Thermodynamik Formelsammlung

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Contents

$$\frac{d}{dt}\left\{U+m\left(\frac{c^2}{2}+gz\right)\right\}=\sum_j\left[\dot{m}_j\left(h+\frac{c^2}{2}+gz\right)_j\right]+\sum_l\left(\dot{Q}_t\right)_l+\sum_i\left(\dot{W}_t\right)_i-p\frac{dV}{dt}$$

1 Nomenklatur

$\mathbf{An}$ = Anergie[J]
$\mathbf{c_s}$ = Schallgeschwindigkeit[m/s]
$\mathbf{c_p}$ = Spezifische Wärmekapazität dp = 0 [J/kg*K]
$\mathbf{c_v}$ = Spezifische Wärmekapazität dv = 0 [J/kg*K]
$\mathbf{E}$ = Energie[J]
$\mathbf{Ex} = -\mathbf{W_{ex}}$ = Exergie[J]
$\mathbf{F}$ = Kraft[N]
$\mathbf{F} = \mathbf{U} - \mathbf{TS}$ = Freie Energie[J]
$\mathbf{f} = \mathbf{u} - \mathbf{Ts}$ = Spezifische freie Energie[J/kg]
$\mathbf{f}$ = Fugazität[Pa]
$\mathbf{G} = \mathbf{H} - \mathbf{TS}$ = Freie Enthalpie[J]
$\mathbf{g} = \mathbf{h} - \mathbf{Ts}$ = Spezifische freie Enthalpie[J/kg]
$\mathbf{g}$ = Erdbeschleunigung[m/s²]
$\mathbf{H} = \mathbf{U} + \mathbf{pV}$ = Enthalpie[J]
$\mathbf{h} = \mathbf{u} + \mathbf{pv}$ = Spezifische Enthalpie[J/kg]
■**Hg** = Molare Reaktionsenthalpie
$\mathbf{K}$ = Konstante des Massenwirkungsgesetztes[-]
$\mathbf{M}$ = Molmasse[kg/mol]
$\dot{\mathbf{m}}$ = Massestrom[kg/s]
$\mathbf{m}'$ = Masse in der flüssigen Phase[kg]
$\mathbf{m}''$ = Masse in der gasförmigen Phase[kg]
$\mathbf{Ma} = \mathbf{c}/\mathbf{c_s}$ = Machzahl[-]
$\mathbf{n} = \mathbf{m}/\mathbf{M}$ = Molzahl[mol]
$\mathbf{n}$ = Polytropenexponent[-]
$\mathbf{P_t}$ = technische Leistung[W]
$\mathbf{Q}$ = Wärme[J]
$\dot{\mathbf{Q}}$ = Wärmestrom[W]
$\mathbf{q}$ = Spezifische Wärme[J/kg]
$\mathbf{r}$ = Spezifische Verdampfungsenthalpie[J/kg]
$\mathbf{R}$ = Gaskonstante[J/(kg K)]
$\mathbf{R_m}$ = Universelle Gaskonstante[J/(mol K)]
$\mathbf{S}$ = Entropie[J/K]
$\mathbf{s}$ = Spezifische Entropie[J/(kg K)]
$\mathbf{T}$ = Temperatur[K]
$\mathbf{t}$ = Zeit[s]
$\mathbf{t}$ = Temperatur[°C]
$\mathbf{T}$ = Sättigungstemperatur[K]
$\mathbf{U}$ = Innere Energie[J]
$\mathbf{u}$ = Spezifische innere Energie [J/kg]
$\mathbf{V}$ = Volumen[m³]
$\mathbf{v}$ = Spezifisches Volumen[m³/kg]
$\mathbf{V_m}$ = Molares Volumen[m³/mol]
$\mathbf{W}$ = Arbeit[J]
$\mathbf{w}$ = Spezifische Arbeit[J/kg]
$\mathbf{W_V}$ = Volumenänderungsarbeit[J]
$\mathbf{W_{el}}$ = Elektrische Arbeit[J]
$\mathbf{W_w}$ = Wellenarbeit[J]
$\mathbf{W_{diss}}$ = Dissipationsarbeit[J]
$\mathbf{W_t}$ = Technische Arbeit[J]
$\mathbf{W_{Virrev}}$ = Arbeitsverlust durch Irreversibilität[J]
$\mathbf{x} = \frac{m''}{m' + m''}$ = Dampfanteil[-]
$\mathbf{x} = \frac{m_{H_2O}}{m_L}$ = Wassergehalt
$\mathbf{Z}$ = Allgemeine extensive Zustandsgrößen[Z]
$\mathbf{z}$ = Allgemeine
β = Isobarer Ausdehnungskoeffizient[1/K]
γ = Isochorer Spannungskoeffizeint[1/K]
$\delta_{\mathbf{T}}$ = Isothermer Drosselkoeffizient[m³/kg]
$\delta_{\mathbf{h}}$ = Isenthalper Drosselkoeffizient[Ks²m/kg]
ε = Leistungsziffer[-]
ε = Verdichtungsverhältnis[-]
$\eta_{\mathbf{th}}$ = Thermischer Wirkungsgrad[-]
$\eta_{\mathbf{mech}}$ = Mechanischer Wirkungsgrad[-]
κ = Adiabaten- oder Isentropenexponent[-]
λ = Reaktionslaufzahl[-]
$\mu_{\mathbf{i}}$ = Chemisches Potential[J/mol]
$\nu_{\mathbf{i}}$ = Stöchiometrische Koeffizienten[-]
$\xi_{\mathbf{i}}$ = Masseanteil[-]
π = Druckverhältnis[-]
ρ = Dichte[kg/m³]
τ = Temperaturverhältnis[-]
ϕ = Relative Feuchte[-]
ϕ = Einspritzverhältnis[-]
ξ = Isothermer Kompressibilitätskoeffizient[m²/N]
■ = Dissipationsenergie[J]
ψ = Spezifische Dissipationsenergie[J]
ψ = Drucksteigerungsverhältnis[-]
$\psi_{\mathbf{i}}$ = Molanteil[-]

2 Grundbegriffe

Systeme

- Abgeschlossenes System - kein Stoff oder Energietransport
- Geschlossenes System - kein Stofftransport
- Adiabates System - kein Δq , aber Masse und Arbeit.
- Offenes System - Stoff und Energietransport
- Stationäres System $\rightarrow \Delta U = 0$

Messgrößen

- Prozessgrößen sind Wegabhängig (eg. Arbeit, Wärme)
- Zustandsgrößen sind Wegunabhängig (eg. Volumen, Druck)
- Extensive Zustandsgrößen sind abhängig von der Masse des Systems (V, m, H, S, F, G, E)
- Intensive Zustandsgrößen sind unabhängig von der Masse des Systems (T, p)

Zustandsgleichungen

- Thermisch $\rightarrow f(p,V,T) = 0$
- Kalorisch $\rightarrow f(U,V,T) = 0, \quad U = U(V,T), \quad u = u(v,T)$

Hauptsätze

0: Temperatur existiert, ihre gleichheit ist notwendige Vorraussetzung für das thermische Gleichgewicht von zwei Systemen.

1: Energie existiert, sie ist für abgeschlossene Systeme konstant.

