

# Thermodynamik Formelsammlung

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$$\frac{d}{dt} \left\{ U + m \left( \frac{c^2}{2} + gz \right) \right\} = \sum_j \left[ \dot{m}_j \left( h + \frac{c^2}{2} + gz \right) \right]_j + \sum_l (\dot{Q}_t)_l + \sum_i (\dot{W}_t)_i - p \frac{dV}{dt}$$

## 1 Nomenklatur

<b>A<sub>n</sub></b> = Anergie[J]	<b>V</b> = Volumen[m <sup>3</sup> ]
<b>c<sub>s</sub></b> = Schallgeschwindigkeit[m/s]	<b>v</b> = Spezifisches Volumen[m <sup>3</sup> /kg]
<b>c<sub>p</sub></b> = Spezifische Wärmekapazität dp = 0 [J/kg*K]	<b>V<sub>m</sub></b> = Molares Volumen[m <sup>3</sup> /mol]
<b>c<sub>v</sub></b> = Spezifische Wärmekapazität dv = 0 [J/kg*K]	<b>W</b> = Arbeit[J]
<b>E</b> = Energie[J]	<b>w</b> = Spezifische Arbeit[J/kg]
<b>Ex</b> = - <b>W<sub>ex</sub></b> = Exergie[J]	<b>W<sub>v</sub></b> = Volumenänderungsarbeit[J]
<b>F</b> = Kraft[N]	<b>W<sub>el</sub></b> = Elektrische Arbeit[J]
<b>F</b> = <b>U</b> - <b>TS</b> = Freie Energie[J]	<b>W<sub>w</sub></b> = Wellenarbeit[J]
<b>f</b> = <b>u</b> - <b>Ts</b> = Spezifische freie Energie[J/kg]	<b>W<sub>diss</sub></b> = Dissipationsarbeit[J]
<b>f</b> = Fugazität[Pa]	<b>W<sub>t</sub></b> = Technische Arbeit[J]
<b>G</b> = <b>H</b> - <b>TS</b> = Freie Enthalpie[J]	<b>W<sub>virrev</sub></b> = Arbeitsverlust durch Irreversibilität[J]
<b>g</b> = <b>h</b> - <b>Ts</b> = Spezifische freie Enthalpie[J/kg]	$x = \frac{m''}{m' + m''}$ = Dampfanteil[-]
<b>g</b> = Erdbeschleunigung[m/s <sup>2</sup> ]	$x = \frac{m_{H_2O}}{m_L}$ = Wassergehalt
<b>H</b> = <b>U</b> + <b>pV</b> = Enthalpie[J]	<b>Z</b> = Allgemeine extensive Zustandsgrößen[Z]
<b>h</b> = <b>u</b> + <b>pv</b> = Spezifische Enthalpie[J/kg]	<b>z</b> = Allgemeine
<b>H<sub>g</sub></b> = Molare Reaktionsenthalpie	<b>β</b> = Isobarer Ausdehnungskoeffizient[1/K]
<b>K</b> = Konstante des Massenwirkungsgesetzes[-]	<b>γ</b> = Isochorer Spannungskoeffizient[1/K]
<b>M</b> = Molmasse[kg/mol]	<b>δ<sub>T</sub></b> = Isothermer Drosselkoeffizient[m <sup>3</sup> /kg]
<b>ṁ</b> = Massestrom[kg/s]	<b>δ<sub>h</sub></b> = Isenthalper Drosselkoeffizient[Ks <sup>2</sup> m/kg]
<b>m'</b> = Masse in der flüssigen Phase[kg]	<b>ε</b> = Leistungsziffer[-]
<b>m''</b> = Masse in der gasförmigen Phase[kg]	<b>ε</b> = Verdichtungsverhältnis[-]
<b>Ma</b> = <b>c</b> / <b>c<sub>s</sub></b> = Machzahl[-]	<b>η<sub>th</sub></b> = Thermischer Wirkungsgrad[-]
<b>n</b> = <b>m</b> / <b>M</b> = Molzahl[mol]	<b>η<sub>mech</sub></b> = Mechanischer Wirkungsgrad[-]
<b>n</b> = Polytropenexponent[-]	<b>κ</b> = Adiabaten- oder Isentropenexponent[-]
<b>P<sub>t</sub></b> = technische Leistung[W]	<b>λ</b> = Reaktionslaufzahl[-]
<b>Q</b> = Wärme[J]	<b>μ<sub>i</sub></b> = Chemisches Potential[J/mol]
<b>Q̇</b> = Wärmestrom[W]	<b>v<sub>i</sub></b> = Stöchiometrische Koeffizienten[-]
<b>q</b> = Spezifische Wärme[J/kg]	<b>ξ<sub>i</sub></b> = Masseanteil[-]
<b>r</b> = Spezifische Verdampfungsenthalpie[J/kg]	<b>π</b> = Druckverhältnis[-]
<b>R</b> = Gaskonstante[J/(kg K)]	<b>ρ</b> = Dichte[kg/m <sup>3</sup> ]
<b>R<sub>m</sub></b> = Universelle Gaskonstante[J/(mol K)]	<b>τ</b> = Temperaturverhältnis[-]
<b>S</b> = Entropie[J/K]	<b>φ</b> = Relative Feuchte[-]
<b>s</b> = Spezifische Entropie[J/(kg K)]	<b>φ</b> = Einspritzverhältnis[-]
<b>T</b> = Temperatur[K]	<b>ξ</b> = Isothermer Kompressibilitätskoeffizient[m <sup>2</sup> /N]
<b>t</b> = Zeit[s]	<b>■</b> = Dissipationsenergie[J]
<b>t</b> = Temperatur[°C]	<b>ψ</b> = Spezifische Dissipationsenergie[J]
<b>T</b> = Sättigungstemperatur[K]	<b>ψ</b> = Drucksteigerungsverhältnis[-]
<b>U</b> = Innere Energie[J]	<b>ψ<sub>i</sub></b> = Molanteil[-]
<b>u</b> = Spezifische innere Energie [J/kg]	

## 2 Grundbegriffe

### Systeme

- Abgeschlossenes System - kein Stoff oder Energietransport
- Geschlossenes System - kein Stofftransport
- Adiabates System - kein  $\Delta q$ , aber Masse und Arbeit.
- Offenes System - Stoff und Energietransport
- Stationäres System  $\rightarrow \Delta U = 0$

### Messgrößen

- Prozessgrößen sind Wegabhängig (eg. Arbeit, Wärme)
- Zustandsgrößen sind Wegunabhängig (eg. Volumen, Druck)
- Extensive Zustandsgrößen sind abhängig von der Masse des Systems (V, m, H, S, F, G, E)
- Intensive Zustandsgrößen sind unabhängig von der Masse des Systems (T, p)

