

$$\frac{d}{dt} \left\{ U + m \left(\frac{c^2}{2} + gz \right) \right\} = \sum_j \left[\dot{m}_j \left(h + \frac{c^2}{2} + gz \right) \right]_j + \sum_l (\dot{Q}_t)_l + \sum_i (\dot{W}_t)_i - p \frac{dV}{dt}$$

1 Nomenklatur

A_n = Anergie[J]
c_s = Schallgeschwindigkeit[m/s]
c_p = Spezifische Wärmekapazität dp = 0 [J/kg*K]
c_v = Spezifische Wärmekapazität dv = 0 [J/kg*K]
E = Energie[J]
Ex = -**W_{ex}** = Exergie[J]
F = Kraft[N]
F = **U** - **TS** = Freie Energie[J]
f = **u** - **Ts** = Spezifische freie Energie[J/kg]
f = Fugazität[Pa]
G = **H** - **TS** = Freie Enthalpie[J]
g = **h** - **Ts** = Spezifische freie Enthalpie[J/kg]
g = Erdbeschleunigung[m/s²]
H = **U** + **pV** = Enthalpie[J]
h = **u** + **pv** = Spezifische Enthalpie[J/kg]
■Hg = Molare Reaktionsenthalpie
K = Konstante des Massenwirkungsgesetzes[-]
M = Molmasse[kg/mol]
ṁ = Massestrom[kg/s]
m' = Masse in der flüssigen Phase[kg]
m'' = Masse in der gasförmigen Phase[kg]
Ma = **c**/**c_s** = Machzahl[-]
n = **m**/**M** = Molzahl[mol]
n = Polytropenexponent[-]
P_t = technische Leistung[W]
Q = Wärme[J]
Q̇ = Wärmestrom[W]
q = Spezifische Wärme[J/kg]
r = Spezifische Verdampfungsenthalpie[J/kg]
R = Gaskonstante[J/(kg K)]
R_m = Universelle Gaskonstante[J/(mol K)]
S = Entropie[J/K]
s = Spezifische Entropie[J/(kg K)]
T = Temperatur[K]
t = Zeit[s]
t = Temperatur[°C]
T = Sättigungstemperatur[K]
U = Innere Energie[J]
u = Spezifische innere Energie [J/kg]

V = Volumen[m³]
v = Spezifisches Volumen[m³/kg]
V_m = Molares Volumen[m³/mol]
W = Arbeit[J]
w = Spezifische Arbeit[J/kg]
W_v = Volumenänderungsarbeit[J]
W_{el} = Elektrische Arbeit[J]
W_w = Wellenarbeit[J]
W_{diss} = Dissipationsarbeit[J]
W_t = Technische Arbeit[J]
W_{virrev} = Arbeitsverlust durch Irreversibilität[J]
x = $\frac{m''}{m' + m''}$ = Dampfanteil[-]
x = $\frac{m_{H_2O}}{m_L}$ = Wassergehalt
Z = Allgemeine extensive Zustandsgrößen[Z]
z = Allgemeine
β = Isobarer Ausdehnungskoeffizient[1/K]
γ = Isochorer Spannungskoeffizient[1/K]
δ_T = Isothermer Drosselkoeffizient[m³/kg]
δ_h = Isenthalper Drosselkoeffizient[Ks²m/kg]
ε = Leistungsziffer[-]
ε = Verdichtungsverhältnis[-]
η_{th} = Thermischer Wirkungsgrad[-]
η_{mech} = Mechanischer Wirkungsgrad[-]
κ = Adiabaten- oder Isentropenexponent[-]
λ = Reaktionslaufzahl[-]
μ_i = Chemisches Potential[J/mol]
v_i = Stöchiometrische Koeffizienten[-]
ξ_i = Masseanteil[-]
π = Druckverhältnis[-]
ρ = Dichte[kg/m³]
τ = Temperaturverhältnis[-]
φ = Relative Feuchte[-]
φ = Einspritzverhältnis[-]
ξ = Isothermer Kompressibilitätskoeffizient[m²/N]
■ = Dissipationsenergie[J]
ψ = Spezifische Dissipationsenergie[J]
ψ = Drucksteigerungsverhältnis[-]
ψ_i = Molanteil[-]

