Thermodynamik Formelsammlung

Jonas Walkling

February 20, 2020

Revision und konzeptionelle Unterstützung: Maximilian Goldapp Fatih Güzel

Contents

	N 114	•
	Nomenklatur	3
	Grundbegriffe	4
3	Basisformeln	4
4	Iso	4
5	Gibbs	4
6	Thermodynamische Beziehungen	4
7	Guggenheim	4
8	Maxwell	5
9	Ideales Gas	5
10	Van-der-Waals	5
11	Drosselung	5
12	Carnot	6
13	Gemische Idealer Gase	6
14	Nassdampf	6
15	Realer Stoff im Nassdampfgebiet	7
16	Maximale Arbeit und Exergie	7
17	Wärmekapazität	7
18	Technische Anwendung	8
19	Clausius-Rankine-Prozess	9
20	Kaltdampfprozess	9
21	Feuchte Luft	10
22	Eindimensionale Strömungsvorgänge	10
23	Chemische Reaktionen	10
24	Stoffwerte einiger Gase	14
25	Stoffdaten einiger Stoffe	14
26	Zahlenwerte feuchte Luft	14
27	Obskure Zusammenhänge	15
28	Dinge die man eigentlich wissen sollte	15

$$\frac{d}{dt}\left\{U+m\left(\frac{c^2}{2}+gz\right)\right\} = \sum_{j}\left[\dot{m}_{j}\left(h+\frac{c^2}{2}+gz\right)_{i}\right] + \sum_{l}\left(\dot{Q}_{l}\right)_{l} + \sum_{i}\left(\dot{W}_{l}\right)_{i} - p\frac{dV}{dt}$$

1 Nomenklatur

 $\mathbf{An} = \text{Anergie}[\mathbf{J}]$

 $c_s = Schallgeschwindigkeit[m/s]$

 $\mathbf{c_p} = \text{Spezifische Wärmekapazität dp} = 0 [J/kg*K]$

 $\mathbf{c_v} = \text{Spezifische Wärmekapazität dv} = 0 [J/kg*K]$

 $\mathbf{E} = \text{Energie}[\mathbf{J}]$

 $\mathbf{E}\mathbf{x} = -\mathbf{W}_{\mathbf{e}\mathbf{x}} = \mathrm{Exergie}[\mathbf{J}]$

 $\mathbf{F} = Kraft[N]$

 $\mathbf{F} = \mathbf{U} - \mathbf{TS} = \text{Freie Energie}[\mathbf{J}]$

 $\mathbf{f} = \mathbf{u} - \mathbf{T}\mathbf{s} = \text{Spezifische freie Energie}[J/kg]$

 $\mathbf{f} = \text{Fugazität}[Pa]$

G = H - TS = Freie Enthalpie[J]

 $\mathbf{g} = \mathbf{h} - \mathbf{T}\mathbf{s} = \text{Spezifische freie Enthalpie}[J/kg]$

 $\mathbf{g} = \text{Erdbeschleunigung}[\text{m/s}^2]$

 $\mathbf{H} = \mathbf{U} + \mathbf{pV} = \text{Enthalpie}[\mathbf{J}]$

 $\mathbf{h} = \mathbf{u} + \mathbf{p}\mathbf{v} = \text{Spezifische Enthalpie}[J/kg]$

 Δ **Hg** = Molare Reaktionsenthalpie

K = Konstante des Massenwirkungsgesetztes[-]

 $\mathbf{M} = \text{Molmasse[kg/mol]}$

 $\dot{\mathbf{m}} = \text{Massenstrom}[\text{kg/s}]$

 $\mathbf{m}' = \text{Masse in der flüssigen Phase[kg]}$

 $\mathbf{m}'' = \text{Masse in der gasförmigen Phase[kg]}$

 $Ma = c/c_s = Machzahl[-]$

 $\mathbf{n} = \mathbf{m}/\mathbf{M} = \text{Molzahl[mol]}$

n = Polytropenexponent[-]

 $\mathbf{P_t} = \text{technische Leistung}[\mathbf{W}]$

 $\mathbf{Q} = \text{W\"{a}rme}[J]$

 $\dot{\mathbf{Q}} = \text{Wärmestrom}[\mathbf{W}]$

q = Spezifische Wärme[J/kg]

 $\mathbf{r} = \text{Spezifische Verdampfungsenthalpie}[J/kg]$

 $\mathbf{R} = \text{Gaskonstante}[J/(\text{kg K})]$

 $\mathbf{R}_{\mathbf{m}} = \text{Universelle Gaskonstante}[J/(\text{mol } K)]$

S = Entropie[J/K]

s = Spezifische Entropie[J/(kg K)]

T = Temperatur[K]

 $\mathbf{t} = \text{Zeit}[s]$

 $\mathbf{t} = \text{Temperatur}[^{\circ}\text{C}]$

T = Sättigungstemperatur[K]

U = Innere Energie[J]

 $\mathbf{u} = \text{Spezifische innere Energie [J/kg]}$

 $V = Volumen[m^3]$

 $\mathbf{v} = \text{Spezifisches Volumen}[\text{m}^3/\text{kg}]$

 $V_m = Molares Volumen[m³/mol]$

 $\mathbf{W} = \text{Arbeit}[J]$

 $\mathbf{w} = \text{Spezifische Arbeit}[J/kg]$

 $\mathbf{W}_{\mathbf{V}} = \text{Volumen}$ änderungsarbeit[J]

 $\mathbf{W_{el}} = \text{Elektrische Arbeit[J]}$

 $\mathbf{W}_{\mathbf{w}} = \text{Wellenarbeit}[\mathbf{J}]$

 $W_{diss} = Dissipations arbeit[J]$

 $\mathbf{W_t} = \text{Technische Arbeit}[\mathbf{J}]$

 $\mathbf{W}_{Virrev} = \text{Arbeits verlust durch Irreversibilität}[J]$

 $\mathbf{x} = \frac{m''}{m' + m''} = \text{Dampfanteil[-]}$

 $\mathbf{x} = \frac{m_{H_2O}}{m_L} = \text{Wassergehalt}$

 $\mathbf{Z} = \text{Allgemeine extensive Zustandsgrößen}[\mathbf{Z}]$

z = Allgemeine

 β = Isobarer Ausdehnungskoeffizient[1/K]

 γ = Isochorer Spannungskoeffizient[1/K]

 $\delta_{\rm T} = {\rm Isothermer\ Drosselkoeffizient[m^3/kg]}$

 $\delta_{\mathbf{h}} = \text{Isenthalper Drosselkoeffizient}[\text{Ks}^2\text{m/kg}]$

 ε = Leistungsziffer[-]

