Thermodynamik Formelsammlung

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January 19, 2020

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Contents

$$\frac{d}{dt}\left\{U+m\left(\frac{c^2}{2}+gz\right)\right\}=\sum_j\left[\dot{m}_j\left(h+\frac{c^2}{2}+gz\right)_j\right]+\sum_l\left(\dot{Q}_t\right)_l+\sum_i\left(\dot{W}_t\right)_i-p\frac{dV}{dt}$$

1 Nomenklatur

$\mathbf{An}$ = Anergie[J]

$\mathbf{c_s}$ = Schallgeschwindigkeit[m/s]

$\mathbf{c_p}$ = Spezifische Wärmekapazität dp = 0 [J/kg*K]

$\mathbf{c_v}$ = Spezifische Wärmekapazität dv = 0 [J/kg*K]

$\mathbf{E}$ = Energie[J]

$\mathbf{Ex}=-\mathbf{W_{ex}}$ = Exergie[J]

$\mathbf{F}$ = Kraft[N]

$\mathbf{F}=\mathbf{U}-\mathbf{TS}$ = Freie Energie[J]

$\mathbf{f}=\mathbf{u}-\mathbf{Ts}$ = Spezifische freie Energie[J/kg]

$\mathbf{f}$ = Fugazität[Pa]

$\mathbf{G}=\mathbf{H}-\mathbf{TS}$ = Freie Enthalpie[J]

$\mathbf{g}=\mathbf{h}-\mathbf{Ts}$ = Spezifische freie Enthalpie[J/kg]

$\mathbf{g}$ = Erdbeschleunigung[m/s²]

$\mathbf{H}=\mathbf{U}+\mathbf{pV}$ = Enthalpie[J]

$\mathbf{h}=\mathbf{u}+\mathbf{pv}$ = Spezifische Enthalpie[J/kg]

$\blacksquare\mathbf{Hg}$ = Molare Reaktionsenthalpie

$\mathbf{K}$ = Konstante des Massenwirkungsgesetztes[-]

$\mathbf{M}$ = Molmasse[kg/mol]

$\dot{\mathbf{m}}$ = Massestrom[kg/s]

$\mathbf{m'}$ = Masse in der flüssigen Phase[kg]

$\mathbf{m''}$ = Masse in der gasförmigen Phase[kg]

$\mathbf{Ma}=\mathbf{c}/\mathbf{c_s}$ = Machzahl[-]

$\mathbf{n}=\mathbf{m}/\mathbf{M}$ = Molzahl[mol]

$\mathbf{n}$ = Polytropenexponent[-]

$\mathbf{P_t}$ = technische Leistung[W]

$\mathbf{Q}$ = Wärme[J]

$\dot{\mathbf{Q}}$ = Wärmestrom[W]

$\mathbf{q}$ = Spezifische Wärme[J/kg]

$\mathbf{r}$ = Spezifische Verdampfungsenthalpie[J/kg]

$\mathbf{R}$ = Gaskonstante[J/(kg K)]

$\mathbf{R_m}$ = Universelle Gaskonstante[J/(mol K)]

$\mathbf{S}$ = Entropie[J/K]

$\mathbf{s}$ = Spezifische Entropie[J/(kg K)]

$\mathbf{T}$ = Temperatur[K]

$\mathbf{t}$ = Zeit[s]

$\mathbf{t}$ = Temperatur[°C]

$\mathbf{T}$ = Sättigungstemperatur[K]

$\mathbf{U}$ = Innere Energie[J]

$\mathbf{u}$ = Spezifische innere Energie [J/kg]

$\mathbf{V}$ = Volumen[m³]

$\mathbf{v}$ = Spezifisches Volumen[m³/kg]

$\mathbf{V_m}$ = Molares Volumen[m³/mol]

$\mathbf{W}$ = Arbeit[J]

$\mathbf{w}$ = Spezifische Arbeit[J/kg]

$\mathbf{W_V}$ = Volumenänderungsarbeit[J]

$\mathbf{W_{el}}$ = Elektrische Arbeit[J]

$\mathbf{W_w}$ = Wellenarbeit[J]

$\mathbf{W_{diss}}$ = Dissipationsarbeit[J]

$\mathbf{W_t}$ = Technische Arbeit[J]

$\mathbf{W_{Virrev}}$ = Arbeitsverlust durch Irreversibilität[J]

$\mathbf{x}=\dfrac{m''}{m'+m''}$ = Dampfanteil[-]

$\mathbf{x}=\dfrac{m_{H_2O}}{m_L}$ = Wassergehalt

$\mathbf{Z}$ = Allgemeine extensive Zustandsgrößen[Z]

$\mathbf{z}$ = Allgemeine

β = Isobarer Ausdehnungskoeffizient[1/K]

γ = Isochorer Spannungskoeffizeint[1/K]

$\delta_{\mathbf{T}}$ = Isothermer Drosselkoeffizient[m³/kg]

$\delta_{\mathbf{h}}$ = Isenthalper Drosselkoeffizient[Ks²m/kg]

ε = Leistungsziffer[-]

ε = Verdichtungsverhältnis[-]

$\eta_{\mathbf{th}}$ = Thermischer Wirkungsgrad[-]

$\eta_{\mathbf{mech}}$ = Mechanischer Wirkungsgrad[-]

κ = Adiabaten- oder Isentropenexponent[-]

λ = Reaktionslaufzahl[-]

$\mu_{\mathbf{i}}$ = Chemisches Potential[J/mol]

$\nu_{\mathbf{i}}$ = Stöchiometrische Koeffizienten[-]

$\xi_{\mathbf{i}}$ = Masseanteil[-]

π = Druckverhältnis[-]

ρ = Dichte[kg/m³]

τ = Temperaturverhältnis[-]

ϕ = Relative Feuchte[-]

ϕ = Einspritzverhältnis[-]

ξ = Isothermer Kompressibilitätskoeffizient[m²/N]

$\blacksquare$ = Dissipationsenergie[J]

ψ = Spezifische Dissipationsenergie[J]

ψ = Drucksteigerungsverhältnis[-]

$\psi_{\mathbf{i}}$ = Molanteil[-]

2 Grundbegriffe

Systeme

- Abgeschlossenes System - kein Stoff oder Energietransport

- Geschlossenes System - kein Stofftransport

- Adiabates System - kein Δq , aber Masse und Arbeit.

- Offenes System - Stoff und Energietransport

- Stationäres System $\rightarrow \Delta U = 0$

Messgrößen

- Prozessgrößen sind Wegabhängig (eg. Arbeit, Wärme)

- Zustandsgrößen sind Wegunabhängig (eg. Volumen, Druck)

- Extensive Zustandsgrößen sind abhängig von der Masse des Systems (V, m, H, S, F, G, E)

- Intensive Zustandsgrößen sind unabhängig von der Masse des Systems (T, p)

Zustandsgleichungen

- Thermisch $\rightarrow f(p,V,T) = 0$

- Kalorisch $\rightarrow f(U,V,T) = 0, \quad U = U(V,T), \quad u = u(v,T)$

Hauptsätze

0: Temperatur existiert, ihre gleichheit ist notwendige Vorraussetzung für das thermische Gleichgewicht.

