

# Thermodynamik Formelsammlung

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$$\frac{d}{dt} \left\{ U + m \left( \frac{c^2}{2} + gz \right) \right\} = \sum_j \left[ \dot{m}_j \left( h + \frac{c^2}{2} + gz \right) \right]_j + \sum_l (\dot{Q}_t)_l + \sum_i (\dot{W}_t)_i - p \frac{dV}{dt}$$

## 1 Nomenklatur

|   |   |
|---|---|
| <b>A<sub>n</sub></b> = Anergie[J]                                   | <b>V</b> = Volumen[m <sup>3</sup> ]   |
| <b>c<sub>s</sub></b> = Schallgeschwindigkeit[m/s]                   | <b>v</b> = Spezifisches Volumen[m <sup>3</sup> /kg]                         |
| <b>c<sub>p</sub></b> = Spezifische Wärmekapazität dp = 0 [J/kg*K]   | <b>V<sub>m</sub></b> = Molares Volumen[m <sup>3</sup> /mol]                 |
| <b>c<sub>v</sub></b> = Spezifische Wärmekapazität dv = 0 [J/kg*K]   | <b>W</b> = Arbeit[J]  |
| <b>E</b> = Energie[J]   | <b>w</b> = Spezifische Arbeit[J/kg]   |
| <b>Ex</b> = - <b>W<sub>ex</sub></b> = Exergie[J]                    | <b>W<sub>v</sub></b> = Volumenänderungsarbeit[J]                            |
| <b>F</b> = Kraft[N]   | <b>W<sub>el</sub></b> = Elektrische Arbeit[J]                               |
| <b>F</b> = <b>U</b> - <b>TS</b> = Freie Energie[J]                  | <b>W<sub>w</sub></b> = Wellenarbeit[J]                                      |
| <b>f</b> = <b>u</b> - <b>Ts</b> = Spezifische freie Energie[J/kg]   | <b>W<sub>diss</sub></b> = Dissipationsarbeit[J]                             |
| <b>f</b> = Fugazität[Pa]  | <b>W<sub>t</sub></b> = Technische Arbeit[J]                                 |
| <b>G</b> = <b>H</b> - <b>TS</b> = Freie Enthalpie[J]                | <b>W<sub>virrev</sub></b> = Arbeitsverlust durch Irreversibilität[J]        |
| <b>g</b> = <b>h</b> - <b>Ts</b> = Spezifische freie Enthalpie[J/kg] | $x = \frac{m''}{m' + m''}$ = Dampfanteil[-]                                 |
| <b>g</b> = Erdbeschleunigung[m/s <sup>2</sup> ]                     | $x = \frac{m_{H_2O}}{m_L}$ = Wassergehalt                                   |
| <b>H</b> = <b>U</b> + <b>pV</b> = Enthalpie[J]                      | <b>Z</b> = Allgemeine extensive Zustandsgrößen[Z]                           |
| <b>h</b> = <b>u</b> + <b>pv</b> = Spezifische Enthalpie[J/kg]       | <b>z</b> = Allgemeine   |
| <b>■H<sub>g</sub></b> = Molare Reaktionsenthalpie                   | <b>β</b> = Isobarer Ausdehnungskoeffizient[1/K]                             |
| <b>K</b> = Konstante des Massenwirkungsgesetzes[-]                  | <b>γ</b> = Isochorer Spannungskoeffizient[1/K]                              |
| <b>M</b> = Molmasse[kg/mol]   | <b>δ<sub>T</sub></b> = Isothermer Drosselkoeffizient[m <sup>3</sup> /kg]    |
| <b>ṁ</b> = Massestrom[kg/s]   | <b>δ<sub>h</sub></b> = Isenthalper Drosselkoeffizient[Ks <sup>2</sup> m/kg] |
| <b>m'</b> = Masse in der flüssigen Phase[kg]                        | <b>ε</b> = Leistungsziffer[-]   |
| <b>m''</b> = Masse in der gasförmigen Phase[kg]                     | <b>ε</b> = Verdichtungsverhältnis[-]  |
| <b>Ma</b> = <b>c</b> / <b>c<sub>s</sub></b> = Machzahl[-]           | <b>η<sub>th</sub></b> = Thermischer Wirkungsgrad[-]                         |
| <b>n</b> = <b>m</b> / <b>M</b> = Molzahl[mol]                       | <b>η<sub>mech</sub></b> = Mechanischer Wirkungsgrad[-]                      |
| <b>n</b> = Polytropenexponent[-]                                    | <b>κ</b> = Adiabaten- oder Isentropenexponent[-]                            |
| <b>P<sub>t</sub></b> = technische Leistung[W]                       | <b>λ</b> = Reaktionslaufzahl[-]   |
| <b>Q</b> = Wärme[J]   | <b>μ<sub>i</sub></b> = Chemisches Potential[J/mol]                          |
| <b>Q̇</b> = Wärmestrom[W]   | <b>v<sub>i</sub></b> = Stöchiometrische Koeffizienten[-]                    |
| <b>q</b> = Spezifische Wärme[J/kg]                                  | <b>ξ<sub>i</sub></b> = Masseanteil[-]                                       |
| <b>r</b> = Spezifische Verdampfungsenthalpie[J/kg]                  | <b>π</b> = Druckverhältnis[-]   |
| <b>R</b> = Gaskonstante[J/(kg K)]                                   | <b>ρ</b> = Dichte[kg/m <sup>3</sup> ]                                       |
| <b>R<sub>m</sub></b> = Universelle Gaskonstante[J/(mol K)]          | <b>τ</b> = Temperaturverhältnis[-]  |
| <b>S</b> = Entropie[J/K]  | <b>φ</b> = Relative Feuchte[-]  |
| <b>s</b> = Spezifische Entropie[J/(kg K)]                           | <b>φ</b> = Einspritzverhältnis[-]   |
| <b>T</b> = Temperatur[K]  | <b>ξ</b> = Isothermer Kompressibilitätskoeffizient[m <sup>2</sup> /N]       |
| <b>t</b> = Zeit[s]  | <b>■</b> = Dissipationsenergie[J]   |
| <b>t</b> = Temperatur[°C]   | <b>ψ</b> = Spezifische Dissipationsenergie[J]                               |
| <b>T</b> = Sättigungstemperatur[K]                                  | <b>ψ</b> = Drucksteigerungsverhältnis[-]                                    |
| <b>U</b> = Innere Energie[J]  | <b>ψ<sub>i</sub></b> = Molanteil[-]   |
| <b>u</b> = Spezifische innere Energie [J/kg]                        |   |

