = \frac{m_{H_2O}}{m_L}$ = Wassergehalt
H = U + pV = Enthalpie[J]	Z = Allgemeine extensive Zustandsgrößen[Z]
h = u + pv = Spezifische Enthalpie[J/kg]	z = Allgemeine
H_g = Molare Reaktionsenthalpie	β = Isobarer Ausdehnungskoeffizient[1/K]
K = Konstante des Massenwirkungsgesetzes[-]	γ = Isochorer Spannungskoeffizient[1/K]
M = Molmasse[kg/mol]	δ_T = Isothermer Drosselkoeffizient[m ³ /kg]
ṁ = Massestrom[kg/s]	δ_h = Isenthalper Drosselkoeffizient[Ks ² m/kg]
m' = Masse in der flüssigen Phase[kg]	ε = Leistungsziffer[-]
m'' = Masse in der gasförmigen Phase[kg]	ε = Verdichtungsverhältnis[-]
Ma = c / c_s = Machzahl[-]	η_{th} = Thermischer Wirkungsgrad[-]
n = m / M = Molzahl[mol]	η_{mech} = Mechanischer Wirkungsgrad[-]
n = Polytropenexponent[-]	κ = Adiabaten- oder Isentropenexponent[-]
P_t = technische Leistung[W]	λ = Reaktionslaufzahl[-]
Q = Wärme[J]	μ_i = Chemisches Potential[J/mol]
Q̇ = Wärmestrom[W]	v_i = Stöchiometrische Koeffizienten[-]
q = Spezifische Wärme[J/kg]	ξ_i = Masseanteil[-]
r = Spezifische Verdampfungsenthalpie[J/kg]	π = Druckverhältnis[-]
R = Gaskonstante[J/(kg K)]	ρ = Dichte[kg/m ³]
R_m = Universelle Gaskonstante[J/(mol K)]	τ = Temperaturverhältnis[-]
S = Entropie[J/K]	φ = Relative Feuchte[-]
s = Spezifische Entropie[J/(kg K)]	φ = Einspritzverhältnis[-]
T = Temperatur[K]	ξ = Isothermer Kompressibilitätskoeffizient[m ² /N]
t = Zeit[s]	■ = Dissipationsenergie[J]
t = Temperatur[°C]	ψ = Spezifische Dissipationsenergie[J]
T = Sättigungstemperatur[K]	ψ = Drucksteigerungsverhältnis[-]
U = Innere Energie[J]	ψ_i = Molanteil[-]
u = Spezifische innere Energie [J/kg]	

2 Grundbegriffe

Systeme

- Abgeschlossenes System - kein Stoff oder Energietransport
- Geschlossenes System - kein Stofftransport
- Adiabates System - kein Δq , aber Masse und Arbeit.
- Offenes System - Stoff und Energietransport
- Stationäres System $\rightarrow \Delta U = 0$

Messgrößen

- Prozessgrößen sind Wegabhängig (eg. Arbeit, Wärme)
- Zustandsgrößen sind Wegunabhängig (eg. Volumen, Druck)
- Extensive Zustandsgrößen sind abhängig von der Masse des Systems (V, m, H, S, F, G, E)
- Intensive Zustandsgrößen sind unabhängig von der Masse des Systems (T, p)

Zustandsgleichungen

- Thermisch $\rightarrow f(p, V, T) = 0$
- Kalorisch $\rightarrow f(U, V, T) = 0, \quad U = U(V, T), \quad u = u(v, T)$

Hauptsätze

- 0: Temperatur existiert, ihre gleichheit ist notwendige Voraussetzung für das thermische Gleichgewicht.
- 1: Energie existiert, sie ist für abgeschlossene Systeme konstant.
- 2: Entropie existiert, sie wird bei allen irreversiblen Prozessen erzeugt. $dS = \frac{\delta Q_{rev}}{T}$
- 3: 0K existiert, bei dieser Temperatur ist die Entropie = 0

3 Basisformeln

$$\begin{aligned}
 H &= U + pV \\
 dS &= \frac{\delta Q_{rev}}{T} \\
 F &= U - TS \\
 G &= H - ST \\
 W &= - \int p dV \\
 dS &= \frac{Q_{rev}}{T} + S_{prod} \\
 dS_{prod} &= \frac{\Psi}{T} \\
 \Psi &= \int_1^2 T dS_{prod} \\
 W_{ir} &= \frac{T_u}{T} \Psi \\
 p_1 &= p_a + \frac{\varphi_1 - \varphi_a}{\varphi_b - \varphi_a} (p_b - p_a)
 \end{aligned}$$

4 Gibbs

$$\begin{aligned}
 dU &= Tds - pdV + \sum_{k=1}^K \mu_k dn_k \\
 dG &= -SdT + Vdp + \sum_{k=1}^K \mu_k dn_k \\
 dH &= TdS + Vdp + \sum_{k=1}^K \mu_k dn_k \\
 dF &= -SdT - pdV + \sum_{k=1}^K \mu_k dn_k \\
 dU &= \left(\frac{\partial U}{\partial S} \right)_V dS + \left(\frac{\partial U}{\partial V} \right)_S dV + \sum_{k=1}^K \left(\frac{\partial U}{\partial n_k} \right)_S dn_k
 \end{aligned}$$

5 Thermodynamische Beziehungen

$$\begin{aligned}
 T &= \left(\frac{\partial U}{\partial S} \right)_V = T(S, V) & -S &= \left(\frac{\partial F}{\partial T} \right)_V = S(T, V) \\
 T &= \left(\frac{\partial H}{\partial S} \right)_p = T(S, p) & -S &= \left(\frac{\partial G}{\partial T} \right)_p = S(T, p) \\
 p &= - \left(\frac{\partial U}{\partial V} \right)_S = p(V, S) & V &= \left(\frac{\partial G}{\partial p} \right)_T = V(p, T) \\
 -p &= \left(\frac{\partial F}{\partial V} \right)_T = p(T, V) & \mu &= \left(\frac{\partial U}{\partial n} \right)_{S, V} = \mu(S, V, n)
 \end{aligned}$$

6 Guggenheim

$$\begin{array}{llll}
 -S & U & V & U = U(S, V) \\
 H & & F & H = H(S, p) \\
 -p & G & T & F = F(T, V) \\
 & & & G = G(T, p)
 \end{array}$$

$$\overbrace{\frac{d}{dt} \left\{ U + m \left(\frac{c^2}{2} + gz \right) \right\}}^{\text{Stationäres System} \rightarrow 0} = \sum_j \overbrace{\left[\dot{m}_j \left(h + \frac{c^2}{2} + gz \right) \right]_j}^{\text{Geschlossenes System} \rightarrow 0} + \overbrace{\sum_l (\dot{Q}_t)_l}^{\text{Kein Wärmestrom} \rightarrow 0} + \overbrace{\sum_i (\dot{W}_t)_i}^{\text{Keine Leistung} \rightarrow 0} - \overbrace{p \frac{dV}{dt}}^{\text{Keine Volumenänderung} \rightarrow 0}$$

