Thermodynamik Formelsammlung

Jonas Walkling

February 24, 2020

Revision und konzeptionelle Unterstützung: Maximilian Goldapp Fatih Güzel

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$$\frac{d}{dt}\left\{U+m\left(\frac{c^2}{2}+gz\right)\right\} = \sum_{j}\left[\dot{m}_{j}\left(h+\frac{c^2}{2}+gz\right)_{i}\right] + \sum_{l}\left(\dot{Q}_{l}\right)_{l} + \sum_{i}\left(\dot{W}_{l}\right)_{i} - p\frac{dV}{dt}$$

1 Nomenklatur

 $\mathbf{An} = \text{Anergie}[\mathbf{J}]$

 $c_s = Schallgeschwindigkeit[m/s]$

 $\mathbf{c_p} = \text{Spezifische Wärmekapazität dp} = 0 [J/kg*K]$

 $\mathbf{c_v} = \text{Spezifische Wärmekapazität dv} = 0 [J/kg*K]$

 $\mathbf{E} = \text{Energie}[\mathbf{J}]$

 $\mathbf{E}\mathbf{x} = -\mathbf{W}_{\mathbf{e}\mathbf{x}} = \mathrm{Exergie}[\mathbf{J}]$

 $\mathbf{F} = Kraft[N]$

 $\mathbf{F} = \mathbf{U} - \mathbf{TS} = \text{Freie Energie}[\mathbf{J}]$

 $\mathbf{f} = \mathbf{u} - \mathbf{T}\mathbf{s} = \text{Spezifische freie Energie}[J/kg]$

 $\mathbf{f} = \text{Fugazität}[Pa]$

G = H - TS = Freie Enthalpie[J]

 $\mathbf{g} = \mathbf{h} - \mathbf{T}\mathbf{s} = \text{Spezifische freie Enthalpie}[J/kg]$

 $\mathbf{g} = \text{Erdbeschleunigung}[\text{m/s}^2]$

 $\mathbf{H} = \mathbf{U} + \mathbf{pV} = \text{Enthalpie}[\mathbf{J}]$

 $\mathbf{h} = \mathbf{u} + \mathbf{p}\mathbf{v} = \text{Spezifische Enthalpie}[J/kg]$

 Δ **Hg** = Molare Reaktionsenthalpie

K = Konstante des Massenwirkungsgesetztes[-]

 $\mathbf{M} = \text{Molmasse[kg/mol]}$

 $\dot{\mathbf{m}} = \text{Massenstrom}[\text{kg/s}]$

 $\mathbf{m}' = \text{Masse in der flüssigen Phase[kg]}$

 $\mathbf{m}'' = \text{Masse in der gasförmigen Phase[kg]}$

 $Ma = c/c_s = Machzahl[-]$

 $\mathbf{n} = \mathbf{m}/\mathbf{M} = \text{Molzahl[mol]}$

n = Polytropenexponent[-]

 $\mathbf{P_t} = \text{technische Leistung}[\mathbf{W}]$

 $\mathbf{Q} = \text{W\"{a}rme}[J]$

 $\dot{\mathbf{Q}} = \text{Wärmestrom}[\mathbf{W}]$

q = Spezifische Wärme[J/kg]

 $\mathbf{r} = \text{Spezifische Verdampfungsenthalpie}[J/kg]$

 $\mathbf{R} = \text{Gaskonstante}[J/(\text{kg K})]$

 $\mathbf{R}_{\mathbf{m}} = \text{Universelle Gaskonstante}[J/(\text{mol } K)]$

S = Entropie[J/K]

s = Spezifische Entropie[J/(kg K)]

T = Temperatur[K]

 $\mathbf{t} = \text{Zeit}[s]$

 $\mathbf{t} = \text{Temperatur}[^{\circ}\text{C}]$

T = Sättigungstemperatur[K]

U = Innere Energie[J]

 $\mathbf{u} = \text{Spezifische innere Energie [J/kg]}$

 $V = Volumen[m^3]$

 $\mathbf{v} = \text{Spezifisches Volumen}[\text{m}^3/\text{kg}]$

 $V_m = Molares Volumen[m³/mol]$

 $\mathbf{W} = \text{Arbeit}[J]$

 $\mathbf{w} = \text{Spezifische Arbeit}[J/kg]$

 $\mathbf{W}_{\mathbf{V}} = \text{Volumen}$ änderungsarbeit[J]

 $\mathbf{W_{el}} = \text{Elektrische Arbeit[J]}$

 $\mathbf{W}_{\mathbf{w}} = \text{Wellenarbeit}[\mathbf{J}]$

 $W_{diss} = Dissipations arbeit[J]$

 $\mathbf{W_t} = \text{Technische Arbeit}[\mathbf{J}]$

 $\mathbf{W}_{Virrev} = \text{Arbeits verlust durch Irreversibilität}[J]$

 $\mathbf{x} = \frac{m''}{m' + m''} = \text{Dampfanteil[-]}$

 $\mathbf{x} = \frac{m_{H_2O}}{m_L} = \text{Wassergehalt}$

 $\mathbf{Z} = \text{Allgemeine extensive Zustandsgrößen}[\mathbf{Z}]$

z = Allgemeine

 β = Isobarer Ausdehnungskoeffizient[1/K]

 γ = Isochorer Spannungskoeffizient[1/K]

 $\delta_{\rm T} = {\rm Isothermer\ Drosselkoeffizient[m^3/kg]}$

 $\delta_{\mathbf{h}} = \text{Isenthalper Drosselkoeffizient}[\text{Ks}^2\text{m/kg}]$

 ε = Leistungsziffer[-]

 $\varepsilon = \text{Verdichtungsverhältnis}[-]$

 $\eta_{\rm th} = \text{Thermischer Wirkungsgrad}[-]$

 $\eta_{\text{mech}} = \text{Mechanischer Wirkungsgrad[-]}$

 $\kappa = \text{Adiabaten- oder Isentropenexponent}[-]$

 $\lambda = \text{Reaktionslaufzahl}[-]$

 μ_i = Chemisches Potential[J/mol]

 v_i = Stöchiometrische Koeffizienten[-]

 $\xi_{\mathbf{i}} = \text{Masseanteil}[-]$

 $\pi = \text{Druckverhältnis}[-]$

 $\rho = \text{Dichte}[\text{kg/m}^3]$

 $\tau =$ Temperaturverhältnis[-]

 ϕ = Relative Feuchte[-]

 $\phi = \text{Einspritzverhältnis}[-]$

 ξ = Isothermer Kompressibilitätskoeffizient[m²/N]

 $\Psi = \text{Dissipationsenergie}[J]$

 $\psi = \text{Spezifische Dissipationsenergie}[J]$

 ψ = Drucksteigerungsverhältnis[-]