2 Grundbegriffe

Systeme

- Abgeschlossenes System - kein Stoff oder Energietransport
- Geschlossenes System - kein Stofftransport
- Adiabates System - kein Wärmetransport, aber asse und arbeit.
- Offenes System - Stoff und Energietransport
- Stationäres System $\rightarrow \Delta U = 0$

Messgrößen

- Prozessgrößen sind Wegabhängig (eg. Arbeit, Wärme)
- Zustandsgrößen sind Wegunabhängig (eg. Volumen, Druck)
- Intensive Zustandsgrößen sind unabhängig von der Masse des Systems (eg. Druck, Temperatur)
- Extensive Zustandsgrößen sind abhängig von der Masse des Systems (eg. Volumen, Masse)

Zustandsgleichungen

- Thermisch $\rightarrow f(p, V, T) = 0$
- Kalorisch $\rightarrow f(U, V, T) = 0, \quad U = U(V, T), \quad u = u(v, T)$

3 Basisformeln

$$H = U + pV, \quad dS = \frac{\delta Q_{rev}}{T}$$

$$F = U - TS, \quad G = H - TS$$

$$W_{V,12} = - \int_1^2 p dV, \quad dS = \frac{Q_{rev}}{T} + S_{prod}$$

$$\Psi_{12} = \int_1^2 T dS_{prod}$$

4 Gibbs-Duhem

$$dU = Tds - pdV + \sum_{k=1}^K \mu_k dn_k$$

$$dG = -SdT + Vdp + \sum_{k=1}^K \mu_k dn_k$$

$$dH = TdS + Vdp + \sum_{k=1}^K \mu_k dn_k$$

$$dF = -SdT - pdV + \sum_{k=1}^K \mu_k dn_k$$

$$dU = \left(\frac{\partial U}{\partial S} \right)_V dS + \left(\frac{\partial U}{\partial V} \right)_S dV + \sum_{k=1}^K \left(\frac{\partial U}{\partial n_k} \right)_S dn_k$$

$$p_1 = p_a + \frac{\varphi_1 - \varphi_a}{\varphi_b - \varphi_a} (p_b - p_a)$$

$$\overbrace{\frac{d}{dt} \left\{ U + m \left(\frac{c^2}{2} + gz \right) \right\}}^{\text{Stationäres System} \rightarrow 0} = \sum_j \overbrace{\left[\dot{m}_j \left(h + \frac{c^2}{2} + gz \right) \right]_j}^{\text{Geschlossenes System} \rightarrow 0} + \overbrace{\sum_l \left(\dot{Q}_l \right)_l}^{\text{Kein Wärmestrom} \rightarrow 0} + \overbrace{\sum_i \left(\dot{W}_i \right)_i}^{\text{Keine Leistung} \rightarrow 0} - \overbrace{p \frac{dV}{dt}}^{\text{Keine Volumenänderung} \rightarrow 0}$$

5 Maxwell

$$\left(\frac{\partial T}{\partial p} \right)_{S, n_j} = \left(\frac{\partial V}{\partial S} \right)_{p, n_j}$$

$$\left(\frac{\partial S}{\partial V} \right)_{T, n_j} = \left(\frac{\partial p}{\partial T} \right)_{V, n_j}$$

$$\left(\frac{\partial S}{\partial p} \right)_{T, n_j} = - \left(\frac{\partial V}{\partial T} \right)_{p, n_j}$$

$$\left(\frac{\partial \mu_i}{\partial T} \right)_{p, n_j} = - \left(\frac{\partial S}{\partial n_i} \right)_{T, p, n_j \neq n_i}$$

$$\left(\frac{\partial \mu_i}{\partial p} \right)_{T, n_j} = \left(\frac{\partial V}{\partial n_i} \right)_{T, p, n_j \neq n_i}$$