7 Maxwell

$$\left(\frac{\partial T}{\partial p}\right)_{S,n_j} = \left(\frac{\partial V}{\partial S}\right)_{p,n_j}$$

$$\left(\frac{\partial S}{\partial V}\right)_{T,n_j} = \left(\frac{\partial p}{\partial T}\right)_{V,n_j}$$

$$\left(\frac{\partial S}{\partial p}\right)_{T,n_j} = -\left(\frac{\partial V}{\partial T}\right)_{p,n_j}$$

$$\left(\frac{\partial \mu_i}{\partial T}\right)_{p,n_j} = -\left(\frac{\partial S}{\partial n_i}\right)_{T,p,n_j \neq n_i}$$

$$\left(\frac{\partial \mu_i}{\partial p}\right)_{T,n_j} = \left(\frac{\partial V}{\partial n_i}\right)_{T,p,n_j \neq n_i}$$

8 Ideales Gas

$$pV = mRT$$

$$pv = RT$$

$$pV = nR_m T$$

$$\beta = \frac{1}{T}$$

$$\gamma = \frac{1}{T}$$

$$\chi = \frac{1}{p}$$

$$\beta = p\gamma\chi$$

$$R_m = 8,3143 \left[\frac{kJ}{kmolK} \right]$$

$$R = c_p - c_v$$

$$R = \frac{R_m}{M}$$

$$U - U_0 = mc_v(T - T_0)$$

$$H - H_0 = mc_p(T - T_0) \quad \leftarrow \text{Für } c_p \text{ und } c_v \text{ const.}$$

$$s - s_0 = R \ln \left(\frac{v}{v_0} \right) + c_v \ln \left(\frac{T}{T_0} \right)$$

$$= c_v \ln \left(\frac{p}{p_0} \right) + c_p \ln \left(\frac{v}{v_0} \right)$$

$$= c_p \ln \left(\frac{T}{T_0} \right) - R \ln \left(\frac{p}{p_0} \right)$$

$$\beta = \frac{1}{T} = \frac{1}{V} \left(\frac{\partial V}{\partial T} \right)_p = \frac{1}{v} \left(\frac{\partial v}{\partial T} \right)_p = -\frac{1}{p} \left(\frac{\partial p}{\partial T} \right)_p$$

$$\gamma = \frac{1}{T} = \frac{1}{p} \left(\frac{\partial p}{\partial T} \right)_v$$

$$\chi = \frac{1}{p} = -\frac{1}{V} \left(\frac{\partial V}{\partial p} \right)_T = -\frac{1}{v} \left(\frac{\partial v}{\partial p} \right)_T$$

$$u_2 - u_1 = \int_{T_1}^{T_2} c_v(T) dT$$

9 Van-der-Waals

$$\left(p + \frac{a}{v^2}\right)(v - b) = RT$$

$$\left(\bar{p} + \frac{3}{\bar{v}^2}\right)(3\bar{v} - 1) = 8\bar{T}$$

$$\bar{p} = \frac{p}{p_K}, \quad \bar{v} = \frac{v}{v_K}, \quad \bar{T} = \frac{T}{T_K}$$

$$p_K = \frac{a}{27b^2}, \quad T_K = \frac{8}{27} \frac{a}{b} \frac{1}{R}, \quad \text{mit } \mathcal{L}$$

$$a = 3p_K v_K^2, \quad b = \frac{v_K}{3}, \quad \frac{p_K v_K}{RT_K} = \frac{3}{8}$$

$$\beta = \frac{(v - b)Rv^2}{RTv^3 - 2a(v - b)^2}$$

$$\gamma = \frac{Rv^2}{RTv^3 - 2a(v - b)^2}$$

$$\chi = \frac{(v - b)^2 v^2}{RTv^3 - 2a(v - b)^2}$$

$$du = \frac{a}{v^2} dv + c_v(T) dT$$

$$u - u_0 = \left(\frac{a}{v_0} - \frac{a}{v} \right) + \int_{T_0}^T c_v(\tilde{T}) d\tilde{T}$$

$$u - u_0 = \left(\frac{a}{v_0} - \frac{a}{v} \right) + c_v(T - T_0) \quad \leftarrow \text{für } c_v = \text{const.}$$

$$c_p - c_v = \frac{Tv\beta^2}{\chi}$$

$$s - s_0 = c_v \ln \left(\frac{T}{T_0} \right) + R \ln \left(\frac{v - b}{v_0 - b} \right)$$

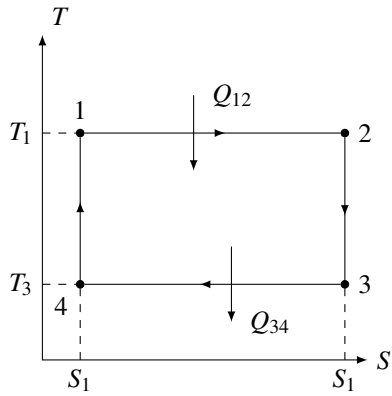
$$h_2 - h_1 = \frac{1}{2}(p_2 - p_1) \left(\frac{1}{\rho_1} + \frac{1}{\rho_2} \right)$$

10 Carnot

$$\eta_{th} = 1 - \frac{-Q_{34}}{Q_{12}} = 1 - \frac{T_3(S_3 - S_4)}{T_1(S_2 - S_1)} = 1 - \frac{T_3}{T_1}$$

$$\frac{Q_{12}}{T_1} + \frac{Q_{34}}{T_3} = 0$$

$$\Delta S_{ges} = -Q_{34} \left(\frac{1}{T_{KK}} - \frac{T_1}{T_3} \frac{1}{T_{HK}} \right)$$



Adiabate Drosselung (ideal):

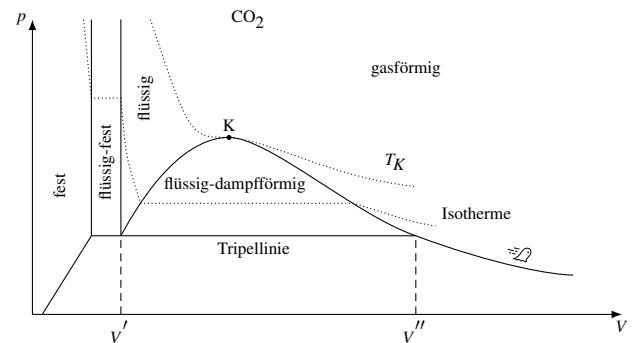
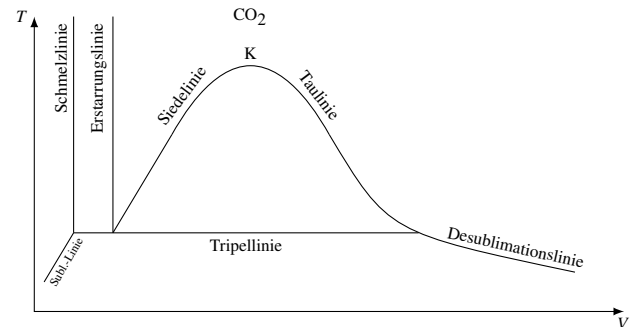
$$h + \frac{c^2}{2} + gz = \text{const.}$$

$$dh = 0$$

Adiabete Drosselung (real):

$$\delta_h = \left(\frac{\partial T}{\partial p} \right)_h = -\frac{v}{c_p} (1 - \beta T)$$