1: Energie existiert, sie ist für abgeschlossene Systeme konstant.

2: Entropie existiert, sie wird bei allen irreversiblen Prozessen erzeugt. $dS = \frac{\delta Q_{rev}}{T}$

3: $0K$ exisitert, bei dieser Temperatur ist die Entropie $= 0$

3 Basisformeln

$$H = U + pV$$

$$dS = \frac{\delta Q_{rev}}{T}$$

$$F = U - TS$$

$$G = H - ST$$

$$W = -\int p\, dV$$

$$dS = \frac{Q_{rev}}{T} + S_{prod}$$

$$dS_{prod} = \frac{\Psi}{T}$$

$$\Psi = \int_1^2 T\, dS_{prod}$$

$$W_{ir} = \frac{T_u}{T}\Psi$$

$$p_1 = p_a + \frac{\varphi_1 - \varphi_a}{\varphi_b - \varphi_a}(p_b - p_a)$$

4 Gibbs

$$dU = T ds - p dV + \sum_{k=1}^{K} \mu_k dn_k$$

$$dG = -S dT + V dp + \sum_{k=1}^{K} \mu_k dn_k$$

$$dH = T dS + V dp + \sum_{k=1}^{K} \mu_k dn_k$$

$$dF = -S dT - p dV + \sum_{k=1}^{K} \mu_k dn_k$$

$$dU = \left(\frac{\partial U}{\partial S}\right)_V dS + \left(\frac{\partial U}{\partial V}\right)_S dV + \sum_{k=1}^{K} \left(\frac{\partial U}{\partial n_k}\right)_S dn_k$$

5 Thermodynamische Beziehungen

$$T = \left(\frac{\partial U}{\partial S}\right)_V = T(S,V) \qquad -S = \left(\frac{\partial F}{\partial T}\right)_V = S(T,V)$$

$$T = \left(\frac{\partial H}{\partial S}\right)_p = T(S,p) \qquad -S = \left(\frac{\partial G}{\partial T}\right)_p = S(T,p)$$

$$p = -\left(\frac{\partial U}{\partial V}\right)_S = p(V,S) \qquad V = \left(\frac{\partial G}{\partial p}\right)_T = V(p,T)$$

$$-p = \left(\frac{\partial F}{\partial V}\right)_T = p(T,V) \qquad \mu = \left(\frac{\partial U}{\partial n}\right)_{S,V} = \mu(S,V,n)$$

6 Guggenheim

-S	U	V	U	= U(S,V)
H		F	H	= H(S,p)
-p	G	T	F	= F(T,V)
			G	= G(T,p)

$$\overbrace{\frac{d}{dt}\left\{ U + m\left(\frac{c^2}{2} + gz\right)\right\}}^{\text{Stationäres System -> 0}} = \overbrace{\sum_j \left[\dot{m}_j\left(h + \frac{c^2}{2} + gz\right)_j\right]}^{\text{Geschlossenes System -> 0}} + \overbrace{\sum_l \left(\dot{Q}_t\right)_l}^{\text{Kein Wärmestrom -> 0}} + \overbrace{\sum_i \left(\dot{W}_t\right)_i}^{\text{Keine Leistung -> 0}} - \overbrace{p\frac{dV}{dt}}^{\text{Keine Volumenänderung -> 0}}$$

7 Maxwell

$$\left(\frac{\partial T}{\partial p}\right)_{S,n_j} = \left(\frac{\partial V}{\partial S}\right)_{p,n_j}$$

$$\left(\frac{\partial S}{\partial V}\right)_{T,n_j} = \left(\frac{\partial p}{\partial T}\right)_{V,n_j}$$

$$\left(\frac{\partial S}{\partial p}\right)_{T,n_j} = -\left(\frac{\partial V}{\partial T}\right)_{p,n_j}$$

$$\left(\frac{\partial \mu_i}{\partial T}\right)_{p,n_j} = -\left(\frac{\partial S}{\partial n_i}\right)_{T,p,n_j \neq n_i}$$

$$\left(\frac{\partial \mu_i}{\partial p}\right)_{T,n_j} = \left(\frac{\partial V}{\partial n_i}\right)_{T,p,n_j \neq n_i}$$

8 Ideales Gas

$$pV = mRT$$
$$pv = RT$$
$$pV = nR_mT$$
$$\beta = \frac{1}{T}$$
$$\gamma = \frac{1}{T}$$
$$\chi = \frac{1}{p}$$
$$\beta = p\gamma\chi$$
$$R_m = 8{,}3143 \left[\frac{kJ}{kmolK}\right]$$
$$R = c_p - c_v$$
$$R = \frac{R_m}{M}$$
$$U - U_0 = mc_v(T - T_0)$$
$$H - H_0 = mc_p(T - T_0) \quad \leftarrow \text{Für } c_p \text{ und } c_v \text{ const.}$$

$$\begin{aligned}
s - s_0 &= R\ln\left(\frac{v}{v_0}\right) + c_v\ln\left(\frac{T}{T_0}\right)\\
&= c_v\ln\left(\frac{p}{p_0}\right) + c_p\ln\left(\frac{v}{v_0}\right)\\
&= c_p\ln\left(\frac{T}{T_0}\right) - R\ln\left(\frac{p}{p_0}\right)
\end{aligned}$$

$$\beta = \frac{1}{T} = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_p = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_p$$

$$\gamma = \frac{1}{T} = \frac{1}{p}\left(\frac{\partial p}{\partial T}\right)_V$$

$$\chi = \frac{1}{p} = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_T$$

$$u_2 - u_1 = \int_{T_1}^{T_2} c_v(T)\,dT$$

9 Van-der-Waals

$$\left(p + \frac{a}{v^2}\right)(v - b) = RT$$

$$\left(\overline{p} + \frac{3}{\overline{v}^2}\right)(3\overline{v} - 1) = 8\overline{T}$$

$$\overline{p} = \frac{p}{p_K}, \quad \overline{v} = \frac{v}{v_K}, \quad \overline{T} = \frac{T}{T_K}$$

$$p_K = \frac{a}{27b^2}, \quad T_K = \frac{8}{27}\frac{a}{b}\frac{1}{R}, \quad \text{☞}$$

$$a = 3p_Kv_K^2, \quad b = \frac{v_K}{3}, \quad \frac{p_Kv_K}{RT_K} = \frac{3}{8}$$

$$\beta = \frac{(v - b)Rv^2}{RTv^3 - 2a(v - b)^2}$$

$$\gamma = \frac{Rv^2}{RTv^2 - a(v - b)}$$

$$\chi = \frac{(v - b)^2v^2}{RTv^3 - 2a(v - b)^2}$$

$$du = \frac{a}{v^2}dv + c_v(T)dT$$

$$u - u_0 = \left(\frac{a}{v_0} - \frac{a}{v}\right) + \int_{T_0}^{T} c_v(\tilde{T})\,d\tilde{T}$$