2: Entropie existiert, sie wird bei allen irreversiblen Prozessen erzeugt. $dS = \frac{\delta Q_{rev}}{T}$

3: $0K$ exisitert, bei dieser Temperatur ist die Entropie $= 0$

3 Basisformeln

$$
\begin{aligned}
H &= U + pV \\
dS &= \frac{\delta Q_{rev}}{T} \\
F &= U - TS \\
G &= H - ST \\
W &= -\int p\, dV
\end{aligned}
$$

$$
\begin{aligned}
dS &= \frac{Q_{rev}}{T} + S_{prod} \\
dS_{prod} &= \frac{\Psi}{T} \\
\Psi &= \int_1^2 T\, dS_{prod} \\
W_{ir} &= \frac{T_u}{T}\Psi \\
p_1 &= p_a + \frac{\varphi_1 - \varphi_a}{\varphi_b - \varphi_a}(p_b - p_a)
\end{aligned}
$$

4 Gibbs

$$
\begin{aligned}
dU &= Tds - pdV + \sum_{k=1}^{K} \mu_k dn_k \\
dG &= -SdT + Vdp + \sum_{k=1}^{K} \mu_k dn_k \\
dH &= TdS + Vdp + \sum_{k=1}^{K} \mu_k dn_k \\
dF &= -SdT - pdV + \sum_{k=1}^{K} \mu_k dn_k \\
dU &= \left(\frac{\partial U}{\partial S}\right)_V dS + \left(\frac{\partial U}{\partial V}\right)_S dV + \sum_{k=1}^{K} \left(\frac{\partial U}{\partial n_k}\right)_S dn_k
\end{aligned}
$$

5 Thermodynamische Beziehungen

$$
\begin{aligned}
T &= \left(\frac{\partial U}{\partial S}\right)_V = T(S,V) & -S &= \left(\frac{\partial F}{\partial T}\right)_V = S(T,V) \\
T &= \left(\frac{\partial H}{\partial S}\right)_p = T(S,p) & -S &= \left(\frac{\partial G}{\partial T}\right)_p = S(T,p) \\
p &= -\left(\frac{\partial U}{\partial V}\right)_S = p(V,S) & V &= \left(\frac{\partial G}{\partial p}\right)_T = V(p,T) \\
-p &= \left(\frac{\partial F}{\partial V}\right)_T = p(T,V) & \mu &= \left(\frac{\partial U}{\partial n}\right)_{S,V} = \mu(S,V,n)
\end{aligned}
$$

6 Guggenheim

-S	U	V	U	= U(S,V)
H		F	H	= H(S,p)
-p	G	T	F	= F(T,V)
			G	= G(T,p)

$$
\overbrace{\frac{d}{dt}\left\{U + m\left(\frac{c^2}{2} + gz\right)\right\}}^{\text{Stationäres System -> 0}} = \overbrace{\sum_j \left[\dot{m}_j \left(h + \frac{c^2}{2} + gz\right)\right]_j}^{\text{Geschlossenes System -> 0}} + \overbrace{\sum_l \left(\dot{Q}_t\right)_l}^{\text{Kein Wärmestrom -> 0}} + \overbrace{\sum_i \left(\dot{W}_t\right)_i}^{\text{Keine Leistung -> 0}} - \overbrace{p\frac{dV}{dt}}^{\text{Keine Volumenänderung -> 0}}
$$

7 Maxwell

$$\left(\frac{\partial T}{\partial p}\right)_{S,n_j} = \left(\frac{\partial V}{\partial S}\right)_{p,n_j}$$

$$\left(\frac{\partial S}{\partial V}\right)_{T,n_j} = \left(\frac{\partial p}{\partial T}\right)_{V,n_j}$$

$$\left(\frac{\partial S}{\partial p}\right)_{T,n_j} = -\left(\frac{\partial V}{\partial T}\right)_{p,n_j}$$

$$\left(\frac{\partial \mu_i}{\partial T}\right)_{p,n_j} = -\left(\frac{\partial S}{\partial n_i}\right)_{T,p,n_j \neq n_i}$$

$$\left(\frac{\partial \mu_i}{\partial p}\right)_{T,n_j} = \left(\frac{\partial V}{\partial n_i}\right)_{T,p,n_j \neq n_i}$$

8 Ideales Gas

$$pV = mRT$$
$$pv = RT$$
$$pV = nR_mT$$
$$\beta = \frac{1}{T}$$
$$\gamma = \frac{1}{T}$$
$$\chi = \frac{1}{p}$$
$$\beta = p\gamma\chi$$
$$R_m = 8,3143 \left[\frac{kJ}{kmolK}\right]$$
$$R = c_p - c_v$$
$$R = \frac{R_m}{M}$$
$$U - U_0 = mc_v(T - T_0)$$
$$H - H_0 = mc_p(T - T_0) \quad \leftarrow \text{Für } c_p \text{ und } c_v \text{ const.}$$

$$\begin{aligned} s - s_0 &= R\ln\left(\frac{v}{v_0}\right) &&+ c_v\ln\left(\frac{T}{T_0}\right) \\ &= c_v\ln\left(\frac{p}{p_0}\right) &&+ c_p\ln\left(\frac{v}{v_0}\right) \\ &= c_p\ln\left(\frac{T}{T_0}\right) &&- R\ln\left(\frac{p}{p_0}\right) \end{aligned}$$

$$\beta = \frac{1}{T} = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_p = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_p$$

$$\gamma = \frac{1}{T} = \frac{1}{p}\left(\frac{\partial p}{\partial T}\right)_V$$

$$\chi = \frac{1}{p} = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_T$$

$$u_2 - u_1 = \int_{T_1}^{T_2} c_v(T)\, dT$$

9 Van-der-Waals

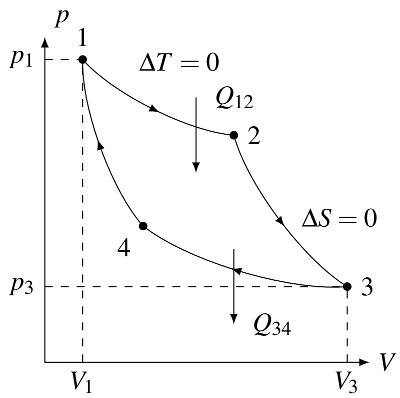
$$\left(p + \frac{a}{v^2}\right)(v - b) = RT$$
$$\left(\overline{p} + \frac{3}{\overline{v}^2}\right)(3\overline{v} - 1) = 8\overline{T}$$
$$\overline{p} = \frac{p}{p_K}, \quad \overline{v} = \frac{v}{v_K}, \quad \overline{T} = \frac{T}{T_K}$$
$$p_K = \frac{a}{27b^2}, \quad T_K = \frac{8}{27}\frac{a}{b}\frac{1}{R},$$
$$a = 3p_Kv_K^2, \quad b = \frac{v_K}{3}, \quad \frac{p_Kv_K}{RT_K} = \frac{3}{8}$$
$$\beta = \frac{(v-b)Rv^2}{RTv^3 - 2a(v-b)^2}$$
$$\gamma = \frac{Rv^2}{RTv^2 - a(v-b)}$$
$$\chi = \frac{(v-b)^2v^2}{RTv^3 - 2a(v-b)^2}$$
$$du = \frac{a}{v^2}dv + c_v(T)dT$$
$$u - u_0 = \left(\frac{a}{v_0} - \frac{a}{v}\right) + \int_{T_0}^{T} c_v(\tilde{T})\, d\tilde{T}$$
$$u - u_0 = \left(\frac{a}{v_0} - \frac{a}{v}\right) + c_v(T - T_0) \leftarrow \text{für } c_v = \text{const.}$$
$$c_p - c_v = \frac{Tv\beta^2}{\chi}$$
$$s - s_0 = c_v\ln\left(\frac{T}{T_0}\right) + R\ln\left(\frac{v-b}{v_0-b}\right)$$
$$h_2 - h_1 = \frac{1}{2}(p_2 - p_1)\left(\frac{1}{\rho_1} + \frac{1}{\rho_2}\right)$$