12 Nassdampf



11 Gemische Idealer Gase

$$\xi_i = \frac{m_i}{m}, \quad \psi_i = \frac{n_i}{n}, \quad p_i = \psi_i p$$

$$\xi_i = \frac{M_i n_i}{\sum_{k=1}^K M_k n_k} = \frac{M_i}{M_G} \psi$$

$$p_i V = m_i R_i T, \quad p_i V = n_i R_m T, \quad p V = m R_G T$$

$$\sum_{k=1}^K p_k = p$$

$$R_G = \frac{1}{m} \sum_{k=1}^K m_k R_k = \sum_{k=1}^K \xi_k R_k$$

$$U_G = \sum_{k=1}^K U_k = \sum_{k=1}^K m_k u_k = \sum_{k=1}^K c_{vk} m_k T \leftarrow c_v = \text{const}$$

$$H_G = \sum_{k=1}^K H_k = \sum_{k=1}^K m_k h_k = \sum_{k=1}^K c_{pk} m_k T \leftarrow c_p = \text{const.}$$

$$c_{vG} = \sum_{k=1}^K c_{vk} \xi_k, \quad c_{pG} = \sum_{k=1}^K c_{pk} \xi_k$$

$$S_2 - S_1 = R_m \left(n \ln n - \sum_{k=1}^K n_k \ln n_k \right)$$

$$v = (1-x)v' + xv''$$

$$v = v' + (v'' - v')x$$

$$u = (1-x)u' + xu''$$

$$u = u' + (u'' - u')x$$

$$h = (1-x)h' + xh''$$

$$h = h' + (h'' - h')x$$

$$s = (1-x)s' + xs''$$

$$s = s' + (s'' - s')x$$

$$r = h'' - h' = T(s'' - s')$$

$$T' = T''$$

$$p' = p''$$

$$g' = g''$$

$$dg' = v' dp' - s' dT'$$

$$dg'' = v'' dp'' - s'' dT''$$

$$dg' = dg''$$

$$\frac{dp}{dT} = \frac{s'' - s'}{v'' - v'}$$

$$\frac{dp}{dT} = \frac{1}{T} \frac{h'' - h'}{v'' - v'}$$

$$\frac{dp}{dT} = \frac{1}{T} \frac{r}{v'' - v'}$$

13 Realer Stoff im Nassdampfgebiet

Isobare Zustandsänderung

$$q_{12} = T(s_2 - s_1)$$

$$= T(s'' - s')(x_2 - x_1)$$

$$w_{V,12} = - \int_1^2 p dv$$

$$= -p(v_2 - v_1) = -p(v'' - v')(x_2 - x_1)$$

Isochore Zustandsänderung

$$q_{12} = u_2 - u_1 = u'_2 + x_2(u''_2 - u'_2) - u'_1 - x_1(u''_1 - u'_1)$$

Adiabate Zustandsänderung

$$w_{V,12} = u_2 - u_1 = u'_2 + x_2(u''_2 - u'_2) - u'_1 - x_1(u''_1 - u'_1)$$

Entropieänderung während des Mischvorgangs

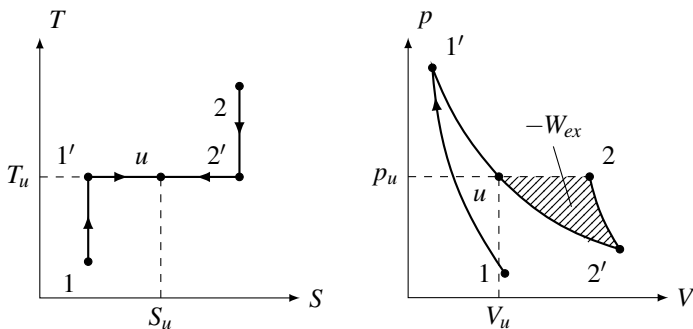
$$S_2 - S_1 = R_m \left(n \ln n - \sum_i n_i \ln n_i \right)$$

14 Maximale Arbeit und Exergie

Maximale nutzbare Arbeit → isentrop, reibungsfrei

1 → 1' : isentrop auf T_u

1' → u : isotherm auf u



$$-\dot{W}_{ex} = -(\dot{W}_t)_{rev} = -\frac{d}{dt} \left(U + m \left(\frac{c^2}{2} + gz \right) + p_u V - T_u S \right) + \sum_{j=1}^K \left(\dot{m}_j \left(h + \frac{c^2}{2} + gz - T_s \right) \right) + \sum_{l=1}^K \left(1 - \frac{T_u}{T} \right) \dot{Q}_l$$

Die Exergie der Enthalpie (offenes, stationäres System)

$$-\dot{W}_{ex,1u} = \dot{m}(h_1 - h_u - T_u(s_1 - s_u))$$

Die Exergie der inneren Energie (geschlossenes, instationäres System)

$$-\dot{W}_{ex} = -\frac{d}{dt} (U + p_u V - T_u S)$$

$$-\dot{W}_{ex,1u} = U_1 - U_u - p_u(V_1 - V_u) - T_u(S_1 - S_u)$$

$$-\dot{W}_{ex,1u} = H_1 - (p_1 - p_u)V_1 - H_u - T_u(S_1 - S_u)$$

Für Ideales Gas

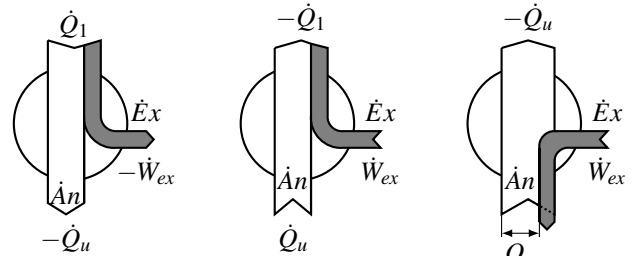
$$-\dot{W}_{ex} = m c_v (T_1 - T_u) + p_u (V_1 - V_u) - T_u m \left(c_p \ln \left(\frac{T_1}{T_u} \right) - R_i \ln \left(\frac{p_1}{p_u} \right) \right)$$

Dampf/Luftdruckkammer

$$-\dot{W}_{ex,1u} = m_1 [u_1 - u_u + p_u(v_1 - v_u) - T_u(s_1 - s_u)]$$

Die Exergie der Wärme (geschlossenes, stationäres System)

$$-\dot{W}_{ex} = \left(1 - \frac{T_u}{T_1} \right) \dot{Q}_1 = \eta_{th,C} \dot{Q}_1$$