 $\psi_i = Molanteil[-]$

2 Grundbegriffe

Systeme

- Abgeschlossenes System kein Stoff oder Energietransport
- Geschlossenes System kein Stofftransport
- Adiabates System kein Δq , aber Masse und Arbeit.
- Offenes System Stoff und Energietransport
- Stationäres System $\rightarrow \Delta U = 0$

Messgrößen

- Prozessgrößen sind wegabhängig (eg. Arbeit, Wärme)
- Zustandsgrößen sind wegunabhängig (eg. Volumen, Druck)
- Extensive Zustandsgrößen sind abhängig von der Masse des Systems (V, m, H, S, F, G, E)
- Intensive Zustandsgrößen sind unabhängig von der Masse des Systems (T, p)

Zustandsgleichungen

- Thermisch $\rightarrow f(p, V, T) = 0$
- Kalorisch $\rightarrow f(U, V, T) = 0$, U = U(V, T), u = u(v, T)

Hauptsätze

- 0: Temperatur existiert, ihre Gleichheit ist notwendige Voraussetzung für das thermische Gleichgewicht von zwei Systemen
- 1: Energie existiert, sie ist für abgeschlossene Systeme konstant.
- 2: Entropie existiert, sie wird bei allen irreversiblen Prozessen erzeugt. $dS = \frac{\delta Q_{rev}}{T}$
- 3: 0K existiert, bei dieser Temperatur ist die Entropie = 0

3 Basisformeln

$$H = U + pV$$

$$dS = \frac{Q_{rev}}{T} + S_{prod}$$

$$dS = \frac{\Phi}{T}$$

$$F = U - TS$$

$$G = H - ST$$

$$W = -\int p \, dV$$

$$dU = mc_v dT$$

$$m = \rho \cdot V$$

$$dS_{prod} = \frac{\Psi}{T}$$

$$\Psi = \int_{1}^{2} T \, dS_{prod}$$

$$W_{ir} = \frac{T_u}{T} \Psi$$

$$W_{V,ir} = T_U \cdot S_{prod}$$

$$p_1 = p_a + \frac{\varphi_1 - \varphi_a}{\varphi_b - \varphi_a} (p_b - p_a)$$

4 Iso



5 Gibbs

$$dU = TdS - pdV + \sum_{k=1}^{K} \mu_k dn_k$$

$$dG = -SdT + Vdp + \sum_{k=1}^{K} \mu_k dn_k$$

$$dH = TdS + Vdp + \sum_{k=1}^{K} \mu_k dn_k$$

$$dF = -SdT - pdV + \sum_{k=1}^{K} \mu_k dn_k$$

$$dU = \left(\frac{\partial U}{\partial S}\right)_V dS + \left(\frac{\partial U}{\partial V}\right)_S dV + \sum_{k=1}^{K} \left(\frac{\partial U}{\partial n_k}\right)_S dn_k$$

6 Thermodynamische Beziehungen

$$T = \left(\frac{\partial U}{\partial S}\right)_{V} = T(S, V) \qquad -S = \left(\frac{\partial F}{\partial T}\right)_{V} = S(T, V)$$

$$T = \left(\frac{\partial H}{\partial S}\right)_{p} = T(S, p) \qquad -S = \left(\frac{\partial G}{\partial T}\right)_{p} = S(T, p)$$

$$p = -\left(\frac{\partial U}{\partial V}\right)_{S} = p(V, S) \qquad V = \left(\frac{\partial G}{\partial p}\right)_{T} = V(p, T)$$

$$-p = \left(\frac{\partial F}{\partial V}\right)_{T} = p(T, V) \qquad \mu = \left(\frac{\partial U}{\partial n}\right)_{S, V} = \mu(S, V, n)$$

7 Guggenheim

$$\underbrace{\frac{d}{dt} \left\{ U + m \left(\frac{c^2}{2} + gz \right) \right\}}_{j} = \underbrace{\sum_{j} \left[\dot{m}_{j} \left(h + \frac{c^2}{2} + gz \right)_{j} \right]}_{Geschlossenes System -> 0} + \underbrace{\sum_{l} \left(\dot{Q}_{t} \right)_{l}}_{Kein Wärmestrom -> 0} + \underbrace{\sum_{l} \left(\dot{W}_{t} \right)_{i}}_{Keine Leistung -> 0} - \underbrace{\sum_{l} \left(\dot{W}_{t} \right)_{i}}_{Keine Volumenänderung -> 0} + \underbrace{\sum_{l} \left(\dot{W}_{t} \right)_{i}}_{Keine$$

8 Maxwell

$$\left(\frac{\partial T}{\partial p}\right)_{S,n_j} = \left(\frac{\partial V}{\partial S}\right)_{p,n_j} \\
\left(\frac{\partial S}{\partial V}\right)_{T,n_j} = \left(\frac{\partial p}{\partial T}\right)_{V,n_j} \\
\left(\frac{\partial S}{\partial p}\right)_{T,n_j} = -\left(\frac{\partial V}{\partial T}\right)_{p,n_j} \\
\left(\frac{\partial \mu_i}{\partial T}\right)_{p,n_j} = -\left(\frac{\partial S}{\partial n_i}\right)_{T,p,n_j \neq n_i} \\
\left(\frac{\partial \mu_i}{\partial p}\right)_{T,n_i} = \left(\frac{\partial V}{\partial n_i}\right)_{T,p,n_i \neq n_i} \\$$

9 Ideales Gas

pV = mRT

$$pv = RT$$

$$pV = nR_mT$$

$$\beta = \frac{1}{T}$$

$$\gamma = \frac{1}{T}$$

$$\chi = \frac{1}{p}$$

$$\beta = p\gamma\chi$$

$$R_m = 8,3143 \left[\frac{kJ}{kmolK} \right]$$

$$R = c_p - c_v$$

$$R = \frac{R_m}{M}$$

$$U - U_0 = mc_v(T - T_0) \quad \leftarrow \text{Für } c_p \text{ und } c_v \text{ const.}$$

$$s - s_0 = R \ln \left(\frac{v}{v_0} \right) \quad + c_v \ln \left(\frac{T}{T_0} \right)$$

$$= c_v \ln \left(\frac{p}{p_0} \right) \quad + c_p \ln \left(\frac{v}{v_0} \right)$$

$$= c_p \ln \left(\frac{T}{T_0} \right) \quad - R \ln \left(\frac{p}{p_0} \right)$$

$$\beta = \frac{1}{T} = \frac{1}{V} \left(\frac{\partial V}{\partial T} \right)_p = \frac{1}{V} \left(\frac{\partial v}{\partial T} \right)_p = -\frac{1}{\rho} \left(\frac{\partial \rho}{\partial T} \right)_p$$

$$\gamma = \frac{1}{T} = \frac{1}{p} \left(\frac{\partial P}{\partial T} \right)_V$$

