Thermodynamik Formelsammlung

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$$\frac{d}{dt}\left\{U+m\left(\frac{c^2}{2}+gz\right)\right\} = \sum_{j}\left[\dot{m}_{j}\left(h+\frac{c^2}{2}+gz\right)_{i}\right] + \sum_{l}\left(\dot{Q}_{l}\right)_{l} + \sum_{i}\left(\dot{W}_{l}\right)_{i} - p\frac{dV}{dt}$$

1 Nomenklatur

 $\mathbf{An} = \text{Anergie}[\mathbf{J}]$

 $c_s = Schallgeschwindigkeit[m/s]$

 $c_{\rm p} = {\rm Spezifische\ W\"{a}rmekapazit\"{a}t\ dp} = 0\ [{\rm J/kg*K}]$

 $\mathbf{c_v} = \text{Spezifische Wärmekapazität dv} = 0 [J/kg*K]$

 $\mathbf{E} = \text{Energie}[\mathbf{J}]$

 $\mathbf{E}\mathbf{x} = -\mathbf{W}_{\mathbf{e}\mathbf{x}} = \mathrm{Exergie}[\mathbf{J}]$

 $\mathbf{F} = Kraft[N]$

 $\mathbf{F} = \mathbf{U} - \mathbf{TS} = \text{Freie Energie}[\mathbf{J}]$

 $\mathbf{f} = \mathbf{u} - \mathbf{T}\mathbf{s} = \text{Spezifische freie Energie}[J/kg]$

 $\mathbf{f} = \text{Fugazität}[Pa]$

G = H - TS = Freie Enthalpie[J]

 $\mathbf{g} = \mathbf{h} - \mathbf{T}\mathbf{s} = \text{Spezifische freie Enthalpie}[J/kg]$

 $\mathbf{g} = \text{Erdbeschleunigung}[\text{m/s}^2]$

 $\mathbf{H} = \mathbf{U} + \mathbf{pV} = \text{Enthalpie}[\mathbf{J}]$

 $\mathbf{h} = \mathbf{u} + \mathbf{p}\mathbf{v} = \text{Spezifische Enthalpie}[\text{J/kg}]$

■**Hg** = Molare Reaktionsenthalpie

K = Konstante des Massenwirkungsgesetztes[-]

 $\mathbf{M} = \text{Molmasse[kg/mol]}$

 $\dot{\mathbf{m}} = \text{Massestrom}[\text{kg/s}]$

 $\mathbf{m}' = \text{Masse in der flüssigen Phase[kg]}$

 $\mathbf{m}'' = \text{Masse in der gasförmigen Phase[kg]}$

 $Ma = c/c_s = Machzahl[-]$

 $\mathbf{n} = \mathbf{m}/\mathbf{M} = \text{Molzahl[mol]}$

n = Polytropenexponent[-]

 $\mathbf{P_t} = \text{technische Leistung}[\mathbf{W}]$

 $\mathbf{Q} = \text{W\"{a}rme}[J]$

 $\dot{\mathbf{Q}} = \text{Wärmestrom}[\mathbf{W}]$

q = Spezifische Wärme[J/kg]

 $\mathbf{r} = \text{Spezifische Verdampfungsenthalpie}[J/kg]$

 $\mathbf{R} = \text{Gaskonstante}[J/(\text{kg K})]$

 $\mathbf{R}_{\mathbf{m}} = \text{Universelle Gaskonstante}[J/(\text{mol } K)]$

S = Entropie[J/K]

s = Spezifische Entropie[J/(kg K)]

T = Temperatur[K]

 $\mathbf{t} = \text{Zeit}[s]$

 $\mathbf{t} = \text{Temperatur}[^{\circ}\text{C}]$

T = Sättigungstemperatur[K]

U = Innere Energie[J]

 $\mathbf{u} = \text{Spezifische innere Energie [J/kg]}$

 $V = Volumen[m^3]$

 $\mathbf{v} = \text{Spezifisches Volumen}[\text{m}^3/\text{kg}]$

 $V_m = Molares Volumen[m³/mol]$

 $\mathbf{W} = \text{Arbeit}[J]$

 $\mathbf{w} = \text{Spezifische Arbeit}[J/kg]$

 $\mathbf{W}_{\mathbf{V}} = \text{Volumen}$ änderungsarbeit[J]