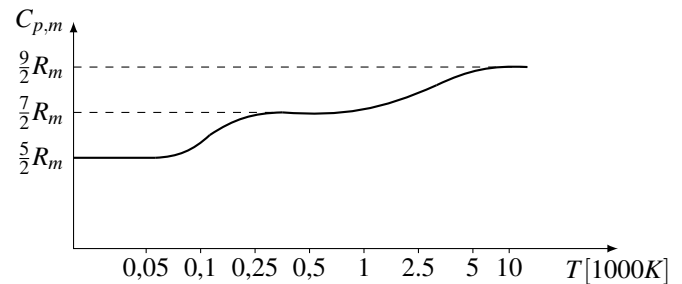


WärmeKraftprozess

Wärmepumpenprozess

Kälteprozess

15 Wärmekapazität



$$C_{v,m} = \frac{1}{\kappa - 1} R_m$$

$$c_v = \frac{1}{\kappa - 1} R_j$$

$$\kappa = \frac{c_p}{c_v}$$

$$R = \frac{R_m}{M}$$

$$C_{p,m} = \frac{\kappa}{\kappa - 1} R_m$$

$$c_p = \frac{\kappa}{\kappa - 1} R_j$$

$$R = c_p - c_v$$

$$R_m = 8,3143 \left[\frac{kJ}{kmolK} \right]$$

$$C_{v,m} = \underbrace{3 + \frac{R_m}{2}}_{\text{Translatorisch}} + \underbrace{\frac{n_{rot} R_m}{2}}_{\text{Rotatorisch}} + \underbrace{R_m (3n_{Atome} - 3 - n_{rot})}_{\text{Vibratorisch}} + \underbrace{C_{v,m, \text{Elektronenanregung}}}_{\text{Relevant ab: } T \approx 10^4 K}$$

16 Technische Anwendung

adiabat ($c_p = \text{const.}$)	$W_{t,12} = mc_p(T_2 - T_1) = \frac{\kappa}{\kappa - 1}(p_2V_2 - p_1V_1)$	$Q_{12} = 0$
reversibel adiabat $\kappa = \text{const.}$	$W_{t,12} = \frac{\kappa}{\kappa - 1}(p_1V_1) \left[\left(\frac{p_2}{p_1} \right)^{\frac{\kappa-1}{\kappa}} - 1 \right]$	$Q_{12} = 0$
irreversibel adiabat als Polytrope $n > \kappa; n, \kappa = \text{const.}$	$W_{t,12} = \frac{\kappa}{\kappa - 1}(p_1V_1) \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right]$	$Q_{12} = 0$
reversibel polytrop $n, \kappa = \text{const.}$	$W_{t,12} = \frac{n}{n-1}(p_2V_2 - p_1V_1)$ $= \frac{n}{n-1}mR(T_2 - T_1)$ $= \frac{n}{n-1}(p_1V_1) \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right]$	$Q_{12} = mc_n(T_2 - T_1)$ $= \frac{n - \kappa}{(n-1)(\kappa-1)}(p_1V_1) \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right]$ $c_n = \frac{n - \kappa}{n-1}c_v$
isotherm	$W_{t,12} = (p_1V_1) \ln \left(\frac{p_2}{p_1} \right)$	$Q_{12} = -W_{t,12}$

Thermischer Wirkungsgrad $\eta_{th} = \frac{-w}{q_{zu}} = \frac{\text{Nutzen}}{\text{Aufwand}} = 1 - \frac{|q_{ab}|}{q_{zu}}$

Isentroper Verdichterwirkungsgrad $\eta_{sV} = \frac{w_{t,12,rev}}{w_{t,12}} = \frac{h_{2,rev} - h_1}{h_2 - h_1} = \frac{T_{2,rev} - T_1}{T_2 - T_1}$
idealer Fall

Isentroper Turbinenwirkungsgrad $\eta_{sT} = \frac{w_{t,12}}{W_{t,12,rev}} = \frac{h_1 - h_2}{h_1 - h_{2,rev}} = \frac{T_1 - T_2}{T_1 - T_{2,rev}}$

Dampfkraftprozess Wirkungsgrad $\eta_{th} = 1 - \frac{|q_{61}|}{q_{23} + q_{34} + q_{45}} = 1 - \frac{h_6 - h_1}{h_5 - h_2}$

Leistungszahl Kältemaschine $\epsilon_{K(A)} = \frac{q_{zu}}{w} = \frac{\dot{Q}_0}{\dot{W}}$

Leistungszahl Kaltluftprozess $\epsilon_K = \frac{1}{\left(\frac{p}{p_0} \right)^{\frac{\kappa-1}{\kappa}} - 1}$

Leistungszahl Kalddampfprozess $\epsilon_K = \frac{q_0}{|q| - q_0} = \frac{q_o}{w_t} = \frac{h_1 - h_6}{h_2 - h_1}$

Linkslaufender Carnotprozess $\epsilon_{carnot} = \frac{T_k}{T_H - T_K}$

Leistungszahl Wärmepumpe $\epsilon_{WP} = \frac{q}{|q| - q_0} = \frac{|q|}{w_t} = \frac{q_{zu}}{w} = \frac{h_2 - h_5}{h_2 - h_1} = 1 + \epsilon_{K(A)}$

Verdichtungsverhältnis $\epsilon = \frac{v_1}{v_2}$

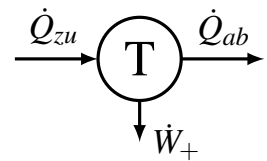
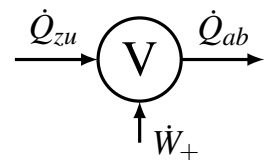
Drucksteigerungsverhältnis $\psi = \frac{p_3}{p_2}$

Einspritzverhältnis $\phi = \frac{v_4}{v_3}$

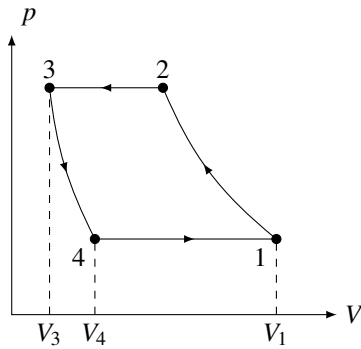
Temperaturverhältnis $\tau = \frac{T_3}{T_1}$