$$u - u_0 = \left(\frac{a}{v_0} - \frac{a}{v}\right) + c_v(T - T_0) \leftarrow \text{für } c_v = \text{const.}$$

$$c_p - c_v = \frac{Tv\beta^2}{\chi}$$

$$s - s_0 = c_v\ln\left(\frac{T}{T_0}\right) + R\ln\left(\frac{v - b}{v_0 - b}\right)$$
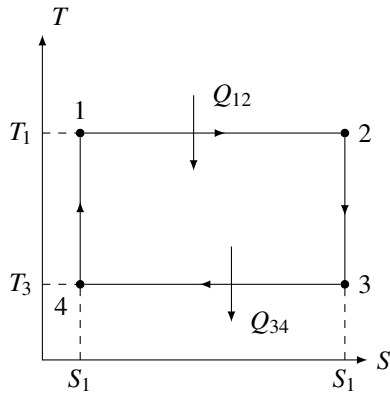
$$h_2 - h_1 = \frac{1}{2}(p_2 - p_1)\left(\frac{1}{\rho_1} + \frac{1}{\rho_2}\right)$$

10 Carnot

$$\eta_{th} = 1 - \frac{-Q_{34}}{Q_{12}} = 1 - \frac{T_3(S_3 - S_4)}{T_1(S_2 - S_1)} = 1 - \frac{T_3}{T_1}$$

$$\frac{Q_{12}}{T_1} + \frac{Q_{34}}{T_3} = 0$$

$$\Delta S_{ges} = -Q_{34}\left(\frac{1}{T_{KK}} - \frac{T_1}{T_3}\frac{1}{T_{HK}}\right)$$
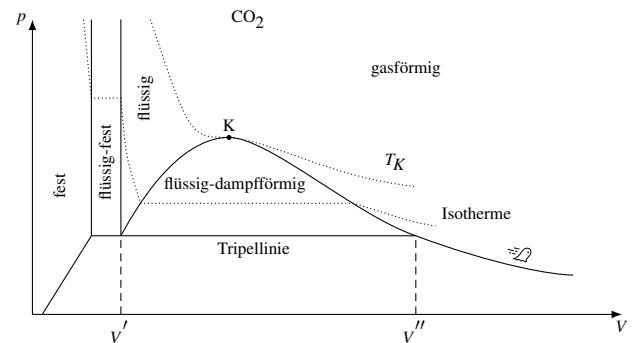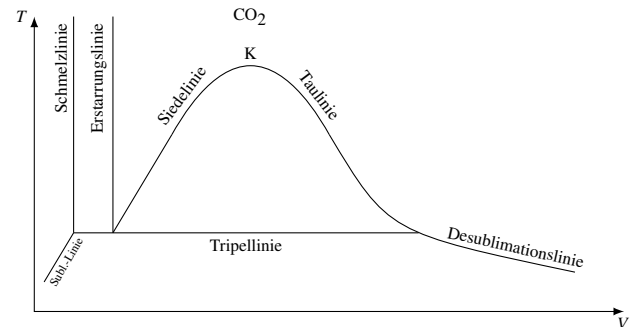
Adiabate Drosselung (ideal):

$$h + \frac{c^2}{2} + gz = \text{const.}$$

$$dh = 0$$

Adiabet Drosselung (real):

$$\delta_h = \left(\frac{\partial T}{\partial p}\right)_h = -\frac{v}{c_p}(1 - \beta T)$$

12 Nassdampf

11 Gemische Idealer Gase

$$\xi_i = \frac{m_i}{m}, \quad \psi_i = \frac{n_i}{n}, \quad p_i = \psi_i p$$

$$\xi_i = \frac{M_i n_i}{\sum_{k=1}^{K} M_k n_k} = \frac{M_i}{M_G}\psi$$

$$p_i V = m_i R_i T, \quad p_i V = n_i R_m T, \quad pV = mR_G T$$

$$\sum_{k=1}^{K} p_k = p$$

$$R_G = \frac{1}{m}\sum_{k=1}^{K} m_k R_k = \sum_{k=1}^{K} \xi_k R_k$$

$$U_G = \sum_{k=1}^{K} U_k = \sum_{k=1}^{K} m_k u_k = \sum_{k=1}^{K} c_{vk} m_k T \leftarrow c_v = \text{const}$$

$$H_G = \sum_{k=1}^{K} H_k = \sum_{k=1}^{K} m_k h_k = \sum_{k=1}^{K} c_{pk} m_k T \leftarrow c_p = \text{const.}$$

$$c_{vG} = \sum_{k=1}^{K} c_{vk}\xi_k, \quad c_{pG} = \sum_{k=1}^{K} c_{pk}\xi_k$$

$$S_2 - S_1 = R_m\left(n\ln n - \sum_{k=1}^{K} n_k \ln n_k\right)$$

$$v = (1-x)v' + xv''$$
$$v = v' + (v'' - v')x$$

$$u = (1-x)u' + xu''$$
$$u = u' + (u'' - u')x$$

$$h = (1-x)h' + xh''$$
$$h = h' + (h'' - h')x$$

$$s = (1-x)s' + xs''$$
$$s = s' + (s'' - s')x$$

$$r = h'' - h' = T(s'' - s')$$

$$T' = T''$$
$$p' = p''$$
$$g' = g''$$
$$dg' = v'\,dp' - s'\,dT'$$
$$dg'' = v''\,dp'' - s''\,dT''$$
$$dg' = dg''$$
$$\frac{dp}{dT} = \frac{s'' - s'}{v'' - v'}$$
$$\frac{dp}{dT} = \frac{1}{T}\frac{h'' - h'}{v'' - v'}$$
$$\frac{dp}{dT} = \frac{1}{T}\frac{r}{v'' - v'}$$

13 Realer Stoff im Nassdampfgebiet

Isobare Zustandsänderung

$$q_{12} = T(s_2 - s_1)$$
$$= T\left(s'' - s'\right)(x_2 - x_1)$$
$$w_{V,12} = -\int_1^2 p \, dv$$
$$= -p(v_2 - v_1) = -p\left(v'' - v'\right)(x_2 - x_1)$$

Isochore Zustandsänderung

$$q_{12} = u_2 - u_1 = u_2' + x_2\left(u_2'' - u_2'\right) - u_1' - x_1\left(u_1'' - u_1'\right)$$

Adiabate Zustandsänderung

$$w_{V,12} = u_2 - u_1 = u_2' + x_2\left(u_2'' - u_2'\right) - u_1' - x_1\left(u_1'' - u_1'\right)$$