 $W_{el} = Elektrische Arbeit[J]$

 $\mathbf{W}_{\mathbf{w}} = \text{Wellenarbeit}[\mathbf{J}]$

 $W_{diss} = Dissipations arbeit[J]$

 $W_t = \text{Technische Arbeit}[J]$

 $\mathbf{W}_{Virrev} = \text{Arbeits verlust durch Irreversibilität}[J]$

 $\mathbf{x} = \frac{m''}{m' + m''} = \text{Dampfanteil[-]}$

 $\mathbf{x} = \frac{m_{H_2O}}{m_L} = \text{Wassergehalt}$

 $\mathbf{Z} = \text{Allgemeine extensive Zustandsgrößen}[\mathbf{Z}]$

z = Allgemeine

 β = Isobarer Ausdehnungskoeffizient[1/K]

 γ = Isochorer Spannungskoeffizeint[1/K]

 $\delta_{\rm T} = {\rm Isothermer\ Drosselkoeffizient[m^3/kg]}$

 $\delta_{\mathbf{h}} = \text{Isenthalper Drosselkoeffizient}[\text{Ks}^2\text{m/kg}]$

 ε = Leistungsziffer[-]

 $\varepsilon = \text{Verdichtungsverhältnis}[-]$

 $\eta_{\rm th} = \text{Thermischer Wirkungsgrad}[-]$

 $\eta_{\text{mech}} = \text{Mechanischer Wirkungsgrad[-]}$

 $\kappa = \text{Adiabaten- oder Isentropenexponent}[-]$

 $\lambda = \text{Reaktionslaufzahl}[-]$

 μ_i = Chemisches Potential[J/mol]

 v_i = Stöchiometrische Koeffizienten[-]

 $\xi_{\mathbf{i}} = \text{Masseanteil}[-]$

 $\pi = \text{Druckverhältnis}[-]$

 $\rho = \text{Dichte}[\text{kg/m}^3]$

 $\tau = \text{Temperaturverhältnis}[-]$

 ϕ = Relative Feuchte[-]

 $\phi = \text{Einspritzverhältnis}[-]$

 ξ = Isothermer Kompressibilitätskoeffizient[m²/N]

 \blacksquare = Dissipationsenergie[J]

 $\psi = \text{Spezifische Dissipationsenergie}[J]$

 ψ = Drucksteigerungsverhältnis[-]

 $\psi_{\mathbf{i}} = \text{Molanteil[-]}$

2 Grundbegriffe

Systeme

- Abgeschlossenes System kein Stoff oder Energietransport
- Geschlossenes System kein Stofftransport
- Adiabates System kein Δq , aber Masse und Arbeit.
- Offenes System Stoff und Energietransport
- Stationäres System $\rightarrow \Delta U = 0$

Messgrößen

- Prozessgrößen sind Wegabhängig (eg. Arbeit, Wärme)
- Zustandsgrößen sind Wegunabhängig (eg. Volumen, Druck)
- Extensive Zustandsgrößen sind abhängig von der Masse des Systems (V, m, H, S, F, G, E)
- Intensive Zustandsgrößen sind unabhängig von der Masse des Systems (T, p)

Zustandsgleichungen

- Thermisch $\rightarrow f(p, V, T) = 0$
- Kalorisch $\rightarrow f(U, V, T) = 0$, U = U(V, T), u = u(v, T)

Hauptsätze

- 0: Temperatur existiert, ihre gleichheit ist notwendige Vorraussetzung für das thermische Gleichgewicht.
- 1: Energie existiert, sie ist für abgeschlossene Systeme konstant.
- 2: Entropie existiert, sie wird bei allen irreversiblen Prozessen erzeugt. $dS = \frac{\delta Q_{rev}}{T}$
- 3: 0K exisitert, bei dieser Temperatur ist die Entropie = 0