 $\varepsilon = \text{Verdichtungsverhältnis}[-]$

 $\eta_{\rm th} = \text{Thermischer Wirkungsgrad}[-]$

 $\eta_{\text{mech}} = \text{Mechanischer Wirkungsgrad[-]}$

 $\kappa = \text{Adiabaten- oder Isentropenexponent}[-]$

 $\lambda = \text{Reaktionslaufzahl}[-]$

 μ_i = Chemisches Potential[J/mol]

 v_i = Stöchiometrische Koeffizienten[-]

 $\xi_{\mathbf{i}} = \text{Masseanteil}[-]$

 $\pi = \text{Druckverhältnis}[-]$

 $\rho = \text{Dichte}[\text{kg/m}^3]$

 $\tau =$ Temperaturverhältnis[-]

 ϕ = Relative Feuchte[-]

 $\phi = \text{Einspritzverhältnis}[-]$

 ξ = Isothermer Kompressibilitätskoeffizient[m²/N]

 $\Psi = \text{Dissipationsenergie}[J]$

 $\psi = \text{Spezifische Dissipationsenergie}[J]$

 ψ = Drucksteigerungsverhältnis[-]

 $\psi_i = Molanteil[-]$

2 Grundbegriffe

Systeme

- Abgeschlossenes System kein Stoff oder Energietransport
- Geschlossenes System kein Stofftransport
- Adiabates System kein Δq , aber Masse und Arbeit.
- Offenes System Stoff und Energietransport
- Stationäres System $\rightarrow \Delta U = 0$

Messgrößen

- Prozessgrößen sind wegabhängig (eg. Arbeit, Wärme)
- Zustandsgrößen sind wegunabhängig (eg. Volumen, Druck)
- Extensive Zustandsgrößen sind abhängig von der Masse des Systems (V, m, H, S, F, G, E)
- Intensive Zustandsgrößen sind unabhängig von der Masse des Systems (T, p)

Zustandsgleichungen

- Thermisch $\rightarrow f(p, V, T) = 0$
- Kalorisch $\rightarrow f(U, V, T) = 0$, U = U(V, T), u = u(v, T)

Hauptsätze

- 0: Temperatur existiert, ihre Gleichheit ist notwendige Voraussetzung für das thermische Gleichgewicht von zwei Systemen
- 1: Energie existiert, sie ist für abgeschlossene Systeme konstant.
- 2: Entropie existiert, sie wird bei allen irreversiblen Prozessen erzeugt. $dS = \frac{\delta Q_{rev}}{T}$
- 3: 0K existiert, bei dieser Temperatur ist die Entropie = 0

3 Basisformeln

$$H = U + pV$$

$$dS = \frac{Q_{rev}}{T} + S_{prod}$$

$$dS = \frac{\Phi}{T}$$

$$F = U - TS$$

$$G = H - ST$$

$$W = -\int p \, dV$$

$$dU = mc_v dT$$

$$m = \rho \cdot V$$

$$dS_{prod} = \frac{\Psi}{T}$$

$$\Psi = \int_{1}^{2} T \, dS_{prod}$$

$$W_{ir} = \frac{T_u}{T} \Psi$$

$$W_{V,ir} = T_U \cdot S_{prod}$$

$$p_1 = p_a + \frac{\varphi_1 - \varphi_a}{\varphi_b - \varphi_a} (p_b - p_a)$$

4 Iso



5 Gibbs

$$dU = TdS - pdV + \sum_{k=1}^{K} \mu_k dn_k$$

$$dG = -SdT + Vdp + \sum_{k=1}^{K} \mu_k dn_k$$

$$dH = TdS + Vdp + \sum_{k=1}^{K} \mu_k dn_k$$

$$dF = -SdT - pdV + \sum_{k=1}^{K} \mu_k dn_k$$

$$dU = \left(\frac{\partial U}{\partial S}\right)_V dS + \left(\frac{\partial U}{\partial V}\right)_S dV + \sum_{k=1}^{K} \left(\frac{\partial U}{\partial n_k}\right)_S dn_k$$

6 Thermodynamische Beziehungen

$$T = \left(\frac{\partial U}{\partial S}\right)_{V} = T(S, V) \qquad -S = \left(\frac{\partial F}{\partial T}\right)_{V} = S(T, V)$$

$$T = \left(\frac{\partial H}{\partial S}\right)_{p} = T(S, p) \qquad -S = \left(\frac{\partial G}{\partial T}\right)_{p} = S(T, p)$$

$$p = -\left(\frac{\partial U}{\partial V}\right)_{S} = p(V, S) \qquad V = \left(\frac{\partial G}{\partial p}\right)_{T} = V(p, T)$$

$$-p = \left(\frac{\partial F}{\partial V}\right)_{T} = p(T, V) \qquad \mu = \left(\frac{\partial U}{\partial n}\right)_{S, V} = \mu(S, V, n)$$

7 Guggenheim

$$\underbrace{\frac{d}{dt} \left\{ U + m \left(\frac{c^2}{2} + gz \right) \right\}}_{j} = \underbrace{\sum_{j} \left[\dot{m}_{j} \left(h + \frac{c^2}{2} + gz \right)_{j} \right]}_{Geschlossenes System -> 0} + \underbrace{\sum_{l} \left(\dot{Q}_{t} \right)_{l}}_{Kein Wärmestrom -> 0} + \underbrace{\sum_{l} \left(\dot{W}_{t} \right)_{i}}_{Keine Leistung -> 0} - \underbrace{\sum_{l} \left(\dot{W}_{t} \right)_{i}}_{Keine Volumenänderung -> 0} + \underbrace{\sum_{l} \left(\dot{W}_{t} \right)_{i}}_{Keine$$

8 Maxwell

$$\left(\frac{\partial T}{\partial p}\right)_{S,n_j} = \left(\frac{\partial V}{\partial S}\right)_{p,n_j} \\
\left(\frac{\partial S}{\partial V}\right)_{T,n_j} = \left(\frac{\partial p}{\partial T}\right)_{V,n_j} \\
\left(\frac{\partial S}{\partial p}\right)_{T,n_j} = -\left(\frac{\partial V}{\partial T}\right)_{p,n_j} \\
\left(\frac{\partial \mu_i}{\partial T}\right)_{p,n_j} = -\left(\frac{\partial S}{\partial n_i}\right)_{T,p,n_j \neq n_i} \\
\left(\frac{\partial \mu_i}{\partial p}\right)_{T,n_i} = \left(\frac{\partial V}{\partial n_i}\right)_{T,p,n_i \neq n_i} \\$$