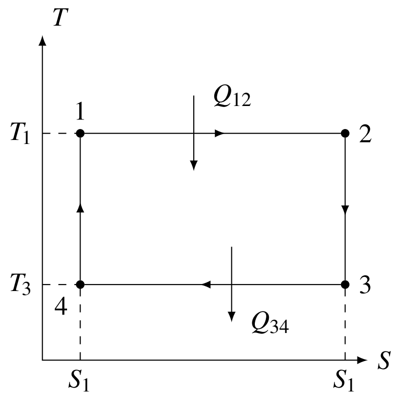
10 Carnot

$$\eta_{th} = 1 - \frac{-Q_{34}}{Q_{12}} = 1 - \frac{T_3(S_3 - S_4)}{T_1(S_2 - S_1)} = 1 - \frac{T_3}{T_1}$$

$$\frac{Q_{12}}{T_1} + \frac{Q_{34}}{T_3} = 0$$

$$\Delta S_{ges} = -Q_{34}\left(\frac{1}{T_{KK}} - \frac{T_1}{T_3}\frac{1}{T_{HK}}\right)$$





Adiabate Drosselung (ideal):

$$h + \frac{c^2}{2} + gz = \text{const.}$$

$$dh = 0$$

Adiabet Drosselung (real):

$$\delta_h = \left(\frac{\partial T}{\partial p}\right)_h = -\frac{v}{c_p}(1 - \beta T)$$

11 Gemische Idealer Gase

$$\xi_i = \frac{m_i}{m}, \quad \psi_i = \frac{n_i}{n}, \quad p_i = \psi_i p$$

$$\xi_i = \frac{M_i n_i}{\sum_{k=1}^{K} M_k n_k} = \frac{M_i}{M_G}\psi$$

$$p_i V = m_i R_i T, \quad p_i V = n_i R_m T, \quad pV = m R_G T$$

$$\sum_{k=1}^{K} p_k = p$$

$$R_G = \frac{1}{m}\sum_{k=1}^{K} m_k R_k = \sum_{k=1}^{K} \xi_k R_k$$

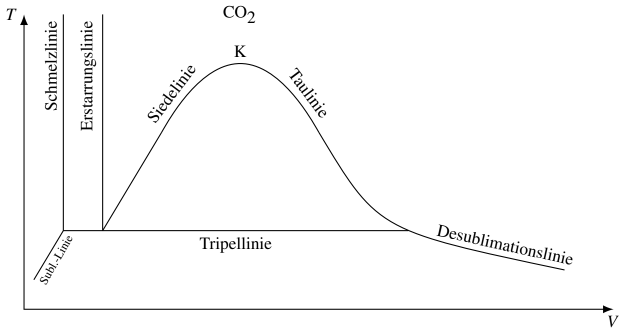
$$U_G = \sum_{k=1}^{K} U_k = \sum_{k=1}^{K} m_k u_k = \sum_{k=1}^{K} c_{vk} m_k T \leftarrow c_v = \text{const}$$

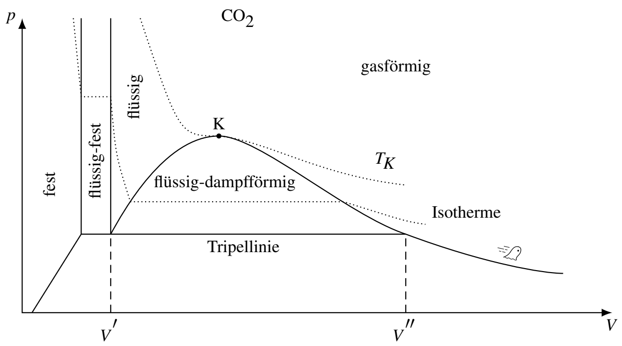
$$H_G = \sum_{k=1}^{K} H_k = \sum_{k=1}^{K} m_k h_k = \sum_{k=1}^{K} c_{pk} m_k T \leftarrow c_p = \text{const.}$$

$$c_{vG} = \sum_{k=1}^{K} c_{vk}\xi_k, \quad c_{pG} = \sum_{k=1}^{K} c_{pk}\xi_k$$

$$S_2 - S_1 = R_m\left(n \ln n - \sum_{k=1}^{K} n_k \ln n_k\right)$$

12 Nassdampf





$$v = (1-x)v' + xv''$$

$$v = v' + (v'' - v')x$$

$$u = (1-x)u' + xu''$$

$$u = u' + (u'' - u')x$$

$$h = (1-x)h' + xh''$$

$$h = h' + (h'' - h')x$$

$$s = (1-x)s' + xs''$$

$$s = s' + (s'' - s')x$$

$$r = h'' - h' = T(s'' - s')$$

$$T' = T''$$

$$p' = p''$$

$$g' = g''$$

$$dg' = v'dp' - s'dT'$$

$$dg'' = v''dp'' - s''dT''$$

$$dg' = dg''$$

$$\frac{dp}{dT} = \frac{s'' - s'}{v'' - v'}$$

$$\frac{dp}{dT} = \frac{1}{T}\frac{h'' - h'}{v'' - v'}$$

$$\frac{dp}{dT} = \frac{1}{T}\frac{r}{v'' - v'}$$

13 Realer Stoff im Nassdampfgebiet

Isobare Zustandsänderung

$$\begin{aligned} q_{12} &= T(s_2 - s_1) \\ &= T\left(s'' - s'\right)(x_2 - x_1) \\ w_{V,12} &= -\int_1^2 p\, dv \\ &= -p(v_2 - v_1) = -p\left(v'' - v'\right)(x_2 - x_1) \end{aligned}$$

Isochore Zustandsänderung

$$q_{12} = u_2 - u_1 = u_2' + x_2\left(u_2'' - u_2'\right) - u_1' - x_1\left(u_1'' - u_1'\right)$$

Adiabate Zustandsänderung

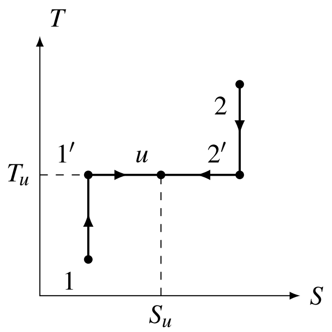
$$w_{V,12} = u_2 - u_1 = u_2' + x_2\left(u_2'' - u_2'\right) - u_1' - x_1\left(u_1'' - u_1'\right)$$

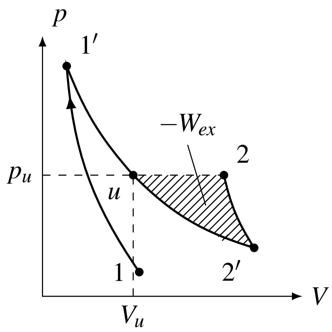
Entropieänderung wärend des Mischvorgangs

$$S_2 - S_2 = R_m\left(n \ln n - \sum_i n_i \ln n_i\right)$$

14 Maximale Arbeit und Exergie

Maxiaml nutzbare Arbeit $\rightarrow$ isentrop, reibungsfrei
$1 \rightarrow 1'$: isentrop auf T_u
$1' \rightarrow u$: isotherm auf u





$$-\dot{W}_{ex} = -(\dot{W}_t)_{rev} = -\frac{d}{dt}\left(U + m\left(\frac{c^2}{2} + gz\right) + p_u V - T_u S\right) + \sum_{j=1}^{K}\left(\dot{m}_j\left(h + \frac{c^2}{2} + gz - T_s\right)\right) + \sum_{l=1}^{K}\left(1 - \frac{T_u}{T}\right)\dot{Q}_l$$