6 Guggenheim

$$\begin{array}{lll} -S & U & V \\ H & & F \\ -p & G & T \end{array} \quad \begin{array}{l} U = U(S, V) \\ H = H(S, p) \\ F = F(T, V) \\ G = G(T, p) \end{array}$$

7 Thermodynamische Beziehungen

$$\begin{array}{ll} T = \left(\frac{\partial U}{\partial S} \right)_V & -S = \left(\frac{\partial F}{\partial T} \right)_V \\ T = \left(\frac{\partial H}{\partial S} \right)_p & -S = \left(\frac{\partial G}{\partial T} \right)_p \\ p = - \left(\frac{\partial U}{\partial V} \right)_S & -p = \left(\frac{\partial F}{\partial V} \right)_T \\ \mu = \left(\frac{\partial U}{\partial n} \right)_{S, V} & V = \left(\frac{\partial G}{\partial p} \right)_T \end{array}$$

8 Ideales Gas

$$pV = mRT, \quad pv = RT, \quad pV = nR_m T$$

$$\beta = \frac{1}{T}, \quad \gamma = \frac{1}{T}, \quad \chi = \frac{1}{p}, \quad \beta = p\gamma\chi$$

$$R_m = 8,3143 \left[\frac{kJ}{kmolK} \right], \quad R = c_p - c_v$$

$$R = \frac{R_m}{M}$$

$$U - U_0 = mc_v(T - T_0)$$

$$H - H_0 = mc_p(T - T_0) \quad \leftarrow \text{Für } c_p \text{ und } c_v \text{ const.}$$

$$s - s_0 = R \ln \left(\frac{v}{v_0} \right) + c_v \ln \left(\frac{T}{T_0} \right)$$

$$= c_v \ln \left(\frac{p}{p_0} \right) + c_p \ln \left(\frac{v}{v_0} \right)$$

$$= c_p \ln \left(\frac{T}{T_0} \right) - R \ln \left(\frac{p}{p_0} \right)$$

$$\beta = \frac{1}{T} = \frac{1}{V} \left(\frac{\partial V}{\partial T} \right)_p = \frac{1}{v} \left(\frac{\partial v}{\partial T} \right)_p = -\frac{1}{\rho} \left(\frac{\partial \rho}{\partial T} \right)_p$$

$$\gamma = \frac{1}{T} = \frac{1}{p} \left(\frac{\partial p}{\partial T} \right)_v$$

$$\chi = \frac{1}{p} = -\frac{1}{V} \left(\frac{\partial V}{\partial p} \right)_T = -\frac{1}{v} \left(\frac{\partial v}{\partial p} \right)_T$$

$$u_2 - u_1 = \int_{T_1}^{T_2} c_v(T) dT$$

9 Van-der-Waals

$$\left(p + \frac{a}{v^2} \right) (v - b) = RT$$

$$\left(\bar{p} + \frac{3}{\bar{v}^2} \right) (3\bar{v} - 1) = 8\bar{T}$$

$$\bar{p} = \frac{p}{p_K}, \quad \bar{v} = \frac{v}{v_K}, \quad \bar{T} = \frac{T}{T_K}$$

$$p_K = \frac{a}{27b^2}, \quad T_K = \frac{8}{27} \frac{a}{bR}, \quad \bar{v} = \frac{3}{8}$$

$$a = 3p_K v_K^2, \quad b = \frac{v_K}{3}, \quad \frac{p_K v_K}{RT_K} = \frac{3}{8}$$

$$\beta = \frac{(v - b)Rv^2}{RTv^3 - 2a(v - b)^2}$$

$$\gamma = \frac{Rv^2}{RTv^2 - a(v - b)}$$

$$\chi = \frac{(v - b)^2 v^2}{RTv^3 - 2a(v - b)^2}$$

$$du = \frac{a}{v^2} dv + c_v(T) dT$$

$$u - u_0 = \left(\frac{a}{v_0} - \frac{a}{v} \right) + \int_{T_0}^T c_v(\tilde{T}) d\tilde{T}$$