9 Ideales Gas

pV = mRT

$$pv = RT$$

$$pV = nR_mT$$

$$\beta = \frac{1}{T}$$

$$\gamma = \frac{1}{T}$$

$$\chi = \frac{1}{p}$$

$$\beta = p\gamma\chi$$

$$R_m = 8,3143 \left[\frac{kJ}{kmolK} \right]$$

$$R = c_p - c_v$$

$$R = \frac{R_m}{M}$$

$$U - U_0 = mc_v(T - T_0) \quad \leftarrow \text{Für } c_p \text{ und } c_v \text{ const.}$$

$$s - s_0 = R \ln \left(\frac{v}{v_0} \right) \quad + c_v \ln \left(\frac{T}{T_0} \right)$$

$$= c_v \ln \left(\frac{p}{p_0} \right) \quad + c_p \ln \left(\frac{v}{v_0} \right)$$

$$= c_p \ln \left(\frac{T}{T_0} \right) \quad - R \ln \left(\frac{p}{p_0} \right)$$

$$\beta = \frac{1}{T} = \frac{1}{V} \left(\frac{\partial V}{\partial T} \right)_p = \frac{1}{V} \left(\frac{\partial v}{\partial T} \right)_p = -\frac{1}{\rho} \left(\frac{\partial \rho}{\partial T} \right)_p$$

$$\gamma = \frac{1}{T} = \frac{1}{p} \left(\frac{\partial P}{\partial T} \right)_V$$

$$\chi = \frac{1}{p} = -\frac{1}{V} \left(\frac{\partial V}{\partial p} \right)_T = -\frac{1}{V} \left(\frac{\partial V}{\partial p} \right)_T$$

$$u_2 - u_1 = \int_{T_1}^{T_2} c_v(T) dT$$

$$U_2 - U_1 = Q_{12} + W_{V,12}$$

10 Van-der-Waals

$$\left(p + \frac{a}{v^2}\right)(v - b) = RT$$

$$\left(\overline{p} + \frac{3}{\overline{v}^2}\right)(3\overline{v} - 1) = 8\overline{T}$$

$$\overline{p} = \frac{p}{p_K}, \quad \overline{v} = \frac{v}{v_K}, \quad \overline{T} = \frac{T}{T_K}$$

$$p_K = \frac{a}{27b^2}, \quad T_K = \frac{8}{27} \frac{a}{b} \frac{1}{R},$$

$$a = 3p_K v_K^2, \quad b = \frac{v_K}{3}, \quad \frac{p_K v_K}{RT_K} = \frac{3}{8}$$

$$\beta = \frac{(v - b)Rv^2}{RTv^3 - 2a(v - b)^2}$$

$$\gamma = \frac{Rv^2}{RTv^3 - 2a(v - b)^2}$$

$$\chi = \frac{(v - b)^2 v^2}{RTv^3 - 2a(v - b)^2}$$

$$du = \frac{a}{v^2} dv + c_v(T) dT$$

$$u - u_0 = \left(\frac{a}{v_0} - \frac{a}{v}\right) + \int_{T_0}^T c_v(\tilde{T}) d\tilde{T}$$

$$u - u_0 = \left(\frac{a}{v_0} - \frac{a}{v}\right) + c_v(T - T_0) \leftarrow \text{für } c_v = \text{const.}$$

$$c_p - c_v = \frac{Tv\beta^2}{\chi}$$

$$s - s_0 = c_v \ln\left(\frac{T}{T_0}\right) + R\ln\left(\frac{v - b}{v_0 - b}\right)$$

$$h_2 - h_1 = \frac{1}{2}(p_2 - p_1)\left(\frac{1}{\rho_1} + \frac{1}{\rho_2}\right)$$

11 Drosselung



$$h + \frac{c^2}{2} + gz = \text{const.}$$

$$dh = 0, \quad T_1 = T_2$$

$$\delta_h = \left(\frac{\partial T}{\partial p}\right)_h = -\frac{v}{c_p}(1 - \beta T)$$

$$\delta_T = \left(\frac{\partial h}{\partial p}\right)_T$$

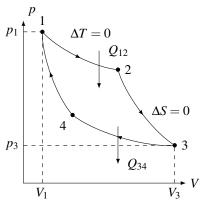
$$s_2 - s_1 = R \ln\left(\frac{v_2}{v_1}\right) = R \ln\left(\frac{p_1}{p_2}\right)$$

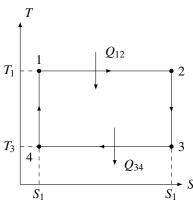
$$\mu_{J-T} = \left(\frac{\partial H}{\partial p}\right)_H \approx \frac{\frac{2a}{RT} - b}{c_{p,m}}$$

$$T_i = \frac{2a}{Rb}$$

12 Carnot

$$\begin{split} &\eta_{th} = 1 - \frac{-Q_{34}}{Q_{12}} = 1 - \frac{T_3(S_3 - S_4)}{T_1(S_2 - S_1)} = 1 - \frac{T_1}{T_3} \\ &\frac{Q_{12}}{T_1} + \frac{Q_{34}}{T_3} = 0 \\ &\Delta S_{ges} = -Q_{34} \left(\frac{1}{T_{KK}} - \frac{T_1}{T_3} \frac{1}{T_{HK}} \right) \end{split}$$



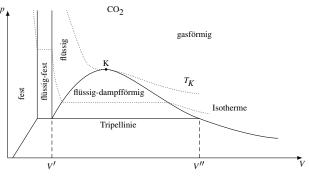


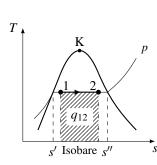
13 Gemische Idealer Gase

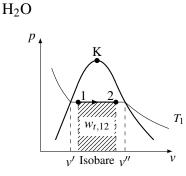
$$\begin{split} \xi_{i} &= \frac{m_{i}}{m}, \quad \psi_{i} = \frac{n_{i}}{n}, \quad p_{i} = \psi_{i}p \\ \xi_{i} &= \frac{M_{i}n_{i}}{\sum_{k=1}^{K} M_{k}n_{k}} = \frac{M_{i}}{M_{G}} \psi \\ p_{i}V &= m_{i}R_{i}T, \quad p_{i}V = n_{i}R_{m}T, \quad pV = mR_{G}T \\ \sum_{k=1}^{K} p_{k} &= p \\ R_{G} &= \frac{1}{m} \sum_{k=1}^{K} m_{k}R_{k} = \sum_{k=1}^{K} \xi_{k}R_{k} \\ U_{G} &= \sum_{k=1}^{K} U_{k} = \sum_{k=1}^{K} m_{k}u_{k} = \sum_{k=1}^{K} c_{vk}m_{k}T \leftarrow c_{v} = \text{const} \\ H_{G} &= \sum_{k=1}^{K} H_{k} = \sum_{k=1}^{K} m_{k}h_{k} = \sum_{k=1}^{K} c_{pk}m_{k}T \leftarrow c_{p} = \text{const.} \\ c_{vG} &= \sum_{k=1}^{K} c_{vk}\xi_{k}, \quad c_{pG} &= \sum_{k=1}^{K} c_{pk}\xi_{k} \\ S_{2} - S_{1} &= R_{m} \left(n \ln n - \sum_{k=1}^{K} n_{k} \ln n_{k} \right) \end{split}$$