Die Exergie der Enthalpie (offens, stationäres System)

$$-\dot{W}_{ex,1u} = \dot{m}(h_1 - h_u - T_u(s_1 - s_u))$$

Die Exergie der inneren Energie (geschlossenes, instationäres System)

$$\begin{aligned} -\dot{W}_{ex} &= -\frac{d}{dt}(U + p_u V - T_u S) \\ -\dot{W}_{ex,1u} &= U_1 - U_u - p_u(V_1 - V_u) - T_u(S_1 - S_u) \\ -\dot{W}_{ex,1u} &= H_1 - (p_1 - p_u)V_1 - H_u - T_u(S_1 - S_u) \end{aligned}$$

Für Ideales Gas

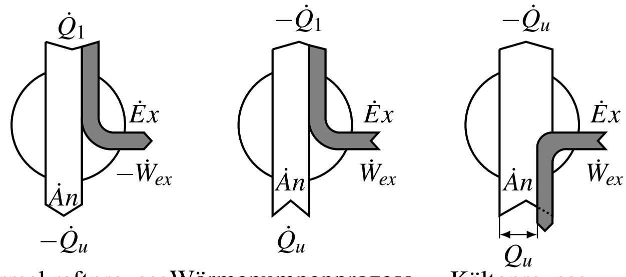
$$-W_{ex} = mc_v(T_1 - T_u) + p_u(V_1 - V_u) - T_u m\left(c_p \ln\left(\frac{T_1}{T_u}\right) - R_i \ln\left(\frac{p_1}{p_u}\right)\right)$$

Dampf/Luftdruckkammer

$$-W_{ex,1u} = m_1[u_1 - u_u + p_u(v_1 - v_u) - T_u(s_1 - s_u)]$$

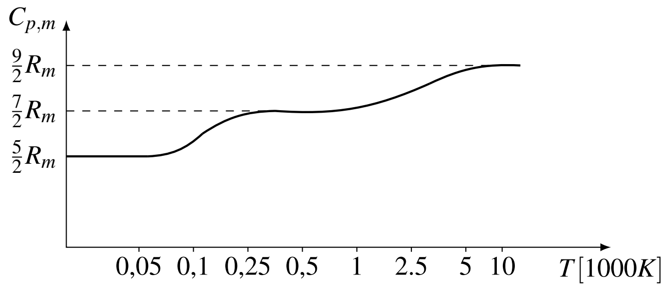
Die Exergie der Wärme (geschlossenes, stationäres System)

$$-\dot{W}_{ex} = \left(1 - \frac{T_u}{T_1}\right)\dot{Q}_1 = \eta_{th,C}\dot{Q}_1$$



Wärmekraftprozess Wärmepumpenprozess Kälteprozess

15 Wärmekapazität



$$C_{v,m} = \frac{1}{\kappa - 1}R_m \qquad C_{p,m} = \frac{\kappa}{\kappa - 1}r_m$$

$$c_v = \frac{1}{\kappa - 1}R_j \qquad c_p = \frac{\kappa}{\kappa - 1}R_j$$

$$\kappa = \frac{c_p}{c_v} \qquad R = c_p - c_v$$

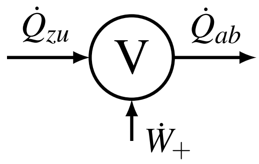
$$R = \frac{R_m}{M} \qquad R_m = 8,3143\left[\frac{kJ}{kmolK}\right]$$

$$C_{v,m} = \underbrace{3 + \frac{R_m}{2}}_{\text{Translatorisch}} + \underbrace{\frac{n_{\text{rot}}R_m}{2}}_{\text{Rotatorisch}} + \underbrace{R_M(3n_{\text{Atome}} - 3 - n_{rot})}_{\text{Vibratorisch}} + \underbrace{C_{v,m,Elektronenanregung}}_{\text{Relevant ab: } T \approx 10^4 K}$$

16 Technische Anwendung

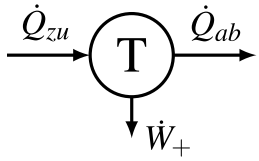
adiabat ($c_p = const.$)	$W_{t,12} = mc_p(T_2 - T_1) = \frac{\kappa}{\kappa - 1}(p_2V_2 - p_1V_1)$	$Q_{12} = 0$
reversibel adiabat $\kappa = const.$	$W_{t,12} = \frac{\kappa}{\kappa - 1}(p_1V_1)\left[\left(\frac{p_2}{p_1}\right)^{\frac{\kappa-1}{\kappa}} - 1\right]$	$Q_{12} = 0$
irreversibel adiabat als Polytrope $n > \kappa; n, \kappa = const.$	$W_{t,12} = \frac{\kappa}{\kappa - 1}(p_1V_1)\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$	$Q_{12} = 0$
reversibel polytrop $n, \kappa = const.$	$W_{t,12} = \frac{n}{n-1}(p_2V_2 - p_1V_1)$	$Q_{12} = mc_n(T_2 - T_1)$
	$= \frac{n}{n-1} mR(T_2 - T_1)$	$= \frac{n-\kappa}{(n-1)(\kappa-1)}(p_1V_1)\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}-1}\right]$
	$= \frac{n}{n-1}(p_1V_1)\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$	$c_n = \frac{n-\kappa}{n-1} cv$
isotherm	$W_{t,12} = (p_1V_1)\ln\left(\frac{p_2}{p_2}\right)$	$Q_{12} = -W_{t,12}$

Thermischer Wirkungsgrad $\eta_{th} = \frac{-w}{q_{zu}} = \frac{\text{Nutzen}}{\text{Aufwand}} = 1 - \frac{|q_{ab}|}{q_{zu}}$



Isentroper Verdichterwirkungsgrad $\eta_{sV} = \frac{w_{t,12,rev}}{w_{t,12}} = \frac{h_{2,rev} - h_1}{h_2 - h_1} = \underbrace{\frac{T_{2,rev} - T_1}{T_2 - T_1}}_{\text{idealer Fall}}$

Isentroper Turbinenwirkungsgrad $\eta_{sT} = \frac{w_{t,12}}{W_{t,12,rev}} = \frac{h_1 - h_2}{h_1 - h_{2,rev}} = \overbrace{\frac{T_1 - T_2}{T_1 - T_{2,rev}}}$



Dampfkraftprozess Wirkungsgrad $n_{th} = 1 - \frac{|q_{61}|}{q_{23} + q34 + q_{45}} = 1 - \frac{h_6 - h_1}{h_5 - h_2}$

Leistungszahl Kältemaschine $\varepsilon_{K(A)} = \frac{q_{zu}}{w} = \frac{\dot{Q}_0}{\dot{W}}$

Leistungszahl Kaltluftprozess $\varepsilon_K = \frac{1}{\left(\frac{p}{p_0}\right)^{\frac{\kappa-1}{\kappa}-1}}$