## 2 Grundbegriffe

### Systeme

- Abgeschlossenes System - kein Stoff oder Energietransport
- Geschlossenes System - kein Stofftransport
- Adiabates System - kein  $\Delta q$ , aber Masse und Arbeit.
- Offenes System - Stoff und Energietransport
- Stationäres System  $\rightarrow \Delta U = 0$

### Messgrößen

- Prozessgrößen sind Wegabhängig (eg. Arbeit, Wärme)
- Zustandsgrößen sind Wegunabhängig (eg. Volumen, Druck)
- Extensive Zustandsgrößen sind abhängig von der Masse des Systems (V, m, H, S, F, G, E)
- Intensive Zustandsgrößen sind unabhängig von der Masse des Systems (T, p)

### Zustandsgleichungen

- Thermisch  $\rightarrow f(p, V, T) = 0$
- Kalorisch  $\rightarrow f(U, V, T) = 0, \quad U = U(V, T), \quad u = u(v, T)$

### Hauptsätze

- 0: Temperatur existiert, ihre Gleichheit ist notwendige Voraussetzung für das thermische Gleichgewicht.
- 1: Energie existiert, sie ist für abgeschlossene Systeme konstant.
- 2: Entropie existiert, sie wird bei allen irreversiblen Prozessen erzeugt.  $dS = \frac{\delta Q_{rev}}{T}$
- 3: 0K existiert, bei dieser Temperatur ist die Entropie = 0

## 3 Basisformeln

$$\begin{aligned}
 H &= U + pV \\
 dS &= \frac{\delta Q_{rev}}{T} \\
 F &= U - TS \\
 G &= H - ST \\
 W &= - \int p dV
 \end{aligned}$$

$$\begin{aligned}
 dS &= \frac{Q_{rev}}{T} + S_{prod} \\
 \Psi &= \int_1^2 T dS_{prod} \\
 W_{ir} &= \frac{T_u}{T} \Psi \\
 p_1 &= p_a + \frac{\varphi_1 - \varphi_a}{\varphi_b - \varphi_a} (p_b - p_a)
 \end{aligned}$$

## 4 Gibbs

$$\begin{aligned}
 dU &= Tds - pdV + \sum_{k=1}^K \mu_k dn_k \\
 dG &= -SdT + Vdp + \sum_{k=1}^K \mu_k dn_k \\
 dH &= TdS + Vdp + \sum_{k=1}^K \mu_k dn_k \\
 dF &= -SdT - pdV + \sum_{k=1}^K \mu_k dn_k \\
 dU &= \left( \frac{\partial U}{\partial S} \right)_V dS + \left( \frac{\partial U}{\partial V} \right)_S dV + \sum_{k=1}^K \left( \frac{\partial U}{\partial n_k} \right)_S dn_k
 \end{aligned}$$

## 5 Thermodynamische Beziehungen

$$\begin{aligned}
 T &= \left( \frac{\partial U}{\partial S} \right)_V = T(S, V) & -S &= \left( \frac{\partial F}{\partial T} \right)_V = S(T, V) \\
 T &= \left( \frac{\partial H}{\partial S} \right)_p = T(S, p) & -S &= \left( \frac{\partial G}{\partial T} \right)_p = S(T, p) \\
 p &= - \left( \frac{\partial U}{\partial V} \right)_S = p(V, S) & V &= \left( \frac{\partial G}{\partial p} \right)_T = V(p, T) \\
 -p &= \left( \frac{\partial F}{\partial V} \right)_T = p(T, V) & \mu &= \left( \frac{\partial U}{\partial n} \right)_{S, V} = \mu(S, V, n)
 \end{aligned}$$

## 6 Guggenheim

$$\begin{array}{ccccc}
 -S & U & V & U & = U(S, V) \\
 H & & F & H & = H(S, p) \\
 -p & G & T & F & = F(T, V) \\
 & & & G & = G(T, p)
 \end{array}$$

$$\overbrace{\frac{d}{dt} \left\{ U + m \left( \frac{c^2}{2} + gz \right) \right\}}^{\text{Stationäres System} \rightarrow 0} = \sum_j \overbrace{\left[ \dot{m}_j \left( h + \frac{c^2}{2} + gz \right) \right]_j}_{\text{Geschlossenes System} \rightarrow 0} + \overbrace{\sum_l (\dot{Q}_t)_l}_{\text{Kein Wärmestrom} \rightarrow 0} + \overbrace{\sum_i (\dot{W}_t)_i}_{\text{Keine Leistung} \rightarrow 0} - \overbrace{p \frac{dV}{dt}}^{\text{Keine Volumenänderung} \rightarrow 0}$$