Verdrichtungsdruckverhältnis $\pi = \frac{p_2}{p_1}$

für Joule-Prozess $\pi_{opt} = \tau^{\frac{\kappa}{2(\kappa-1)}}$



Kolbenverdichter



V_1 = Maximales Zylindervolumen

V_2 = Volumen nach Verdichtung

V_3 =

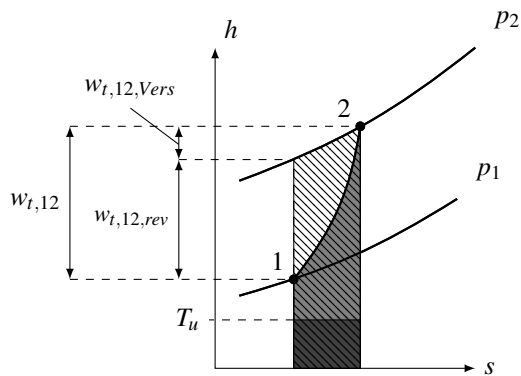
V_4 = Schädlicher Raum

$$\mu = \frac{V_1 - V_4}{V_1 - V_3}, \quad \epsilon_s = \frac{V_3}{V_1 - V_3}$$

$$\mu = 1 - \epsilon_s \left[\left(\frac{p_2}{p_1} \right)^{\frac{1}{n}} - 1 \right]$$

$$\begin{aligned} W_{t,12} &= \int_1^2 V dp \\ &= \underbrace{p_2 V_2}_{\text{Ausschiebearbeit}} - \underbrace{p_1 V_1}_{\text{Einschiebearbeit}} - \int_1^2 p dV \\ &= \frac{n}{n-1} p_1 (V_1 - V_4) \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right] \end{aligned}$$

Turboverdichter



$$\text{hatched area: } w_{t,12,Verl} = \int_{2,rev}^2 T ds$$

$$\text{grey area: } \psi_{12} = \int_1^2 T ds$$

$$\text{black area: } w_{V,12,irr} = T_u \Delta s$$

Verdichter Wirkungsgrad

$$\eta_{sV} = \frac{w_{t,12,rev}}{w_{t,12}} = \frac{h_{2,rev} - h_1}{h_2 - h_1}$$

Verdichter wirkungsgrad (Ideales Gas, $c_p = \text{const.}$)

$$\eta_{sV} = \frac{T_{2,rev} - T_1}{T_2 - T_1}$$

Technische Verlustarbeit

$$\begin{aligned} w_{t,Verl,12} &= w_{t,12} - w_{t,12,rev} = h_2 - h_{2,rev} \\ &= \int_{2,rev}^2 T|_{p_2=\text{const.}} ds \end{aligned}$$

17 Eindimensionale Strömungsvorgänge

$$\begin{aligned}\chi &= \frac{1}{p} = -\frac{1}{V} \left(\frac{\partial V}{\partial p} \right)_T \\ c_S^2 &= \left(\frac{\partial p}{\partial \rho} \right)_S \\ c_S^2 &= \left(\frac{R}{c_v} + 1 \right) \left(v^2 \frac{RT}{(v-b)^2} \right) - \frac{2a}{v} \leftarrow VdW \\ c_S^2 &= \kappa RT \leftarrow ideal \\ Ma &= \frac{c}{c_S} \\ \frac{T_0}{T} &= 1 + \frac{\kappa-1}{2} \frac{c^2}{\kappa RT} = 1 + \frac{\kappa-1}{2} Ma^2 \\ \frac{p_0}{p} &= \left(\frac{T_0}{T} \right)^{\frac{\kappa}{\kappa-1}} = \left(1 + \frac{\kappa-1}{2} Ma^2 \right)^{\frac{\kappa}{\kappa-1}} \\ \frac{\rho_0}{\rho} &= \left(\frac{T_0}{T} \right)^{\frac{\kappa-1}{\kappa}} = \left(1 + \frac{\kappa-1}{2} Ma^2 \right)^{\frac{\kappa-1}{\kappa}} \\ \left(\frac{A}{A^*} \right)^2 &= \frac{1}{Ma^2} \left[\frac{2}{\kappa+1} \left(1 + \frac{\kappa-1}{2} Ma^2 \right) \right]^{\frac{\kappa+1}{\kappa-1}}\end{aligned}$$

$$h_2 - h_1 = \frac{1}{2}(p_2 - p_1) \left(\frac{1}{\rho_1} + \frac{1}{\rho_2} \right) = (p_2 - p_1) \frac{1}{2}(v_1 + v_2)$$

Stoßbeziehungen für ein ideales Gas

$$\begin{aligned}\frac{p_2}{p_1} &= \frac{2\kappa Ma^2 - (\kappa-1)}{\kappa+1} \\ \frac{\rho_2}{\rho_1} &= \frac{(\kappa+1)Ma^2}{2 + (\kappa-1)Ma^2} \\ \frac{T_2}{T_1} &= \frac{[2\kappa Ma^2 - (\kappa-1)][2 + (\kappa-1)Ma^2]}{(\kappa+1)^2} Ma^2 \\ Ma_2^2 &= \frac{(\kappa-1)(Ma_1^2 - 1) + (\kappa+1)}{2\kappa(Ma_1^2 - 1) + (\kappa+1)}\end{aligned}$$

Entropie über den senkrechten Verdichtungsstoß

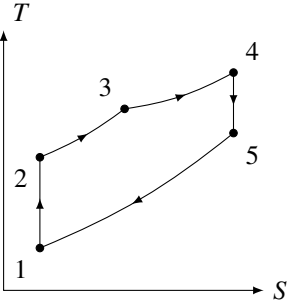
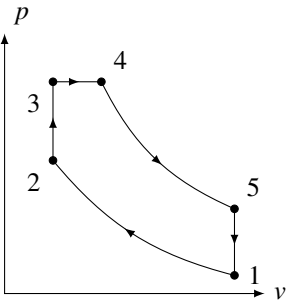
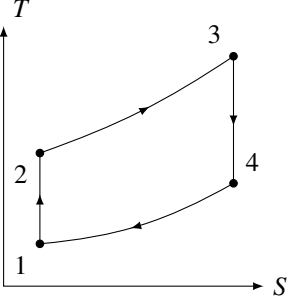
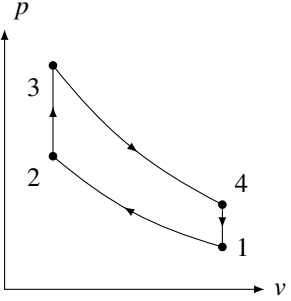
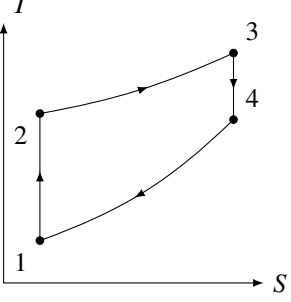
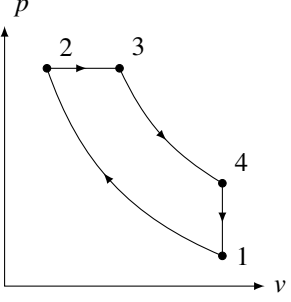
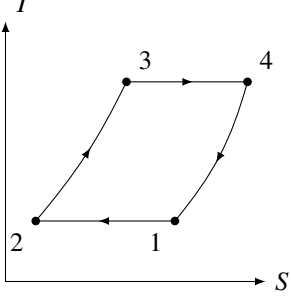
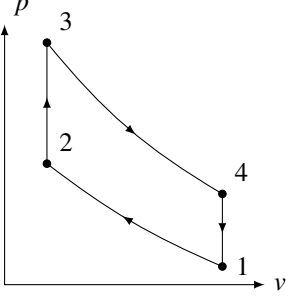
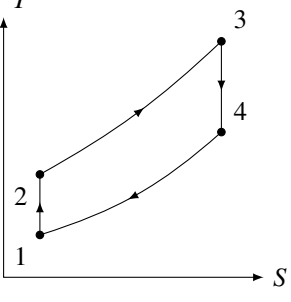
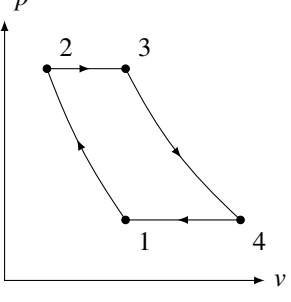
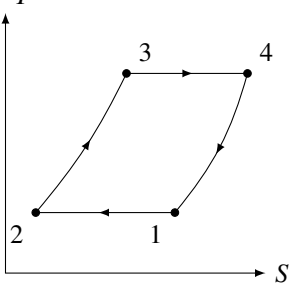
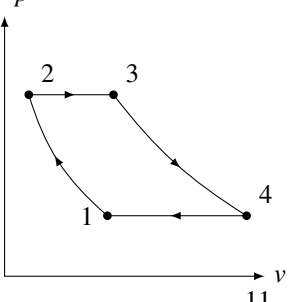
$$\begin{aligned}s_2 - s_1 &= c_v \ln \left(\frac{T_2}{T_1} \right) + R \ln \left(\frac{v_2}{v_1} \right) \\ &= c_p \ln \left(\frac{T_2}{T_1} \right) + R \ln \left(\frac{p_2}{p_1} \right)\end{aligned}$$