### Zustandsgleichungen

- Thermisch  $\rightarrow f(p, V, T) = 0$
- Kalorisch  $\rightarrow f(U, V, T) = 0, \quad U = U(V, T), \quad u = u(v, T)$

### Hauptsätze

- 0: Temperatur existiert, ihre Gleichheit ist notwendige Voraussetzung für das thermische Gleichgewicht.
- 1: Energie existiert, sie ist für abgeschlossene Systeme konstant.
- 2: Entropie existiert, sie wird bei allen irreversiblen Prozessen erzeugt.  $dS = \frac{\delta Q_{rev}}{T}$
- 3: 0K existiert, bei dieser Temperatur ist die Entropie = 0

## 3 Basisformeln

$$\begin{aligned}
 H &= U + pV \\
 dS &= \frac{\delta Q_{rev}}{T} \\
 F &= U - TS \\
 G &= H - ST \\
 W &= - \int p dV
 \end{aligned}
 \quad
 \begin{aligned}
 dS &= \frac{Q_{rev}}{T} + S_{prod} \\
 \Psi &= \int_1^2 T dS_{prod} \\
 W_{ir} &= \frac{T_u}{T} \Psi \\
 p_1 &= p_a + \frac{\varphi_1 - \varphi_a}{\varphi_b - \varphi_a} (p_b - p_a)
 \end{aligned}$$

## 4 Gibbs

$$\begin{aligned}
 dU &= Tds - pdV + \sum_{k=1}^K \mu_k dn_k \\
 dG &= -SdT + Vdp + \sum_{k=1}^K \mu_k dn_k \\
 dH &= TdS + Vdp + \sum_{k=1}^K \mu_k dn_k \\
 dF &= -SdT - pdV + \sum_{k=1}^K \mu_k dn_k \\
 dU &= \left( \frac{\partial U}{\partial S} \right)_V dS + \left( \frac{\partial U}{\partial V} \right)_S dV + \sum_{k=1}^K \left( \frac{\partial U}{\partial n_k} \right)_S dn_k
 \end{aligned}$$

## 5 Thermodynamische Beziehungen

$$\begin{aligned}
 T &= \left( \frac{\partial U}{\partial S} \right)_V = T(S, V) & -S &= \left( \frac{\partial F}{\partial T} \right)_V = S(T, V) \\
 T &= \left( \frac{\partial H}{\partial S} \right)_p = T(S, p) & -S &= \left( \frac{\partial G}{\partial T} \right)_p = S(T, p) \\
 p &= - \left( \frac{\partial U}{\partial V} \right)_S = p(V, S) & V &= \left( \frac{\partial G}{\partial p} \right)_T = V(p, T) \\
 -p &= \left( \frac{\partial F}{\partial V} \right)_T = p(T, V) & \mu &= \left( \frac{\partial U}{\partial n} \right)_{S, V} = \mu(S, V, n)
 \end{aligned}$$

## 6 Guggenheim

$$\begin{array}{ccccc}
 -S & U & V & U & = U(S, V) \\
 H & & F & H & = H(S, p) \\
 -p & G & T & F & = F(T, V) \\
 & & & G & = G(T, p)
 \end{array}$$

$$\overbrace{\frac{d}{dt} \left\{ U + m \left( \frac{c^2}{2} + gz \right) \right\}}^{\text{Stationäres System} \rightarrow 0} = \sum_j \overbrace{\left[ \dot{m}_j \left( h + \frac{c^2}{2} + gz \right) \right]_j}_{\text{Geschlossenes System} \rightarrow 0} + \overbrace{\sum_l (\dot{Q}_t)_l}_{\text{Kein Wärmestrom} \rightarrow 0} + \overbrace{\sum_i (\dot{W}_t)_i}_{\text{Keine Leistung} \rightarrow 0} - \overbrace{p \frac{dV}{dt}}^{\text{Keine Volumenänderung} \rightarrow 0}$$

## 7 Maxwell

$$\left(\frac{\partial T}{\partial p}\right)_{S,n_j} = \left(\frac{\partial V}{\partial S}\right)_{p,n_j}$$

$$\left(\frac{\partial S}{\partial V}\right)_{T,n_j} = \left(\frac{\partial p}{\partial T}\right)_{V,n_j}$$

$$\left(\frac{\partial S}{\partial p}\right)_{T,n_j} = -\left(\frac{\partial V}{\partial T}\right)_{p,n_j}$$

$$\left(\frac{\partial \mu_i}{\partial T}\right)_{p,n_j} = -\left(\frac{\partial S}{\partial n_i}\right)_{T,p,n_j \neq n_i}$$

$$\left(\frac{\partial \mu_i}{\partial p}\right)_{T,n_j} = \left(\frac{\partial V}{\partial n_i}\right)_{T,p,n_j \neq n_i}$$

## 8 Ideales Gas

$$pV = mRT$$

$$pv = RT$$

$$pV = nR_m T$$

$$\beta = \frac{1}{T}$$

$$\gamma = \frac{1}{T}$$

$$\chi = \frac{1}{p}$$

$$\beta = p\gamma\chi$$

$$R_m = 8,3143 \left[ \frac{kJ}{kmolK} \right]$$

$$R = c_p - c_v$$

$$R = \frac{R_m}{M}$$

$$U - U_0 = mc_v(T - T_0)$$

$$H - H_0 = mc_p(T - T_0) \quad \leftarrow \text{Für } c_p \text{ und } c_v \text{ const.}$$

$$s - s_0 = R \ln \left( \frac{v}{v_0} \right) + c_v \ln \left( \frac{T}{T_0} \right)$$

$$= c_v \ln \left( \frac{p}{p_0} \right) + c_p \ln \left( \frac{v}{v_0} \right)$$

$$= c_p \ln \left( \frac{T}{T_0} \right) - R \ln \left( \frac{p}{p_0} \right)$$

$$\beta = \frac{1}{T} = \frac{1}{V} \left( \frac{\partial V}{\partial T} \right)_p = \frac{1}{v} \left( \frac{\partial v}{\partial T} \right)_p = -\frac{1}{p} \left( \frac{\partial p}{\partial T} \right)_p$$

$$\gamma = \frac{1}{T} = \frac{1}{p} \left( \frac{\partial p}{\partial T} \right)_v$$

$$\chi = \frac{1}{p} = -\frac{1}{V} \left( \frac{\partial V}{\partial p} \right)_T = -\frac{1}{v} \left( \frac{\partial v}{\partial p} \right)_T$$