4 Gibbs

$$dU = Tds - pdV + \sum_{k=1}^{K} \mu_k dn_k$$

$$dG = -SdT + Vdp + \sum_{k=1}^{K} \mu_k dn_k$$

$$dH = TdS + Vdp + \sum_{k=1}^{K} \mu_k dn_k$$

$$dF = -SdT - pdV + \sum_{k=1}^{K} \mu_k dn_k$$

$$dU = \left(\frac{\partial U}{\partial S}\right)_V dS + \left(\frac{\partial U}{\partial V}\right)_S dV + \sum_{k=1}^{K} \left(\frac{\partial U}{\partial n_k}\right)_S dn_k$$

5 Thermodynamische Beziehungen

$$T = \left(\frac{\partial U}{\partial S}\right)_{V} = T(S, V) \qquad -S = \left(\frac{\partial F}{\partial T}\right)_{V} = S(T, V)$$

$$T = \left(\frac{\partial H}{\partial S}\right)_{p} = T(S, p) \qquad -S = \left(\frac{\partial G}{\partial T}\right)_{p} = S(T, p)$$

$$p = -\left(\frac{\partial U}{\partial V}\right)_{S} = p(V, S) \qquad V = \left(\frac{\partial G}{\partial p}\right)_{T} = V(p, T)$$

$$-p = \left(\frac{\partial F}{\partial V}\right)_{T} = p(T, V) \qquad \mu = \left(\frac{\partial U}{\partial n}\right)_{S, V} = \mu(S, V, n)$$

3 Basisformeln

$$dS = \frac{Q_{rev}}{T} + S_{prod}$$

$$dS = \frac{\delta Q_{rev}}{T}$$

$$F = U - TS$$

$$G = \frac{H - ST}{I}$$

$$W = -\int p \, dV$$

$$dS = \frac{Q_{rev}}{T} + S_{prod}$$

$$\Psi = \int_{1}^{2} T \, dS_{prod}$$

$$W_{ir} = \frac{T_{u}}{T} \Psi$$

$$p_{1} = p_{a} + \frac{\varphi_{1} - \varphi_{a}}{\varphi_{b} - \varphi_{a}} (p_{b} - p_{c})$$

6 Guggenheim

$$W_{ir} = \frac{T_u}{T} \Psi$$

$$p_1 = p_a + \frac{\varphi_1 - \varphi_a}{\varphi_b - \varphi_a} (p_b - p_a)$$

$$-S \quad U \quad V \quad U = U(S, V)$$

$$H \quad F \quad H = H(S, p)$$

$$-p \quad G \quad T \quad F = F(T, V)$$

$$G = G(T, p)$$

$$\underbrace{\frac{d}{dt}\left\{U+m\left(\frac{c^{2}}{2}+gz\right)\right\}}_{\text{Stationäres System -> 0}} = \underbrace{\sum_{j}\left[\dot{m}_{j}\left(h+\frac{c^{2}}{2}+gz\right)_{j}\right]}_{\text{Geschlossenes System -> 0}} + \underbrace{\sum_{l}\left(\dot{Q}_{t}\right)_{l}}_{\text{Keine Leistung -> 0}} + \underbrace{\sum_{l}\left(\dot{W}_{t}\right)_{i}}_{\text{Keine Volumenänderung -> 0}} + \underbrace{\sum_{l}\left(\dot{W}_{t$$