$$u - u_0 = \left(\frac{a}{v_0} - \frac{a}{v} \right) + c_v(T - T_0) \leftarrow \text{für } c_v = \text{const.}$$

$$c_p - c_v = \frac{Tv\beta^2}{\chi}$$

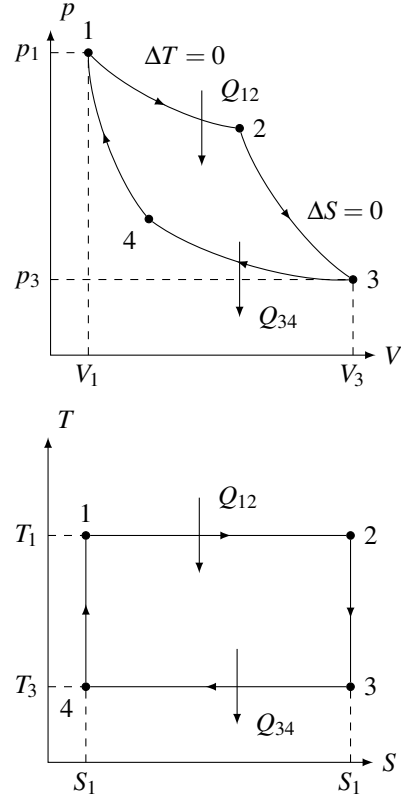
$$s - s_0 = c_v \ln \left(\frac{T}{T_0} \right) + R \ln \left(\frac{v - b}{v_0 - b} \right)$$

10 Carnot

$$\eta_{th} = 1 - \frac{-Q_{34}}{Q_{12}} = 1 - \frac{T_3(S_3 - S_4)}{T_1(S_2 - S_1)} = 1 - \frac{T_3}{T_1}$$

$$\frac{Q_{12}}{T_1} + \frac{Q_{34}}{T_3} = 0$$

$$\Delta S_{ges} = -Q_{34} \left(\frac{1}{T_{KK}} - \frac{T_1}{T_3} \frac{1}{T_{HK}} \right)$$



11 Gemische Idealer Gase

$$\xi_i = \frac{m_i}{m}, \quad \psi_i = \frac{n_i}{n}, \quad p_i = \psi_i p$$

$$\xi_i = \frac{M_i n_i}{\sum_{k=1}^K M_k n_k} = \frac{M_i}{M_G} \psi$$

$$p_i V = m_i R_i T, \quad p_i V = n_i R_m T, \quad pV = m R_G T$$

$$\sum_{k=1}^K p_k = p$$

$$R_G = \frac{1}{m} \sum_{k=1}^K m_k R_k = \sum_{k=1}^K \xi_k R_k$$

$$U_G = \sum_{k=1}^K U_k = \sum_{k=1}^K m_k u_k = \sum_{k=1}^K c_{vk} m_k T \leftarrow c_v = \text{const}$$

$$H_G = \sum_{k=1}^K H_k = \sum_{k=1}^K m_k h_k = \sum_{k=1}^K c_{pk} m_k T \leftarrow c_p = \text{const.}$$