## 7 Maxwell

$$\left(\frac{\partial T}{\partial p}\right)_{S,n_j} = \left(\frac{\partial V}{\partial S}\right)_{p,n_j}$$

$$\left(\frac{\partial S}{\partial V}\right)_{T,n_j} = \left(\frac{\partial p}{\partial T}\right)_{V,n_j}$$

$$\left(\frac{\partial S}{\partial p}\right)_{T,n_j} = -\left(\frac{\partial V}{\partial T}\right)_{p,n_j}$$

$$\left(\frac{\partial \mu_i}{\partial T}\right)_{p,n_j} = -\left(\frac{\partial S}{\partial n_i}\right)_{T,p,n_j \neq n_i}$$

$$\left(\frac{\partial \mu_i}{\partial p}\right)_{T,n_j} = \left(\frac{\partial V}{\partial n_i}\right)_{T,p,n_j \neq n_i}$$

## 8 Ideales Gas

$$pV = mRT$$

$$pv = RT$$

$$pV = nR_m T$$

$$\beta = \frac{1}{T}$$

$$\gamma = \frac{1}{T}$$

$$\chi = \frac{1}{p}$$

$$\beta = p\gamma\chi$$

$$R_m = 8,3143 \left[ \frac{kJ}{kmolK} \right]$$

$$R = c_p - c_v$$

$$R = \frac{R_m}{M}$$

$$U - U_0 = mc_v(T - T_0)$$

$$H - H_0 = mc_p(T - T_0) \quad \leftarrow \text{Für } c_p \text{ und } c_v \text{ const.}$$

$$s - s_0 = R \ln \left( \frac{v}{v_0} \right) + c_v \ln \left( \frac{T}{T_0} \right)$$

$$= c_v \ln \left( \frac{p}{p_0} \right) + c_p \ln \left( \frac{v}{v_0} \right)$$

$$= c_p \ln \left( \frac{T}{T_0} \right) - R \ln \left( \frac{p}{p_0} \right)$$

$$\beta = \frac{1}{T} = \frac{1}{V} \left( \frac{\partial V}{\partial T} \right)_p = \frac{1}{v} \left( \frac{\partial v}{\partial T} \right)_p = -\frac{1}{p} \left( \frac{\partial p}{\partial T} \right)_p$$

$$\gamma = \frac{1}{T} = \frac{1}{p} \left( \frac{\partial p}{\partial T} \right)_v$$

$$\chi = \frac{1}{p} = -\frac{1}{V} \left( \frac{\partial V}{\partial p} \right)_T = -\frac{1}{v} \left( \frac{\partial v}{\partial p} \right)_T$$

$$u_2 - u_1 = \int_{T_1}^{T_2} c_v(T) dT$$

## 9 Van-der-Waals

$$\left( p + \frac{a}{v^2} \right) (v - b) = RT$$

$$\left( \bar{p} + \frac{3}{\bar{v}^2} \right) (3\bar{v} - 1) = 8\bar{T}$$

$$\bar{p} = \frac{p}{p_K}, \quad \bar{v} = \frac{v}{v_K}, \quad \bar{T} = \frac{T}{T_K}$$

$$p_K = \frac{a}{27b^2}, \quad T_K = \frac{8}{27} \frac{a}{b} \frac{1}{R}, \quad \frac{p_K v_K}{RT_K} = \frac{3}{8}$$

$$a = 3p_K v_K^2, \quad b = \frac{v_K}{3}, \quad \frac{p_K v_K}{RT_K} = \frac{3}{8}$$

$$\beta = \frac{(v - b)Rv^2}{RTv^3 - 2a(v - b)^2}$$

$$\gamma = \frac{Rv^2}{RTv^2 - a(v - b)}$$

$$\chi = \frac{(v - b)^2 v^2}{RTv^3 - 2a(v - b)^2}$$

$$du = \frac{a}{v^2} dv + c_v(T) dT$$

$$u - u_0 = \left( \frac{a}{v_0} - \frac{a}{v} \right) + \int_{T_0}^T c_v(\tilde{T}) d\tilde{T}$$

$$u - u_0 = \left( \frac{a}{v_0} - \frac{a}{v} \right) + c_v(T - T_0) \quad \leftarrow \text{für } c_v = \text{const.}$$

$$c_p - c_v = \frac{Tv\beta^2}{\chi}$$

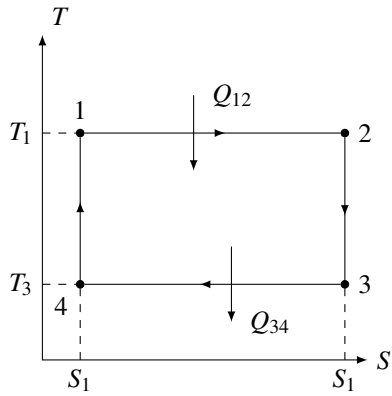
$$s - s_0 = c_v \ln \left( \frac{T}{T_0} \right) + R \ln \left( \frac{v - b}{v_0 - b} \right)$$