7 Maxwell

$$\left(\frac{\partial T}{\partial p}\right)_{S,n_j} = \left(\frac{\partial V}{\partial S}\right)_{p,n_j} \\
\left(\frac{\partial S}{\partial V}\right)_{T,n_j} = \left(\frac{\partial p}{\partial T}\right)_{V,n_j} \\
\left(\frac{\partial S}{\partial p}\right)_{T,n_j} = -\left(\frac{\partial V}{\partial T}\right)_{p,n_j} \\
\left(\frac{\partial \mu_i}{\partial T}\right)_{p,n_j} = -\left(\frac{\partial S}{\partial n_i}\right)_{T,p,n_j \neq n_i} \\
\left(\frac{\partial \mu_i}{\partial p}\right)_{T,n_j} = \left(\frac{\partial V}{\partial n_i}\right)_{T,p,n_j \neq n_i} \\
\left(\frac{\partial W}{\partial n_i}\right)_{T,p,n_j \neq n_i} = \left(\frac{\partial V}{\partial n_i}\right)_{T,p,n_j \neq n_i} \\
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\left(\frac{\partial W}{\partial N_i}\right)_{T,p,n_j \neq n_i} = \left(\frac{\partial W}{\partial N_i}\right)_{T,p,n_j \neq n_i} \\
\left(\frac{\partial W}{\partial N_i}\right)_{T,p,n_j \neq$$

8 Ideales Gas

$$pV = mRT$$

$$pV = nR_mT$$

$$\beta = \frac{1}{T}$$

$$\gamma = \frac{1}{T}$$

$$\chi = \frac{1}{p}$$

$$\beta = p\gamma\chi$$

$$R_m = 8,3143 \left[\frac{kJ}{kmolK} \right]$$

$$R = c_p - c_v$$

$$R = \frac{R_m}{M}$$

$$U - U_0 = mc_v(T - T_0)$$

$$H - H_0 = mc_p(T - T_0) \leftarrow \text{Für } c_p \text{ und } c_v \text{ const.}$$

$$s - s_0 = R \ln \left(\frac{p}{v_0} \right) + c_v \ln \left(\frac{T}{T_0} \right)$$

$$= c_v \ln \left(\frac{p}{p_0} \right) + c_p \ln \left(\frac{v}{v_0} \right)$$

$$= c_p \ln \left(\frac{T}{T_0} \right) - R \ln \left(\frac{p}{p_0} \right)$$

$$\beta = \frac{1}{T} = \frac{1}{V} \left(\frac{\partial V}{\partial T} \right)_p = \frac{1}{V} \left(\frac{\partial v}{\partial T} \right)_p = -\frac{1}{\rho} \left(\frac{\partial \rho}{\partial T} \right)_p$$

$$\gamma = \frac{1}{T} = \frac{1}{p} \left(\frac{\partial P}{\partial T} \right)_v$$

$$\chi = \frac{1}{p} = -\frac{1}{V} \left(\frac{\partial V}{\partial p} \right)_T = -\frac{1}{V} \left(\frac{\partial v}{\partial p} \right)_T$$

$$u_2 - u_1 = \int_{T_1}^{T_2} c_v(T) dT$$

9 Van-der-Waals

$$\left(p + \frac{a}{v^2}\right)(v - b) = RT$$

$$\left(\overline{p} + \frac{3}{\overline{v}^2}\right)(3\overline{v} - 1) = 8\overline{T}$$

$$\overline{p} = \frac{p}{p_K}, \quad \overline{v} = \frac{v}{v_K}, \quad \overline{T} = \frac{T}{T_K}$$

$$p_K = \frac{a}{27b^2}, \quad T_K = \frac{8}{27}\frac{a}{b}\frac{1}{R}, \quad \overline{z}$$

$$a = 3p_K v_K^2, \quad b = \frac{v_K}{3}, \quad \frac{p_K v_K}{RT_K} = \frac{3}{8}$$

$$\beta = \frac{(v - b)Rv^2}{RTv^3 - 2a(v - b)^2}$$

$$\gamma = \frac{Rv^2}{RTv^3 - 2a(v - b)^2}$$

$$\chi = \frac{(v - b)^2 v^2}{RTv^3 - 2a(v - b)^2}$$

$$du = \frac{a}{v^2}dv + c_v(T)dT$$

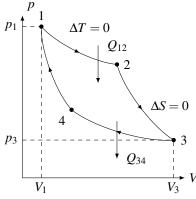
$$u - u_0 = \left(\frac{a}{v_0} - \frac{a}{v}\right) + \int_{T_0}^T c_v(\tilde{T}) d\tilde{T}$$