Leistungszahl Kaltdampfprozess $\varepsilon_K = \frac{q_0}{|q| - q_0} = \frac{q_o}{w_t} = \frac{h_1 - h_6}{h_2 - h_1}$

Linkslaufender Carnotprozess $\varepsilon_{carnot} = \frac{T_k}{T_H - T_K}$

Leistungszahl Wärmepumpe $\varepsilon_{WP} = \frac{q}{|q| - q_0} = \frac{|q|}{w_t} = \frac{q_{zu}}{w} = \frac{h_2 - h_5}{h_2 - h_1} = 1 + \varepsilon_{K(A)}$

Verdichtungsverhältnis $\varepsilon = \frac{v_1}{v_2}$

Drucksteigerungsverhältniss $\psi = \frac{p_3}{p_2}$

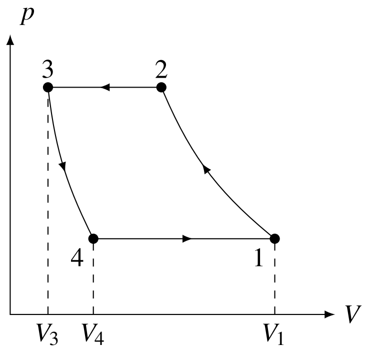
Einspriztverhältniss $\varphi = \frac{v_4}{v_3}$

Temperaturverhältnis $\tau = \frac{T_3}{T_1}$

Verdrichtungsdruckverhältnis $\pi = \; = \frac{p_2}{p_1}$

für Joule-Prozess $\pi_{opt} = \tau^{\frac{\kappa}{2(\kappa-1)}}$

Kolbenverdichter



$V1 =$ Maximales Zylindervolumen

$V2 =$ Volumen nach Verdichtung

$V3 =$

$V4 =$ Schädlicher Raum

$$\mu = \frac{V_1 - V_4}{V_1 - V3}, \qquad \varepsilon_S = \frac{V_3}{V_1 - V_3}$$

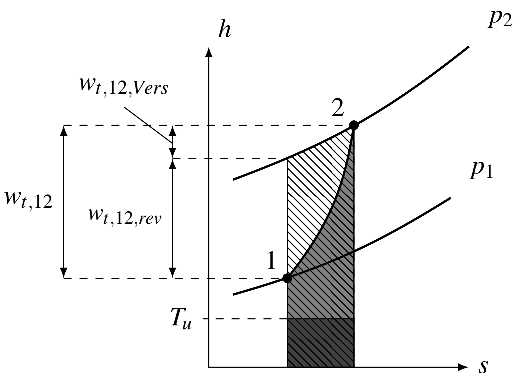
$$\mu = 1 - \varepsilon_S \left[\left(\frac{p_2}{p_1} \right)^{\frac{1}{n}} - 1 \right]$$

$$W_{t,12} = \int_1^2 V\, dp$$

$$= \underbrace{p_2 V_2}_{Ausschiebearbeit} - \underbrace{p_1 V_1}_{Einschiebearbeit} - \int_1^2 p\, dV$$

$$= \frac{n}{n-1} p_1 (V_1 - V_4) \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - \right]$$

Turboverdichter



$w_{t,12,Verl} = \int_{2,rev}^{2} T\, ds$

$\psi_{12} = \int_1^2 T\, ds$

$w_{V,12,irr} = T_u \Delta s$

Verdichter Wirkungsgrad

$$\eta_{sV} = \frac{w_{t,12,rev}}{w_{t,12}} = \frac{h_{2,rev} - h_1}{h_2 - h_1}$$

Verdichter wirkungsgrad (Ideales Gas, $c_p = \text{const.}$)

$$\eta_{sV} = \frac{T_{2,rev} - T_1}{T_2 - T_1}$$

Technische Verlustarbeit

$$w_{t,Verl,12} = w_{t,12} - w_{t,12,rev} = h_2 - h_{2,rev}$$

$$= \int_{2,rev}^{2} T|_{p_2=const.}\, ds$$

17 Eindimensionale Strömungsvorgänge

$$\chi = \frac{1}{p} = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T$$
$$c_S^2 = \left(\frac{\partial p}{\partial \rho}\right)_S$$
$$c_S^2 = \left(\frac{R}{c_v}+1\right)\left(v^2\frac{RT}{(v-b)^2}\right) - \frac{2a}{v} \leftarrow VdW$$
$$c_S^2 = \kappa RT \leftarrow ideal$$
$$Ma = \frac{c}{c_S}$$
$$\frac{T_0}{T} = 1+\frac{\kappa-1}{2}\frac{c^2}{\kappa RT} = 1+\frac{\kappa-1}{2}Ma^2$$
$$\frac{p_0}{p} = \left(\frac{T_0}{T}\right)^{\frac{\kappa}{\kappa-1}} = \left(1+\frac{\kappa-1}{2}Ma^2\right)^{\frac{\kappa}{\kappa-1}}$$
$$\frac{\rho_0}{\rho} = \left(\frac{T_0}{T}\right)^{\frac{\kappa-1}{\kappa}} = \left(1+\frac{\kappa-1}{2}Ma^2\right)^{\frac{\kappa-1}{\kappa}}$$
$$\left(\frac{A}{A^*}\right)^2 = \frac{1}{Ma^2}\left[\frac{2}{\kappa+1}\left(1+\frac{\kappa-1}{2}Ma^2\right)\right]^{\frac{\kappa+1}{\kappa-1}}$$

$$h_2-h_1 = \frac{1}{2}(p_2-p_1)\left(\frac{1}{\rho_1}+\frac{1}{\rho_2}\right) = (p_2-p_1)\frac{1}{2}(v_1+v_2)$$

Stoßbeziehungen für ein ideales Gas

$$\frac{p_2}{p_1} = \frac{2\kappa Ma^2-(\kappa-1)}{\kappa+1}$$
$$\frac{\rho_2}{\rho_1} = \frac{(\kappa+1)Ma^2}{2+(\kappa-1)Ma^2}$$
$$\frac{T_2}{T_1} = \frac{\left[2\kappa Ma^2-(\kappa-1\right]\left[2+(\kappa-1)Ma^2\right]}{(\kappa+1)^2}Ma^2$$
$$Ma_2^2 = \frac{(\kappa-1)(Ma_1^2-1)+(\kappa+1)}{2\kappa(Ma_1^2-1)+(\kappa+1)}$$

Entropie über den senkrechten Verdichtungsstoß

$$s_2-s_1 = c_v\ln\left(\frac{T_2}{T_1}\right)+R\ln\left(\frac{v_2}{v_1}\right)$$
$$= c_p\ln\left(\frac{T_2}{T_1}\right)+R\ln\left(\frac{p_2}{p_1}\right)$$

18 Feuchte Luft

$$x = \frac{m_{H_2O}}{m_L}$$
$$x = x_{D(ampf)}+x_{W(asser)}+x_{E(is)}$$
$$\varphi = \frac{p_D}{p_s}$$
$$x_D = \frac{m_d}{m_L} = \frac{R_L}{R_D}\frac{p_D}{p_L} = \frac{R_L}{R_D}\frac{p_D}{p-p_D} = 0.622\frac{p_D}{p-p_D}$$
$$x_s = \frac{m_{D,max}}{m_L} = 0.622\frac{p_s}{p-p_s} \rightarrow \text{für } \varphi = 1$$
$$\rho = \frac{p}{R_{gesT}} = \frac{1+x}{R_L+xR_D}\frac{p}{T}$$
$$R_{ges} = \frac{R_L+xR_D}{1+x}$$
$$h = c_{pL}t+x_D(c_{pD}t+r_D)+x_Wc_Wt+x_E(c_Et-r_E)$$