$$u_2 - u_1 = \int_{T_1}^{T_2} c_v(T) dT$$

## 9 Van-der-Waals

$$\left(p + \frac{a}{v^2}\right)(v - b) = RT$$

$$\left(\bar{p} + \frac{3}{\bar{v}^2}\right)(3\bar{v} - 1) = 8\bar{T}$$

$$\bar{p} = \frac{p}{p_K}, \quad \bar{v} = \frac{v}{v_K}, \quad \bar{T} = \frac{T}{T_K}$$

$$p_K = \frac{a}{27b^2}, \quad T_K = \frac{8}{27} \frac{a}{b} \frac{1}{R}, \quad \text{mit } \mathcal{L}$$

$$a = 3p_K v_K^2, \quad b = \frac{v_K}{3}, \quad \frac{p_K v_K}{RT_K} = \frac{3}{8}$$

$$\beta = \frac{(v - b)Rv^2}{RTv^3 - 2a(v - b)^2}$$

$$\gamma = \frac{Rv^2}{RTv^3 - 2a(v - b)^2}$$

$$\chi = \frac{(v - b)^2 v^2}{RTv^3 - 2a(v - b)^2}$$

$$du = \frac{a}{v^2} dv + c_v(T) dT$$

$$u - u_0 = \left( \frac{a}{v_0} - \frac{a}{v} \right) + \int_{T_0}^T c_v(\tilde{T}) d\tilde{T}$$

$$u - u_0 = \left( \frac{a}{v_0} - \frac{a}{v} \right) + c_v(T - T_0) \quad \leftarrow \text{für } c_v = \text{const.}$$

$$c_p - c_v = \frac{Tv\beta^2}{\chi}$$

$$s - s_0 = c_v \ln \left( \frac{T}{T_0} \right) + R \ln \left( \frac{v - b}{v_0 - b} \right)$$

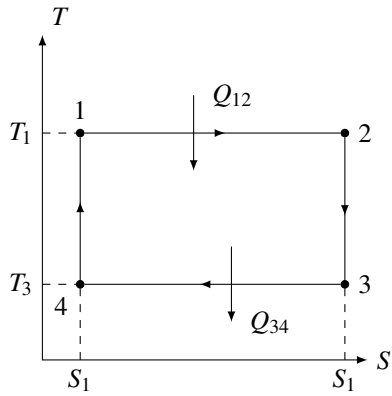
$$h_2 - h_1 = \frac{1}{2}(p_2 - p_1) \left( \frac{1}{\rho_1} + \frac{1}{\rho_2} \right)$$

## 10 Carnot

$$\eta_{th} = 1 - \frac{-Q_{34}}{Q_{12}} = 1 - \frac{T_3(S_3 - S_4)}{T_1(S_2 - S_1)} = 1 - \frac{T_3}{T_1}$$

$$\frac{Q_{12}}{T_1} + \frac{Q_{34}}{T_3} = 0$$

$$\Delta S_{ges} = -Q_{34} \left( \frac{1}{T_{KK}} - \frac{T_1}{T_3} \frac{1}{T_{HK}} \right)$$



Adiabate Drosselung (ideal):

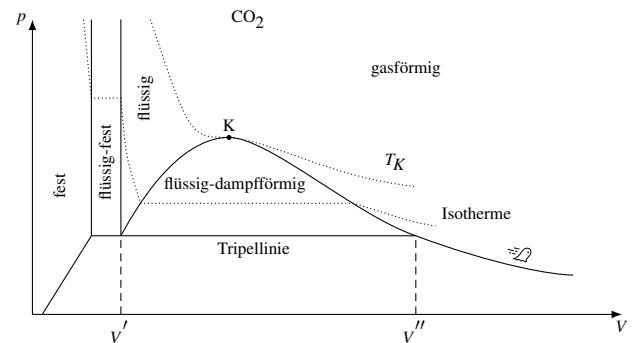
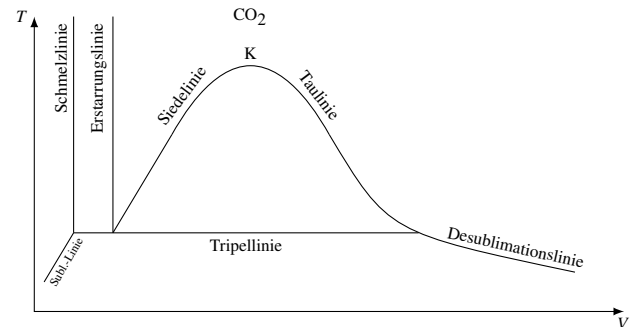
$$h + \frac{c^2}{2} + gz = \text{const.}$$

$$dh = 0$$

Adiabet Drosselung (real):

$$\delta_h = \left( \frac{\partial T}{\partial p} \right)_h = -\frac{v}{c_p} (1 - \beta T)$$

## 12 Nassdampf



## 11 Gemische Idealer Gase

$$\xi_i = \frac{m_i}{m}, \quad \psi_i = \frac{n_i}{n}, \quad p_i = \psi_i p$$

$$\xi_i = \frac{M_i n_i}{\sum_{k=1}^K M_k n_k} = \frac{M_i}{M_G} \psi$$

$$p_i V = m_i R_i T, \quad p_i V = n_i R_m T, \quad p V = m R_G T$$

$$\sum_{k=1}^K p_k = p$$

$$R_G = \frac{1}{m} \sum_{k=1}^K m_k R_k = \sum_{k=1}^K \xi_k R_k$$

$$U_G = \sum_{k=1}^K U_k = \sum_{k=1}^K m_k u_k = \sum_{k=1}^K c_{vk} m_k T \leftarrow c_v = \text{const}$$

$$H_G = \sum_{k=1}^K H_k = \sum_{k=1}^K m_k h_k = \sum_{k=1}^K c_{pk} m_k T \leftarrow c_p = \text{const.}$$