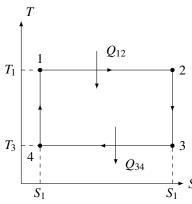
$$u - u_0 = \left(\frac{a}{v_0} - \frac{a}{v}\right) + c_v(T - T_0) \leftarrow \text{für } c_v = \text{const.}$$

$$c_p - c_v = \frac{Tv\beta^2}{\chi}$$

$$s - s_0 = c_v \ln\left(\frac{T}{T_0}\right) + R\ln\left(\frac{v - b}{v_0 - b}\right)$$

$$\begin{split} &\eta_{th} = 1 - \frac{-Q_{34}}{Q_{12}} = 1 - \frac{T_3(S_3 - S_4)}{T_1(S_2 - S_1)} = 1 - \frac{T_3}{T_1} \\ &\frac{Q_{12}}{T_1} + \frac{Q_{34}}{T_3} = 0 \\ &\Delta S_{ges} = -Q_{34} \left(\frac{1}{T_{KK}} - \frac{T_1}{T_3} \frac{1}{T_{HK}} \right) \end{split}$$





11 Gemische Idealer Gase

$$\xi_{i} = \frac{m_{i}}{m}, \quad \psi_{i} = \frac{n_{i}}{n}, \quad p_{i} = \psi_{i}p$$

$$\xi_{i} = \frac{M_{i}n_{i}}{\sum_{k=1}^{K} M_{k}n_{k}} = \frac{M_{i}}{M_{G}} \psi$$

$$p_{i}V = m_{i}R_{i}T, \quad p_{i}V = n_{i}R_{m}T, \quad pV = mR_{G}T$$

$$\sum_{k=1}^{K} p_{k} = p$$

$$R_{G} = \frac{1}{m} \sum_{k=1}^{K} m_{k}R_{k} = \sum_{k=1}^{K} \xi_{k}R_{k}$$

$$U_{G} = \sum_{k=1}^{K} U_{k} = \sum_{k=1}^{K} m_{k}u_{k} = \sum_{k=1}^{K} c_{vk}m_{k}T \leftarrow c_{v} = \text{const}$$

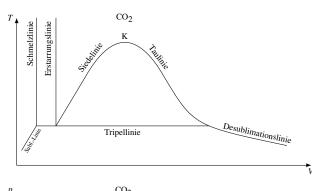
$$H_{G} = \sum_{k=1}^{K} H_{k} = \sum_{k=1}^{K} m_{k}h_{k} = \sum_{k=1}^{K} c_{pk}m_{k}T \leftarrow c_{p} = \text{const}.$$

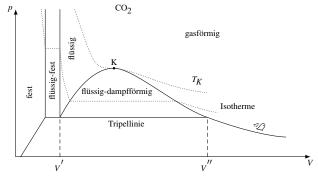
$$c_{vG} = \sum_{k=1}^{K} c_{vk}\xi_{k}, \quad c_{pG} = \sum_{k=1}^{K} c_{pk}\xi_{k}$$

$$S_{2} - S_{1} = R_{m} \left(n \ln n - \sum_{k=1}^{K} n_{k} \ln n_{k} \right)$$

Adiabate Drosselung (ideal): $h + \frac{c^2}{2} + gz = \text{const.}$ dh = 0 Adiabet Drosselung (real): $\delta_h = \left(\frac{\partial T}{\partial p}\right)_h = -\frac{v}{c_p}(1 - \beta T)$

12 Nassdampf





$$v = (1 - x)v' + xv''$$

$$v = v' + (v'' - v')x$$

$$u = (1 - x)u' + xu''$$

$$u = u' + (u'' - u')x$$

$$dg' = v''dp' - s'dT'$$

$$dg'' = v''dp'' - s''dT''$$

$$dg' = dg''$$

$$df = \frac{1}{T} \frac{h'' - h'}{v'' - v'}$$

$$s = s' + (s'' - s')x$$

$$\frac{dp}{dT} = \frac{1}{T} \frac{r}{v'' - v'}$$

$$r = h'' - h' = T(s'' - s')$$

13 Realer Stoff im Nassdampfgebiet

Isobare Zustandsänderung

$$q_{12} = T(s_2 - s_1)$$

$$= T(s'' - s')(x_2 - x_1)$$

$$w_{V,12} = -\int_1^2 p \, dv$$

$$= -p(v_2 - v_1) = -p(v'' - v')(x_2 - x_1)$$

Isochore Zustandsänderung

$$q_{12} = u_2 - u_1 = u_2' + x_2 \left(u_2'' - u_2'\right) - u_1' - x_1 \left(u_1'' - u_1'\right)$$