## 10 Carnot

$$\eta_{th} = 1 - \frac{-Q_{34}}{Q_{12}} = 1 - \frac{T_3(S_3 - S_4)}{T_1(S_2 - S_1)} = 1 - \frac{T_3}{T_1}$$

$$\frac{Q_{12}}{T_1} + \frac{Q_{34}}{T_3} = 0$$

$$\Delta S_{ges} = -Q_{34} \left( \frac{1}{T_{KK}} - \frac{T_1}{T_3} \frac{1}{T_{HK}} \right)$$



Adiabate Drosselung (ideal):

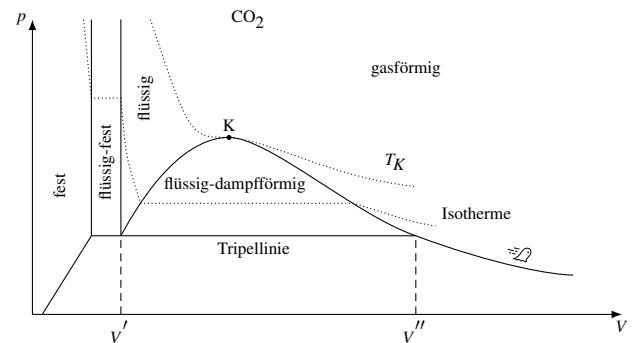
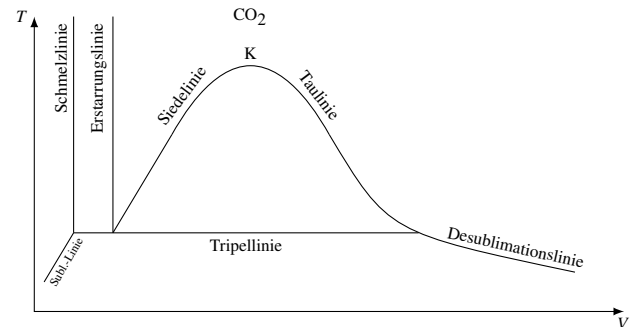
$$h + \frac{c^2}{2} + gz = \text{const.}$$

$$dh = 0$$

Adiabete Drosselung (real):

$$\delta_h = \left( \frac{\partial T}{\partial p} \right)_h = -\frac{v}{c_p} (1 - \beta T)$$

## 12 Nassdampf



## 11 Gemische Idealer Gase

$$\xi_i = \frac{m_i}{m}, \quad \psi_i = \frac{n_i}{n}, \quad p_i = \psi_i p$$

$$\xi_i = \frac{M_i n_i}{\sum_{k=1}^K M_k n_k} = \frac{M_i}{M_G} \psi$$

$$p_i V = m_i R_i T, \quad p_i V = n_i R_m T, \quad p V = m R_G T$$

$$\sum_{k=1}^K p_k = p$$

$$R_G = \frac{1}{m} \sum_{k=1}^K m_k R_k = \sum_{k=1}^K \xi_k R_k$$

$$U_G = \sum_{k=1}^K U_k = \sum_{k=1}^K m_k u_k = \sum_{k=1}^K c_{vk} m_k T \leftarrow c_v = \text{const}$$

$$H_G = \sum_{k=1}^K H_k = \sum_{k=1}^K m_k h_k = \sum_{k=1}^K c_{pk} m_k T \leftarrow c_p = \text{const.}$$