18 Feuchte Luft

$$\begin{aligned}x &= \frac{m_{H_2O}}{m_L} \\ x &= x_{D(ampf)} + x_{W(asser)} + x_{E(is)} \\ \phi &= \frac{p_D}{p_s} \\ x_D &= \frac{m_d}{m_L} = \frac{R_L}{R_D} \frac{p_D}{p_L} = \frac{R_L}{R_D} \frac{p_D}{p - p_D} = 0.622 \frac{p_D}{p - p_D} \\ x_s &= \frac{m_{D,max}}{m_L} = 0.622 \frac{p_s}{p - p_s} \rightarrow \text{für } \phi = 1 \\ \rho &= \frac{p}{R_{gesT}} = \frac{1+x}{R_L + xR_D} \frac{p}{T} \\ R_{ges} &= \frac{R_L + xR_D}{1+x} \\ h &= c_{pL}t + x_D(c_{pD}t + r_D) + x_W c_{Wt} + x_E(c_{Et} - r_E)\end{aligned}$$

19 Chemische Reaktionen

$$\begin{aligned}\frac{dn_1}{v_1} &= \frac{dn_2}{v_2} = \dots = d\lambda = .const \\ \sum_{k=1}^K \mu_k dn_k &= \sum_{k=1}^K \mu_k (v_k d\lambda) = \sum_{k=1}^K \mu_k v_k = 0 \\ \mu_i &= \left(\frac{\partial U}{\partial n_i} \right)_{S,V} = \left(\frac{\partial H}{\partial n_i} \right)_{S,p} = \left(\frac{\partial F}{\partial n_i} \right)_{T,V} = \left(\frac{\partial G}{\partial n_i} \right)_{T,p} \\ \mu(p,T) &= \mu(p^+, T) + R_m T \ln \left(\frac{p}{p^+} \right) \\ \text{Massenwirkungsgesetz} \\ \prod_{k=1}^K \psi_k^{v_k} &= \exp - \frac{1}{R_m T} \sum_{k=1}^K v_k \mu_{0k}(p,T) \\ &= \exp - \frac{1}{R_m T} \sum_{k=1}^K v_k G_{m,k}(p,T) \\ \text{Gleichgewichtskonstante} \\ K(p,T) &= \prod_{k=1}^K \psi_k^{v_k} \\ K(p_2,T) &= K(p_1,T) \left(\frac{p_1}{p_2} \right)^{\sum v_k} \\ \ln \left(\frac{K(p,T_2)}{K(p,T_1)} \right) &= \frac{\Delta H_R}{R_m} \left(\frac{1}{T_1} - \frac{1}{T_2} \right) = \frac{\Delta H_R}{R_m} \frac{T_2 - T_1}{T_1 T_2} \\ \Delta H_R &= \sum_{k=1}^K v_k H_{m,k}\end{aligned}$$

		<p>Seiliger Prozess</p> $\eta_{th} = 1 - \frac{\varphi^{\kappa} \psi - 1}{\varepsilon^{\kappa-1} [\psi - 1 + \kappa \psi (\varphi - 1)]}$ $\varepsilon = \frac{v_1}{v_2} \quad \psi = \frac{p_3}{p_2} \quad \varphi = \frac{v_4}{v}$
		<p>Otto Prozess</p> $\eta_{th} = 1 - \frac{1}{\varepsilon^{\kappa-1}}$ $\varepsilon = \frac{v_1}{v_2}$
		<p>Diesel Prozess</p> $\eta_{th} = 1 - \frac{\varphi^{\kappa} - 1}{\varepsilon^{\kappa-1} \kappa (\varphi - 1)}$ $\varepsilon = \frac{v_1}{v_2} \quad \varphi = \frac{v_4}{v}$
		<p>Stirling Prozess</p> $\eta_{th} = 1 - \frac{ q_{12} }{q_{34}} = \frac{RT_1 \ln \left(\frac{v_1}{v_2} \right)}{RT_3 \ln \left(\frac{v_4}{v_3} \right)} = 1 - \frac{T_1}{T_3}$
		<p>Joule Prozess</p> $\eta_{th} = 1 - \frac{T_1}{T_2} = 1 - \left(\frac{p_1}{p_2} \right)^{\frac{\kappa-1}{\kappa}} = 1 - \left(\frac{1}{\pi} \right)^{\frac{\kappa-1}{\kappa}}$ $\pi = \frac{p_2}{p_1}$
		<p>Ericsson Prozess</p> $\eta_{th} = 1 - \frac{ q_{12} }{q_{34}} = \frac{RT_1 \ln \left(\frac{p_1}{p_2} \right)}{RT_3 \ln \left(\frac{p_4}{p_3} \right)} = 1 - \frac{T_1}{T_3}$