$$c_{vG} = \sum_{k=1}^K c_{vk} \xi_k, \quad c_{pG} = \sum_{k=1}^K c_{pk} \xi_k$$

$$S_2 - S_1 = R_m \left( n \ln n - \sum_{k=1}^K n_k \ln n_k \right)$$

$$v = (1-x)v' + xv''$$

$$v = v' + (v'' - v')x$$

$$u = (1-x)u' + xu''$$

$$u = u' + (u'' - u')x$$

$$h = (1-x)h' + xh''$$

$$h = h' + (h'' - h')x$$

$$s = (1-x)s' + xs''$$

$$s = s' + (s'' - s')x$$

$$r = h'' - h' = T(s'' - s')$$

$$T' = T''$$

$$p' = p''$$

$$g' = g''$$

$$dg' = v' dp' - s' dT'$$

$$dg'' = v'' dp'' - s'' dT''$$

$$dg' = dg''$$

$$\frac{dp}{dT} = \frac{s'' - s'}{v'' - v'}$$

$$\frac{dp}{dT} = \frac{1}{T} \frac{h'' - h'}{v'' - v'}$$

$$\frac{dp}{dT} = \frac{1}{T} \frac{r}{v'' - v'}$$

## 13 Realer Stoff im Nassdampfgebiet

Isobare Zustandsänderung

$$q_{12} = T(s_2 - s_1)$$

$$= T(s'' - s')(x_2 - x_1)$$

$$w_{V,12} = - \int_1^2 p dv$$

$$= -p(v_2 - v_1) = -p(v'' - v')(x_2 - x_1)$$

Isochore Zustandsänderung

$$q_{12} = u_2 - u_1 = u'_2 + x_2(u''_2 - u'_2) - u'_1 - x_1(u''_1 - u'_1)$$

Adiabate Zustandsänderung

$$w_{V,12} = u_2 - u_1 = u'_2 + x_2(u''_2 - u'_2) - u'_1 - x_1(u''_1 - u'_1)$$

Entropieänderung während des Mischvorgangs

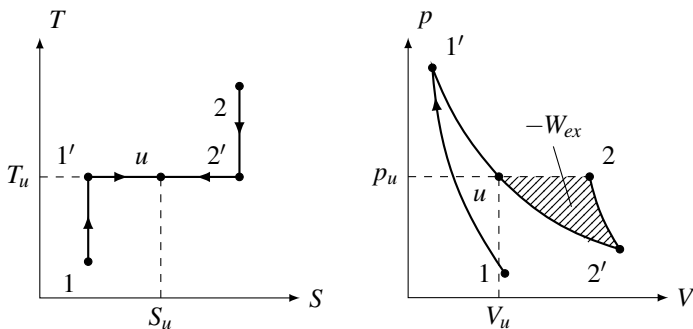
$$S_2 - S_1 = R_m \left( n \ln n - \sum_i n_i \ln n_i \right)$$

## 14 Maximale Arbeit und Exergie

Maximale nutzbare Arbeit → isentrop, reibungsfrei

1 → 1': isentrop auf  $T_u$

1' → u: isotherm auf  $u$



$$-\dot{W}_{ex} = -(\dot{W}_t)_{rev} = -\frac{d}{dt} \left( U + m \left( \frac{c^2}{2} + gz \right) + p_u V - T_u S \right) + \sum_{j=1}^K \left( \dot{m}_j \left( h + \frac{c^2}{2} + gz - T_s \right) \right) + \sum_{l=1}^K \left( 1 - \frac{T_u}{T} \right) \dot{Q}_l$$

Die Exergie der Enthalpie (offenes, stationäres System)

$$-\dot{W}_{ex,1u} = \dot{m}(h_1 - h_u - T_u(s_1 - s_u))$$

Die Exergie der inneren Energie (geschlossenes, instationäres System)

$$-\dot{W}_{ex} = -\frac{d}{dt} (U + p_u V - T_u S)$$

$$-\dot{W}_{ex,1u} = U_1 - U_u - p_u(V_1 - V_u) - T_u(S_1 - S_u)$$

$$-\dot{W}_{ex,1u} = H_1 - (p_1 - p_u)V_1 - H_u - T_u(S_1 - S_u)$$

Für Ideales Gas

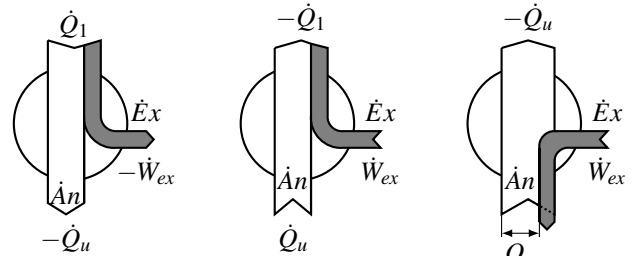
$$-\dot{W}_{ex} = m c_v (T_1 - T_u) + p_u (V_1 - V_u) - T_u m \left( c_p \ln \left( \frac{T_1}{T_u} \right) - R_i \ln \left( \frac{p_1}{p_u} \right) \right)$$

Dampf/Luftdruckkammer

$$-\dot{W}_{ex,1u} = m_1 [u_1 - u_u + p_u (v_1 - v_u) - T_u (s_1 - s_u)]$$

Die Exergie der Wärme (geschlossenes, stationäres System)

$$-\dot{W}_{ex} = \left( 1 - \frac{T_u}{T_1} \right) \dot{Q}_1 = \eta_{th,C} \dot{Q}_1$$

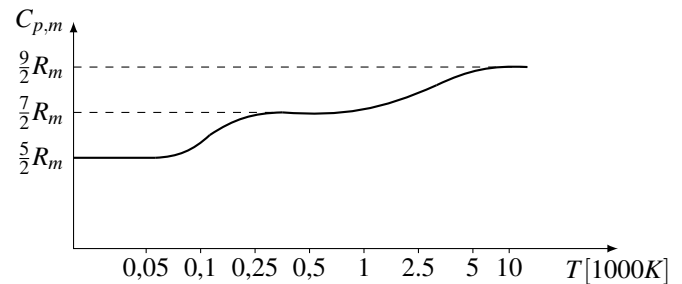


Wärmekraftprozess

Wärmepumpenprozess

Kälteprozess

## 15 Wärmekapazität



$$C_{v,m} = \frac{1}{\kappa - 1} R_m$$

$$c_v = \frac{1}{\kappa - 1} R_j$$

$$\kappa = \frac{c_p}{c_v}$$

$$R = \frac{R_m}{M}$$

$$C_{p,m} = \frac{\kappa}{\kappa - 1} R_m$$

$$c_p = \frac{\kappa}{\kappa - 1} R_j$$

$$R = c_p - c_v$$

$$R_m = 8,3143 \left[ \frac{kJ}{kmolK} \right]$$

$$C_{v,m} = \underbrace{3 + \frac{R_m}{2}}_{\text{Translatorisch}} + \underbrace{\frac{n_{rot} R_m}{2}}_{\text{Rotatorisch}} + \underbrace{R_m (3n_{Atome} - 3 - n_{rot})}_{\text{Vibratorisch}} + \underbrace{C_{v,m, \text{Elektronenanregung}}}_{\text{Relevant ab: } T \approx 10^4 K}$$