$$\chi = \frac{1}{p} = -\frac{1}{V} \left(\frac{\partial V}{\partial p} \right)_T = -\frac{1}{V} \left(\frac{\partial V}{\partial p} \right)_T$$

$$u_2 - u_1 = \int_{T_1}^{T_2} c_v(T) dT$$

$$U_2 - U_1 = Q_{12} + W_{V,12}$$

10 Van-der-Waals

$$\left(p + \frac{a}{v^2}\right)(v - b) = RT$$

$$\left(\overline{p} + \frac{3}{\overline{v}^2}\right)(3\overline{v} - 1) = 8\overline{T}$$

$$\overline{p} = \frac{p}{p_K}, \quad \overline{v} = \frac{v}{v_K}, \quad \overline{T} = \frac{T}{T_K}$$

$$p_K = \frac{a}{27b^2}, \quad T_K = \frac{8}{27} \frac{a}{b} \frac{1}{R},$$

$$a = 3p_K v_K^2, \quad b = \frac{v_K}{3}, \quad \frac{p_K v_K}{RT_K} = \frac{3}{8}$$

$$\beta = \frac{(v - b)Rv^2}{RTv^3 - 2a(v - b)^2}$$

$$\gamma = \frac{Rv^2}{RTv^3 - 2a(v - b)^2}$$

$$\chi = \frac{(v - b)^2 v^2}{RTv^3 - 2a(v - b)^2}$$

$$du = \frac{a}{v^2} dv + c_v(T) dT$$

$$u - u_0 = \left(\frac{a}{v_0} - \frac{a}{v}\right) + \int_{T_0}^T c_v(\tilde{T}) d\tilde{T}$$

$$u - u_0 = \left(\frac{a}{v_0} - \frac{a}{v}\right) + c_v(T - T_0) \leftarrow \text{für } c_v = \text{const.}$$

$$c_p - c_v = \frac{Tv\beta^2}{\chi}$$

$$s - s_0 = c_v \ln\left(\frac{T}{T_0}\right) + R\ln\left(\frac{v - b}{v_0 - b}\right)$$

$$h_2 - h_1 = \frac{1}{2}(p_2 - p_1)\left(\frac{1}{\rho_1} + \frac{1}{\rho_2}\right)$$

11 Drosselung



$$h + \frac{c^2}{2} + gz = \text{const.}$$

$$dh = 0, \quad T_1 = T_2$$

$$\delta_h = \left(\frac{\partial T}{\partial p}\right)_h = -\frac{v}{c_p}(1 - \beta T)$$

$$\delta_T = \left(\frac{\partial h}{\partial p}\right)_T$$

$$s_2 - s_1 = R \ln\left(\frac{v_2}{v_1}\right) = R \ln\left(\frac{p_1}{p_2}\right)$$

$$\mu_{J-T} = \left(\frac{\partial H}{\partial p}\right)_H \approx \frac{\frac{2a}{RT} - b}{c_{p,m}}$$

$$T_i = \frac{2a}{Rb}$$

12 Carnot

$$\begin{split} &\eta_{th} = 1 - \frac{-Q_{34}}{Q_{12}} = 1 - \frac{T_3(S_3 - S_4)}{T_1(S_2 - S_1)} = 1 - \frac{T_1}{T_3} \\ &\frac{Q_{12}}{T_1} + \frac{Q_{34}}{T_3} = 0 \\ &\Delta S_{ges} = -Q_{34} \left(\frac{1}{T_{KK}} - \frac{T_1}{T_3} \frac{1}{T_{HK}} \right) \end{split}$$





13 Gemische Idealer Gase

$$\begin{split} \xi_{i} &= \frac{m_{i}}{m}, \quad \psi_{i} = \frac{n_{i}}{n}, \quad p_{i} = \psi_{i}p \\ \xi_{i} &= \frac{M_{i}n_{i}}{\sum_{k=1}^{K} M_{k}n_{k}} = \frac{M_{i}}{M_{G}} \psi \\ p_{i}V &= m_{i}R_{i}T, \quad p_{i}V = n_{i}R_{m}T, \quad pV = mR_{G}T \\ \sum_{k=1}^{K} p_{k} &= p \\ R_{G} &= \frac{1}{m} \sum_{k=1}^{K} m_{k}R_{k} = \sum_{k=1}^{K} \xi_{k}R_{k} \\ U_{G} &= \sum_{k=1}^{K} U_{k} = \sum_{k=1}^{K} m_{k}u_{k} = \sum_{k=1}^{K} c_{vk}m_{k}T \leftarrow c_{v} = \text{const} \\ H_{G} &= \sum_{k=1}^{K} H_{k} = \sum_{k=1}^{K} m_{k}h_{k} = \sum_{k=1}^{K} c_{pk}m_{k}T \leftarrow c_{p} = \text{const.} \\ c_{vG} &= \sum_{k=1}^{K} c_{vk}\xi_{k}, \quad c_{pG} &= \sum_{k=1}^{K} c_{pk}\xi_{k} \\ S_{2} - S_{1} &= R_{m} \left(n \ln n - \sum_{k=1}^{K} n_{k} \ln n_{k} \right) \end{split}$$

14 Nassdampf













$$\frac{dp}{dT} = \frac{s'' - s'}{v'' - v'} = \frac{1}{T} \frac{h'' - h'}{v'' - v'}$$

$$r = h'' - h' = T(s'' - s')$$

$$v = (1 - x)v' + xv''$$

$$v = v' + (v'' - v')x$$

$$u = (1 - x)u' + xu''$$

$$u = u' + (u'' - u')x$$

$$h = (1 - x)h' + xh''$$

$$h = h' + (h'' - h')x$$

$$s = (1 - x)s' + xs''$$

$$s = s' + (s'' - s')x$$

$$\frac{dp}{dT} = \frac{1}{T} \frac{r}{v'' - v'}$$

$$F = K + 2 - P$$

$$T' = T''$$

$$p' = p''$$

$$g' = g''$$

$$dg' = v'dp' - s'dT'$$

$$dg'' = v''dp'' - s''dT''$$

$$dg' = dg''$$

15 Realer Stoff im Nassdampfgebiet

Isobare Zustandsänderung

$$q_{12} = T(s_2 - s_1)$$

$$= T(s'' - s')(x_2 - x_1)$$

$$w_{V,12} = -\int_1^2 p \, dv$$

$$= -p(v_2 - v_1) = -p(v'' - v')(x_2 - x_1)$$

Isochore Zustandsänderung

$$q_{12} = u_2 - u_1 = u'_2 + x_2 \left(u''_2 - u'_2 \right) - u'_1 - x_1 \left(u''_1 - u'_1 \right)$$

Adiabate Zustandsänderung

$$w_{V,12} = u_2 - u_1 = u'_2 + x_2 \left(u''_2 - u'_2 \right) - u'_1 - x_1 \left(u''_1 - u'_1 \right)$$