Adiabate Zustandsänderung

$$w_{V,12} = u_2 - u_1 = u'_2 + x_2 \left(u''_2 - u'_2\right) - u'_1 - x_1 \left(u''_1 - u'_1\right)$$

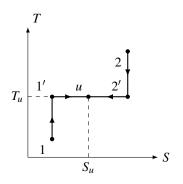
Entropieänderung wärend des Mischvorgangs

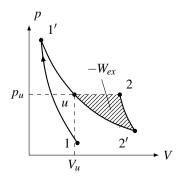
$$S_2 - S_2 = R_m \left(n \ln n - \sum_i n_i \ln n_i \right)$$

14 Maximale Arbeit und Exergie

Maxiaml nutzbare Arbeit → isentrop, reibungsfrei

 $1 \rightarrow 1'$: isentrop auf T_u $1' \rightarrow u$: isotherm auf u





$$-\dot{W}_{ex} = -(\dot{W}_{t})_{rev} = -\frac{d}{dt} \left(U + m \left(\frac{c^{2}}{2} + gz \right) + p_{u}V - T_{u}S \right) + \sum_{i=1}^{K} \left(\dot{m}_{j} \left(h + \frac{c^{2}}{2} + gz - T_{s} \right) \right) + \sum_{l=1}^{K} \left(1 - \frac{T_{u}}{T} \right) \dot{Q}_{l}$$

Die Exergie der Enthalpie (offens, stationäres System)

$$-\dot{W}_{ex,1u} = \dot{m}(h_1 - h_u - T_u(s_1 - s_u))$$

Die Exergie der inneren Energie (geschlossenes, instationäres System)

$$-\dot{W}_{ex} = -\frac{d}{dt}(U + p_u V - T_u S)$$

$$-\dot{W}_{ex,1u} = U_1 - U_u - p_u(V_1 - V_u) - T_u(S_1 - S_u)$$

Für Ideales Gas

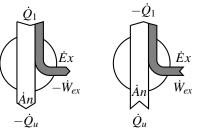
$$-W_{ex} = mc_v(T_1 - T_u) + p_u(V_1 - V_u) - T_u m \left(c_p \ln\left(\frac{T_1}{T_u}\right) - R_i \ln\left(\frac{p_1}{p_u}\right)\right)$$

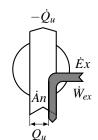
Dampf/Luftdruckkammer

$$-W_{ex,1u} = m_1[u_1 - u_u + p_u(v_1 - v_u) - T_u(s_1 - s_u)]$$

Die Exergie der Wärme (geschlossenes, stationäres System)

$$-\dot{W}_{ex} = \left(1 - \frac{T_u}{T_1}\right)\dot{Q}_1 = \eta_{th,C}\dot{Q}_1$$

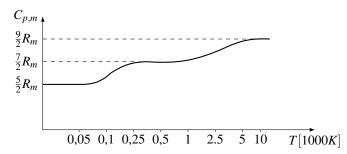




WärmekraftprozessWärmepumpenprozess

Kälteprozess

15 Wärmekapazität



$$C_{v,m} = \frac{1}{\kappa - 1} R_m, \quad C_{p,m} = \frac{\kappa}{\kappa - 1} r_m$$