## 16 Technische Anwendung

adiabat ( $c_p = \text{const.}$ )	$W_{t,12} = mc_p(T_2 - T_1) = \frac{\kappa}{\kappa - 1}(p_2V_2 - p_1V_1)$	$Q_{12} = 0$
reversibel adiabat $\kappa = \text{const.}$	$W_{t,12} = \frac{\kappa}{\kappa - 1}(p_1V_1) \left[ \left( \frac{p_2}{p_1} \right)^{\frac{\kappa-1}{\kappa}} - 1 \right]$	$Q_{12} = 0$
irreversibel adiabat als Polytrope $n > \kappa; n, \kappa = \text{const.}$	$W_{t,12} = \frac{\kappa}{\kappa - 1}(p_1V_1) \left[ \left( \frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right]$	$Q_{12} = 0$
reversibel polytrop $n, \kappa = \text{const.}$	$W_{t,12} = \frac{n}{n-1}(p_2V_2 - p_1V_1)$ $= \frac{n}{n-1}mR(T_2 - T_1)$ $= \frac{n}{n-1}(p_1V_1) \left[ \left( \frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right]$	$Q_{12} = mc_n(T_2 - T_1)$ $= \frac{n-\kappa}{(n-1)(\kappa-1)}(p_1V_1) \left[ \left( \frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right]$ $c_n = \frac{n-\kappa}{n-1}c_v$
isotherm	$W_{t,12} = (p_1V_1) \ln \left( \frac{p_2}{p_1} \right)$	$Q_{12} = -W_{t,12}$

$$\text{Isentroper Verdichterwirkungsgrad} \quad \eta_{sV} = \frac{w_{t,12,rev}}{w_{t,12}} = \frac{h_{2,rev} - h_1}{h_2 - h_1}$$

$$\text{Isentroper Turbinenwirkungsgrad} \quad \eta_{sT} = \frac{w_{t,12}}{w_{t,12,rev}} = \frac{h_1 - h_2}{h_1 - h_{2,rev}}$$

$$\text{Thermischer Wirkungsgrad} \quad \eta_{th} = \frac{-w}{q_{zu}} = 1 - \frac{|q_{ab}|}{q_{zu}}$$

$$\text{Dampfkraftprozess Wirkungsgrad} \quad \eta_{th} = 1 - \frac{|q_{61}|}{q_{23} + q_{34} + q_{45}} = 1 - \frac{h_6 - h_1}{h_5 - h_2}$$

$$\text{Leistungszahl Kältemaschine} \quad \varepsilon_{K(A)} = \frac{q_{zu}}{w}$$

$$\text{Leistungszahl Kaltluftprozess} \quad \varepsilon_K = \frac{1}{\left( \frac{p}{p_0} \right)^{\frac{\kappa-1}{\kappa}} - 1}$$

$$\text{Leistungszahl Kaltdampfprozess} \quad \varepsilon_K = \frac{q_0}{|q| - q_0} = \frac{q_o}{w_t} = \frac{h_1 - h_6}{h_2 - h_1}$$

$$\text{Leistungszahl Wärmepumpe} \quad \varepsilon_{WP} = \frac{q}{|q| - q_0} = \frac{|q|}{w_t} = \frac{q_{zu}}{w} = \frac{h_2 - h_5}{h_2 - h_1} = 1 + \varepsilon_{K(A)}$$

$$\text{Verdichtungsverhältnis} \quad \varepsilon = \frac{v_1}{v_2}$$

$$\text{Drucksteigerungsverhältniss} \quad \psi = \frac{p_3}{p_2}$$

$$\text{Einspritzverhältnis} \quad \varphi = \frac{v_4}{v_3}$$

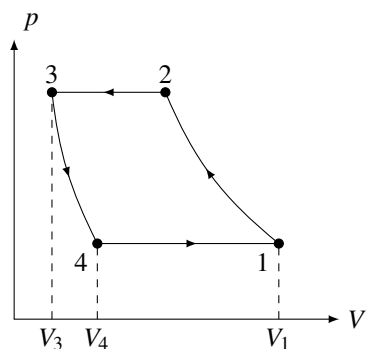
$$\text{Temperaturverhältnis} \quad \tau = \frac{T_3}{T_1}$$

$$\text{Verdrichtungsdruckverhältnis} \quad \pi = \frac{p_2}{p_1}$$

$$\text{für Joule} \quad \pi_{opt} = \tau^{\frac{\kappa}{2(\kappa-1)}}$$



## 16.1 Kolbenverdichter



$V_1$  = Maximales Zylindervolumen

$V_2$  = Volumen nach Verdichtung

$V_3$  =

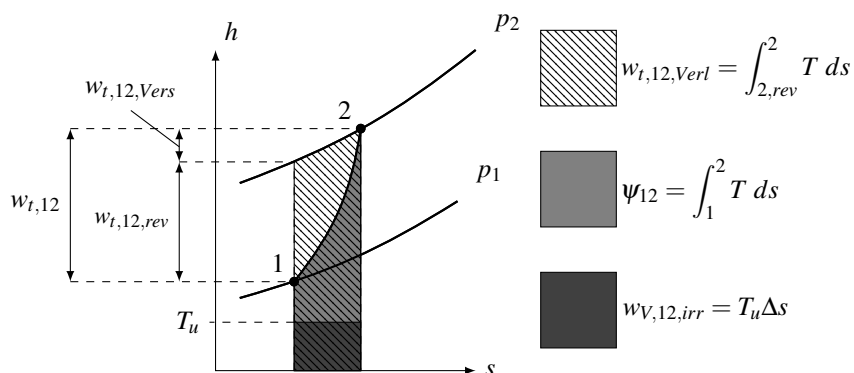
$V_4$  = Schädlicher Raum

$$\mu = \frac{V_1 - V_4}{V_1 - V_3}, \quad \varepsilon_s = \frac{V_3}{V_1 - V_3}$$

$$\mu = 1 - \varepsilon_s \left[ \left( \frac{p_2}{p_1} \right)^{\frac{1}{n}} - 1 \right]$$

$$\begin{aligned} W_{t,12} &= \int_1^2 V dp \\ &= \underbrace{p_2 V_2}_{\text{Ausschiebearbeit}} - \underbrace{p_1 V_1}_{\text{Einschiebearbeit}} - \int_1^2 p dV \\ &= \frac{n}{n-1} p_1 (V_1 - V_4) \left[ \left( \frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right] \end{aligned}$$

## 16.2 Turboverdichter



Verdichter Wirkungsgrad

$$\eta_{sV} = \frac{w_{t,12,rev}}{w_{t,12}} = \frac{h_{2,rev} - h_1}{h_2 - h_1}$$

Verdichter wirkungsgrad (Ideales Gas,  $c_p = \text{const.}$ )

$$\eta_{sV} = \frac{T_{2,rev} - T_1}{T_2 - T_1}$$

Technische Verlustarbeit

$$\begin{aligned} w_{t,Verl,12} &= w_{t,12} - w_{t,12,rev} = h_2 - h_{2,rev} \\ &= \int_{2,rev}^2 T|_{p_2=\text{const.}} ds \end{aligned}$$