$$c_{vG} = \sum_{k=1}^K c_{vk} \xi_k, \quad c_{pG} = \sum_{k=1}^K c_{pk} \xi_k$$

$$S_2 - S_1 = R_m \left( n \ln n - \sum_{k=1}^K n_k \ln n_k \right)$$

$$v = (1-x)v' + xv''$$

$$v = v' + (v'' - v')x$$

$$u = (1-x)u' + xu''$$

$$u = u' + (u'' - u')x$$

$$h = (1-x)h' + xh''$$

$$h = h' + (h'' - h')x$$

$$s = (1-x)s' + xs''$$

$$s = s' + (s'' - s')x$$

$$r = h'' - h' = T(s'' - s')$$

$$T' = T''$$

$$p' = p''$$

$$g' = g''$$

$$dg' = v' dp' - s' dT'$$

$$dg'' = v'' dp'' - s'' dT''$$

$$dg' = dg''$$

$$\frac{dp}{dT} = \frac{s'' - s'}{v'' - v'}$$

$$\frac{dp}{dT} = \frac{1}{T} \frac{h'' - h'}{v'' - v'}$$

$$\frac{dp}{dT} = \frac{1}{T} \frac{r}{v'' - v'}$$

## 13 Realer Stoff im Nassdampfgebiet

Isobare Zustandsänderung

$$q_{12} = T(s_2 - s_1)$$

$$= T(s'' - s')(x_2 - x_1)$$

$$w_{V,12} = - \int_1^2 p dv$$

$$= -p(v_2 - v_1) = -p(v'' - v')(x_2 - x_1)$$

Isochore Zustandsänderung

$$q_{12} = u_2 - u_1 = u'_2 + x_2(u''_2 - u'_2) - u'_1 - x_1(u''_1 - u'_1)$$

Adiabate Zustandsänderung

$$w_{V,12} = u_2 - u_1 = u'_2 + x_2(u''_2 - u'_2) - u'_1 - x_1(u''_1 - u'_1)$$

Entropieänderung während des Mischvorgangs

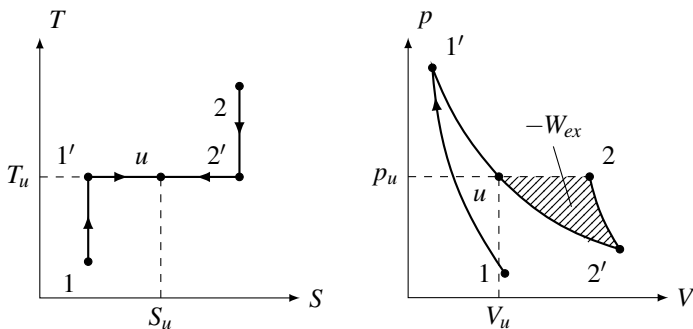
$$S_2 - S_1 = R_m \left( n \ln n - \sum_i n_i \ln n_i \right)$$

## 14 Maximale Arbeit und Exergie

Maximale nutzbare Arbeit → isentrop, reibungsfrei

1 → 1': isentrop auf  $T_u$

1' → u: isotherm auf  $u$



$$-\dot{W}_{ex} = -(\dot{W}_t)_{rev} = -\frac{d}{dt} \left( U + m \left( \frac{c^2}{2} + gz \right) + p_u V - T_u S \right) + \sum_{j=1}^K \left( \dot{m}_j \left( h + \frac{c^2}{2} + gz - T_s \right) \right) + \sum_{l=1}^K \left( 1 - \frac{T_u}{T} \right) \dot{Q}_l$$

Die Exergie der Enthalpie (offenes, stationäres System)

$$-\dot{W}_{ex,1u} = \dot{m}(h_1 - h_u - T_u(s_1 - s_u))$$

Die Exergie der inneren Energie (geschlossenes, instationäres System)

$$-\dot{W}_{ex} = -\frac{d}{dt} (U + p_u V - T_u S)$$

$$-\dot{W}_{ex,1u} = U_1 - U_u - p_u(V_1 - V_u) - T_u(S_1 - S_u)$$

$$-\dot{W}_{ex,1u} = H_1 - (p_1 - p_u)V_1 - H_u - T_u(S_1 - S_u)$$

Für Ideales Gas

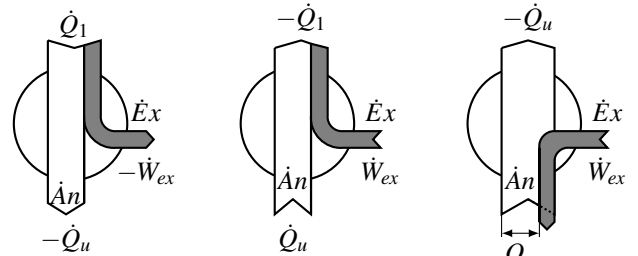
$$-W_{ex} = mc_v(T_1 - T_u) + p_u(V_1 - V_u) - T_u m \left( c_p \ln \left( \frac{T_1}{T_u} \right) - R_i \ln \left( \frac{p_1}{p_u} \right) \right)$$

Dampf/Luftdruckkammer

$$-W_{ex,1u} = m_1[u_1 - u_u + p_u(v_1 - v_u) - T_u(s_1 - s_u)]$$

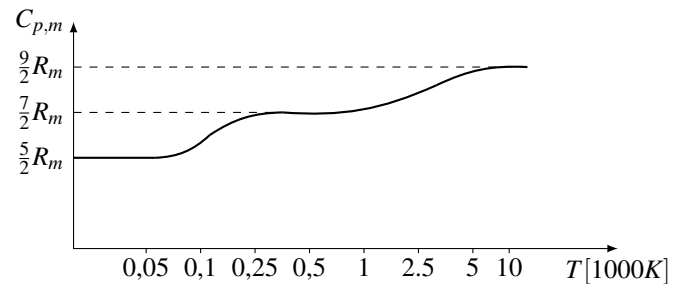
Die Exergie der Wärme (geschlossenes, stationäres System)

$$-\dot{W}_{ex} = \left( 1 - \frac{T_u}{T_1} \right) \dot{Q}_1 = \eta_{th,C} \dot{Q}_1$$



Wärme-Kraft-Prozess    Wärmepumpenprozess    Kälteprozess

## 15 Wärmekapazität



$$C_{v,m} = \frac{1}{\kappa - 1} R_m, \quad C_{p,m} = \frac{\kappa}{\kappa - 1} R_m$$

$$c_v = \frac{1}{\kappa - 1} R_j, \quad c_p = \frac{\kappa}{\kappa - 1} R_j$$

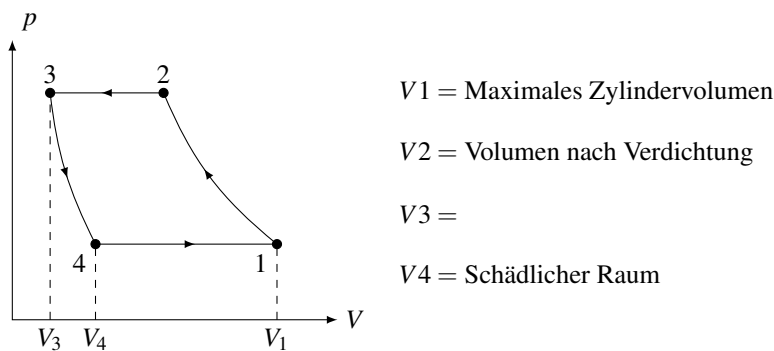
$$\kappa = \frac{c_p}{c_v}, \quad c_p - c_v = R$$

$$C_{v,m} = \underbrace{3 + \frac{R_m}{2}}_{\text{Translatorisch}} + \underbrace{\frac{n_{\text{rot}} R_m}{2}}_{\text{Rotatorisch}} + \underbrace{R_m(3n_{\text{Atome}} - 3 - n_{\text{rot}})}_{\text{Vibratorisch}} + \underbrace{C_{v,m, \text{Elektronenanregung}}}_{\text{Relevant ab: } T \approx 10^4 K}$$