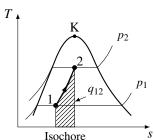
14 Nassdampf

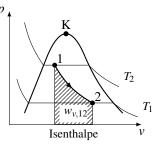












$$\frac{dp}{dT} = \frac{s'' - s'}{v'' - v'} = \frac{1}{T} \frac{h'' - h'}{v'' - v'}$$

$$r = h'' - h' = T(s'' - s')$$

$$v = (1 - x)v' + xv''$$

$$v = v' + (v'' - v')x$$

$$u = (1 - x)u' + xu''$$

$$u = u' + (u'' - u')x$$

$$h = (1 - x)h' + xh''$$

$$h = h' + (h'' - h')x$$

$$s = (1 - x)s' + xs''$$

$$s = s' + (s'' - s')x$$

$$\frac{dp}{dT} = \frac{1}{T} \frac{r}{v'' - v'}$$

$$F = K + 2 - P$$

$$T' = T''$$

$$p' = p''$$

$$g' = g''$$

$$dg' = v'dp' - s'dT'$$

$$dg'' = v''dp'' - s''dT''$$

$$dg' = dg''$$

15 Realer Stoff im Nassdampfgebiet

Isobare Zustandsänderung

$$q_{12} = T(s_2 - s_1)$$

$$= T(s'' - s')(x_2 - x_1)$$

$$w_{V,12} = -\int_1^2 p \, dv$$

$$= -p(v_2 - v_1) = -p(v'' - v')(x_2 - x_1)$$

Isochore Zustandsänderung

$$q_{12} = u_2 - u_1 = u'_2 + x_2 \left(u''_2 - u'_2 \right) - u'_1 - x_1 \left(u''_1 - u'_1 \right)$$

Adiabate Zustandsänderung

$$w_{V,12} = u_2 - u_1 = u'_2 + x_2 \left(u''_2 - u'_2 \right) - u'_1 - x_1 \left(u''_1 - u'_1 \right)$$

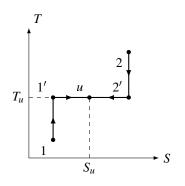
EntropieÄnderung während des Mischvorgangs

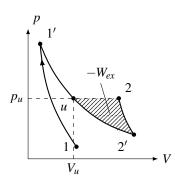
$$S_2 - S_1 = R_m \left(n \ln n - \sum_i n_i \ln n_i \right)$$

16 Maximale Arbeit und Exergie

Maximal nutzbare Arbeit → isentrop, reibungsfrei

 $1 \rightarrow 1'$: isentrop auf T_u $1' \rightarrow u$: isotherm auf u





$$-\dot{W}_{ex} = -(\dot{W}_{t})_{rev} = -\frac{d}{dt} \left(U + m \left(\frac{c^{2}}{2} + gz \right) + p_{u}V - T_{u}S \right) + \sum_{j=1}^{K} \left(\dot{m}_{j} \left(h + \frac{c^{2}}{2} + gz - T_{s} \right) \right) + \sum_{l=1}^{K} \left(1 - \frac{T_{u}}{T} \right) \dot{Q}_{l}$$

Die Exergie der Enthalpie (offenes, stationäres System)

$$-\dot{W}_{ex,1u} = \dot{m}(h_1 - h_u - T_u(s_1 - s_u))$$

Die Exergie der inneren Energie (geschlossenes, instationäres System)

$$\begin{split} -\dot{W}_{ex} &= -\frac{d}{dt}(U + p_u V - T_u S) \\ -\dot{W}_{ex,1u} &= U_1 - U_u - p_u (V_1 - V_u) - T_u (S_1 - S_u) \\ -\dot{W}_{ex,1u} &= H_1 - (p_1 - p_u) V_1 - H_u - T_u (S_1 - S_u) \end{split}$$

Für Ideales Gas

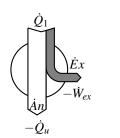
$$\begin{split} -W_{ex} &= mc_v(T_1 - T_u) + p_u(V_1 - V_u) - T_u m \left(c_p \ln\left(\frac{T_1}{T_u}\right) - R_i \ln\left(\frac{p_1}{p_u}\right)\right) \\ -W_{ex} &= m \left[c_p(T_1 - T_u) - T_u c_p \ln\left(\frac{T_1}{T_u}\right)\right] \leftarrow \text{isobar} \end{split}$$

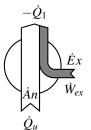
Dampf/Luftdruckkammer

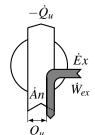
$$-W_{ex,1u} = m_1[u_1 - u_u + p_u(v_1 - v_u) - T_u(s_1 - s_u)]$$

Die Exergie der Wärme (geschlossenes, stationäres System)

$$-\dot{W}_{ex} = \left(1 - \frac{T_u}{T_1}\right)\dot{Q}_1 = \eta_{th,C}\dot{Q}_1$$



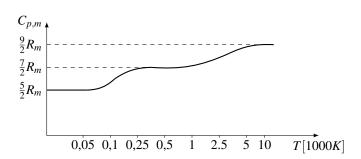




WärmekraftprozessWärmepumpenprozess

Kälteprozess

17 Wärmekapazität



$$C_{v,m} = \frac{1}{\kappa - 1} R_m \qquad C_{p,m} = \frac{\kappa}{\kappa - 1} r_m$$

$$c_v = \frac{1}{\kappa - 1} R_j \qquad c_p = \frac{\kappa}{\kappa - 1} R_j$$

$$\kappa = \frac{c_p}{c_v} \qquad R = c_p - c_v$$

$$R = \frac{R_m}{M} \qquad R_m = 8,3143 \left[\frac{kJ}{kmolK} \right]$$

$$C_{m,v} = rac{f}{2}R_m = rac{f_{trans} + f_{rot} + f_{vib}}{2}R_m$$
 $C_{m,p} = rac{f+2}{2}R$
 $\kappa = rac{f+2}{f}$

 $f_{trans} = 3$ (für die 3 Translatorischen Freiheitsgrade) $f_{rot} \in \{0,2,3\}$ {Einatomig, Linear, Verzweigt} $f_{vib} = 2 \cdot l$, l = 1 Normalschwingungen der Atomkerne (Kann für komplexere Moleküle auch > 1 sein.)