Entropieänderung wärend des Mischvorgangs

$$S_2 - S_2 = R_m\left(n \ln n - \sum_i n_i \ln n_i\right)$$

14 Maximale Arbeit und Exergie

Maxiaml nutzbare Arbeit → isentrop, reibungsfrei
$1 \to 1'$: isentrop auf T_u
$1' \to u$: isotherm auf u
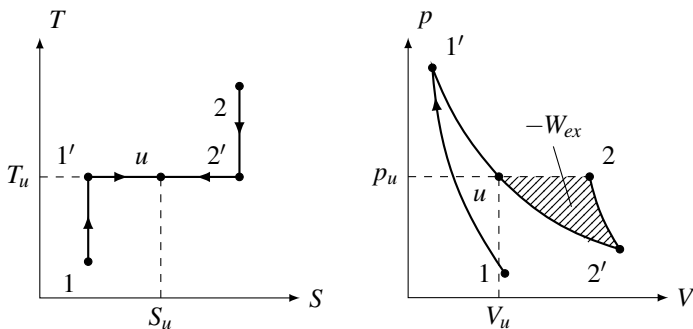
$$-\dot{W}_{ex} = -(\dot{W}_t)_{rev} = -\frac{d}{dt}\left(U + m\left(\frac{c^2}{2} + gz\right) + p_u V - T_u S\right)$$
$$+ \sum_{j=1}^K \left(\dot{m}_j\left(h + \frac{c^2}{2} + gz - T_s\right)\right) + \sum_{l=1}^K \left(1 - \frac{T_u}{T}\right)\dot{Q}_l$$

Die Exergie der Enthalpie (offens, stationäres System)

$$-\dot{W}_{ex,1u} = \dot{m}(h_1 - h_u - T_u(s_1 - s_u))$$

Die Exergie der inneren Energie (geschlossenes, instationäres System)

$$-\dot{W}_{ex} = -\frac{d}{dt}(U + p_u V - T_u S)$$
$$-\dot{W}_{ex,1u} = U_1 - U_u - p_u(V_1 - V_u) - T_u(S_1 - S_u)$$
$$-\dot{W}_{ex,1u} = H_1 - (p_1 - p_u)V_1 - H_u - T_u(S_1 - S_u)$$
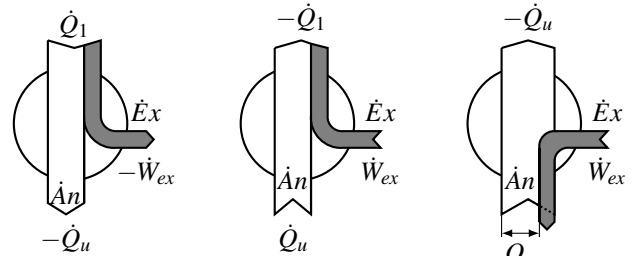
Für Ideales Gas

$$-W_{ex} = mc_v(T_1 - T_u) + p_u(V_1 - V_u) - T_u m\left(c_p \ln\left(\frac{T_1}{T_u}\right) - R_i \ln\left(\frac{p_1}{p_u}\right)\right)$$

Dampf/Luftdruckkammer

$$-W_{ex,1u} = m_1[u_1 - u_u + p_u(v_1 - v_u) - T_u(s_1 - s_u)]$$

Die Exergie der Wärme (geschlossenes, stationäres System)

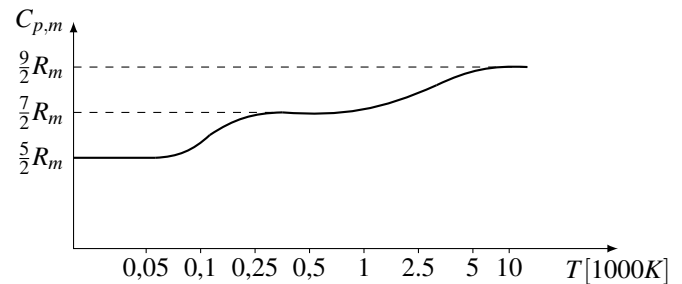$$-\dot{W}_{ex} = \left(1 - \frac{T_u}{T_1}\right)\dot{Q}_1 = \eta_{th,C}\dot{Q}_1$$

Wärmekraftprozess Wärmepumpenprozess Kälteprozess

15 Wärmekapazität

$$C_{v,m} = \frac{1}{\kappa - 1}R_m \qquad C_{p,m} = \frac{\kappa}{\kappa - 1}r_m$$
$$c_v = \frac{1}{\kappa - 1}R_j \qquad c_p = \frac{\kappa}{\kappa - 1}R_j$$
$$\kappa = \frac{c_p}{c_v} \qquad R = c_p - c_v$$
$$R = \frac{R_m}{M} \qquad R_m = 8,3143\left[\frac{kJ}{kmolK}\right]$$

$$C_{v,m} = \underbrace{3 + \frac{R_m}{2}}_{\text{Translatorisch}} + \underbrace{\frac{n_{rot}R_m}{2}}_{\text{Rotatorisch}} + \underbrace{R_M(3n_{\text{Atome}} - 3 - n_{rot})}_{\text{Vibratorisch}}$$
$$+ \underbrace{C_{v,m,Elektronenanregung}}_{\text{Relevant ab: } T \approx 10^4 K}$$

16 Technische Anwendung

adiabat $(c_p = const.)$	$W_{t,12} = mc_p(T_2 - T_1) = \dfrac{\kappa}{\kappa-1}(p_2V_2 - p_1V_1)$	$Q_{12} = 0$
reversibel adiabat $\kappa = const.$	$W_{t,12} = \dfrac{\kappa}{\kappa-1}(p_1V_1)\left[\left(\dfrac{p_2}{p_1}\right)^{\frac{\kappa-1}{\kappa}} - 1\right]$	$Q_{12} = 0$
irreversibel adiabat als Polytrope $n > \kappa; n, \kappa = const.$	$W_{t,12} = \dfrac{\kappa}{\kappa-1}(p_1V_1)\left[\left(\dfrac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$	$Q_{12} = 0$
reversibel polytrop $n, \kappa = const.$	$W_{t,12} = \frac{n}{n-1}(p_2V_2 - p_1V_1)$	$Q_{12} = mc_n(T_2 - T_1)$
	$= \frac{n}{n-1}mR(T_2 - T_1)$	$= \frac{n-\kappa}{(n-1)(\kappa-1)}(p_1V_1)\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}-1}\right]$
	$= \frac{n}{n-1}(p_1V_1)\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$	$c_n = \frac{n-\kappa}{n-1}cv$
isotherm	$W_{t,12} = (p_1V_1)\ln\left(\frac{p_2}{p_2}\right)$	$Q_{12} = -W_{t,12}$