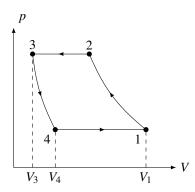
$$c_v = \frac{1}{\kappa - 1} R_j, \qquad c_p = \frac{\kappa}{\kappa - 1} R_j$$

$$\kappa = \frac{c_p}{c_v}, \qquad c_p - c_v = R$$

$$C_{v,m} = \underbrace{3 + \frac{R_m}{2}}_{\text{Translatorisch}} + \underbrace{\frac{n_{\text{rot}}R_m}{2}}_{\text{Rotatorisch}} + \underbrace{\frac{R_M(3n_{\text{Atome}} - 3 - n_{rot})}{\text{Vibratorisch}}}_{\text{Vibratorisch}}$$

16 Technische Anwendung

16.1 Kolbenverdichter



V1 = Maximales Zylindervolumen

V2 = Volumen nach Verdichtung

V3 =

V4 = Schädlicher Raum

$$\mu = \frac{V_1 - V_4}{V_1 - V_3}, \qquad \varepsilon_S = \frac{V_3}{V_1 - V_3}$$

$$\mu = 1 - \varepsilon_S \left[\left(\frac{p_2}{p_1} \right)^{\frac{1}{n}} - 1 \right]$$

$$W_{t,12} = \int_1^2 V \, dp$$

$$= \underbrace{p_2 V_2}_{Ausschiebearbeit} - \underbrace{p_1 V_1}_{Einschiebearbeit} - \int_1^2 p \, dV$$

$$= \frac{n}{n-1} p_1 (V_1 - V_4) \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - \right]$$

Verdichter Wirkungsgrad

$$\eta_{sV} = \frac{w_{t,12,rev}}{w_{t,12}} = \frac{h_{2,rev} - h_1}{h_2 - h_1}$$

Verdichter wirkungsgrad (Ideales Gas, $c_p = \text{const.}$)

$$n_{sV} = \frac{T_{2,rev} - T_1}{T_2 - T_1}$$

Technische Verlustarbeit

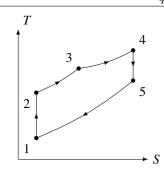
$$w_{t,Verl,12} = w_{t,12} - w_{t,12,rev} = h_2 - h_{2,rev}$$

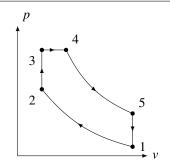
= $\int_{2,rev}^{2} T|_{p_2 = const.} ds$

Kreisprozesse mit idealen Gasen als Arbeitsmedium

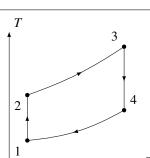
$$\eta_{th} = 1 - \frac{|q_{ab}|}{q_{zu}}, \quad \varepsilon_K = \frac{q_z u}{w}, \quad \varepsilon_{WP} = \frac{|q_{ab}|}{w}, \quad \varepsilon = \frac{v_1}{v_2}, \quad \psi = \frac{p3}{p2}, \quad \varphi = \frac{v_4}{v}$$

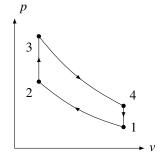
Seiliger Prozess



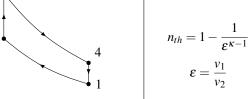


 $\eta_{th} = 1 - \frac{\varphi^{\kappa} \psi - 1}{\varepsilon^{\kappa - 1} \left[\psi - 1 + \kappa \psi \left(\varphi - 1 \right) \right]}$

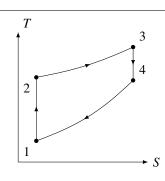


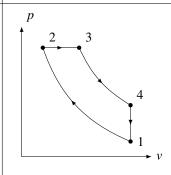


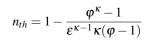
Otto Prozess

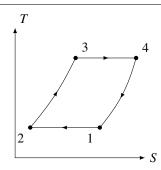


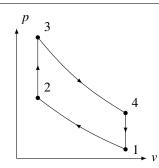






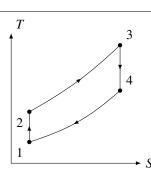


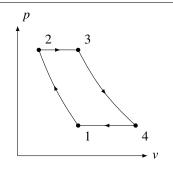




$n_{th}1 - \frac{|q_{12}|}{q_{34}} = \frac{RT_1 \ln\left(\frac{\nu_1}{\nu_2}\right)}{RT_3 \ln\left(\frac{\nu_4}{\nu_3}\right)} = 1 - \frac{T_1}{T_3}$