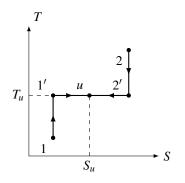
Entropie Änderung während des Mischvorgangs

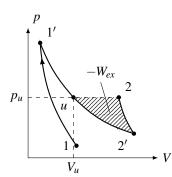
$$S_2 - S_1 = R_m \left(n \ln n - \sum_i n_i \ln n_i \right)$$

16 Maximale Arbeit und Exergie

Maximal nutzbare Arbeit → isentrop, reibungsfrei

 $1 \rightarrow 1'$: isentrop auf T_u $1' \rightarrow u$: isotherm auf u





$$-\dot{W}_{ex} = -(\dot{W}_{t})_{rev} = -\frac{d}{dt} \left(U + m \left(\frac{c^{2}}{2} + gz \right) + p_{u}V - T_{u}S \right) + \sum_{i=1}^{K} \left(\dot{m}_{j} \left(h + \frac{c^{2}}{2} + gz - T_{s} \right) \right) + \sum_{l=1}^{K} \left(1 - \frac{T_{u}}{T} \right) \dot{Q}_{l}$$

Die Exergie der Enthalpie (offenes, stationäres System)

$$-\dot{W}_{ex,1u} = \dot{m}(h_1 - h_u - T_u(s_1 - s_u))$$

Die Exergie der inneren Energie (geschlossenes, instationäres System)

$$\begin{split} -\dot{W}_{ex} &= -\frac{d}{dt}(U + p_u V - T_u S) \\ -\dot{W}_{ex,1u} &= U_1 - U_u - p_u (V_1 - V_u) - T_u (S_1 - S_u) \\ -\dot{W}_{ex,1u} &= H_1 - (p_1 - p_u) V_1 - H_u - T_u (S_1 - S_u) \end{split}$$

Für Ideales Gas

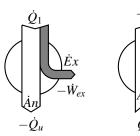
$$\begin{split} -W_{ex} &= mc_v(T_1 - T_u) + p_u(V_1 - V_u) - T_u m \left(c_p \ln\left(\frac{T_1}{T_u}\right) - R_i \ln\left(\frac{p_1}{p_u}\right)\right) \\ -W_{ex} &= m \left[c_p(T_1 - T_u) - T_u c_p \ln\left(\frac{T_1}{T_u}\right)\right] \leftarrow \text{isobar} \end{split}$$

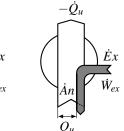
Dampf/Luftdruckkammer

$$-W_{ex,1u} = m_1[u_1 - u_u + p_u(v_1 - v_u) - T_u(s_1 - s_u)]$$

Die Exergie der Wärme (geschlossenes, stationäres System)

$$-\dot{W}_{ex} = \left(1 - \frac{T_u}{T_1}\right)\dot{Q}_1 = \eta_{th,C}\dot{Q}_1$$

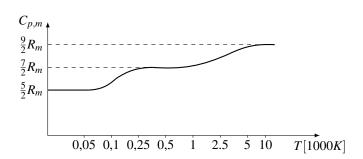




WärmekraftprozessWärmepumpenprozess

Kälteprozess

17 Wärmekapazität



$$C_{v,m} = \frac{1}{\kappa - 1} R_m \qquad C_{p,m} = \frac{\kappa}{\kappa - 1} R_m$$

$$c_v = \frac{1}{\kappa - 1} R_j \qquad c_p = \frac{\kappa}{\kappa - 1} R_j$$

$$\kappa = \frac{c_p}{c_v} \qquad R = c_p - c_v$$

$$R = \frac{R_m}{M} \qquad R_m = 8,3143 \left[\frac{kJ}{kmolK} \right]$$

$$C_{m,v} = rac{f}{2}R_m = rac{f_{trans} + f_{rot} + f_{vib}}{2}R_m$$
 $C_{m,p} = rac{f+2}{2}R$
 $\kappa = rac{f+2}{f}$

 $f_{trans} = 3$ (für die 3 Translatorischen Freiheitsgrade) $f_{rot} \in \{0,2,3\}$ {Einatomig, Linear, Verzweigt} $f_{vib} = 2 \cdot l$, l = 1 Normalschwingungen der Atomkerne (Kann für komplexere Moleküle auch > 1 sein.)

18 Technische Anwendung

adiabat
$$(c_p = const.)$$
 $W_{t,12} = mc_p(T_2 - T_1) = \frac{\kappa}{\kappa - 1}(p_2V_2 - p_1V_1)$ $Q_{12} = 0$

reversibel adiabat $\kappa = const.$ $W_{t,12} = \frac{\kappa}{\kappa - 1}(p_1V_1) \left[\left(\frac{p_2}{p_1} \right)^{\frac{\kappa - 1}{\kappa}} - 1 \right]$ $Q_{12} = 0$

irreversibel adiabat als Polytrope $n > \kappa; n, \kappa = const.$ $W_{t,12} = \frac{\kappa}{\kappa - 1}(p_1V_1) \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right]$ $Q_{12} = 0$

reversibel polytrop $n, \kappa = const.$ $W_{t,12} = \frac{n}{n-1}(p_2V_2 - p_1V_1)$ $Q_{12} = mc_n(T_2 - T_1)$

$$= \frac{n}{n-1}mR(T_2 - T_1)$$

$$= \frac{n-\kappa}{(n-1)(\kappa-1)}(p_1V_1) \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right]$$

$$= \frac{n}{n-1}(p_1V_1) \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right]$$
 $c_n = \frac{n-\kappa}{n-1}cv$

isotherm $W_{t,12} = (p_1V_1) \ln \left(\frac{p_2}{p_1} \right)$ $Q_{12} = -W_{t,12}$

Thermischer Wirkungsgrad
$$\eta_{th} = \frac{-w}{q_{zu}} = \frac{\text{Nutzen}}{\text{Aufwand}} = 1 - \frac{|q_{ab}|}{q_{zu}}$$

Isentroper Verdichterwirkungsgrad $\eta_{tV} = \frac{w_{t,12,rev}}{w_{t,12}} = \frac{h_{2,rev} - h_1}{h_2 - h_1} = \frac{T_{2,rev} - T_1}{T_2 - T_1}$

idealer Fall

Isentroper Turbinenwirkungsgrad $\eta_{tT} = \frac{w_{t,12}}{w_{t,12,rev}} = \frac{h_1 - h_2}{h_1 - h_{2,rev}} = \frac{T_1 - T_2}{T_1 - T_{2,rev}}$

Dampfkraftprozess Wirkungsgrad $\eta_{th} = 1 - \frac{|q_{61}|}{q_{23} + q_{34} + q_{45}} = 1 - \frac{h_6 - h_1}{h_5 - h_2}$