18 Technische Anwendung

adiabat
$$(c_p = const.)$$
 $W_{t,12} = mc_p(T_2 - T_1) = \frac{\kappa}{\kappa - 1}(p_2V_2 - p_1V_1)$ $Q_{12} = 0$

reversibel adiabat $\kappa = const.$ $W_{t,12} = \frac{\kappa}{\kappa - 1}(p_1V_1) \left[\left(\frac{p_2}{p_1} \right)^{\frac{\kappa - 1}{\kappa}} - 1 \right]$ $Q_{12} = 0$

irreversibel adiabat als Polytrope $n > \kappa; n, \kappa = const.$ $W_{t,12} = \frac{\kappa}{\kappa - 1}(p_1V_1) \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right]$ $Q_{12} = 0$

reversibel polytrop $n, \kappa = const.$ $W_{t,12} = \frac{n}{n-1}(p_2V_2 - p_1V_1)$ $Q_{12} = mc_n(T_2 - T_1)$

$$= \frac{n}{n-1}mR(T_2 - T_1)$$

$$= \frac{n-\kappa}{(n-1)(\kappa-1)}(p_1V_1) \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right]$$

$$= \frac{n}{n-1}(p_1V_1) \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right]$$
 $c_n = \frac{n-\kappa}{n-1}cv$

isotherm $W_{t,12} = (p_1V_1) \ln \left(\frac{p_2}{p_2} \right)$ $Q_{12} = -W_{t,12}$

Thermischer Wirkungsgrad
$$\eta_{th} = \frac{-w}{q_{zu}} = \frac{\text{Nutzen}}{\text{Aufwand}} = 1 - \frac{|q_{ab}|}{q_{zu}}$$

Isentroper Verdichterwirkungsgrad $\eta_{tV} = \frac{w_{t,12,rev}}{w_{t,12}} = \frac{h_{2,rev} - h_1}{h_2 - h_1} = \frac{T_{2,rev} - T_1}{T_2 - T_1}$

idealer Fall

Isentroper Turbinenwirkungsgrad $\eta_{tT} = \frac{w_{t,12}}{w_{t,12,rev}} = \frac{h_1 - h_2}{h_1 - h_{2,rev}} = \frac{T_1 - T_2}{T_1 - T_{2,rev}}$

Dampfkraftprozess Wirkungsgrad $\eta_{th} = 1 - \frac{|q_{61}|}{q_{23} + q_{34} + q_{45}} = 1 - \frac{h_6 - h_1}{h_5 - h_2}$

Leistungszahl Kältemaschine $\varepsilon_{K(A)} = \frac{q_{0u}}{w} = \frac{\dot{Q}_0}{\dot{W}}$

Leistungszahl Kaltdampfprozess $\varepsilon_{K} = \frac{q_0}{|q| - q_0} = \frac{q_0}{w_t} = \frac{\dot{h}_1 - h_6}{\dot{h}_2 - h_1}$

Linkslaufender Carnotprozess $\varepsilon_{WP} = \frac{q}{|q| - q_0} = \frac{|q_1|}{w_t} = \frac{q_{2u}}{w} = \frac{\dot{h}_2 - h_5}{\dot{h}_2 - h_1} = 1 + \varepsilon_{K(A)}$

Kälteleistung Wärmepumpe $\dot{Q}_0 = \dot{m}(h_2 - h_5)$

Leistungszahl Kaltluftprozess $\dot{Q}_0 = \dot{m}(h_1 - h_6)$

Arbeit der Enthalpie $W_t = Q = mdh = mcpdT$

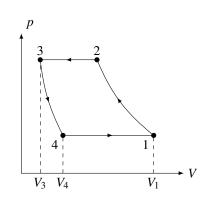
Verdichtungsverhältnis $\psi = p_3/p_2$

Einspritzverhältnis $\psi = v_1/v_2$

Temperaturverhältnis $\tau = q_2/p_1$

Für Joule-Prozess $\pi_{opt} = \tau^{Y_{1N-1}}$

Kolbenverdichter



V1 = Maximales Zylindervolumen

V2 = Volumen nach Verdichtung

V3 =

V4 = Schädlicher Raum

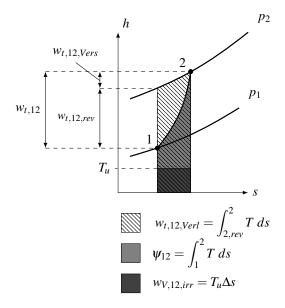
$$\mu = \frac{V_1 - V_4}{V_1 - V_3}, \qquad \varepsilon_S = \frac{V_3}{V_1 - V_3}$$

$$\mu = 1 - \varepsilon_S \left[\left(\frac{p_2}{p_1} \right)^{\frac{1}{n}} - 1 \right]$$

$$= \underbrace{p_2 V_2}_{Ausschiebearbeit} - \underbrace{p_1 V_1}_{Einschiebearbeit} - \int_1^2 p \, dV$$

$$= \frac{n}{n - 1} p_1 (V_1 - V_4) \left[\left(\frac{p_2}{p_1} \right)^{\frac{n - 1}{n}} - \right]$$

Turboverdichter



Verdichter Wirkungsgrad

$$\eta_{sV} = \frac{w_{t,12,rev}}{w_{t,12}} = \frac{h_{2,rev} - h_1}{h_2 - h_1}$$

Verdichter wirkungsgrad (Ideales Gas, $c_p = \text{const.}$)

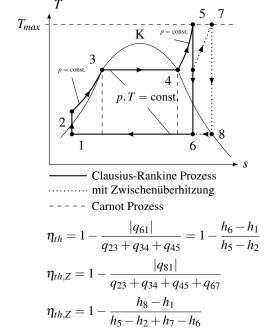
$$\eta_{sV} = \frac{T_{2,rev} - T_1}{T_2 - T_1}$$

Technische Verlustarbeit

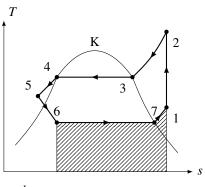
$$w_{t,Verl,12} = w_{t,12} - w_{t,12,rev} = h_2 - h_{2,rev}$$

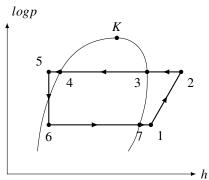
= $\int_{2,rev}^{2} T|_{p_2 = const.} ds$