Ideales Gas

	Isothermo	Isobare	Isochore	Isentrop	Polytrope
konstant:	T	p	v	$\delta q = 0$	$p v^n$
-	-	-	-	$p_1 v_1^\kappa = p_2 v_2^\kappa$	$v_1^n = p_2 v_2^n$
	$p_1 p_2 = p_2 v_2$	$\frac{v_1}{v_2} = \frac{T_1}{T_2}$	$\frac{p_1}{T_1} = \frac{p_2}{T_2}$	$T_1 v_1^{\kappa-1} = T_2 v_2^{\kappa-1}$	$T_1 v_1^{n-1} = T_2 v_2^{n-1}$
-	-	-	$\neq \varnothing$	$\frac{T_1^{\frac{\kappa-1}{\kappa}}}{p_1} = \frac{T_2^{\frac{\kappa-1}{\kappa}}}{p_2}$	$\frac{T_1^{\frac{n-1}{n}}}{p_1} = \frac{T_2^{\frac{n-1}{n}}}{p_2}$
p, v	$p = \frac{p_1 v_1}{v}$	$p = p_1$	$v = v_1$	$p = \frac{p_1 v_1^\kappa}{v^\kappa}$	$p = \frac{p_1 v_1^n}{v^n}$
p, T	$p = \frac{p_1 v_1}{v}$	$p = p_1$	$p = \frac{p_1 T}{T_1}$	$p = \frac{p_1 T^{\frac{\kappa}{\kappa-1}}}{T_1^{\frac{\kappa}{\kappa-1}}}$	$p = \frac{p_1 T^{\frac{n}{n-1}}}{T_1^{\frac{n}{n-1}}}$
v, T	$T = T_1$	$v = \frac{v_1 T}{T_1}$	$v = v_1$	$T = \frac{T_1 v_1^{\kappa-1}}{v^{\kappa-1}}$	$T = \frac{T_1 v_1^{n-1}}{v^{n-1}}$
q_{12}	$= p_1 v_1 \ln \frac{p_1}{p_2}$	$= c_p (T_2 - T_1)$	$= c_v (T_2 - T_1)$	$= 0$	$= c_v \frac{n-\kappa}{n-1} (T_2 - T_1)$
$w_{V,12}$	$= -q_{12}$	$= -p_1 (v_2 - v_1)$	$= 0$	$= \frac{p_1 v_1}{\kappa-1} \left[\left(\frac{v_1}{v_1} \right)^{\kappa-1} - 1 \right]$	$= \frac{p_1 v_1}{n-1} \left[\left(\frac{v_1}{v_2} \right)^{n-1} - 1 \right]$
$s_2 - s_1$	$= R \ln \left(\frac{p_1}{p_2} \right)$	$= c_p \ln \left(\frac{T_2}{T_1} \right)$	$= c_v \ln \left(\frac{T_2}{T_1} \right)$	$= 0$	$= c_v \frac{n-\kappa}{n-1} \ln \left(\frac{T_2}{T_1} \right)$

Van-De-Waals-Gas

	Isotherme	Isobare	Isochore	Isentrop
konst.	T	p	v	$\delta = 0$
	$(p_1 + \frac{a}{v_1^2})(v_1 - b)$ $= (p_2 + \frac{a}{v_2^2})(v_2 - b)$	$\frac{RT_1}{v_1 - b} - \frac{a}{v_1^2} = \frac{RT_2}{v_2 - b} - \frac{a}{v_2^2}$	$\frac{p_1 + \frac{a}{v_1^2}}{T_1} = \frac{p_2 + \frac{a}{v_2^2}}{T_2}$	$(p_1 + \frac{a}{v_1^2})(v_1 - b)^{\frac{c_v + R}{c_v}}$ $= (p_2 + \frac{a}{v_2^2})(v_2 - b)^{\frac{c_v + R}{c_v}}$, $T_1(v_1 - b)^{R/c_v} = T_2(v_2 - b)^{R/c_v}$
p, v	$p = (p + \frac{a}{v^2})\frac{v_u}{v - b} - \frac{a}{v^2}$	$p = p_1$	$v = v_1$	$p = -\frac{a}{v^2} + (p_1 + \frac{a}{v_1^2})\left(\frac{v_1 - b}{v - b}\right)^{\frac{c_v + R}{R}}$
p, T	$T = T_1$	$p = p_1$	$p = \frac{T}{T_1}(p_1 + \frac{a}{v_1^2}) - \frac{a}{v^2}$	$p = -\frac{a}{v^2} + (p_1 + \frac{a}{v_1^2})\left(\frac{T}{T_1}\right)^{\frac{c_v + R}{R}}$
v, T	$T = T_1$	$T = T_1 \frac{v - b}{v_1 - b} + \frac{a}{R}(v - b)\left(\frac{1}{v^2} - \frac{1}{v_1^2}\right)$	$v = v_1$	$T = T_1 \left(\frac{v_1 - b}{v - b}\right)^{\frac{R}{c_v}}$
q_{12}	$= RT_1 \ln \left(\frac{v_2 - b}{v_1 - b}\right)$	$= \frac{a}{v_1} - \frac{a}{v_2} + c_v(T_2 - T_1)$	$= c_v(T_2 - T_1)$	$= 0$
$w_{V,12}$	$= -RT_1 \ln \left(\frac{v_2 - b}{v_1 - b}\right) + \frac{a}{v_1} - \frac{a}{v_2}$	$= -p_1(v_2 - v_1)$	$= 0$	$= \frac{a}{v_1} - \frac{a}{v_2} + c_v(T_2 - T_1)$
$s_2 - s_1$	$= R \ln \left(\frac{v_2 - b}{v_1 - b}\right)$	$= c_v \ln \left(\frac{T_2}{T_1}\right) + R \ln \left(\frac{v_2 - b}{v_1 - b}\right)$	$= c_v \ln \left(\frac{T_2}{T_1}\right)$	$= 0$

20 Stoffwerte einiger Gase

Bezeichnung	Symbol	Molmasse [kg/kmol]	Gaskonstante [J/(kg K)]	Dichte [kg/m ³]	c_p [J/(kg K)]	c_v [J/(kg,K)]	κ
Acetylen	C_2H_2	26.038	319.3	1.16	1616	1278	1.26
Ammoniak	NH_3	17.031	488.2	0.76	2056	1526	1.35
Argon	Ar	39.948	208.1	1.76	519	309	1.68
Äthan	C_2H_6	30.070	276.5	1.34	1650	1355	1.22
Butan	C_4H_{10}	58.124	143.0	2.67	1599	1410	1.13
Chlor	Cl_2	56.108	117.3	3.17	473	343	1.38
Chlorwasserstoff	HCl	70.906	228.0	1.62	795	556	1.43
Helium	He	4.003	2077.0	0.18	5200	3124	1.66
Kohlendioxid	CO_2	44.010	188.9	1.95	816	618	1.32
Kohlenmonoxid	CO	28.010	296.8	1.23	1038	739	1.40
Luft	~	28.964	287.1	1.28	1006	718	1.40
Methan	CH_4	16.043	518.3	0.71	2165	1638	1.32
Propan	C_3H_8	44.097	188.5	1.99	1549	1331	1.16
Sauerstoff	O_2	31.999	259.8	1.41	909	647	1.40
Stickstoff	N_2	28.013	296.8	1.23	1038	739	1.40
Wasserstoff	H_2	2.016	4124.2	0.09	14050	9926	1.42
Xenon	Xe	131.300	63.3	5.82	159	93	1.71