## 16 Technische Anwendung

|  |   |  |
|--|---|--|
| adiabat<br>( $c_p = \text{const.}$ )   | $W_{t,12} = mc_p(T_2 - T_1) = \frac{\kappa}{\kappa - 1}(p_2 V_2 - p_1 V_1)$   | $Q_{12} = 0$   |
| reversibel adiabat<br>$\kappa = \text{const.}$                                   | $W_{t,12} = \frac{\kappa}{\kappa - 1}(p_1 V_1) \left[ \left( \frac{p_2}{p_1} \right)^{\frac{\kappa-1}{\kappa}} - 1 \right]$   | $Q_{12} = 0$   |
| irreversibel adiabat<br>als Polytrope<br>$n > \kappa; n, \kappa = \text{const.}$ | $W_{t,12} = \frac{\kappa}{\kappa - 1}(p_1 V_1) \left[ \left( \frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right]$   | $Q_{12} = 0$   |
| reversibel polytrop<br>$n, \kappa = \text{const.}$                               | $W_{t,12} = \frac{n}{n-1}(p_2 V_2 - p_1 V_1)$<br>$= \frac{n}{n-1}mR(T_2 - T_1)$<br>$= \frac{n}{n-1}(p_1 V_1) \left[ \left( \frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right]$ | $Q_{12} = mc_n(T_2 - T_1)$<br>$= \frac{n-\kappa}{(n-1)(\kappa-1)}(p_1 V_1) \left[ \left( \frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right]$<br>$c_n = \frac{n-\kappa}{n-1}c_v$ |
| isotherm   | $W_{t,12} = (p_1 V_1) \ln \left( \frac{p_2}{p_1} \right)$   | $Q_{12} = -W_{t,12}$   |

### 16.1 Kolbenverdichter



$$\begin{aligned}
 \mu &= \frac{V_1 - V_4}{V_1 - V_3}, & \varepsilon_s &= \frac{V_3}{V_1 - V_3} \\
 \mu &= 1 - \varepsilon_s \left[ \left( \frac{p_2}{p_1} \right)^{\frac{1}{n}} - 1 \right] \\
 W_{t,12} &= \int_1^2 V dp \\
 &= \underbrace{p_2 V_2}_{\text{Ausschiebearbeit}} - \underbrace{p_1 V_1}_{\text{Einschiebearbeit}} - \int_1^2 p dV \\
 &= \frac{n}{n-1} p_1 (V_1 - V_4) \left[ \left( \frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right]
 \end{aligned}$$

Verdichter Wirkungsgrad

$$\eta_{sV} = \frac{w_{t,12,rev}}{w_{t,12}} = \frac{h_{2,rev} - h_1}{h_2 - h_1}$$

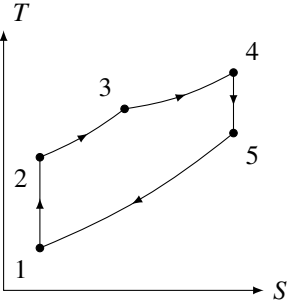
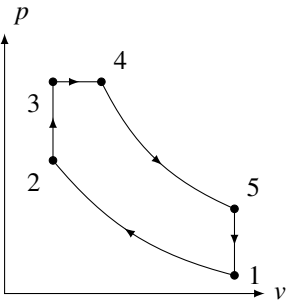
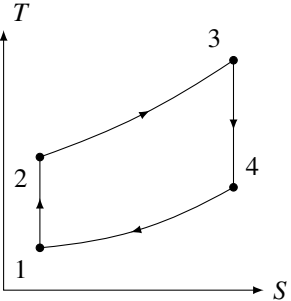
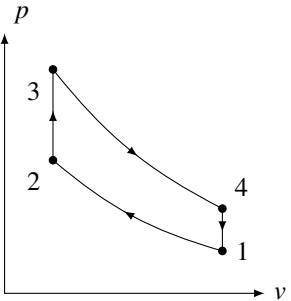
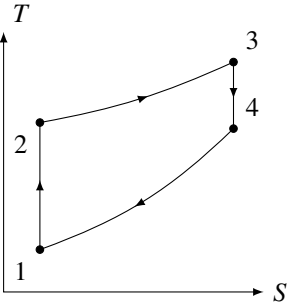
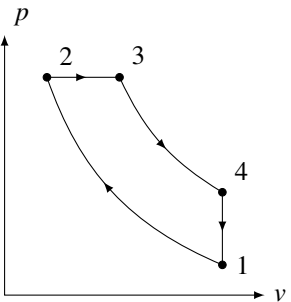
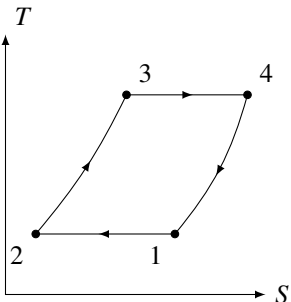
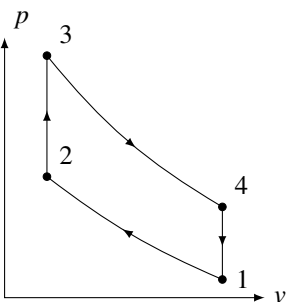
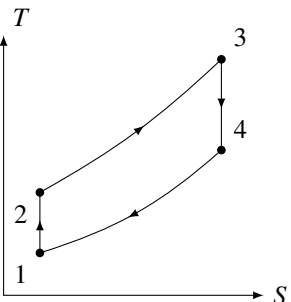
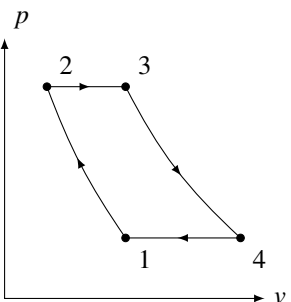
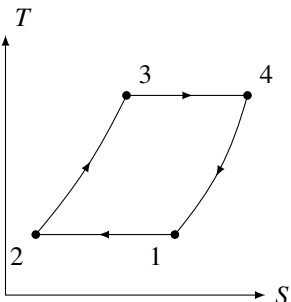
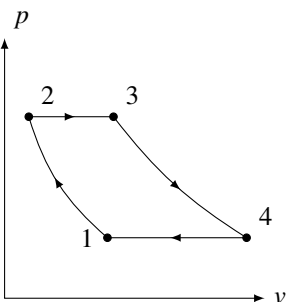
Verdichter wirkungsgrad (Ideales Gas,  $c_p = \text{const.}$ )