19 Clausius-Rankine-Prozess



20 Kaltdampfprozess





21 Feuchte Luft

$$x = \frac{m_{H_2O}}{m_L}$$

$$x = x_{D(ampf)} + x_{W(asser)} + x_{E(is)}$$

$$\varphi = \frac{p_D}{p_s}$$

$$x_D = \frac{m_d}{m_L} = \frac{R_L}{R_D} \frac{p_D}{p_L} = \frac{R_L}{R_D} \frac{p_D}{p - p_D} = 0.622 \frac{p_D}{p - p_D}$$

$$x_s = \frac{m_{D,max}}{m_L} = 0.622 \frac{p_s}{p - p_s} \rightarrow \text{für } \varphi = 1$$

$$p_s = \frac{x_s \cdot p}{0.622 + x_s}$$

$$x_s(t_{min}) = \frac{M_{H_2O}}{M_L} \frac{p_s^{min}(t_{min})}{p_1 - p_s^{min}(t_{min})}$$

$$p = p_L + p_D$$

$$\rho = \frac{p}{R_{gesT}} = \frac{1 + x}{R_L + xR_D} \frac{p}{T}$$

$$R_{ges} = \frac{R_L + xR_D}{1 + x}$$

$$h = c_{pL}t + x_D(c_{pD}t + r_D) + x_W c_W t + x_E(c_E t - r_E)$$

$$\frac{M_L}{M_{H_2O}} = \frac{18,02}{18,02} \frac{kg/kmol}{kg/kmol}$$

$$\frac{M_{H_2O}}{R_L} = \frac{1,0287}{0.287} \frac{kJ/(kg K)}{kJ/(kg K)}$$

$$\frac{c_{pD}}{c_{pD}} = \frac{1,92}{1,92} \frac{kJ/(kg K)}{kJ/(kg K)}$$

$$\frac{c_{pD}}{c_E} = \frac{1,92}{2,11} \frac{kJ/(kg K)}{kJ/(kg K)}$$

$$\frac{c_E}{c_E} = \frac{2,11}{2,11} \frac{kJ/(kg K)}{kJ/(kg K)}$$

$$\frac{kJ/(kg K)}{kJ/(kg K)}$$

$$\frac{c_E}{c_E} = \frac{2,11}{3,34} \frac{kJ/(kg K)}{kJ/(kg K)}$$

22 Eindimensionale Strömungsvorgänge

$$\begin{split} \chi &= \frac{1}{p} = -\frac{1}{V} \left(\frac{\partial V}{\partial p} \right)_T \\ c_S^2 &= \left(\frac{\partial p}{\partial \rho} \right)_S \\ c_S^2 &= \left(\frac{R}{c_v} + 1 \right) \left(v^2 \frac{RT}{(v - b)^2} \right) - \frac{2a}{v} \leftarrow V dW \\ c_S^2 &= \kappa RT \leftarrow i deal \\ Ma &= \frac{c}{c_S} \\ \frac{T_0}{T} &= 1 + \frac{\kappa - 1}{2} \frac{c^2}{\kappa RT} = 1 + \frac{\kappa - 1}{2} M a^2 \\ \frac{p_0}{p} &= \left(\frac{T_0}{T} \right)^{\frac{\kappa}{\kappa - 1}} = \left(1 + \frac{\kappa - 1}{2} M a^2 \right)^{\frac{\kappa}{\kappa - 1}} \\ \frac{\rho_0}{\rho} &= \left(\frac{T_0}{T} \right)^{\frac{\kappa - 1}{\kappa}} = \left(1 + \frac{\kappa - 1}{2} M a^2 \right)^{\frac{\kappa - 1}{\kappa}} \\ \left(\frac{A}{A^*} \right)^2 &= \frac{1}{M a^2} \left[\frac{2}{\kappa + 1} \left(1 + \frac{\kappa - 1}{2} M a^2 \right) \right]^{\frac{\kappa + 1}{\kappa - 1}} \end{split}$$

$$h_2 - h_1 = \frac{1}{2}(p_2 - p_1)\left(\frac{1}{\rho_1} + \frac{1}{\rho_2}\right) = (p_2 - p_1)\frac{1}{2}(v_1 + v_2)$$

Stoßbeziehungen für ein ideales Gas

$$\begin{split} \frac{p_2}{p_1} &= \frac{2\kappa Ma^2 - (\kappa - 1)}{\kappa + 1} \\ \frac{\rho_2}{\rho_1} &= \frac{(\kappa + 1)Ma^2}{2 + (\kappa - 1)Ma^2} \\ \frac{T_2}{T_1} &= \frac{\left[2\kappa Ma^2 - (\kappa - 1)\left[2 + (\kappa - 1)Ma^2\right]}{(\kappa + 1)^2}Ma^2 \\ Ma_2^2 &= \frac{(\kappa - 1)(Ma_1^2 - 1) + (\kappa + 1)}{2\kappa(Ma_1^2 - 1) + (\kappa + 1)} \end{split}$$

Entropie über den senkrechten Verdichtungsstoß

$$\begin{aligned} s_2 - s_1 &= c_v \ln \left(\frac{T_2}{T_1}\right) + R \ln \left(\frac{v_2}{v_1}\right) \\ &= c_p \ln \left(\frac{T_2}{T_1}\right) + R \ln \left(\frac{p_2}{p_1}\right) \end{aligned}$$

23 Chemische Reaktionen

$$\frac{dn_1}{v_1} = \frac{dn_2}{v_2} = \dots = d\lambda = .const$$

$$\sum_{k=1}^K \mu_k dn_k = \sum_{k=1}^K \mu_k (v_k d\lambda) = \sum_{k=1}^K \mu_k v_k = 0$$

$$\mu_i = \left(\frac{\partial U}{\partial n_i}\right)_{S,V} = \left(\frac{\partial H}{\partial n_i}\right)_{S,p} = \left(\frac{\partial F}{\partial n_i}\right)_{T,V} = \left(\frac{\partial G}{\partial n_i}\right)_{T,p}$$

$$\mu(p,T) = \mu(p^+,T) + R_m T \ln\left(\frac{p}{p^+}\right)$$

Massenwirkungsgesetz

$$\prod_{k=1}^{K} \psi_k^{\nu_k} = exp - \frac{1}{R_m T} \sum_{k=1}^{K} \nu_k \mu_{0k}(p, T)$$
$$= exp - \frac{1}{R_m T} \sum_{k=1}^{K} \nu_k G_{m,k}(p, T)$$