## 17 Eindimensionale Strömungsvorgänge

$$\chi = \frac{1}{p} = -\frac{1}{V} \left( \frac{\partial V}{\partial p} \right)_T$$

$$c_S^2 = \left( \frac{\partial p}{\partial \rho} \right)_S$$

$$c_S^2 = \left( \frac{R}{c_v} + 1 \right) \left( v^2 \frac{RT}{(v-b)^2} \right) - \frac{2a}{v} \leftarrow VdW$$

$$c_S^2 = \kappa RT \leftarrow ideal$$

$$Ma = \frac{c}{c_S}$$

$$\frac{T_0}{T} = 1 + \frac{\kappa-1}{2} \frac{c^2}{\kappa RT} = 1 + \frac{\kappa-1}{2} Ma^2$$

$$\frac{p_0}{p} = \left( \frac{T_0}{T} \right)^{\frac{\kappa}{\kappa-1}} = \left( 1 + \frac{\kappa-1}{2} Ma^2 \right)^{\frac{\kappa}{\kappa-1}}$$

$$\frac{\rho_0}{\rho} = \left( \frac{T_0}{T} \right)^{\frac{\kappa-1}{\kappa}} = \left( 1 + \frac{\kappa-1}{2} Ma^2 \right)^{\frac{\kappa-1}{\kappa}}$$

$$\left( \frac{A}{A^*} \right)^2 = \frac{1}{Ma^2} \left[ \frac{2}{\kappa+1} \left( 1 + \frac{\kappa-1}{2} Ma^2 \right) \right]^{\frac{\kappa+1}{\kappa-1}}$$

$$h_2 - h_1 = \frac{1}{2}(p_2 - p_1) \left( \frac{1}{\rho_1} + \frac{1}{\rho_2} \right) = (p_2 - p_1) \frac{1}{2}(v_1 + v_2)$$

Stoßbeziehungen für ein ideales Gas

$$\frac{p_2}{p_1} = \frac{2\kappa Ma^2 - (\kappa - 1)}{\kappa + 1}$$

$$\frac{\rho_2}{\rho_1} = \frac{(\kappa + 1)Ma^2}{2 + (\kappa - 1)Ma^2}$$

$$\frac{T_2}{T_1} = \frac{[2\kappa Ma^2 - (\kappa - 1)][2 + (\kappa - 1)Ma^2]}{(\kappa + 1)^2} Ma^2$$

$$Ma_2^2 = \frac{(\kappa - 1)(Ma_1^2 - 1) + (\kappa + 1)}{2\kappa(Ma_1^2 - 1) + (\kappa + 1)}$$

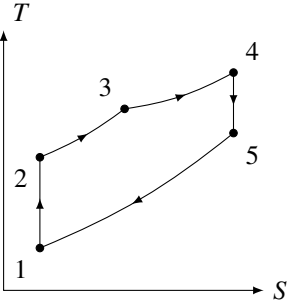
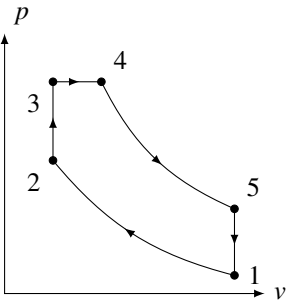
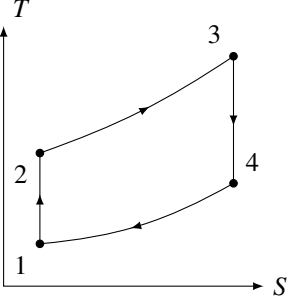
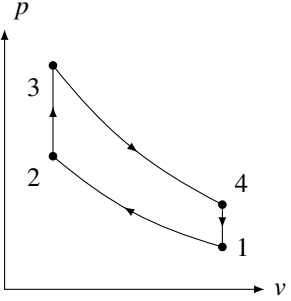
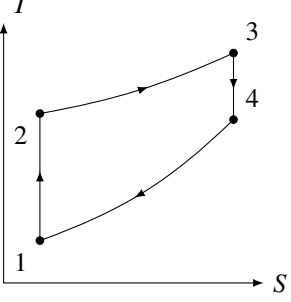
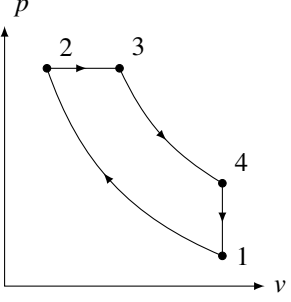
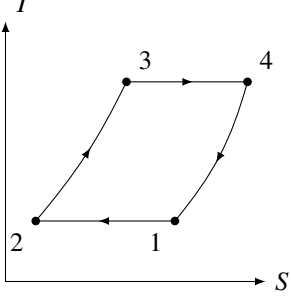
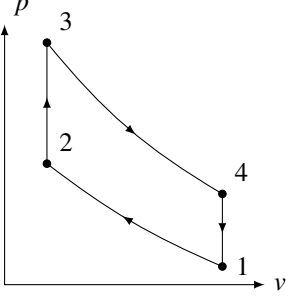
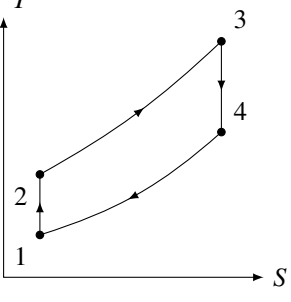
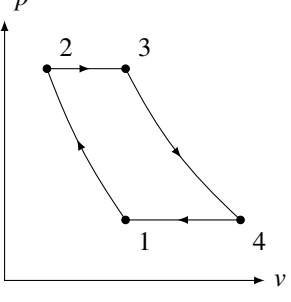
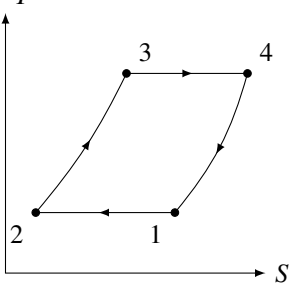
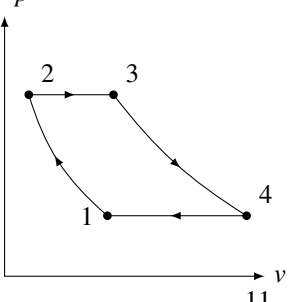
Entropie über den senkrechten Verdichtungsstoß

$$s_2 - s_1 = c_v \ln \left( \frac{T_2}{T_1} \right) + R \ln \left( \frac{v_2}{v_1} \right)$$

$$= c_p \ln \left( \frac{T_2}{T_1} \right) + R \ln \left( \frac{p_2}{p_1} \right)$$