$$c_{vG} = \sum_{k=1}^K c_{vk} \xi_k, \quad c_{pG} = \sum_{k=1}^K c_{pk} \xi_k$$

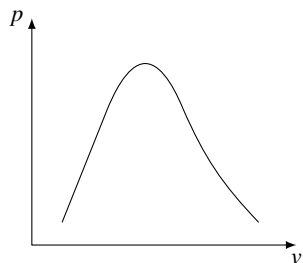
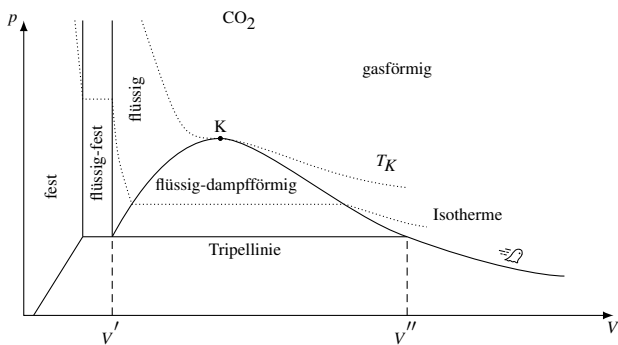
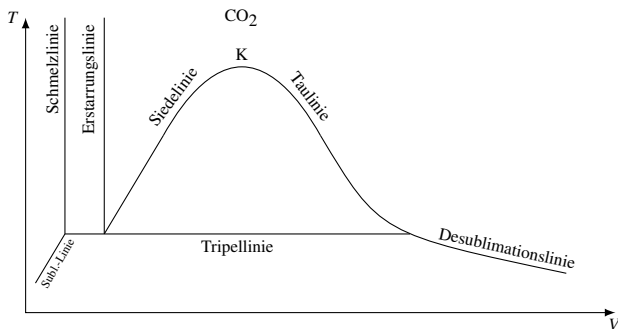
$$S_2 - S_1 = R_m \left(n \ln n - \sum_{k=1}^K n_k \ln n_k \right)$$

13 Realer Stoff im Nassdampfgebiet

Adiabate Drosselung (ideal): $h + \frac{c^2}{2} + gz = \text{const.}$
 $dh = 0$

Adiabete Drosselung (real): $\delta_h = \left(\frac{\partial T}{\partial p} \right)_h = -\frac{v}{c_p} (1 - \beta T)$

12 Nassdampf



$$v = (1-x)v' + xv''$$

$$v = v' + (v'' - v')x$$

$$u = (1-x)u' + xu''$$

$$u = u' + (u'' - u')x$$

$$h = (1-x)h' + xh''$$

$$h = h' + (h'' - h')x$$

$$s = (1-x)s' + xs''$$

$$s = s' + (s'' - s')x$$

$$r = h'' - h' = T(s'' - s')$$

$$T' = T''$$

$$p' = p''$$

$$g' = g''$$

$$dg' = v' dp' - s' dT'$$

$$dg'' = v'' dp'' - s'' dT''$$

$$dg' = dg''$$

$$\frac{dp}{dT} = \frac{s'' - s'}{v'' - v'}$$

$$\frac{dp}{dT} = \frac{1}{T} \frac{h'' - h'}{v'' - v'}$$

$$\frac{dp}{dT} = \frac{1}{T} \frac{r}{v'' - v'}$$

Isobare Zustandsänderung

$$q_{12} = T(s_2 - s_1) = T(s'' - s')(x_2 - x_1)$$

$$w_{V,12} = - \int_1^2 p dv = -p(v_2 - v_1) = -p(v'' - v')(x_2 - x_1)$$

Isochore Zustandsänderung

$$q_{12} = u_2 - u_1 = u'_2 + x_2(u''_2 - u'_2) - u'_1 - x_1(u''_1 - u'_1)$$

Adiabete Zustandsänderung

$$w_{V,12} = u_2 - u_1 = u'_2 + x_2(u''_2 - u'_2) - u'_1 - x_1(u''_1 - u'_1)$$

Entropieänderung während des Mischvorgangs

$$S_2 - S_1 = R_m \left(n \ln n - \sum_i n_i \ln n_i \right) \quad (1)$$

14 Maximale Arbeit und Exergie

$$-\dot{W}_{ex} = -(\dot{W}_t)_{rev} = -\frac{d}{dt} \left(U + m \left(\frac{c^2}{2} + gz \right) + p_u V - T_u S \right) + \sum_{j=1}^K \left(\dot{m}_j \left(h + \frac{c^2}{2} + gz - T_s \right) \right) + \sum_{l=1}^K \left(1 - \frac{T_u}{T} \right) \dot{Q}_l \quad (2)$$

Die Exergie der Enthalpie (offens, stationäres System)

$$-\dot{W}_{ex,1u} = \dot{m}(h_1 - h_u - T_u(s_1 - s_u)) \quad (3)$$

Die Exergie der inneren Energie (geschlossenes, instationäres System)