Gleichgewichtkonstante

$$K(p,T) = \prod_{k=1}^{K} \psi_k^{\nu_k}$$

$$K(p_2,T) = K(p_1,T) \left(\frac{p_1}{p_2}\right)^{\sum \nu_k}$$

$$\ln\left(\frac{K(p,T_2)}{K(p,T_1)}\right) = \frac{\Delta H_R}{R_m} \left(\frac{1}{T_1} - \frac{1}{T_2}\right) = \frac{\Delta H_R}{R_m} \frac{T_2 - T_1}{T_1 T_2}$$

$$\Delta H_R = \sum_{k=1}^{K} \nu_k H_{m,k}$$



Ideales Gas

Isobare $ \begin{array}{c} p \\ -\\ -\\ -\\ -\\ p = p_1 \end{array} $ $ \begin{array}{c} \frac{v_1}{v_2} = \frac{T_1}{T_2} \\ -\\ p = p_1 \end{array} $ $ \begin{array}{c} p = p_1 \\ p = p_1 \end{array} $ $ \begin{array}{c} v = \frac{v_1}{T_1}T \\ = c_p(T_2 - T_1) \end{array} $ $ = -p_1(v_2 - v_1) $	1	Isotherm T $p_1v_1 = p_2v_2$ $p = \frac{p_1v_1}{v}$ $p = \frac{p_1v_1}{v}$ $T = T_1$ $T = T_1$ $= p_1v_1 \ln \frac{p_1}{p_2}$ $= -q_{12}$
V_2 – $\left(\frac{T_2}{T_1}\right)$	$=-p_1(r)$ $=c_p \ln \left(\frac{r}{r}\right)$	$\left(\frac{p_1}{p_2}\right)$

Van-Der-Waals-Gas

Isentrop	$\delta = 0$	$(p_1 + rac{a}{v^2})(v_1 - b)rac{c_V + R}{c_V} = (p + rac{a}{v^2})(v_2 - b)rac{c_V + R}{c_V}, \ T_1(v_1 - b)^{R/c_V} = T_2(v_2 - b)^{R/c_V}$	$p = -\frac{a}{v^2} + \left(p_1 + \frac{a}{v^2}\right) \left(\frac{v_1 - b}{v_m}\right)^{\frac{v_V + R}{R}}$	$p=-rac{a}{v^2}+(p_1+rac{a}{v^2})\left(rac{T}{T_1} ight)^{rac{c_V+R}{R}}$	$T = T_1 \left(rac{v_1 - b}{v - b} ight)^{rac{R}{c_V}}$	0 =	$= \frac{a}{v_1} - \frac{a}{v_2} + c_v (T_2 - T_1)$	0 =
Isochore	^	$\frac{p_1 + \frac{a_2}{v_1^2}}{T_1} = \frac{p_2 + \frac{a_2}{v_1^2}}{T_2}$	$\nu = \nu_1$	$p = \frac{T}{T_1}(p_1 + \frac{a}{v^2}) - \frac{a}{v_1^2}$	$\nu = \nu_1$		0 =	$=c_{ u}\ln\left(rac{T_{2}}{T_{1}} ight)$
Isobare	d	$rac{RT_1}{v_1 - b} - rac{a}{v_1^2} = rac{RT_2}{v - b} - rac{a}{v_2^2}$	$p = p_1$	$p = p_1$	$T = T_1 \frac{v - b}{v_1 - b} + \frac{a}{R}(v - b) \left(\frac{1}{v^2} - \frac{1}{v_1^2}\right)$	$= \left = rac{a}{ u_1} - rac{a}{ u_2} + c_ u (T_2 - T_1) + p_1 (u_2 - u_1) \right = c_ u (T_2 - T_1)$	$=-p_1(\nu_2-\nu_1)$	$=c_{\scriptscriptstyle V} \ln \left(rac{T_2}{T_1} ight) + R \ln \left(rac{ u_2 - b}{ u_1 - b} ight)$
Isotherme	T	$(p_1 + rac{a}{v^2})(v_1 - b) = (p_2 + rac{a}{v^2})(v_2 - b)$	$p=(p+rac{a}{v^2})rac{v_u}{v-b}-rac{a}{v^2}$	$T=T_1$	$T=T_1$	$=RT_1\ln\left(rac{ u_2-b}{ u_1-b} ight)$	$= -RT_1 \ln \left(\frac{v_2 - b}{v_1 - b} \right) + \frac{a}{v_1} - \frac{a}{v_2}$	$S_2 - S_1 = R \ln \left(\frac{v_2 - b}{v_1 - b} \right)$
	const.		p, ν	p,T	ν, T	<i>q</i> 12	WV,12	$S_2 - S_1$

24 Stoffwerte einiger Gase

Bezeichnung	Symbol	Molmasse	Gaskonstante	Dichte	c_p	c_v	κ
		[kg/kmol]	[J/(kg K)]	$[kg/m^3]$	[J/(kg K)]	[J/(kg,K)]	
Acetylen	C_2H_2	26.038	319.3	1.16	1616	1278	1.26
Ammoniak	NH_3	17.031	488.2	0.76	2056	1526	1.35
Argon	Ar	39.948	208.1	1.76	519	309	1.68
Äthan	C_2H_6	30.070	276.5	1.34	1650	1355	1.22
Butan	C_4H_10	58.124	143.0	2.67	1599	1410	1.13
Chlor	C_l2	56.108	117.3	3.17	473	343	1.38
Chlorwasserstoff	HCl	70.906	228.0	1.62	795	556	1.43
Helium	He	4.003	2077.0	0.18	5200	3124	1.66
Kohlendioxid	CO_2	44.010	188.9	1.95	816	618	1.32
Kohlenmonoxid	CO	28.010	296.8	1.23	1038	739	1.40
Luft	_	28.964	287.1	1.28	1006	718	1.40
Methan	CH_4	16.043	518.3	0.71	2165	1638	1.32
Propan	C_3H_8	44.097	188.5	1.99	1549	1331	1.16
Sauerstoff	O_2	31.999	259.8	1.41	909	647	1.40
Stickstoff	N_2	28.013	296.8	1.23	1038	739	1.40
Wasserstoff	H_2	2.016	4124.2	0.09	14050	9926	1.42
Xenon	Xe	131.300	63.3	5.82	159	93	1.71
Ideales Gas			8.3143				

25 Stoffdaten einiger Stoffe

Name	chemische	Molmasse	Normal-	kritische	kritischer
Name	Formel	[kg/kmol]	Siedepunkt [°C]	Temperatur [°C]	Druck [MPa]
Wasserstoff	H_2	2.02	-252.9	-240.0	1.32
Helium	He	4.00	-268.9	-268.0	0.23
Ammoniak	NH_3	17.03	-33.3	132.3	11.33
Wasser	H_2O	18.02	100.0	373.9	22.06
	78%				
Luft	$N_221\%$	28.96	-194.2	-140.4	3.84
	$O_2.1\%Ar.+$				
Kohlendioxid	CO_2	44.01	-78.4	31.0	7.38
Methan	CH_4	16.04	-161.5	-82.6	4.60
Äthan	C_2H_6	30.07	-88.6	32.2	4.87
Propan	C_3H_8	44.10	-42.1	96.7	4.25
R134a	CH_2FCF_3	102.03	-26.1	101.1	4.06