## 18 Feuchte Luft

$$x = \frac{m_{H_2O}}{m_L}$$

		<p>Seiliger Prozess</p> $\eta_{th} = 1 - \frac{\varphi^{\kappa} \psi - 1}{\varepsilon^{\kappa-1} [\psi - 1 + \kappa \psi (\varphi - 1)]}$ $\varepsilon = \frac{v_1}{v_2} \quad \psi = \frac{p_3}{p_2} \quad \varphi = \frac{v_4}{v}$
		<p>Otto Prozess</p> $\eta_{th} = 1 - \frac{1}{\varepsilon^{\kappa-1}}$ $\varepsilon = \frac{v_1}{v_2}$
		<p>Diesel Prozess</p> $\eta_{th} = 1 - \frac{\varphi^{\kappa} - 1}{\varepsilon^{\kappa-1} \kappa (\varphi - 1)}$ $\varepsilon = \frac{v_1}{v_2} \quad \varphi = \frac{v_4}{v}$
		<p>Stirling Prozess</p> $\eta_{th} = 1 - \frac{ q_{12} }{q_{34}} = \frac{RT_1 \ln \left( \frac{v_1}{v_2} \right)}{RT_3 \ln \left( \frac{v_4}{v_3} \right)} = 1 - \frac{T_1}{T_3}$
		<p>Joule Prozess</p> $\eta_{th} = 1 - \frac{T_1}{T_2} = 1 - \left( \frac{p_1}{p_2} \right)^{\frac{\kappa-1}{\kappa}} = 1 - \left( \frac{1}{\pi} \right)^{\frac{\kappa-1}{\kappa}}$ $\pi = \frac{p_2}{p_1}$
		<p>Ericsson Prozess</p> $\eta_{th} = 1 - \frac{ q_{12} }{q_{34}} = \frac{RT_1 \ln \left( \frac{p_1}{p_2} \right)}{RT_3 \ln \left( \frac{p_4}{p_3} \right)} = 1 - \frac{T_1}{T_3}$

# Ideales Gas

	Isothermo	Isobare	Isochore	Isentrop	Polytrope
konstant:	T	p	v	$\delta q = 0$	$p v^n$
-	-	-	-	$p_1 v_1^\kappa = p_2 v_2^\kappa$	$v_1^n = p_2 v_2^n$
	$p_1 p_2 = p_2 v_2$	$\frac{v_1}{v_2} = \frac{T_1}{T_2}$	$\frac{p_1}{T_1} = \frac{p_2}{T_2}$	$T_1 v_1^{\kappa-1} = T_2 v_2^{\kappa-1}$	$T_1 v_1^{n-1} = T_2 v_2^{n-1}$
-	-	-	$\neq \varnothing$	$\frac{T_1^{\frac{\kappa-1}{\kappa}}}{p_1} = \frac{T_2^{\frac{\kappa-1}{\kappa}}}{p_2}$	$\frac{T_1^{\frac{n-1}{n}}}{p_1} = \frac{T_2^{\frac{n-1}{n}}}{p_2}$
$p, v$	$p = \frac{p_1 v_1}{v}$	$p = p_1$	$v = v_1$	$p = \frac{p_1 v_1^\kappa}{v^\kappa}$	$p = \frac{p_1 v_1^n}{v^n}$
$p, T$	$p = \frac{p_1 v_1}{v}$	$p = p_1$	$p = \frac{p_1 T}{T_1}$	$p = \frac{p_1 T^{\frac{\kappa}{\kappa-1}}}{T_1^{\frac{\kappa}{\kappa-1}}}$	$p = \frac{p_1 T^{\frac{n}{n-1}}}{T_1^{\frac{n}{n-1}}}$
$v, T$	$T = T_1$	$v = \frac{v_1 T}{T_1}$	$v = v_1$	$T = \frac{T_1 v_1^{\kappa-1}}{v^{\kappa-1}}$	$T = \frac{T_1 v_1^{n-1}}{v^{n-1}}$
$q_{12}$	$= p_1 v_1 \ln \frac{p_1}{p_2}$	$= c_p (T_2 - T_1)$	$= c_v (T_2 - T_1)$	$= 0$	$= c_v \frac{n-\kappa}{n-1} (T_2 - T_1)$
$w_{V,12}$	$= -q_{12}$	$= -p_1 (v_2 - v_1)$	$= 0$	$= \frac{p_1 v_1}{\kappa-1} \left[ \left( \frac{v_1}{v_1} \right)^{\kappa-1} - 1 \right]$	$= \frac{p_1 v_1}{n-1} \left[ \left( \frac{v_1}{v_2} \right)^{n-1} - 1 \right]$
$s_2 - s_1$	$= R \ln \left( \frac{p_1}{p_2} \right)$	$= c_p \ln \left( \frac{T_2}{T_1} \right)$	$= c_v \ln \left( \frac{T_2}{T_1} \right)$	$= 0$	$= c_v \frac{n-\kappa}{n-1} \ln \left( \frac{T_2}{T_1} \right)$

# Van-De-Waals-Gas

	Isotherme	Isobare	Isochore	Isentrop
konst.	T	p	v	$\delta = 0$
	$(p_1 + \frac{a}{v_1^2})(v_1 - b)$ $= (p_2 + \frac{a}{v_2^2})(v_2 - b)$	$\frac{RT_1}{v_1 - b} - \frac{a}{v_1^2} = \frac{RT_2}{v_2 - b} - \frac{a}{v_2^2}$	$\frac{p_1 + \frac{a}{v_1^2}}{T_1} = \frac{p_2 + \frac{a}{v_2^2}}{T_2}$	$(p_1 + \frac{a}{v_1^2})(v_1 - b)^{\frac{c_v + R}{c_v}}$ $= (p_2 + \frac{a}{v_2^2})(v_2 - b)^{\frac{c_v + R}{c_v}},$ $T_1(v_1 - b)^{R/c_v} = T_2(v_2 - b)^{R/c_v}$
$p, v$	$p = (p + \frac{a}{v^2})\frac{v_u}{v - b} - \frac{a}{v^2}$	$p = p_1$	$v = v_1$	$p = -\frac{a}{v^2} + (p_1 + \frac{a}{v^2})\left(\frac{v_1 - b}{v_m}\right)^{\frac{v_1 + R}{R}}$
$p, T$	$T = T_1$	$p = p_1$	$p = \frac{T}{T_1}(p_1 + \frac{a}{v^2}) - \frac{a}{v_1^2}$	$p = -\frac{a}{v^2} + (p_1 + \frac{a}{v^2})\left(\frac{T}{T_1}\right)^{\frac{c_v + R}{R}}$
$v, T$	$T = T_1$	$T = T_1 \frac{v - b}{v_1 - b} + \frac{a}{R}(v - b)\left(\frac{1}{v^2} - \frac{1}{v_1^2}\right)$	$v = v_1$	$T = T_1 \left(\frac{v_1 - b}{v - b}\right)^{\frac{R}{c_v}}$
$q_{12}$	$= RT_1 \ln \left(\frac{v_2 - b}{v_1 - b}\right)$	$= \frac{a}{v_1} - \frac{a}{v_2} + c_v(T_2 - T_1) + p_1(v_2 - v_1)$	$= c_v(T_2 - T_1)$	$= 0$
$w_{V,12}$	$= -RT_1 \ln \left(\frac{v_2 - b}{v_1 - b}\right) + \frac{a}{v_1} - \frac{a}{v_2}$	$= -p_1(v_2 - v_1)$	$= 0$	$= \frac{a}{v_1} - \frac{a}{v_2} + c_v(T_2 - T_1)$
$s_2 - s_1$	$= R \ln \left(\frac{v_2 - b}{v_1 - b}\right)$	$= c_v \ln \left(\frac{T_2}{T_1}\right) + R \ln \left(\frac{v_2 - b}{v_1 - b}\right)$	$= c_v \ln \left(\frac{T_2}{T_1}\right)$	$= 0$