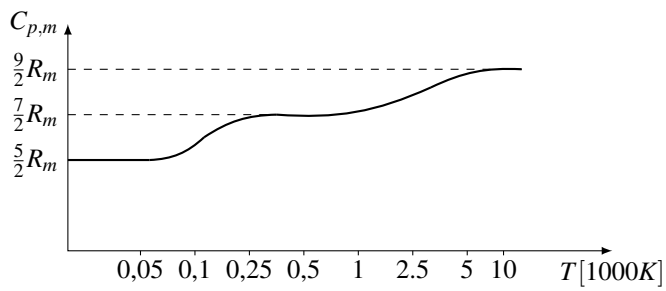
$$-\dot{W}_{ex} = -\frac{d}{dt} (U + p_u V - T_u S) \quad (4)$$

$$-\dot{W}_{ex,1u} = U_1 - U_u - p_u(V_1 - V_u) - T_u(S_1 - S_u) \quad (5)$$

Die Exergie der Wärme (geschlossenes, stationäres System)

$$-\dot{W}_{ex} = \left(1 - \frac{T_u}{T_1} \right) \dot{Q}_1 = \eta_{th,C} \dot{Q}_1 \quad (6)$$

15 Wärmekapazität



$$C_{v,m} = \frac{1}{\kappa - 1} R_m, \quad C_{p,m} = \frac{\kappa}{\kappa - 1} R_m$$

$$c_v = \frac{1}{\kappa - 1} R_j, \quad c_p = \frac{\kappa}{\kappa - 1} R_j$$

$$\kappa = \frac{c_p}{c_v}, \quad c_p - c_v = R$$

$$C_{v,m} = \underbrace{3 + \frac{R_m}{2}}_{\text{Translatorisch}} + \underbrace{\frac{n_{\text{rot}} R_m}{2}}_{\text{Rotatorisch}} + \underbrace{R_M (3n_{\text{Atome}} - 3 - n_{\text{rot}})}_{\text{Vibratorisch}} + \underbrace{C_{v,m, \text{Elektronenanregung}}}_{\text{Relevant ab: } T \approx 10^4 K}$$

Ideales Gas

	Isothermo	Isobare	Isochore	Isentrop	Polytrope
konstant:	T	p	v	$\delta q = 0$	$p v^n$
	-	-	-	$p_1 v_1^\kappa = p_2 v_2^\kappa$	$v_1^n = p_2 v_2^n$
	$p_1 p_2 = p_2 v_2$	$\frac{v_1}{v_2} = \frac{T_1}{T_2}$	$\frac{p_1}{T_1} = \frac{p_2}{T_2}$	$T_1 v_1^{\kappa-1} = T_2 v_2^{\kappa-1}$	$T_1 v_1^{n-1} = T_2 v_2^{n-1}$
	-	-	$\neq \mathcal{Q}$	$\frac{T_1^{\frac{\kappa-1}{\kappa}}}{p_1} = \frac{T_2^{\frac{\kappa-1}{\kappa}}}{p_2}$	$\frac{T_1^{\frac{n-1}{n}}}{p_1} = \frac{T_2^{\frac{n-1}{n}}}{p_2}$
p, v	$p = \frac{p_1 v_1}{v}$	$p = p_1$	$v = v_1$	$p = \frac{p_1 v_1^\kappa}{v^\kappa}$	$p = \frac{p_1 v_1^n}{v^n}$
p, T	$p = \frac{p_1 v_1}{v}$	$p = p_1$	$p = \frac{p_1}{T_1} T$	$p = \frac{p_1^{\frac{\kappa}{\kappa-1}} T^{\frac{\kappa}{\kappa-1}}}{T_1^{\frac{\kappa}{\kappa-1}}}$	$p = \frac{p_1^{\frac{n}{n-1}} T^{\frac{n}{n-1}}}{T_1^{\frac{n}{n-1}}}$
v, T	$T = T_1$	$v = \frac{v_1}{T_1} T$	$v = v_1$	$T = \frac{T_1 v_1^{\kappa-1}}{v^{\kappa-1}}$	$T = \frac{T_1 v_1^{n-1}}{v^{n-1}}$
q_{12}	$= p_1 v_1 \ln \frac{p_1}{p_2}$	$= c_p (T_2 - T_1)$	$= c_v (T_2 - T_1)$	$= 0$	$= c_v \frac{n-\kappa}{n-1} (T_2 - T_1)$
$w_{V,12}$	$= -q_{12}$	$= -p_1 (v_2 - v_1)$	$= 0$	$= \frac{p_1 v_1}{k-1} \left[\left(\frac{v_1}{v_1} \right)^{k-1} - 1 \right]$	$= \frac{p_1 v_1}{n-1} \left[\left(\frac{v_1}{v_2} \right)^{n-1} - 1 \right]$
$s_2 - s_1$	$s_2 - s_1 = R \ln \left(\frac{p_1}{p_2} \right)$	$= c_p \ln \left(\frac{T_2}{T_1} \right)$	$= c_v \ln \left(\frac{T_2}{T_1} \right)$	$= 0$	$= c_v \frac{n-\kappa}{n-1} \ln \left(\frac{T_2}{T_1} \right)$