$$\eta_{sV} = \frac{T_{2,rev} - T_1}{T_2 - T_1}$$

Technische Verlustarbeit

$$\begin{aligned}
 w_{t,Verl,12} &= w_{t,12} - w_{t,12,rev} = h_2 - h_{2,rev} \\
 &= \int_{2,rev}^2 T|_{p_2=\text{const.}} ds
 \end{aligned}$$



|   |   |   |
|---|---|---|
|     |     | <p>Seiliger Prozess</p> $\eta_{th} = 1 - \frac{\phi^{\kappa} \psi - 1}{\varepsilon^{\kappa-1} [\psi - 1 + \kappa \psi (\phi - 1)]}$ $\varepsilon = \frac{v_1}{v_2} \quad \psi = \frac{p_3}{p_2} \quad \phi = \frac{v_4}{v}$ |
|    |    | <p>Otto Prozess</p> $\eta_{th} = 1 - \frac{1}{\varepsilon^{\kappa-1}}$ $\varepsilon = \frac{v_1}{v_2}$  |
|   |   | <p>Diesel Prozess</p> $\eta_{th} = 1 - \frac{\phi^{\kappa} - 1}{\varepsilon^{\kappa-1} \kappa (\phi - 1)}$ $\varepsilon = \frac{v_1}{v_2} \quad \phi = \frac{v_4}{v}$   |
|  |  | <p>Stirling Prozess</p> $\eta_{th} 1 - \frac{ q_{12} }{q_{34}} = \frac{RT_1 \ln \left( \frac{v_1}{v_2} \right)}{RT_3 \ln \left( \frac{v_4}{v_3} \right)} = 1 - \frac{T_1}{T_3}$   |
|  |  | <p>Joule Prozess</p> $\eta_{th} = 1 - \frac{T_1}{T_2} = 1 - \left( \frac{p_1}{p_2} \right)^{\frac{\kappa-1}{\kappa}} = 1 - \left( \frac{1}{\pi} \right)^{\frac{\kappa-1}{\kappa}}$ $\pi = \frac{p_2}{p_1}$                  |
|  |  | <p>Ericsson Prozess</p> $\eta_{th} 1 - \frac{ q_{12} }{q_{34}} = \frac{RT_1 \ln \left( \frac{p_1}{p_2} \right)}{RT_3 \ln \left( \frac{p_4}{p_3} \right)} = 1 - \frac{T_1}{T_3}$   |