Thermischer Wirkungsgrad	η_{th}	$= \dfrac{-w}{q_{zu}} = \dfrac{\text{Nutzen}}{\text{Aufwand}}$	$= 1 - \dfrac{|q_{ab}|}{q_{zu}}$
Isentroper Verdichterwirkungsgrad	η_{sV}	$= \dfrac{w_{t,12,rev}}{w_{t,12}}$	$= \dfrac{h_{2,rev} - h_1}{h_2 - h_1}$
Isentroper Turbinenwirkungsgrad	η_{sT}	$= \dfrac{w_{t,12}}{W_{t,12,rev}}$	$= \dfrac{h_1 - h_2}{h_1 - h_{2,rev}}$
Dampfkraftprozess Wirkungsgrad	n_{th}	$= 1 - \dfrac{|q_{61}|}{q_{23} + q34 + q_{45}}$	$= 1 - \dfrac{h_6 - h_1}{h_5 - h_2}$
Leistungszahl Kältemaschine	$\varepsilon_{K(A)}$	$= \dfrac{q_{zu}}{w} = \dfrac{\dot{Q}_0}{\dot{W}}$	
Leistungszahl Kaltluftprozess	ε_K	$= \dfrac{1}{\left(\frac{p}{p_0}\right)^{\frac{\kappa-1}{\kappa}} - 1}$	
Leistungszahl Kaltdampfprozess	ε_K	$= \dfrac{q_0}{|q| - q_0} = \dfrac{q_o}{w_t}$	$= \dfrac{h_1 - h_6}{h_2 - h_1}$
Linkslaufender Carnotprozess	ε_{carnot}	$= \dfrac{T_k}{T_H - T_K}$	
Leistungszahl Wärmepumpe	ε_{WP}	$= \dfrac{q}{|q| - q_0} = \dfrac{|q|}{w_t} = \dfrac{q_{zu}}{w}$	$= \dfrac{h_2 - h_5}{h_2 - h_1} = 1 + \varepsilon_{K(A)}$
Verdichtungsverhältnis	ε	$= \dfrac{v_1}{v_2}$	
Drucksteigerungsverhältniss	ψ	$= \dfrac{p_3}{p_2}$	
Einspriztverhältniss	φ	$= \dfrac{v_4}{v_3}$	
Temperaturverhältnis	τ	$= \dfrac{T_3}{T_1}$	
Verdrichtungsdruckverhältnis	$\pi =$	$= \dfrac{p_2}{p_1}$	
für Joule-Prozess	π_{opt}	$= \tau^{\frac{\kappa}{2(\kappa-1)}}$	
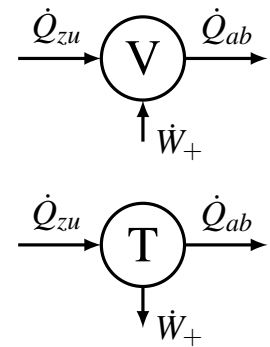
Kolbenverdichter
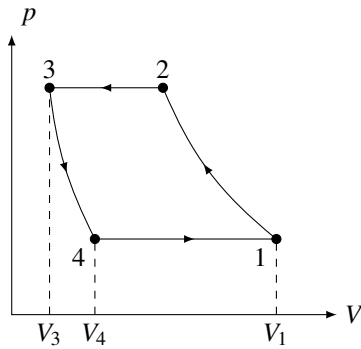
$V1$ = Maximales Zylindervolumen

$V2$ = Volumen nach Verdichtung
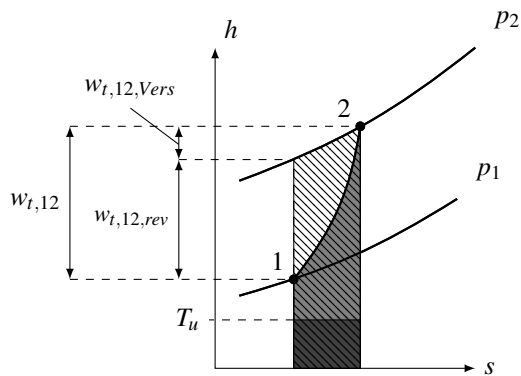
$V3$ =

$V4$ = Schädlicher Raum

$$\mu = \frac{V_1 - V_4}{V_1 - V3}, \qquad \varepsilon_S = \frac{V_3}{V_1 - V_3}$$

$$\mu = 1 - \varepsilon_S \left[\left(\frac{p_2}{p_1} \right)^{\frac{1}{n}} - 1 \right]$$

$$W_{t,12} = \int_1^2 V \, dp$$

$$= \underbrace{p_2 V_2}_{Ausschiebearbeit} - \underbrace{p_1 V_1}_{Einschiebearbeit} - \int_1^2 p \, dV$$

$$= \frac{n}{n-1} p_1 (V_1 - V_4) \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - \right]$$

Turboverdichter

$$w_{t,12,Verl} = \int_{2,rev}^2 T \, ds$$

$$\psi_{12} = \int_1^2 T \, ds$$

$$w_{V,12,irr} = T_u \Delta s$$

Verdichter Wirkungsgrad

$$\eta_{sV} = \frac{w_{t,12,rev}}{w_{t,12}} = \frac{h_{2,rev} - h_1}{h_2 - h_1}$$

Verdichter wirkungsgrad (Ideales Gas, c_p = const.)

$$\eta_{sV} = \frac{T_{2,rev} - T_1}{T_2 - T_1}$$

Technische Verlustarbeit

$$w_{t,Verl,12} = w_{t,12} - w_{t,12,rev} = h_2 - h_{2,rev}$$

$$= \int_{2,rev}^2 T|_{p_2=const.} \, ds$$

17 Eindimensionale Strömungsvorgänge

$$\chi = \frac{1}{p} = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T$$

$$c_S^2 = \left(\frac{\partial p}{\partial \rho}\right)_S$$

$$c_S^2 = \left(\frac{R}{c_v}+1\right)\left(v^2\frac{RT}{(v-b)^2}\right)-\frac{2a}{v} \leftarrow VdW$$

$$c_S^2 = \kappa RT \leftarrow ideal$$

$$Ma = \frac{c}{c_S}$$

$$\frac{T_0}{T} = 1+\frac{\kappa-1}{2}\frac{c^2}{\kappa RT} = 1+\frac{\kappa-1}{2}Ma^2$$

$$\frac{p_0}{p} = \left(\frac{T_0}{T}\right)^{\frac{\kappa}{\kappa-1}} = \left(1+\frac{\kappa-1}{2}Ma^2\right)^{\frac{\kappa}{\kappa-1}}$$

$$\frac{\rho_0}{\rho} = \left(\frac{T_0}{T}\right)^{\frac{\kappa-1}{\kappa}} = \left(1+\frac{\kappa-1}{2}Ma^2\right)^{\frac{\kappa-1}{\kappa}}$$

$$\left(\frac{A}{A^*}\right)^2 = \frac{1}{Ma^2}\left[\frac{2}{\kappa+1}\left(1+\frac{\kappa-1}{2}Ma^2\right)\right]^{\frac{\kappa+1}{\kappa-1}}$$

$$h_2-h_1 = \frac{1}{2}(p_2-p_1)\left(\frac{1}{\rho_1}+\frac{1}{\rho_2}\right) = (p_2-p_1)\frac{1}{2}(v_1+v_2)$$