Van-Der-Waals-Gas

	Isotherme	Isobare	Isochore	Isentrop
konst.	T	p	v	$\delta = 0$
	$(p_1 + \frac{a}{v_1^2})(v_1 - b)$ $= (p_2 + \frac{a}{v_2^2})(v_2 - b)$	$\frac{RT_1}{v_1 - b} - \frac{a}{v_1^2} = \frac{RT_2}{v_2 - b} - \frac{a}{v_2^2}$	$\frac{p_1 + \frac{a}{v_1^2}}{T_1} = \frac{p_2 + \frac{a}{v_2^2}}{T_2}$	$(p_1 + \frac{a}{v_1^2})(v_1 - b)^{\frac{c_v + R}{c_v}}$ $= (p_2 + \frac{a}{v_2^2})(v_2 - b)^{\frac{c_v + R}{c_v}}$, $T_1(v_1 - b)^{R/c_v} = T_2(v_2 - b)^{R/c_v}$
p, v	$p = (p + \frac{a}{v^2})\frac{v_u}{v - b} - \frac{a}{v^2}$	$p = p_1$	$v = v_1$	$p = -\frac{a}{v^2} + (p_1 + \frac{a}{v^2})\left(\frac{v_1 - b}{v_m}\right)^{\frac{v_v + R}{R}}$
p, T	$T = T_1$	$p = p_1$	$p = \frac{T}{T_1}(p_1 + \frac{a}{v^2}) - \frac{a}{v_1^2}$	$p = -\frac{a}{v^2} + (p_1 + \frac{a}{v^2})\left(\frac{T}{T_1}\right)^{\frac{c_v + R}{R}}$
v, T	$T = T_1$	$T = T_1 \frac{v - b}{v_1 - b} + \frac{a}{R}(v - b)\left(\frac{1}{v^2} - \frac{1}{v_1^2}\right)$	$v = v_1$	$T = T_1 \left(\frac{v_1 - b}{v - b}\right)^{\frac{R}{c_v}}$
q_{12}	$= RT_1 \ln \left(\frac{v_2 - b}{v_1 - b}\right)$	$= \frac{a}{v_1} - \frac{a}{v_2} + c_v(T_2 - T_1) + p_1(v_2 - v_1)$	$= c_v(T_2 - T_1)$	$= 0$
$w_{V,12}$	$= -RT_1 \ln \left(\frac{v_2 - b}{v_1 - b}\right) + \frac{a}{v_1} - \frac{a}{v_2}$	$= -p_1(v_2 - v_1)$	$= 0$	$= \frac{a}{v_1} - \frac{a}{v_2} + c_v(T_2 - T_1)$
$s_2 - s_1$	$= R \ln \left(\frac{v_2 - b}{v_1 - b}\right)$	$= c_v \ln \left(\frac{T_2}{T_1}\right) + R \ln \left(\frac{v_2 - b}{v_1 - b}\right)$	$= c_v \ln \left(\frac{T_2}{T_1}\right)$	$= 0$