Leistungszahl Kältemaschine $\varepsilon_{K(A)} = \frac{q_{0u}}{w} = \frac{\dot{Q}_0}{\dot{W}}$

Leistungszahl Kaltdampfprozess $\varepsilon_{K} = \frac{q_0}{|q| - q_0} = \frac{q_0}{w_t} = \frac{\dot{h}_1 - h_6}{\dot{h}_2 - h_1}$

Linkslaufender Carnotprozess $\varepsilon_{WP} = \frac{q}{|q| - q_0} = \frac{|q_0|}{w_t} = \frac{\dot{q}_{ab}}{\dot{w}} = \frac{\dot{h}_2 - \dot{h}_5}{\dot{h}_2 - \dot{h}_1} = 1 + \varepsilon_{K(A)}$

Kälteleistung Wärmepumpe $\dot{Q}_0 = \dot{m}(h_2 - h_5)$

Leistungszahl Kaltluftprozess $\dot{Q}_0 = \dot{m}(h_1 - h_6)$

Arbeit der Enthalpie $W_t = Q = mdh = mcpdT$

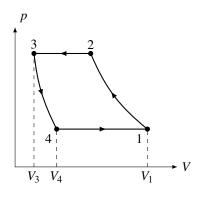
Verdichtungsverhältnis $\psi = p_3/p_2$

Einspritzverhältniss $\psi = v_1/v_2$

Temperaturverhältnis $\tau = q_2/p_1$

Für Joule-Prozess $\pi_{opt} = \tau^{\chi_{tC-1}}$

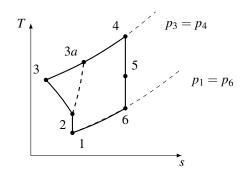
19 Kolbenverdichter



$$\mu = \frac{V_1 - V_4}{V_1 - V_3}, \qquad \varepsilon_S = \frac{V_3}{V_1 - V_3}, \quad \mu = 1 - \varepsilon_S \left[\left(\frac{p_2}{p_1} \right)^{\frac{1}{n}} - 1 \right]$$

$$W_t = \frac{n}{n - 1} p_1 (V_1 - V_4) \left[\left(\frac{p_2}{p_1} \right)^{\frac{n - 1}{n}} - \right]$$

20 Strahltriebwerk

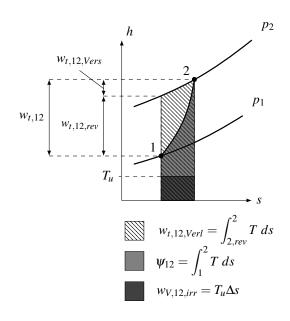


Anströmgeschwindigkeit:

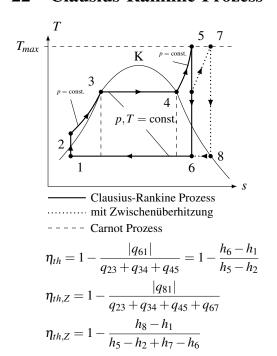
$$\frac{1}{2}c_1^2 = c_p(T_2 - T_1)$$

$$w_{2-3} = -w_{4-5} = c_p(T_3 - T_2)$$

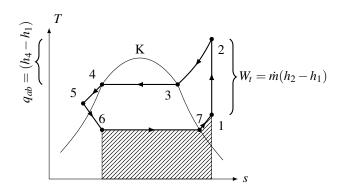
21 Turboverdichter

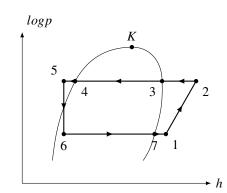


22 Clausius-Rankine-Prozess

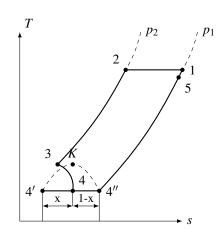


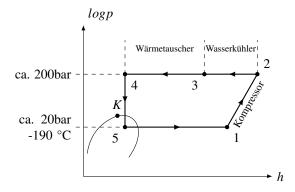
23 Kaltdampfprozess





24 Luftverflüssigung nach Linde





$$h_2 = (q - x)h_{4'} + xh_5$$

$$(1 - x) = \frac{h_5 - h_2}{h_5 - h_{4'}} \le \frac{h_1 - h_2}{h_1 - h_{4'}} \left[\frac{\text{kg Flüssigkeit}}{\text{kg Ansaugluft}} \right]$$

25 Feuchte Luft

$$x = \frac{m_{H_2O}}{m_L}$$

$$x = x_{D(ampf)} + x_{W(asser)} + x_{E(is)}$$

$$\varphi = \frac{p_D}{p_s}$$

$$x_D = \frac{m_D}{m_L} = \frac{R_L}{R_D} \frac{p_D}{p_L} = \frac{R_L}{R_D} \frac{p_D}{p - p_D} = 0.622 \frac{p_D}{p - p_D}$$

$$x_s = \frac{m_{D,max}}{m_L} = 0.622 \frac{p_s}{p - p_s} \rightarrow \text{für } \varphi = 1$$

$$p_s = \frac{x_s \cdot p}{0.622 + x_s}$$

$$x_s(t_{min}) = \frac{M_{H_2O}}{M_L} \frac{p_{s}^{min}(t_{min})}{p_1 - p_{s}^{min}(t_{min})}$$