19 Chemische Reaktionen

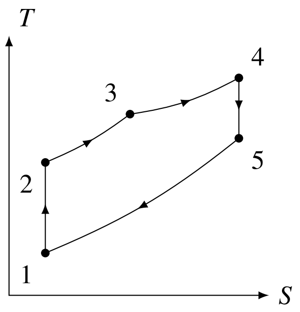
$$\frac{dn_1}{\nu_1} = \frac{dn_2}{\nu_2} = \ldots = d\lambda = .const$$
$$\sum_{k=1}^{K}\mu_k dn_k = \sum_{k=1}^{K}\mu_k(\nu_k d\lambda) = \sum_{k=1}^{K}\mu_k\nu_k = 0$$
$$\mu_i = \left(\frac{\partial U}{\partial n_i}\right)_{S,V} = \left(\frac{\partial H}{\partial n_i}\right)_{S,p} = \left(\frac{\partial F}{\partial n_i}\right)_{T,V} = \left(\frac{\partial G}{\partial n_i}\right)_{T,p}$$
$$\mu(p,T) = \mu(p^+,T)+R_mT\ln\left(\frac{p}{p^+}\right)$$

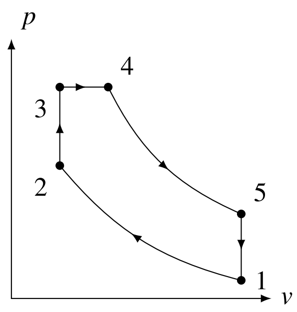
Massenwirkungsgesetz

$$\prod_{k=1}^{K}\psi_k^{\nu_k} = exp-\frac{1}{R_mT}\sum_{k=1}^{K}\nu_k\mu_{0k}(p,T)$$
$$= exp-\frac{1}{R_mT}\sum_{k=1}^{K}\nu_k G_{m,k}(p,T)$$

Gleichgewichtkonstante

$$K(p,T) = \prod_{k=1}^{K}\psi_k^{\nu_k}$$
$$K(p_2,T) = K(p_1,T)\left(\frac{p_1}{p_2}\right)^{\Sigma\,\nu_k}$$
$$\ln\left(\frac{K(p,T_2)}{K(p,T_1)}\right) = \frac{\Delta H_R}{R_m}\left(\frac{1}{T_1}-\frac{1}{T_2}\right) = \frac{\Delta H_R}{R_m}\frac{T_2-T_1}{T_1T_2}$$
$$\Delta H_R = \sum_{k=1}^{K}\nu_k H_{m,k}$$



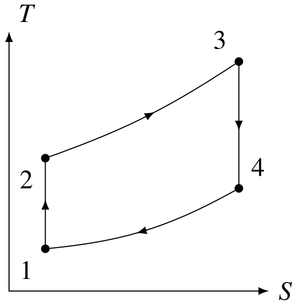


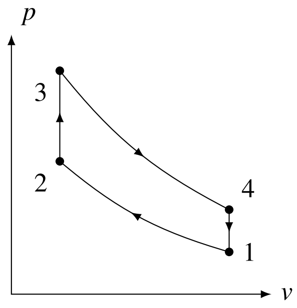
Seiliger Prozess

$$\eta_{th} = \frac{|q_{ab}|}{q_{zu}} = 1 - \frac{u_5 - u_1}{u_3 - u_2 + h_4 - h_3}$$

$$\eta_{th} = 1 - \frac{\varphi^{\kappa}\psi - 1}{\varepsilon^{\kappa-1}\left[\psi - 1 + \kappa\psi\left(\varphi - 1\right)\right]}$$

$$\varepsilon = \tfrac{v_1}{v_2} \qquad \psi = \tfrac{p3}{p2} \qquad \varphi = \tfrac{v_4}{v}$$



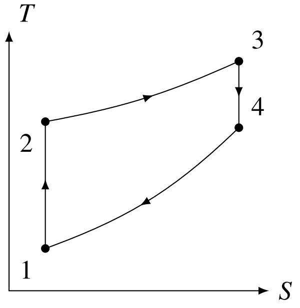


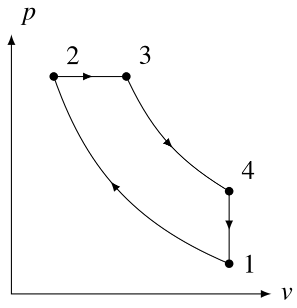
Otto Prozess

Gleichraumverbrennung

$$\eta_{th} = 1 - \frac{1}{\varepsilon^{\kappa-1}}$$

$$\varepsilon = \frac{v_1}{v_2}$$



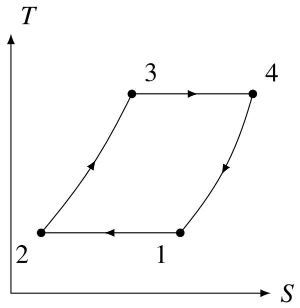


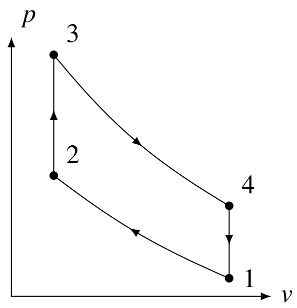
Diesel Prozess

Gleichdruckverbrennung

$$n_{th} = 1 - \frac{\varphi^{\kappa} - 1}{\varepsilon^{\kappa-1}\kappa(\varphi - 1)}$$

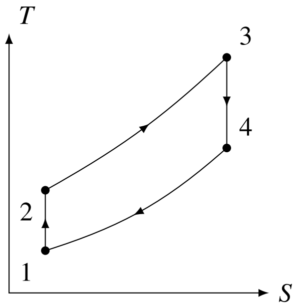
$$\varepsilon = \tfrac{v_1}{v_2} \qquad \varphi = \tfrac{v_4}{v}$$

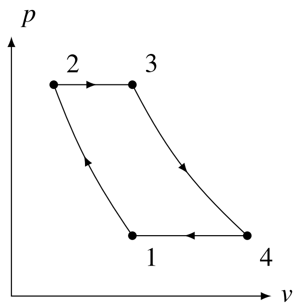




Stirling Prozess

$$n_{th} = 1 - \frac{|q_{12}|}{q_{34}} = \frac{RT_1 \ln\left(\frac{v_1}{v_2}\right)}{RT_3 \ln\left(\frac{v_4}{v_3}\right)} = 1 - \frac{T_1}{T_3}$$

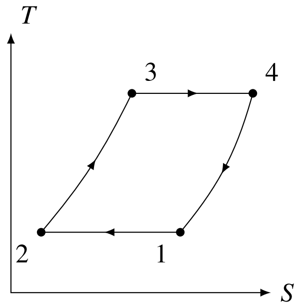


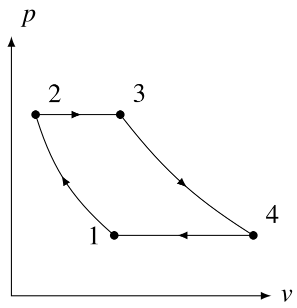


Joule Prozess

$$\eta_{th} = 1 - \frac{T_1}{T_2} = 1 - \left(\frac{p_1}{p_2}\right)^{\frac{\kappa-1}{\kappa}} = 1 - \left(\frac{1}{\pi}\right)^{\frac{\kappa-1}{\kappa}}$$

$$\pi = \tfrac{p_2}{p_1}$$





Ericsson Prozess

$$n_{th} 1 - \frac{|q_{12}|}{q_{34}} = \frac{RT_1 \ln\left(\frac{p_1}{p_2}\right)}{RT_3 \ln\left(\frac{p_4}{p_3}\right)} = 1 - \frac{T_1}{T_3}$$