# Ideales Gas

|             | Isothermo  | Isobare                                    | Isochore                                   | Isentrop  | Polytrope   |
|-------------|--|--|--|---|---|
| konstant:   | T  | p  | v  | $\delta q = 0$  | $p v^n$   |
|             | -  | -  | -  | $p_1 v_1^\kappa = p_2 v_2^\kappa$   | $v_1^n = p_2 v_2^n$   |
|             | $p_1 p_2 = p_2 v_2$                                | $\frac{v_1}{v_2} = \frac{T_1}{T_2}$        | $\frac{p_1}{T_1} = \frac{p_2}{T_2}$        | $T_1 v_1^{\kappa-1} = T_2 v_2^{\kappa-1}$   | $T_1 v_1^{n-1} = T_2 v_2^{n-1}$   |
|             | -  | -  | $\neq \mathcal{Q}$                         | $\frac{T_1^{\frac{\kappa-1}{\kappa}}}{p_1} = \frac{T_2^{\frac{\kappa-1}{\kappa}}}{p_2}$               | $\frac{T_1^{\frac{n-1}{n}}}{p_1} = \frac{T_2^{\frac{n-1}{n}}}{p_2}$             |
| $p, v$      | $p = \frac{p_1 v_1}{v}$                            | $p = p_1$                                  | $v = v_1$                                  | $p = \frac{p_1 v_1^\kappa}{v^\kappa}$   | $p = \frac{p_1 v_1^n}{v^n}$   |
| $p, T$      | $p = \frac{p_1 v_1}{v}$                            | $p = p_1$                                  | $p = \frac{p_1}{T_1} T$                    | $p = \frac{p_1^{\frac{\kappa}{\kappa-1}} T^{\frac{\kappa}{\kappa-1}}}{T_1^{\frac{\kappa}{\kappa-1}}}$ | $p = \frac{p_1^{\frac{n}{n-1}} T^{\frac{n}{n-1}}}{T_1^{\frac{n}{n-1}}}$         |
| $v, T$      | $T = T_1$  | $v = \frac{v_1}{T_1} T$                    | $v = v_1$                                  | $T = \frac{T_1 v_1^{\kappa-1}}{v^{\kappa-1}}$   | $T = \frac{T_1 v_1^{n-1}}{v^{n-1}}$   |
| $q_{12}$    | $= p_1 v_1 \ln \frac{p_1}{p_2}$                    | $= c_p (T_2 - T_1)$                        | $= c_v (T_2 - T_1)$                        | $= 0$   | $= c_v \frac{n-\kappa}{n-1} (T_2 - T_1)$  |
| $w_{V,12}$  | $= -q_{12}$  | $= -p_1 (v_2 - v_1)$                       | $= 0$                                      | $= \frac{p_1 v_1}{k-1} \left[ \left( \frac{v_1}{v_1} \right)^{k-1} - 1 \right]$                       | $= \frac{p_1 v_1}{n-1} \left[ \left( \frac{v_1}{v_2} \right)^{n-1} - 1 \right]$ |
| $s_2 - s_1$ | $s_2 - s_1 = R \ln \left( \frac{p_1}{p_2} \right)$ | $= c_p \ln \left( \frac{T_2}{T_1} \right)$ | $= c_v \ln \left( \frac{T_2}{T_1} \right)$ | $= 0$   | $= c_v \frac{n-\kappa}{n-1} \ln \left( \frac{T_2}{T_1} \right)$                 |

# Van-De-der-Waals-Gas

|             | Isotherme  | Isobare  | Isochore  | Isentrop  |
|-------------|--|--|---|---|
| konst.      | T  | p  | v   | $\delta = 0$  |
|             | $(p_1 + \frac{a}{v_1^2})(v_1 - b)$<br>$= (p_2 + \frac{a}{v_2^2})(v_2 - b)$         | $\frac{RT_1}{v_1 - b} - \frac{a}{v_1^2} = \frac{RT_2}{v_2 - b} - \frac{a}{v_2^2}$                | $\frac{p_1 + \frac{a}{v_1^2}}{T_1} = \frac{p_2 + \frac{a}{v_2^2}}{T_2}$ | $(p_1 + \frac{a}{v_1^2})(v_1 - b)^{\frac{c_v + R}{c_v}}$<br>$= (p_2 + \frac{a}{v_2^2})(v_2 - b)^{\frac{c_v + R}{c_v}}$ ,<br>$T_1(v_1 - b)^{R/c_v} = T_2(v_2 - b)^{R/c_v}$ |
| $p, v$      | $p = (p + \frac{a}{v^2})\frac{v_u}{v - b} - \frac{a}{v^2}$                         | $p = p_1$  | $v = v_1$   | $p = -\frac{a}{v^2} + (p_1 + \frac{a}{v^2})\left(\frac{v_1 - b}{v_m}\right)^{\frac{v_v + R}{R}}$  |
| $p, T$      | $T = T_1$  | $p = p_1$  | $p = \frac{T}{T_1}(p_1 + \frac{a}{v^2}) - \frac{a}{v_1^2}$              | $p = -\frac{a}{v^2} + (p_1 + \frac{a}{v^2})\left(\frac{T}{T_1}\right)^{\frac{c_v + R}{R}}$  |
| $v, T$      | $T = T_1$  | $T = T_1 \frac{v - b}{v_1 - b} + \frac{a}{R}(v - b)\left(\frac{1}{v^2} - \frac{1}{v_1^2}\right)$ | $v = v_1$   | $T = T_1 \left(\frac{v_1 - b}{v - b}\right)^{\frac{R}{c_v}}$  |
| $q_{12}$    | $= RT_1 \ln \left(\frac{v_2 - b}{v_1 - b}\right)$                                  | $= \frac{a}{v_1} - \frac{a}{v_2} + c_v(T_2 - T_1) + p_1(v_2 - v_1)$                              | $= c_v(T_2 - T_1)$  | $= 0$   |
| $w_{V,12}$  | $= -RT_1 \ln \left(\frac{v_2 - b}{v_1 - b}\right) + \frac{a}{v_1} - \frac{a}{v_2}$ | $= -p_1(v_2 - v_1)$  | $= 0$   | $= \frac{a}{v_1} - \frac{a}{v_2} + c_v(T_2 - T_1)$  |
| $s_2 - s_1$ | $= R \ln \left(\frac{v_2 - b}{v_1 - b}\right)$                                     | $= c_v \ln \left(\frac{T_2}{T_1}\right) + R \ln \left(\frac{v_2 - b}{v_1 - b}\right)$            | $= c_v \ln \left(\frac{T_2}{T_1}\right)$                                | $= 0$   |