$$p = p_L + p_D$$

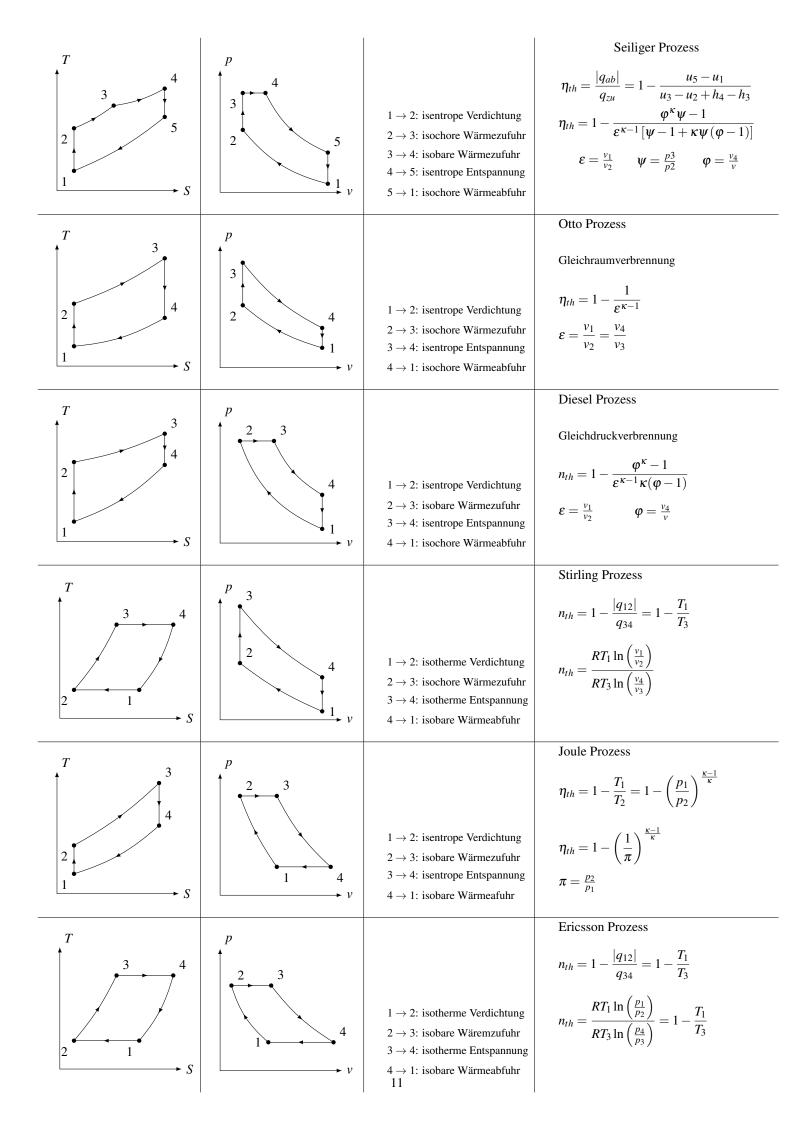
$$\rho = \frac{p}{R_{gesT}} = \frac{1 + x}{R_L + xR_D} \frac{p}{T}$$

$$R_{ges} = \frac{R_L + xR_D}{1 + x}$$

 $h = c_{pL}t + x_D(c_{pD}t + r_D) + x_Wc_Wt + x_E(c_Et - r_E)$ Molare Verdampfungsenthalpie von Wasser (0 °C- 200 °C):

$$H_{v} = \left(50, 9 - 0,9298 \frac{T}{1000} - 65, 19 \left(\frac{T}{1000}\right)^{2}\right) \frac{kJ}{mol}$$

$$M_{L} = \begin{pmatrix} 28,96 & \text{kg/kmol} \\ M_{H_{2}O} & 18,02 & \text{kg/kmol} \\ R_{L} & 0,287 & \text{kJ/(kg K)} \end{pmatrix}$$



Ideales Gas

	Isothermo	Isobare	Isochore	Isentrop	Polytrope
konstant:	T (n = 1)	p (n = 0)	$(\infty \leftarrow n)$ v	$\delta q = 0 \ (n = \kappa)$	pv^n
	I	ı	I	$p_1 v_1^K = p_2 v_2^K$	$v_1^n = p_2 v_2^n$
	$p_1v_1 = p_2v_2$	$\frac{v_1}{v_2} = \frac{T_1}{T_2}$	$\frac{p_1}{T_1} = \frac{p_2}{T_2}$		
	ı	ı	ı	$\frac{T_1^{\frac{K}{K-1}}}{p_1} = \frac{T_2^{\frac{K}{K-1}}}{p_2}$	$\frac{T_1^{\frac{n}{n-1}}}{p_1} = \frac{T_2^{\frac{n}{n-1}}}{p_2}$
p, ν	$p = \frac{p_1 v_1}{v}$	$p = p_1$	$\nu = \nu_1$	$p=rac{p_1 v_1^K}{v^K}$	$p = \frac{p_1 v_1^n}{v^n}$
p,T	$p = \frac{p_1 v_1}{v}$	$p = p_1$	$p=rac{p_1}{T_1}T$	$\left(\frac{R}{1}\right) \frac{K}{K-1}$	$p = p_1 \left(\frac{T}{T_1}\right)^{\frac{n}{n-1}}$
u, T	$T=T_1$	$ u = rac{ u_1}{T_1} T$	$\nu = \nu_1$		$T = T_1 \left(rac{ u_1}{ u} ight)^{n-1}$
<i>q</i> 12	$= p_1 v_1 \ln \frac{p_1}{p_2}$	$=c_p(T_2-T_1)$	$=c_{\nu}(T_2-T_1)$	0 =	$= c_{\nu} \frac{n - \kappa}{n - 1} (T_2 - T_1)$
$W_{V,12}$	$=-q_{12}$	$=-p_1(\nu_2-\nu_1)$	0 =	$= \frac{p_1 v_1}{k-1} \left[\left(\frac{v_1}{v_2} \right)^{K-1} - 1 \right]$	$= \frac{p_1 v_1}{n-1} \left[\left(\frac{v_1}{v_2} \right)^{n-1} - 1 \right]$
$s_2 - s_1$	$=R \ln \left(rac{p_1}{p_2} ight)$	$=c_p \ln \left(rac{T_2}{T_1} ight)$	$=c_{ u}\ln\left(rac{T_{2}}{T_{1}} ight)$	0 =	$=c_{rac{n-\kappa}{n-1}}\ln\left(rac{T_{2}}{T_{1}} ight)$

Van-Der-Waals-Gas

	Isotherme	Isobare	Isochore	Isentrop
konstant:	T	d	Λ	$\delta = 0$
	$(p_1 + \frac{a}{v^2})(v_1 - b)$ = $(p_2 + \frac{a}{v^2})(v_2 - b)$	$\frac{RT_1}{v_1 - b} - \frac{a}{v_1^2} = \frac{RT_2}{v - b} - \frac{a}{v_2^2}$	$\frac{p_1 + \frac{a}{\sqrt{2}}}{T_1} = \frac{p_2 + \frac{a}{\sqrt{2}}}{T_2}$	$(p_1 + rac{a}{v^2})(v_1 - b)rac{c_V + R}{c_V} \ = (p + rac{a}{v^2})(v_2 - b)rac{c_V + R}{c_V}, \ T_1(v_1 - b)^{R/c_V} = T_2(v_2 - b)^{R/c_V}$
p, ν	$p = \left(p + \frac{a}{v^2}\right) \frac{v_u}{v - b} - \frac{a}{v^2}$	$p = p_1$	$\nu = \nu_1$	$p = -\frac{a}{v^2} + \left(p_1 + \frac{a}{v^2}\right) \left(\frac{v_1 - b}{v_m}\right)^{\frac{v_v + R}{R}}$
p,T	$T=T_1$	$p = p_1$	$p = \frac{T}{T_1}(p_1 + \frac{a}{v^2}) - \frac{a}{v_1^2}$	$p=-rac{a}{v^2}+(p_1+rac{a}{v^2})\left(rac{T}{T_1} ight)^{rac{c_V+R}{R}}$
ν, T	$T=T_1$	$T = T_1 \frac{v - b}{v_1 - b} + \frac{a}{R} (v - b) \left(\frac{1}{v^2} - \frac{1}{v_1^2} \right)$	$\nu = \nu_1$	$T = T_1 \left(rac{ u_1 - b}{ u - b} ight)^{rac{R}{C_V}}$
912	$=RT_1\ln\left(rac{ u_2-b}{ u_1-b} ight)$	$=rac{a}{ u_1}-rac{a}{ u_2}+c_ u(T_2-T_1)+p_1(u_2- u_1)\ =c_ u(T_2-T_1)$		0 =
WV,12	$=-RT_1 \ln \left(\frac{v_2-b}{v_1-b}\right) + \frac{a}{v_1} - \frac{a}{v_2}$	$=-p_1(v_2-v_1)$	= 0	$=rac{a}{ u_1}-rac{a}{ u_2}+c_ u(T_2-T_1)$
$s_2 - s_1$	$=R\ln\left(rac{ u_2-b}{ u_1-b} ight)$	$=c_{ u}\ln\left(rac{T_{2}}{T_{1}} ight)+R\ln\left(rac{ u_{2}-b}{ u_{1}-b} ight)$	$=c_ u \ln\left(rac{T_2}{T_1} ight)$	0 =