Stoßbeziehungen für ein ideales Gas

$$\frac{p_2}{p_1} = \frac{2\kappa Ma^2 - (\kappa-1)}{\kappa+1}$$

$$\frac{\rho_2}{\rho_1} = \frac{(\kappa+1)Ma^2}{2+(\kappa-1)Ma^2}$$

$$\frac{T_2}{T_1} = \frac{[2\kappa Ma^2-(\kappa-1]\,[2+(\kappa-1)Ma^2]}{(\kappa+1)^2}Ma^2$$

$$Ma_2^2 = \frac{(\kappa-1)(Ma_1^2-1)+(\kappa+1)}{2\kappa(Ma_1^2-1)+(\kappa+1)}$$

Entropie über den senkrechten Verdichtungsstoß

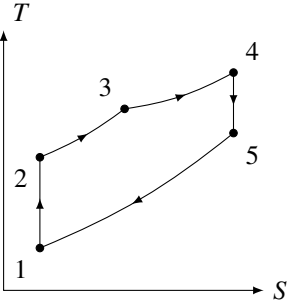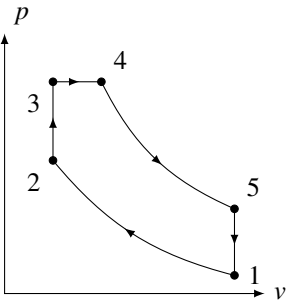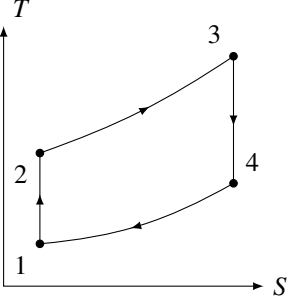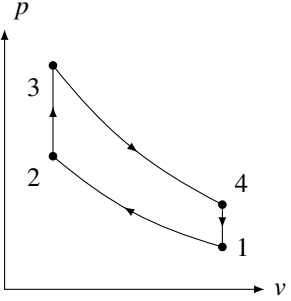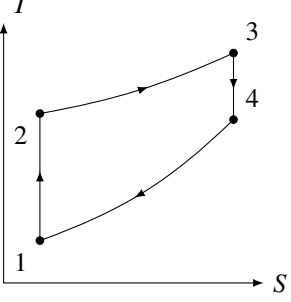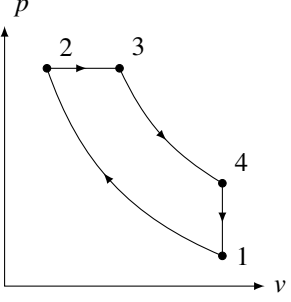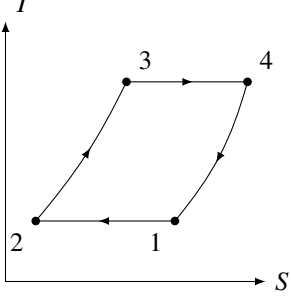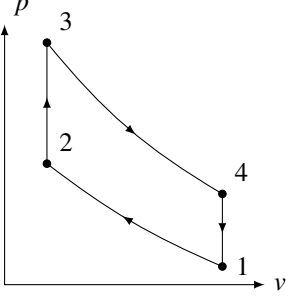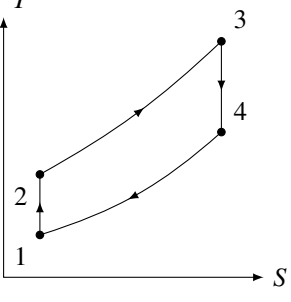$$s_2-s_1 = c_v\ln\left(\frac{T_2}{T_1}\right)+R\ln\left(\frac{v_2}{v_1}\right)$$

$$= c_p\ln\left(\frac{T_2}{T_1}\right)+R\ln\left(\frac{p_2}{p_1}\right)$$

18 Feuchte Luft

$$x = \frac{m_{H_2O}}{m_L}$$

$$x = x_{D(ampf)}+x_{W(asser)}+x_{E(is)}$$

$$\varphi = \frac{p_D}{p_s}$$

$$x_D = \frac{m_d}{m_L} = \frac{R_L}{R_D}\frac{p_D}{p_L} = \frac{R_L}{R_D}\frac{p_D}{p-p_D} = 0.622\frac{p_D}{p-p_D}$$

$$x_s = \frac{m_{D,max}}{m_L} = 0.622\frac{p_s}{p-p_s} \rightarrow \text{für } \varphi=1$$

$$\rho = \frac{p}{R_{gesT}} = \frac{1+x}{R_L+xR_D}\frac{p}{T}$$

$$R_{ges} = \frac{R_L+xR_D}{1+x}$$

$$h = c_{pL}t+x_D(c_{pD}t+r_D)+x_W c_W t+x_E(c_E t-r_E)$$

19 Chemische Reaktionen

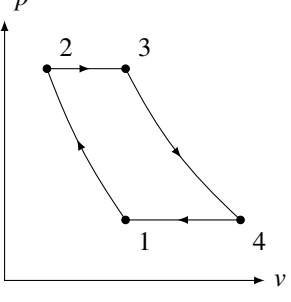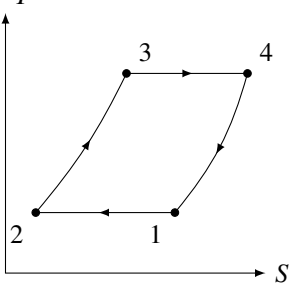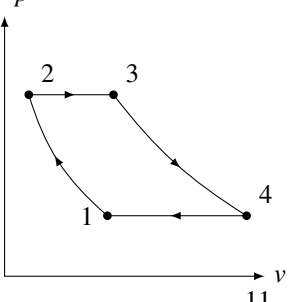$$\frac{dn_1}{\nu_1} = \frac{dn_2}{\nu_2} = \ldots = d\lambda = .const$$

$$\sum_{k=1}^{K}\mu_k dn_k = \sum_{k=1}^{K}\mu_k(\nu_k d\lambda) = \sum_{k=1}^{K}\mu_k\nu_k = 0$$

$$\mu_i = \left(\frac{\partial U}{\partial n_i}\right)_{S,V} = \left(\frac{\partial H}{\partial n_i}\right)_{S,p} = \left(\frac{\partial F}{\partial n_i}\right)_{T,V} = \left(\frac{\partial G}{\partial n_i}\right)_{T,p}$$

$$\mu(p,T) = \mu(p^+,T)+R_m T\ln\left(\frac{p}{p^+}\right)$$

Massenwirkungsgesetz

$$\prod_{k=1}^{K}\psi_k^{\nu_k} = exp-\frac{1}{R_m T}\sum_{k=1}^{K}\nu_k\mu_{0k}(p,T)$$