21 Stoffdaten einiger Stoffe

Name Name	chemische Formel	Molmasse [kg/kmol]	Normal- siedepunkt [°C]	kritische Temperatur [°C]	kritischer Druck [MPa]
Wasserstoff	H_2	2.02	-252.9	-240.0	1.32
Helium	He	4.00	-268.9	-268.0	0.23
Ammoniak	NH_3	17.03	-33.3	132.3	11.33
Wasser	H_2O	18.02	100.0	373.9	22.06
Luft	78% N_2 21%	28.96	-194.2	-140.4	3.84
	O_2 .1%Ar.+				
Kohlendioxid	CO_2	44.01	-78.4	31.0	7.38
Methan	CH_4	16.04	-161.5	-82.6	4.60
Äthan	C_2H_6	30.07	-88.6	32.2	4.87
Propan	C_3H_8	44.10	-42.1	96.7	4.25
R134a	CH_2FCF_3	102.03	-26.1	101.1	4.06

22 Zahlenwerte feuchte Luft

Bezeichnung	Formelzeichen	Zahlenwert	Dimension
Molmasse der Luft	ML	28,96	kg/ kmol
Molmasse des Wassers	MH ₂ O	18,02	kg/ kmol
spezifische Gaskonstante der Luft	RL	0,287	kJ/ (kg K)
spezifische Gaskonstante des Dampfes	RD	0,461	kJ/ (kg K)
spezifische Wärmekapazität der Luft	cpL	1,006	kJ/ (kg K)
spezifische Wärmekapazität des Dampfes	cpD	1,92	kJ/ (kg K)
spezifische Wärmekapazität des Wassers	cW	4,182	kJ/ (kg K)
spezifische Wärmekapazität des Eises	cE	2,1	kJ/ (kg K)
Verdampfungsenthalpie des Wassers bei 0 °C	rD	2500	kJ/ kg
Schmelzenthalpie des Eises bei 0 °C	rE	334	kJ/ kg

23 Obskure Zusammenhänge

$$\begin{aligned}
 dV &= \left(\frac{\partial V}{\partial T} \right)_p dT + \left(\frac{\partial V}{\partial p} \right)_{T,n} dp + \sum_{k=1}^K \left(\frac{\partial V}{\partial n_k} \right)_{T,p} dn_k \\
 dS &= \left(\frac{nC_{p,m}}{T} \right) dT - \left(\frac{\partial V}{\partial T} \right)_{p,n} dp + \sum_{k=1}^K \left(\frac{\partial \mu_k}{\partial T} \right)_{p,n} dn_k \\
 dU &= \left[nC_{p,m} - p \left(\frac{\partial V}{\partial T} \right)_{p,n} \right] dT - \left[p \left(\frac{\partial V}{\partial p} \right)_{T,n} + T \left(\frac{\partial V}{\partial T} \right)_{p,n} \right] dp + \sum_{k=1}^K \left[\mu_k - T \left(\frac{\partial \mu_k}{\partial T} \right)_{p,n} - p \left(\frac{\partial V}{\partial n_k} \right)_{T,p,n} \right] dn_k \\
 dH &= nC_{p,m} dT + \left[VT \left(\frac{\partial V}{\partial T} \right)_{p,n} \right] + \sum_{k=1}^K \left[\mu_k - T \left(\frac{\partial \mu_k}{\partial T} \right)_{p,n} \right] dn_k \\
 dF &= - \left[S + p \left(\frac{\partial V}{\partial T} \right)_{p,n} \right] dT - p \left(\frac{\partial V}{\partial p} \right)_{T,n} dp + \sum_{k=1}^K \left[\mu_k - p \left(\frac{\partial V}{\partial n_k} \right)_{T,p} \right] dn_k \\
 \left(\frac{\partial C_{p,m}}{\partial p} \right)_{T,\psi_j} &= T \frac{\partial}{\partial p} \left[\left(\frac{\partial S_m}{\partial T} \right)_{p,\psi_j} \right]_{T,\psi_j} = T \frac{\partial}{\partial T} \left[\left(\frac{\partial S_m}{\partial p} \right)_{T,\psi_j} \right]_{p,\psi_j} = -T \frac{\partial}{\partial T} \left[\left(\frac{\partial V_m}{\partial T} \right)_{p,\psi_j} \right]_{p,\psi_j} = -T \left(\frac{\partial^2 V_m}{\partial T^2} \right)_{p,\psi_j} \\
 C_{p,m} &= (C_{p,m})_{\text{ideales Gas}} - T \int_0^p \left(\frac{\partial^2 V_m}{\partial T^2} \right)_{p,\psi_j} d\tilde{p} \\
 C_{v,m} &= (C_{v,m})_{\text{ideales Gas}} - T \int_0^{V_m} \left(\frac{\partial^2 p}{\partial T^2} \right)_{p,\psi_j} d\tilde{V}
 \end{aligned}$$

24 Dinge die man eigentlich wissen sollte

$$1J = 1W = 1Nm$$

$$E_{kin} = \frac{1}{2}mc^2$$

$$E_{rot} = \frac{1}{2}I\omega^2$$

$$E_{Feder} = \frac{1}{2}kx^2$$

$$E_{pot} = mgz$$

$$E_{Kondensator} = \frac{1}{2}C \left(\frac{Q_e}{C} \right)^2$$

$$E_{Spule} = \frac{1}{2}LI^2$$

$$E_{Elektrisch} = UA$$

$$10^1 = 1$$

$$10^1 = 10$$

$$10^2 = 100$$

$$10^3 = 1000$$

$$10^4 = 10\,000$$

$$10^5 = 100\,000$$

$$10^6 = 1\,000\,000$$

$$10^7 = 10\,000\,000$$

$$10^8 = 100\,000\,000$$

$$10^9 = 1\,000\,000\,000$$

$$10^{10} = 10\,000\,000\,000$$

$$10^{11} = 100\,000\,000\,000$$

$$10^{12} = 1\,000\,000\,000\,000$$

$$10^{-1} = 0.1$$

$$10^{-2} = 0.01$$

$$10^{-4} = 0.001$$

$$10^{-4} = 0.000\,1$$

$$10^{-5} = 0.000\,01$$

$$10^{-6} = 0.000\,001$$

$$10^{-7} = 0.000\,000\,1$$

$$10^{-8} = 0.000\,000\,01$$

$$10^{-9} = 0.000\,000\,001$$

$$10^{-10} = 0.000\,000\,000\,1$$

$$10^{-11} = 0.000\,000\,000\,01$$

$$10^{-12} = 0.000\,000\,000\,001$$

$$m^2 \quad dm^2 \quad cm^2 \quad mm^2$$

$$1 \quad 10^2 \quad 10^4 \quad 10^6$$

$$10^{-2} \quad 1 \quad 10^2 \quad 10^4$$

$$10^{-4} \quad 10^{-2} \quad 1 \quad 10^2$$

$$10^{-6} \quad 10^{-4} \quad 10^2 \quad 1$$

$$m^3 \quad dm^3 \quad cm^3 \quad mm^3$$

$$1 \quad 10^3 \quad 10^6 \quad 10^9$$

$$10^{-3} \quad 1 \quad 10^3 \quad 10^6$$

$$10^{-6} \quad 10^{-3} \quad 1 \quad 10^3$$

$$10^{-9} \quad 10^{-6} \quad 10^3 \quad 1$$