Ideales Gas

	Isothermo	Isobare	Isochore	Isentrop	Polytrope
konstant:	T	p	v	$\delta q = 0$	pv^n
	-	-	-	$p_1 v_1^\kappa = p_2 v_2^\kappa$	$v_1^n = p_2 v_2^n$
	$p_1 p_2 = p_2 v_2$	$\frac{v_1}{v_2} = \frac{T_1}{T_2}$	$\frac{p_1}{T_1} = \frac{p_2}{T_2}$	$T_1 v_1^{\kappa-1} = T_2 v_2^{\kappa-1}$	$T_1 v_1^{n-1} = T_2 v_2^{n-1}$
	-	-		$\frac{T_1^{\frac{\kappa}{\kappa-1}}}{p_1} = \frac{T_2^{\frac{\kappa}{\kappa-1}}}{p_2}$	$\frac{T_1^{\frac{n}{n-1}}}{p_1} = \frac{T_2^{\frac{n}{n-1}}}{p_2}$
p, v	$p = \frac{p_1 v_1}{v}$	$p = p_1$	$v = v_1$	$p = \frac{p_1 v_1^\kappa}{v^\kappa}$	$p = \frac{p_1 v_1^n}{v^n}$
p, T	$p = \frac{p_1 v_1}{v}$	$p = p_1$	$p = \frac{p_1}{T_1} T$	$p = \frac{p_1}{T_1^{\frac{\kappa}{\kappa-1}}} T^{\frac{\kappa}{\kappa-1}}$	$p = \frac{p_1}{T_1^{\frac{n}{n-1}}} T^{\frac{n}{n-1}}$
v, T	$T = T_1$	$v = \frac{v_1}{T_1} T$	$v = v_1$	$T = \frac{T_1 v_1^{\kappa-1}}{v^{\kappa-1}}$	$T = \frac{T_1 v_1^{n-1}}{v^{n-1}}$
q_{12}	$= p_1 v_1 \ln \frac{p_1}{p_2}$	$= c_p (T_2 - T_1)$	$= c_v (T_2 - T_1)$	$= 0$	$= c_v \frac{n-\kappa}{n-1} (T_2 - T_1)$
$w_{V,12}$	$= -q_{12}$	$= -p_1 (v_2 - v_1)$	$= 0$	$= \frac{p_1 v_1}{k-1} \left[\left(\frac{v_1}{v_1} \right)^{\kappa-1} - 1 \right]$	$= \frac{p_1 v_1}{n-1} \left[\left(\frac{v_1}{v_2} \right)^{n-1} - 1 \right]$
$s_2 - s_1$	$= R \ln \left(\frac{p_1}{p_2} \right)$	$= c_p \ln \left(\frac{T_2}{T_1} \right)$	$= c_v \ln \left(\frac{T_2}{T_1} \right)$	$= 0$	$= c_v \frac{n-\kappa}{n-1} \ln \left(\frac{T_2}{T_1} \right)$

Van-Der-Waals-Gas

	Isotherme	Isobare	Isochore	Isentrop
konst.	T	p	v	$\delta = 0$
	$(p_1 + \frac{a}{v^2})(v_1 - b)$ $= (p_2 + \frac{a}{v^2})(v_2 - b)$	$\frac{RT_1}{v_1 - b} - \frac{a}{v_1^2} = \frac{RT_2}{v - b} - \frac{a}{v_2^2}$	$\frac{p_1 + \frac{a}{v_1^2}}{T_1} = \frac{p_2 + \frac{a}{v_1^2}}{T_2}$	$(p_1 + \frac{a}{v^2})(v_1 - b)^{\frac{c_V + R}{c_V}}$ $= (p + \frac{a}{v^2})(v_2 - b)^{\frac{c_V + R}{c_V}}$, $T_1(v_1 - b)^{R/c_V} = T_2(v_2 - b)^{R/c_V}$
p, v	$p = (p + \frac{a}{v^2})\frac{v_u}{v - b} - \frac{a}{v^2}$	$p = p_1$	$v = v_1$	$p = -\frac{a}{v^2} + (p_1 + \frac{a}{v^2})\left(\frac{v_1 - b}{v_m}\right)^{\frac{v_V + R}{R}}$
p, T	$T = T_1$	$p = p_1$	$p = \frac{T}{T_1}(p_1 + \frac{a}{v^2}) - \frac{a}{v_1^2}$	$p = -\frac{a}{v^2} + (p_1 + \frac{a}{v^2})\left(\frac{T}{T_1}\right)^{\frac{c_V + R}{R}}$
v, T	$T = T_1$	$T = T_1\frac{v - b}{v_1 - b} + \frac{a}{R}(v - b)\left(\frac{1}{v^2} - \frac{1}{v_1^2}\right)$	$v = v_1$	$T = T_1\left(\frac{v_1 - b}{v - b}\right)^{\frac{R}{c_V}}$
q_{12}	$= RT_1 \ln\left(\frac{v_2 - b}{v_1 - b}\right)$	$= \frac{a}{v_1} - \frac{a}{v_2} + c_v(T_2 - T_1) + p_1(v_2 - v_1)$	$= c_v(T_2 - T_1)$	$= 0$
$w_{V,12}$	$= -RT_1 \ln\left(\frac{v_2 - b}{v_1 - b}\right) + \frac{a}{v_1} - \frac{a}{v_2}$	$= -p_1(v_2 - v_1)$	$= 0$	$= \frac{a}{v_1} - \frac{a}{v_2} + c_v(T_2 - T_1)$
$s_2 - s_1$	$= R\ln\left(\frac{v_2 - b}{v_1 - b}\right)$	$= c_v \ln\left(\frac{T_2}{T_1}\right) + R\ln\left(\frac{v_2 - b}{v_1 - b}\right)$	$= c_v \ln\left(\frac{T_2}{T_1}\right)$	$= 0$

20 Stoffwerte einiger Gase

Bezeichnung	Symbol	Molmasse [kg/kmol]	Gaskonstante [J/(kg K)]	Dichte [kg/m3]	c_p [J/(kg K)]	c_v [J/(kg,K)]	κ
Acetylen	C_2H_2	26.038	319.3	1.16	1616	1278	1.26
Ammoniak	NH_3	17.031	488.2	0.76	2056	1526	1.35
Argon	Ar	39.948	208.1	1.76	519	309	1.68
Äthan	C_2H_6	30.070	276.5	1.34	1650	1355	1.22
Butan	C_4H_10	58.124	143.0	2.67	1599	1410	1.13
Chlor	C_l2	56.108	117.3	3.17	473	343	1.38
Chlorwasserstoff	HCl	70.906	228.0	1.62	795	556	1.43
Helium	He	4.003	2077.0	0.18	5200	3124	1.66
Kohlendioxid	CO_2	44.010	188.9	1.95	816	618	1.32
Kohlenmonoxid	CO	28.010	296.8	1.23	1038	739	1.40
Luft	˘	28.964	287.1	1.28	1006	718	1.40
Methan	CH_4	16.043	518.3	0.71	2165	1638	1.32
Propan	C_3H_8	44.097	188.5	1.99	1549	1331	1.16
Sauerstoff	O_2	31.999	259.8	1.41	909	647	1.40
Stickstoff	N_2	28.013	296.8	1.23	1038	739	1.40
Wasserstoff	H_2	2.016	4124.2	0.09	14050	9926	1.42
Xenon	Xe	131.300	63.3	5.82	159	93	1.71