26 Zahlenwerte feuchte Luft

Bezeichnung	Formelzeichen	Zahlenwert	Dimension
Molmasse der Luft	ML	28,96	kg/ kmol
Molmasse des Wassers	MH2O	18,02	kg/ kmol
spezifische Gaskonstante der Luft	RL	0,287	kJ/ (kg K)
spezifische Gaskonstante des Dampfes	RD	0,461	kJ/ (kg K)
spezifische Wärmekapazität der Luft	cpL	1,006	kJ/ (kg K)
spezifische Wärmekapazität des Dampfes	cpD	1,92	kJ/ (kg K)
spezifische Wärmekapazität des Wassers	cW	4,182	kJ/ (kg K)
spezifische Wärmekapazität des Eises	cЕ	2,1	kJ/ (kg K)
Verdampfungsenthalpie des Wassers bei 0 °C	rD	2500	kJ/ kg
Schmelzenthalpie des Eises bei 0 °C	rE	334	kJ/ kg

27 Obskure Zusammenhänge

$$dV = \left(\frac{\partial V}{\partial T}\right)_{p} dT + \left(\frac{\partial V}{\partial p}\right)_{T,n} + \sum_{k=1}^{K} \left(\frac{\partial V}{\partial n_{k}}\right)_{T,p} dkn_{k}$$

$$dS = \left(\frac{nC_{p,m}}{T}\right) dT - \left(\frac{\partial V}{\partial T}\right)_{p,n} dp + \sum_{k=1}^{K} \left(\frac{\partial \mu_{k}}{\partial T}\right)_{p,n} dn_{k}$$

$$dU = \left[nC_{p,m} - p\left(\frac{\partial V}{\partial T}\right)_{p,n}\right] dT - \left[p\left(\frac{\partial V}{\partial p}\right)_{T,n} + T\left(\frac{\partial V}{\partial T}\right)_{p,n}\right] dp + \sum_{k=1}^{K} \left[\mu_{k} - T\left(\frac{\partial \mu_{k}}{\partial T}\right)_{p,n} - p\left(\frac{\partial V}{\partial n_{k}}\right)_{T,p,n}\right] dn_{k}$$

$$dH = nC_{p,m}dT + \left[VT\left(\frac{\partial V}{\partial T}\right)_{p,n}\right] + \sum_{k=1}^{K} \left[\mu_{k} - T\left(\frac{\partial \mu_{k}}{\partial T}\right)_{p,n}\right] dn_{k}$$

$$dF = -\left[S + p\left(\frac{\partial V}{\partial T}\right)_{p,n}\right] dT - p\left(\frac{\partial V}{\partial p}\right)_{T,n} dp + \sum_{k=1}^{K} \left[\mu_{k} - p\left(\frac{\partial V}{\partial n_{k}}\right)_{T,p}\right] dn_{k}$$

$$\left(\frac{\partial C_{p,m}}{\partial p}\right)_{T,\psi_{j}} = T\frac{\partial}{\partial p} \left[\left(\frac{\partial S_{m}}{\partial T}\right)_{p,\psi_{j}}\right]_{T,\psi_{j}} = T\frac{\partial}{\partial T} \left[\left(\frac{\partial S_{m}}{\partial p}\right)_{T,\psi_{j}}\right]_{p,\psi_{j}} = -T\frac{\partial}{\partial T} \left[\left(\frac{\partial V_{m}}{\partial T}\right)_{p,\psi_{j}}\right]_{p,\psi_{j}} = -T\left(\frac{\partial^{2}V_{m}}{\partial T^{2}}\right)_{p,\psi_{j}}$$

$$C_{p,m} = (C_{p,m})_{\text{ideales Gas}} - T\int_{0}^{V_{m}} \left(\frac{\partial^{2}V_{m}}{\partial T^{2}}\right)_{p,\psi_{j}} d\tilde{p}$$

$$C_{v,m} = (C_{v,m})_{\text{ideales Gas}} - T\int_{0}^{V_{m}} \left(\frac{\partial^{2}P}{\partial T^{2}}\right)_{p,\psi_{j}} d\tilde{V}$$

1J = 1W = 1Nm

28 Dinge die man eigentlich wissen sollte

 $n = \frac{m}{M} = \text{Teichenanzahl} = \frac{\text{Masse}}{\text{Mol}}$

$$E_{kin} = \frac{1}{2}mc^2$$

$$E_{rot} = \frac{1}{2}I\omega^2$$

$$E_{rot} = \frac{1}{2}L\omega^2$$

$$E_{rot} = \frac{1}{2}kx^2$$

$$10^2 = 100 \qquad 10^{-1} = 0.1$$

$$E_{Feder} = \frac{1}{2}kx^2$$

$$E_{pot} = mgz$$

$$10^3 = 1000 \qquad 10^{-4} = 0.001$$

$$E_{pot} = mgz$$

$$10^5 = 100\,000 \qquad 10^{-4} = 0.000\,1$$

$$E_{Spule} = \frac{1}{2}LI^2$$

$$10^6 = 1000\,000 \qquad 10^{-5} = 0.000\,001$$

$$E_{Spule} = \frac{1}{2}LI^2$$

$$E_{Elektrisch} = UA$$

$$10^7 = 10\,000\,000 \qquad 10^{-7} = 0.000\,000\,1$$

$$E_{Elektrisch} = UA$$

$$10^9 = 1000\,000\,000 \qquad 10^{-9} = 0.000\,000\,001$$

$$10^9 = 1000\,000\,000 \qquad 10^{-10} = 0.000\,000\,001$$

$$m^2 \qquad dm^2 \qquad cm^2 \qquad mm^2$$

$$10^1 = 10^2 \qquad 10^4 \qquad 10^6$$

$$dm^2 \qquad 10^{-2} \qquad 1 \qquad 10^2 \qquad 10^4$$

$$10^1 = 100\,000\,000\,000 \qquad 10^{-11} = 0.000\,000\,000\,01$$

$$dm^2 \qquad 10^{-2} \qquad 1 \qquad 10^2 \qquad 10^4$$

$$cm^2 \qquad 10^{-4} \qquad 10^{-2} \qquad 1 \qquad 10^2$$

$$mm^2 \qquad 10^{-6} \qquad 10^{-4} \qquad 10^2 \qquad 1$$

$$m^3 \qquad dm^3 \qquad cm^3 \qquad mm^3$$

$$m^3 \qquad 1 \qquad 10^3 \qquad 10^6 \qquad 10^9$$

$$dm^3 \qquad 10^{-3} \qquad 1 \qquad 10^3 \qquad 10^6$$

$$cm^3 \qquad 10^{-6} \qquad 10^{-3} \qquad 1 \qquad 10^3$$

$$mm^3 \qquad 10^{-9} \qquad 10^{-6} \qquad 10^{-3} \qquad 1 \qquad 10^3$$