## 19 Stoffwerte einiger Gase

Bezeichnung	Symbol	Molmasse [kg/kmol]	Gaskonstante [J/(kg K)]	Dichte [kg/m <sup>3</sup> ]	$c_p$ [J/(kg K)]	$c_v$ [J/(kg, K)]	$\kappa$
Acetylen	$C_2H_2$	26.038	319.3	1.16	1616	1278	1.26
Ammoniak	$NH_3$	17.031	488.2	0.76	2056	1526	1.35
Argon	$Ar$	39.948	208.1	1.76	519	309	1.68
Äthan	$C_2H_6$	30.070	276.5	1.34	1650	1355	1.22
Butan	$C_4H_{10}$	58.124	143.0	2.67	1599	1410	1.13
Chlor	$Cl_2$	56.108	117.3	3.17	473	343	1.38
Chlorwasserstoff	$HCl$	70.906	228.0	1.62	795	556	1.43
Helium	$He$	4.003	2077.0	0.18	5200	3124	1.66
Kohlendioxid	$CO_2$	44.010	188.9	1.95	816	618	1.32
Kohlenmonoxid	$CO$	28.010	296.8	1.23	1038	739	1.40
Luft	$\sim$	28.964	287.1	1.28	1006	718	1.40
Methan	$CH_4$	16.043	518.3	0.71	2165	1638	1.32
Propan	$C_3H_8$	44.097	188.5	1.99	1549	1331	1.16
Sauerstoff	$O_2$	31.999	259.8	1.41	909	647	1.40
Stickstoff	$N_2$	28.013	296.8	1.23	1038	739	1.40
Wasserstoff	$H_2$	2.016	4124.2	0.09	14050	9926	1.42
Xenon	$Xe$	131.300	63.3	5.82	159	93	1.71

## 20 Stoffdaten einiger Stoffe

Name	chemische Formel	Molmasse [kg/kmol]	Normal- siedepunkt [°C]	kritische Temperatur [°C]	kritischer Druck [MPa]
Wasserstoff	$H_2$	2.02	-252.9	-240.0	1.32
Helium	$He$	4.00	-268.9	-268.0	0.23
Ammoniak	$NH_3$	17.03	-33.3	132.3	11.33
Wasser	$H_2O$	18.02	100.0	373.9	22.06
Luft	78% $N_2$ 21%	28.96	-194.2	-140.4	3.84
	$O_2$ .1%Ar.+				
Kohlendioxid	$CO_2$	44.01	-78.4	31.0	7.38
Methan	$CH_4$	16.04	-161.5	-82.6	4.60
Äthan	$C_2H_6$	30.07	-88.6	32.2	4.87
Propan	$C_3H_8$	44.10	-42.1	96.7	4.25
R134a	$CH_2FCF_3$	102.03	-26.1	101.1	4.06

## 21 Obskure Zusammenhänge

$$\begin{aligned}
dV &= \left( \frac{\partial V}{\partial T} \right)_p dT + \left( \frac{\partial V}{\partial p} \right)_{T,n} dp + \sum_{k=1}^K \left( \frac{\partial V}{\partial n_k} \right)_{T,p} dn_k \\
dS &= \left( \frac{nC_{p,m}}{T} \right) dT - \left( \frac{\partial V}{\partial T} \right)_{p,n} dp + \sum_{k=1}^K \left( \frac{\partial \mu_k}{\partial T} \right)_{p,n} dn_k \\
dU &= \left[ nC_{p,m} - p \left( \frac{\partial V}{\partial T} \right)_{p,n} \right] dT - \left[ p \left( \frac{\partial V}{\partial p} \right)_{T,n} + T \left( \frac{\partial V}{\partial T} \right)_{p,n} \right] dp + \sum_{k=1}^K \left[ \mu_k - T \left( \frac{\partial \mu_k}{\partial T} \right)_{p,n} - p \left( \frac{\partial V}{\partial n_k} \right)_{T,p,n} \right] dn_k \\
dH &= nC_{p,m} dT + \left[ VT \left( \frac{\partial V}{\partial T} \right)_{p,n} \right] + \sum_{k=1}^K \left[ \mu_k - T \left( \frac{\partial \mu_k}{\partial T} \right)_{p,n} \right] dn_k \\
dF &= - \left[ S + p \left( \frac{\partial V}{\partial T} \right)_{p,n} \right] dT - p \left( \frac{\partial V}{\partial p} \right)_{T,n} dp + \sum_{k=1}^K \left[ \mu_k - p \left( \frac{\partial V}{\partial n_k} \right)_{T,p} \right] dn_k \\
\left( \frac{\partial C_{p,m}}{\partial p} \right)_{T,\psi_j} &= T \frac{\partial}{\partial p} \left[ \left( \frac{\partial S_m}{\partial T} \right)_{p,\psi_j} \right]_{T,\psi_j} = T \frac{\partial}{\partial T} \left[ \left( \frac{\partial S_m}{\partial p} \right)_{T,\psi_j} \right]_{p,\psi_j} = -T \frac{\partial}{\partial T} \left[ \left( \frac{\partial V_m}{\partial T} \right)_{p,\psi_j} \right]_{p,\psi_j} = -T \left( \frac{\partial^2 V_m}{\partial T^2} \right)_{p,\psi_j} \\
C_{p,m} &= (C_{p,m})_{\text{ideales Gas}} - T \int_0^p \left( \frac{\partial^2 V_m}{\partial T^2} \right)_{p,\psi_j} d\tilde{p} \\
C_{v,m} &= (C_{v,m})_{\text{ideales Gas}} - T \int_0^{V_m} \left( \frac{\partial^2 p}{\partial T^2} \right)_{p,\psi_j} d\tilde{V}
\end{aligned}$$