21 Stoffdaten einiger Stoffe

Name Name	chemische Formel	Molmasse [kg/kmol]	Normal-siedepunkt [°C]	kritische Temperatur [°C]	kritischer Druck [MPa]
Wasserstoff	H_2	2.02	-252.9	-240.0	1.32
Helium	He	4.00	-268.9	-268.0	0.23
Ammoniak	NH_3	17.03	-33.3	132.3	11.33
Wasser	H_2O	18.02	100.0	373.9	22.06
Luft	78% $N_221\%$ $O_2.1\%Ar.+$	28.96	-194.2	-140.4	3.84
Kohlendioxid	CO_2	44.01	-78.4	31.0	7.38
Methan	CH_4	16.04	-161.5	-82.6	4.60
Äthan	C_2H_6	30.07	-88.6	32.2	4.87
Propan	C_3H_8	44.10	-42.1	96.7	4.25
R134a	CH_2FCF_3	102.03	-26.1	101.1	4.06

22 Zahlenwerte feuchte Luft

Bezeichnung	Formelzeichen	Zahlenwert	Dimension
Molmasse der Luft	ML	28,96	kg/ kmol
Molmasse des Wassers	MH2O	18,02	kg/ kmol
spezifische Gaskonstante der Luft	RL	0,287	kJ/ (kg K)
spezifische Gaskonstante des Dampfes	RD	0,461	kJ/ (kg K)
spezifische Wärmekapazität der Luft	cpL	1,006	kJ/ (kg K)
spezifische Wärmekapazität des Dampfes	cpD	1,92	kJ/ (kg K)
spezifische Wärmekapazität des Wassers	cW	4,182	kJ/ (kg K)
spezifische Wärmekapazität des Eises	cE	2,1	kJ/ (kg K)
Verdampfungsenthalpie des Wassers bei 0 °C	rD	2500	kJ/ kg
Schmelzenthalpie des Eises bei 0 °C	rE	334	kJ/ kg

23 Obskure Zusammenhänge

$$dV = \left(\frac{\partial V}{\partial T}\right)_p dT + \left(\frac{\partial V}{\partial p}\right)_{T,n} + \sum_{k=1}^{K} \left(\frac{\partial V}{\partial n_k}\right)_{T,p} dkn_k$$

$$dS = \left(\frac{nC_{p,m}}{T}\right) dT - \left(\frac{\partial V}{\partial T}\right)_{p,n} dp + \sum_{k=1}^{K} \left(\frac{\partial \mu_k}{\partial T}\right)_{p,n} dn_k$$

$$dU = \left[nC_{p,m} - p\left(\frac{\partial V}{\partial T}\right)_{p,n}\right] dT - \left[p\left(\frac{\partial V}{\partial p}\right)_{T,n} + T\left(\frac{\partial V}{\partial T}\right)_{p,n}\right] dp + \sum_{k=1}^{K} \left[\mu_k - T\left(\frac{\partial \mu_k}{\partial T}\right)_{p,n} - p\left(\frac{\partial V}{\partial n_k}\right)_{T,p,n}\right] dn_k$$

$$dH = nC_{p,m}dT + \left[VT\left(\frac{\partial V}{\partial T}\right)_{p,n}\right] + \sum_{k=1}^{K} \left[\mu_k - T\left(\frac{\partial \mu_k}{\partial T}\right)_{p,n}\right] dn_k$$

$$dF = -\left[S + p\left(\frac{\partial V}{\partial T}\right)_{p,n}\right] dT - p\left(\frac{\partial V}{\partial p}\right)_{T,n} dp + \sum_{k=1}^{K} \left[\mu_k - p\left(\frac{\partial V}{\partial n_k}\right)_{T,p}\right] dn_k$$

$$\left(\frac{\partial C_{p,m}}{\partial p}\right)_{T,\psi_j} = T\frac{\partial}{\partial p}\left[\left(\frac{\partial S_m}{\partial T}\right)_{p,\psi_j}\right]_{T,\psi_j} = T\frac{\partial}{\partial T}\left[\left(\frac{\partial S_m}{\partial p}\right)_{T,\psi_j}\right]_{p,\psi_j} = -T\frac{\partial}{\partial T}\left[\left(\frac{\partial V_m}{\partial T}\right)_{p,\psi_j}\right]_{p,\psi_j} = -T\left(\frac{\partial^2 V_m}{\partial T^2}\right)_{p,\psi_j}$$

$$C_{p,m} = (C_{p,m})_{\text{ideales Gas}} - T\int_0^p \left(\frac{\partial^2 V_m}{\partial T^2}\right)_{p,\psi_j} d\tilde{p}$$

$$C_{v,m} = (C_{v,m})_{\text{ideales Gas}} - T\int_0^{V_m} \left(\frac{\partial^2 p}{\partial T^2}\right)_{p,\psi_j} d\tilde{V}$$

24 Dinge die man eigentlich wissen sollte

$$1J = 1W = 1Nm$$

$$E_{kin} = \frac{1}{2}mc^2$$

$$E_{rot} = \frac{1}{2}I\omega^2$$

$$E_{Feder} = \frac{1}{2}kx^2$$

$$E_{pot} = mgz$$

$$E_{Kondensator} = \frac{1}{2}C\left(\frac{Q_e}{C}\right)^2$$

$$E_{Spule} = \frac{1}{2}LI^2$$

$$E_{Elelektrisch} = UA$$

$10^1 = 1$	
$10^1 = 10$	$10^{-1} = 0.1$
$10^2 = 100$	$10^{-2} = 0.01$
$10^3 = 1000$	$10^{-4} = 0.001$
$10^4 = 10\,000$	$10^{-4} = 0.000\,1$
$10^5 = 100\,000$	$10^{-5} = 0.000\,01$
$10^6 = 1000\,000$	$10^{-6} = 0.000\,001$
$10^7 = 10\,000\,s000$	$10^{-7} = 0.000\,000\,1$
$10^8 = 100\,000\,000$	$10^{-8} = 0.000\,000\,01$
$10^9 = 1000\,000\,000$	$10^{-9} = 0.000\,000\,001$
$10^10 = 10\,000\,000\,000$	$10^{-10} = 0.000\,000\,000\,1$
$10^11 = 100\,000\,000\,000$	$10^{-11} = 0.000\,000\,000\,01$
$10^12 = 1000\,000\,000\,000$	$10^{-12} = 0.000\,000\,000\,001$

m^2	dm^2	cm^2	mm^2
1	10^2	10^4	10^6
10^{-2}	1	10^2	10^4
10^{-4}	10^{-2}	1	10^2
10^{-6}	10^{-4}	10^2	1

m^3	dm^3	cm^3	mm^3
1	10^3	10^6	10^9
10^{-3}	1	10^3	10^6
10^{-6}	10^{-3}	1	10^3
10^{-9}	10^{-6}	10^3	1