$$= exp-\frac{1}{R_m T}\sum_{k=1}^{K}\nu_k G_{m,k}(p,T)$$

Gleichgewichtkonstante

$$K(p,T) = \prod_{k=1}^{K}\psi_k^{\nu_k}$$

$$K(p_2,T) = K(p_1,T)\left(\frac{p_1}{p_2}\right)^{\Sigma\nu_k}$$

$$\ln\left(\frac{K(p,T_2)}{K(p,T_1)}\right) = \frac{\Delta H_R}{R_m}\left(\frac{1}{T_1}-\frac{1}{T_2}\right) = \frac{\Delta H_R}{R_m}\frac{T_2-T_1}{T_1 T_2}$$

$$\Delta H_R = \sum_{k=1}^{K}\nu_k H_{m,k}$$

T-S Diagram	p-v Diagram	Process
		Seiliger Prozess $$\eta_{th} = 1 - \frac{\varphi^{\kappa}\psi - 1}{\varepsilon^{\kappa-1}\left[\psi - 1 + \kappa\psi(\varphi - 1)\right]}$$ $$\varepsilon = \frac{v_1}{v_2} \qquad \psi = \frac{p3}{p2} \qquad \varphi = \frac{v_4}{v}$$
		Otto Prozess $$\eta_{th} = 1 - \frac{1}{\varepsilon^{\kappa-1}}$$ $$\varepsilon = \frac{v_1}{v_2}$$
		Diesel Prozess $$n_{th} = 1 - \frac{\varphi^{\kappa} - 1}{\varepsilon^{\kappa-1}\kappa(\varphi - 1)}$$ $$\varepsilon = \frac{v_1}{v_2} \qquad \varphi = \frac{v_4}{v}$$
		Stirling Prozess $$n_{th} = 1 - \frac{|q_{12}|}{q_{34}} = \frac{RT_1 \ln\left(\frac{v_1}{v_2}\right)}{RT_3 \ln\left(\frac{v_4}{v_3}\right)} = 1 - \frac{T_1}{T_3}$$
		Joule Prozess $$\eta_{th} = 1 - \frac{T_1}{T_2} = 1 - \left(\frac{p_1}{p_2}\right)^{\frac{\kappa-1}{\kappa}} = 1 - \left(\frac{1}{\pi}\right)^{\frac{\kappa-1}{\kappa}}$$ $$\pi = \frac{p_2}{p_1}$$
		Ericsson Prozess $$n_{th}1 - \frac{|q_{12}|}{q_{34}} = \frac{RT_1 \ln\left(\frac{p_1}{p_2}\right)}{RT_3 \ln\left(\frac{p_4}{p_3}\right)} = 1 - \frac{T_1}{T_3}$$

11

Ideales Gas

	Isothermo	Isobare	Isochore	Isentrop	Polytrope
konstant:	T	p	v	$\delta q = 0$	pv^n
	-	-	-	$p_1 v_1^{\kappa} = p_2 v_2^{\kappa}$	$v_1^n = p_2 v_2^n$
	$p_1 v_1 = p_2 v_2$	$\frac{v_1}{v_2} = \frac{T_1}{T_2}$	$\frac{p_1}{T_1} = \frac{p_2}{T_2}$	$T_1 v_1^{\kappa-1} = T_2 v_2^{\kappa-1}$	$T_1 v_1^{n-1} = T_2 v_2^{n-1}$
	-	-	$=\Omega$	$\frac{T_1^{\frac{\kappa}{\kappa-1}}}{p_1} = \frac{T_2^{\frac{\kappa}{\kappa-1}}}{p_2}$	$\frac{T_1^{\frac{n}{n-1}}}{p_1} = \frac{T_2^{\frac{n}{n-1}}}{p_2}$
p, v	$p = \frac{p_1 v_1}{v}$	$p = p_1$	$v = v_1$	$p = \frac{p_1 v_1^{\kappa}}{v^{\kappa}}$	$p = \frac{p_1 v_1^n}{v^n}$
p, T	$p = \frac{p_1 v_1}{v}$	$p = p_1$	$p = \frac{p_1}{T_1} T$	$p = \frac{p_1}{T_1^{\frac{\kappa}{\kappa-1}}} T^{\frac{\kappa}{\kappa-1}}$	$p = \frac{p_1}{T_1^{\frac{n}{n-1}}} T^{\frac{n}{n-1}}$
v, T	$T = T_1$	$v = \frac{v_1}{T_1} T$	$v = v_1$	$T = \frac{T_1 v_1^{\kappa-1}}{v^{\kappa-1}}$	$T = \frac{T_1 v_1^{n-1}}{v^{n-1}}$
q_{12}	$= p_1 v_1 \ln \frac{p_1}{p_2}$	$= c_p(T_2 - T_1)$	$= c_v(T_2 - T_1)$	$= 0$	$= c_v \frac{n-\kappa}{n-1}(T_2 - T_1)$
$w_{V,12}$	$= -q_{12}$	$= -p_1(v_2 - v_1)$	$= 0$	$= \frac{p_1 v_1}{k-1}\left[\left(\frac{v_1}{v}\right)^{\kappa-1} - 1\right]$	$= \frac{p_1 v_1}{n-1}\left[\left(\frac{v_1}{v_2}\right)^{n-1} - 1\right]$
$s_2 - s_1$	$= R \ln\left(\frac{p_1}{p_2}\right)$	$= c_p \ln\left(\frac{T_2}{T_1}\right)$	$= c_v \ln\left(\frac{T_2}{T_1}\right)$	$= 0$	$= c_v \frac{n-\kappa}{n-1} \ln\left(\frac{T_2}{T_1}\right)$