26 Stoffwerte einiger Gase

Bezeichnung	Symbol	Molmasse	Gaskonstante	Dichte	c_p	c_v	κ
		[kg/kmol]	[J/(kg K)]	$[kg/m^3]$	[J/(kg K)]	[J/(kg,K)]	
Acetylen	C_2H_2	26.038	319.3	1.16	1616	1278	1.26
Ammoniak	NH_3	17.031	488.2	0.76	2056	1526	1.35
Argon	Ar	39.948	208.1	1.76	519	309	1.68
Äthan	C_2H_6	30.070	276.5	1.34	1650	1355	1.22
Butan	C_4H_10	58.124	143.0	2.67	1599	1410	1.13
Chlor	C_l2	56.108	117.3	3.17	473	343	1.38
Chlorwasserstoff	HCl	70.906	228.0	1.62	795	556	1.43
Helium	He	4.003	2077.0	0.18	5200	3124	1.66
Kohlendioxid	CO_2	44.010	188.9	1.95	816	618	1.32
Kohlenmonoxid	CO	28.010	296.8	1.23	1038	739	1.40
Luft	_	28.964	287.1	1.28	1006	718	1.40
Methan	CH_4	16.043	518.3	0.71	2165	1638	1.32
Propan	C_3H_8	44.097	188.5	1.99	1549	1331	1.16
Sauerstoff	O_2	31.999	259.8	1.41	909	647	1.40
Stickstoff	N_2	28.013	296.8	1.23	1038	739	1.40
Wasserstoff	H_2	2.016	4124.2	0.09	14050	9926	1.42
Xenon	Xe	131.300	63.3	5.82	159	93	1.71
Ideales Gas			8.3143				

27 Stoffdaten einiger Stoffe

Name	chemische	Molmasse	Normal-	kritische	kritischer
Name	Formel	[kg/kmol]	Siedepunkt [°C]	Temperatur [°C]	Druck [MPa]
Wasserstoff	H_2	2.02	-252.9	-240.0	1.32
Helium	He	4.00	-268.9	-268.0	0.23
Ammoniak	NH_3	17.03	-33.3	132.3	11.33
Wasser	H_2O	18.02	100.0	373.9	22.06
	78%				
Luft	$N_221\%$	28.96	-194.2	-140.4	3.84
	$O_2.1\%Ar.+$				
Kohlendioxid	CO_2	44.01	-78.4	31.0	7.38
Methan	CH_4	16.04	-161.5	-82.6	4.60
Äthan	C_2H_6	30.07	-88.6	32.2	4.87
Propan	C_3H_8	44.10	-42.1	96.7	4.25
R134a	CH_2FCF_3	102.03	-26.1	101.1	4.06

28 Zahlenwerte feuchte Luft

Bezeichnung	Formelzeichen	Zahlenwert	Dimension
Molmasse der Luft	ML	28,96	kg/ kmol
Molmasse des Wassers	MH2O	18,02	kg/ kmol
spezifische Gaskonstante der Luft	RL	0,287	kJ/ (kg K)
spezifische Gaskonstante des Dampfes	RD	0,461	kJ/ (kg K)
spezifische Wärmekapazität der Luft	cpL	1,006	kJ/ (kg K)
spezifische Wärmekapazität des Dampfes	cpD	1,92	kJ/ (kg K)
spezifische Wärmekapazität des Wassers	cW	4,182	kJ/ (kg K)
spezifische Wärmekapazität des Eises	cЕ	2,1	kJ/ (kg K)
Verdampfungsenthalpie des Wassers bei 0 °C	rD	2500	kJ/ kg
Schmelzenthalpie des Eises bei 0 °C	rE	334	kJ/ kg
	1		

29 Obskure Zusammenhänge

Aus Anhang B

$$dV = \left(\frac{\partial V}{\partial T}\right)_{n} dT + \left(\frac{\partial V}{\partial p}\right)_{T,n} + \sum_{k=1}^{K} \left(\frac{\partial V}{\partial n_{k}}\right)_{T,n} dk n_{k} \tag{1}$$

$$dS = \left(\frac{nC_{p,m}}{T}\right)dT - \left(\frac{\partial V}{\partial T}\right)_{p,n}dp + \sum_{k=1}^{K} \left(\frac{\partial \mu_k}{\partial T}\right)_{p,n}dn_k \tag{2}$$

$$dU = \left[nC_{p,m} - p \left(\frac{\partial V}{\partial T} \right)_{p,n} \right] dT - \left[p \left(\frac{\partial V}{\partial p} \right)_{T,n} + T \left(\frac{\partial V}{\partial T} \right)_{p,n} \right] dp + \sum_{k=1}^{K} \left[\mu_k - T \left(\frac{\partial \mu_k}{\partial T} \right)_{p,n} - p \left(\frac{\partial V}{\partial n_k} \right)_{T,p,n} \right] dn_k \quad (3)$$

$$dH = nC_{p,m}dT + \left[VT\left(\frac{\partial V}{\partial T}\right)_{p,n}\right] + \sum_{k=1}^{K} \left[\mu_k - T\left(\frac{\partial \mu_k}{\partial T}\right)_{p,n}\right] dn_k \tag{4}$$

$$dF = -\left[S + p\left(\frac{\partial V}{\partial T}\right)_{p,n}\right]dT - p\left(\frac{\partial V}{\partial p}\right)_{T,n}dp + \sum_{k=1}^{K} \left[\mu_k - p\left(\frac{\partial V}{\partial n_k}\right)_{T,p}\right]dn_k \tag{5}$$