Van-Der-Waals-Gas

konst.	Isotherme	Isobare	Isochore	Isentrop
	T	p	v	$\delta=0$
	$(p_1+\frac{a}{v^2})(v_1-b)$ $=(p_2+\frac{a}{v^2})(v_2-b)$	$\frac{RT_1}{v_1-b}-\frac{a}{v_1^2}=\frac{RT_2}{v-b}-\frac{a}{v_2^2}$	$\frac{p_1+\frac{a}{v_1^2}}{T_1}=\frac{p_2+\frac{a}{v_1^2}}{T_2}$	$(p_1+\frac{a}{v^2})(v_1-b)^{\frac{c_v+R}{c_v}}$ $=(p+\frac{a}{v^2})(v_2-b)^{\frac{c_v+R}{c_v}}$, $T_1(v_1-b)^{R/c_v}=T_2(v_2-b)^{R/c_v}$
p,v	$p=(p+\frac{a}{v^2})\frac{v_u-b}{v-b}-\frac{a}{v^2}$	$p=p_1$	$v=v_1$	$p=-\frac{a}{v^2}+(p_1+\frac{a}{v_1^2})\left(\frac{v_1-b}{v_m}\right)^{\frac{v_v+R}{R}}$
p,T	$T=T_1$	$p=p_1$	$p=\frac{T}{T_1}(p_1+\frac{a}{v_1^2})-\frac{a}{v_1^2}$	$p=-\frac{a}{v^2}+(p_1+\frac{a}{v_1^2})\left(\frac{T}{T_1}\right)^{\frac{c_v+R}{R}}$
v,T	$T=T_1$	$T=T_1\frac{v-b}{v_1-b}+\frac{a}{R}(v-b)\left(\frac{1}{v^2}-\frac{1}{v_1^2}\right)$	$v=v_1$	$T=T_1\left(\frac{v_1-b}{v-b}\right)^{\frac{R}{c_v}}$
q_{12}	$=RT_1\ln\left(\frac{v_2-b}{v_1-b}\right)$	$=\frac{a}{v_1}-\frac{a}{v_2}+c_v(T_2-T_1)+p_1(v_2-v_1)$	$=c_v(T_2-T_1)$	$=0$
$w_{V,12}$	$=-RT_1\ln\left(\frac{v_2-b}{v_1-b}\right)+\frac{a}{v_1}-\frac{a}{v_2}$	$=-p_1(v_2-v_1)$	$=0$	$=\frac{a}{v_1}-\frac{a}{v_2}+c_v(T_2-T_1)$
s_2-s_1	$=R\ln\left(\frac{v_2-b}{v_1-b}\right)$	$=c_v\ln\left(\frac{T_2}{T_1}\right)+R\ln\left(\frac{v_2-b}{v_1-b}\right)$	$=c_v\ln\left(\frac{T_2}{T_1}\right)$	$=0$

20 Stoffwerte einiger Gase

Bezeichnung	Symbol	Molmasse [kg/kmol]	Gaskonstante [J/(kg K)]	Dichte [kg/m3]	c_p [J/(kg K)]	c_v [J/(kg,K)]	κ
Acetylen	C_2H_2	26.038	319.3	1.16	1616	1278	1.26
Ammoniak	NH_3	17.031	488.2	0.76	2056	1526	1.35
Argon	Ar	39.948	208.1	1.76	519	309	1.68
Äthan	C_2H_6	30.070	276.5	1.34	1650	1355	1.22
Butan	C_4H_10	58.124	143.0	2.67	1599	1410	1.13
Chlor	C_l2	56.108	117.3	3.17	473	343	1.38
Chlorwasserstoff	HCl	70.906	228.0	1.62	795	556	1.43
Helium	He	4.003	2077.0	0.18	5200	3124	1.66
Kohlendioxid	CO_2	44.010	188.9	1.95	816	618	1.32
Kohlenmonoxid	CO	28.010	296.8	1.23	1038	739	1.40
Luft	$\breve{}$	28.964	287.1	1.28	1006	718	1.40
Methan	CH_4	16.043	518.3	0.71	2165	1638	1.32
Propan	C_3H_8	44.097	188.5	1.99	1549	1331	1.16
Sauerstoff	O_2	31.999	259.8	1.41	909	647	1.40
Stickstoff	N_2	28.013	296.8	1.23	1038	739	1.40
Wasserstoff	H_2	2.016	4124.2	0.09	14050	9926	1.42
Xenon	Xe	131.300	63.3	5.82	159	93	1.71

21 Stoffdaten einiger Stoffe

Name Name	chemische Formel	Molmasse [kg/kmol]	Normal-siedepunkt [°C]	kritische Temperatur [°C]	kritischer Druck [MPa]
Wasserstoff	H_2	2.02	-252.9	-240.0	1.32
Helium	He	4.00	-268.9	-268.0	0.23
Ammoniak	NH_3	17.03	-33.3	132.3	11.33
Wasser	H_2O	18.02	100.0	373.9	22.06
Luft	78% N_2 21% O_2 .1%$Ar.+$	28.96	-194.2	-140.4	3.84
Kohlendioxid	CO_2	44.01	-78.4	31.0	7.38
Methan	CH_4	16.04	-161.5	-82.6	4.60
Äthan	C_2H_6	30.07	-88.6	32.2	4.87
Propan	C_3H_8	44.10	-42.1	96.7	4.25
R134a	CH_2FCF_3	102.03	-26.1	101.1	4.06

22 Obskure Zusammenhänge

$$dV = \left(\frac{\partial V}{\partial T}\right)_p dT + \left(\frac{\partial V}{\partial p}\right)_{T,n} + \sum_{k=1}^{K} \left(\frac{\partial V}{\partial n_k}\right)_{T,p} dk n_k$$

$$dS = \left(\frac{nC_{p,m}}{T}\right) dT - \left(\frac{\partial V}{\partial T}\right)_{p,n} dp + \sum_{k=1}^{K} \left(\frac{\partial \mu_k}{\partial T}\right)_{p,n} dn_k$$

$$dU = \left[nC_{p,m} - p\left(\frac{\partial V}{\partial T}\right)_{p,n}\right] dT - \left[p\left(\frac{\partial V}{\partial p}\right)_{T,n} + T\left(\frac{\partial V}{\partial T}\right)_{p,n}\right] dp + \sum_{k=1}^{K} \left[\mu_k - T\left(\frac{\partial \mu_k}{\partial T}\right)_{p,n} - p\left(\frac{\partial V}{\partial n_k}\right)_{T,p,n}\right] dn_k$$

$$dH = nC_{p,m} dT + \left[VT\left(\frac{\partial V}{\partial T}\right)_{p,n}\right] + \sum_{k=1}^{K} \left[\mu_k - T\left(\frac{\partial \mu_k}{\partial T}\right)_{p,n}\right] dn_k$$

$$dF = -\left[S + p\left(\frac{\partial V}{\partial T}\right)_{p,n}\right] dT - p\left(\frac{\partial V}{\partial p}\right)_{T,n} dp + \sum_{k=1}^{K} \left[\mu_k - p\left(\frac{\partial V}{\partial n_k}\right)_{T,p}\right] dn_k$$

$$\left(\frac{\partial C_{p,m}}{\partial p}\right)_{T,\psi_j} = T\frac{\partial}{\partial p}\left[\left(\frac{\partial S_m}{\partial T}\right)_{p,\psi_j}\right]_{T,\psi_j} = T\frac{\partial}{\partial T}\left[\left(\frac{\partial S_m}{\partial p}\right)_{T,\psi_j}\right]_{p,\psi_j} = -T\frac{\partial}{\partial T}\left[\left(\frac{\partial V_m}{\partial T}\right)_{p,\psi_j}\right]_{p,\psi_j} = -T\left(\frac{\partial^2 V_m}{\partial T^2}\right)_{p,\psi_j}$$

$$C_{p,m} = (C_{p,m})_{\text{ideales Gas}} - T\int_0^p \left(\frac{\partial^2 V_m}{\partial T^2}\right)_{p,\psi_j} d\tilde{p}$$

$$C_{v,m} = (C_{v,m})_{\text{ideales Gas}} - T\int_0^{V_m} \left(\frac{\partial^2 p}{\partial T^2}\right)_{p,\psi_j} d\tilde{V}$$