$$\left(\frac{\partial C_{p,m}}{\partial p}\right)_{T,\psi_{j}} = T \frac{\partial}{\partial p} \left[\left(\frac{\partial S_{m}}{\partial T}\right)_{p,\psi_{j}} \right]_{T,\psi_{i}} = T \frac{\partial}{\partial T} \left[\left(\frac{\partial S_{m}}{\partial p}\right)_{T,\psi_{j}} \right]_{p,\psi_{i}} = -T \frac{\partial}{\partial T} \left[\left(\frac{\partial V_{m}}{\partial T}\right)_{p,\psi_{j}} \right]_{p,\psi_{i}} = -T \left(\frac{\partial^{2} V_{m}}{\partial T^{2}}\right)_{p,\psi_{j}} \tag{6}$$

$$C_{p,m} = (C_{p,m})_{\text{ideales Gas}} - T \int_0^p \left(\frac{\partial^2 V_m}{\partial T^2}\right)_{p,\psi_i} d\tilde{p} \tag{7}$$

$$C_{\nu,m} = (C_{\nu,m})_{\text{ideales Gas}} - T \int_0^{V_m} \left(\frac{\partial^2 p}{\partial T^2}\right)_{p,\Psi_i} d\tilde{V}$$
(8)

(9)

1J = 1W = 1Nm

 mm^2

 10^{6}

 10^{4}

 10^{2}

 10^{-4}

 10^{-6}

 10^{-4}

 mm^2

30 Dinge, die man eigentlich wissen sollte

$$n = \frac{m}{M} = \text{Teichenanzahl} = \frac{\text{Masse}}{\text{Mol}}$$

				m°	dm°	cm	mm°
	Area	Umfang	m^3	1	10^{3}	10^{6}	10^{9}
Kreis:	$u=2r\pi$	$A = r^2 \pi$	dm^3	10^{-3}	1	10^{3}	10^{6}
Kreissektor	u = 2r + b	$A = \frac{r^2 \pi \alpha}{2\pi} = \frac{b \cdot r}{2}$	cm^3	10^{-6}	10^{-3}	1	10^3
	, -	2π 2	mm^3	10^{-9}	10^{-6}	10^{3}	1

31 Eindimensionale Strömungsvorgänge

$$\begin{split} \chi &= \frac{1}{p} = -\frac{1}{V} \left(\frac{\partial V}{\partial p} \right)_T \\ c_S^2 &= \left(\frac{\partial p}{\partial \rho} \right)_S \\ c_S^2 &= \left(\frac{R}{c_v} + 1 \right) \left(v^2 \frac{RT}{(v - b)^2} \right) - \frac{2a}{v} \leftarrow V dW \\ c_S^2 &= \kappa RT \leftarrow i deal \\ Ma &= \frac{c}{c_S} \\ \frac{T_0}{T} &= 1 + \frac{\kappa - 1}{2} \frac{c^2}{\kappa RT} = 1 + \frac{\kappa - 1}{2} M a^2 \\ \frac{p_0}{p} &= \left(\frac{T_0}{T} \right)^{\frac{\kappa}{\kappa - 1}} = \left(1 + \frac{\kappa - 1}{2} M a^2 \right)^{\frac{\kappa}{\kappa - 1}} \\ \frac{\rho_0}{\rho} &= \left(\frac{T_0}{T} \right)^{\frac{\kappa - 1}{\kappa}} = \left(1 + \frac{\kappa - 1}{2} M a^2 \right)^{\frac{\kappa - 1}{\kappa}} \\ \left(\frac{A}{A^*} \right)^2 &= \frac{1}{M a^2} \left[\frac{2}{\kappa + 1} \left(1 + \frac{\kappa - 1}{2} M a^2 \right) \right]^{\frac{\kappa + 1}{\kappa - 1}} \end{split}$$

$$h_2 - h_1 = \frac{1}{2}(p_2 - p_1)\left(\frac{1}{\rho_1} + \frac{1}{\rho_2}\right) = (p_2 - p_1)\frac{1}{2}(v_1 + v_2)$$

Stoßbeziehungen für ein ideales Gas

$$\begin{split} \frac{p_2}{p_1} &= \frac{2\kappa Ma^2 - (\kappa - 1)}{\kappa + 1} \\ \frac{\rho_2}{\rho_1} &= \frac{(\kappa + 1)Ma^2}{2 + (\kappa - 1)Ma^2} \\ \frac{T_2}{T_1} &= \frac{\left[2\kappa Ma^2 - (\kappa - 1)\left[2 + (\kappa - 1)Ma^2\right]}{(\kappa + 1)^2}Ma^2 \\ Ma_2^2 &= \frac{(\kappa - 1)(Ma_1^2 - 1) + (\kappa + 1)}{2\kappa(Ma_1^2 - 1) + (\kappa + 1)} \end{split}$$

Entropie über den senkrechten Verdichtungsstoß

$$s_2 - s_1 = c_v \ln\left(\frac{T_2}{T_1}\right) + R \ln\left(\frac{v_2}{v_1}\right)$$
$$= c_p \ln\left(\frac{T_2}{T_1}\right) + R \ln\left(\frac{p_2}{p_1}\right)$$

32 Chemische Reaktionen

$$\frac{dn_1}{v_1} = \frac{dn_2}{v_2} = \dots = d\lambda = .const$$

$$\sum_{k=1}^K \mu_k dn_k = \sum_{k=1}^K \mu_k (v_k d\lambda) = \sum_{k=1}^K \mu_k v_k = 0$$

$$\mu_i = \left(\frac{\partial U}{\partial n_i}\right)_{S,V} = \left(\frac{\partial H}{\partial n_i}\right)_{S,p} = \left(\frac{\partial F}{\partial n_i}\right)_{T,V} = \left(\frac{\partial G}{\partial n_i}\right)_{T,p}$$

$$\mu(p,T) = \mu(p^+,T) + R_m T \ln\left(\frac{p}{p^+}\right)$$

Massenwirkungsgesetz

$$\prod_{k=1}^{K} \psi_{k}^{v_{k}} = exp - \frac{1}{R_{m}T} \sum_{k=1}^{K} v_{k} \mu_{0k}(p, T)$$
$$= exp - \frac{1}{R_{m}T} \sum_{k=1}^{K} v_{k} G_{m,k}(p, T)$$

Gleichgewichtkonstante

$$K(p,T) = \prod_{k=1}^{K} \psi_k^{\nu_k}$$

$$K(p_2,T) = K(p_1,T) \left(\frac{p_1}{p_2}\right)^{\sum \nu_k}$$

$$\ln\left(\frac{K(p,T_2)}{K(p,T_1)}\right) = \frac{\Delta H_R}{R_m} \left(\frac{1}{T_1} - \frac{1}{T_2}\right) = \frac{\Delta H_R}{R_m} \frac{T_2 - T_1}{T_1 T_2}$$

$$\Delta H_R = \sum_{k=1}^{K} \nu_k H_{m,k}$$