Stirling Prozess





Joule Prozess

$$\eta_{th} = 1 - \frac{T_1}{T_2} = 1 - \left(\frac{p_1}{p_2}\right)^{\frac{\kappa - 1}{\kappa}}$$

Ideales Gas

	Isothermo	Isobare	Isochore	Isentrop	Polytrope
konstant:	T	d	Λ	$\delta q = 0$	pv^n
	ı	ı	I	$p_1 v_1^K = p_2 v_2^K$	$v_1^n = p_2 v_2^n$
	$p_1p_2 = p_2v_2$	$\frac{v_1}{v_2} = \frac{T_1}{T_2}$	$\frac{p_1}{T_1} = \frac{p_2}{T_2}$		$T_1 \nu_1^{n-1} = T_2 \nu_2^{n-1}$
	ı	1	$\mathcal{D}_{\bar{z}}$	$\frac{T_1^{\frac{K}{K-1}}}{p_1} = \frac{T_2^{\frac{K}{K-1}}}{p_2}$	$\frac{T_1^{n-1}}{p_1} = \frac{T_2^{n-1}}{p_2}$
p, ν	$p = \frac{p_1 v_1}{v}$	$p = p_1$	$\nu = \nu_1$	$p = \frac{p_1 v_1^{\kappa}}{v^{\kappa}}$	$p=rac{p_1 v_1^n}{v^n}$
p,T	$p = \frac{p_1 v_1}{v}$	$p = p_1$	$p = rac{p_1}{T_1}T$	$p = \frac{p_1}{T_1^{\frac{K}{K-1}}} T^{\frac{K}{K-1}}$	$p = rac{p_1}{T_1^{n-1}} T^{rac{n}{n-1}}$
ν, T	$T = T_1$	$ u = rac{ u_1}{T_1} T $	$\nu = \nu_1$	$T=rac{T_1 v_1^{\mathcal{K}-1}}{v^{\mathcal{K}-1}}$	$T = \frac{T_1 v_1^{n-1}}{v^{n-1}}$
q 12	$= p_1 v_1 \ln \frac{p_1}{p_2}$	$=c_p(T_2-T_1)$	$=c_{ u}(T_2-T_1)$	0 =	$=c_{\nu \frac{n-\kappa}{n-1}}(T_2-T_1)$
WV,12	$=-q_{12}$	$=-p_1(\nu_2-\nu_1)$	0 =	$= \frac{p_1 v_1}{k-1} \left[\left(\frac{v_1}{v_1} \right)^{K-1} - 1 \right]$	$= \frac{p_1 \nu_1}{n-1} \left[\left(\frac{\nu_1}{\nu_2} \right)^{n-1} - 1 \right]$
$s_2 - s_1$	$s_2 - s_1 = R \ln \left(\frac{p_1}{p_2} \right)$	$\left \; ight = c_p \ln \left(rac{T_2}{T_1} ight)$	$=c_ u \ln \left(rac{T_2}{T_1} ight) \;$	0 =	$=c_{rac{n-\kappa}{n-1}}\ln\left(rac{T_2}{T_1} ight)$

Van-Der-Waals-Gas

	Isotherme	Isobare	Isochore	Isentrop
konst.		Ь	Λ	o = 0
	$(p_1 + rac{a}{ u^2})(u_1 - b) = (p_2 + rac{a}{ u^2})(u_2 - b)$	$\frac{RT_1}{v_1 - b} - \frac{a}{v_1^2} = \frac{RT_2}{v - b} - \frac{a}{v_2^2}$	$\frac{p_1 + \frac{a}{\sqrt{2}}}{T_1} = \frac{p_2 + \frac{a}{\sqrt{2}}}{T_2}$	$(p_1 + rac{a}{v^2})(v_1 - b)rac{c_{v+R}}{c_v} = (p + rac{a}{v^2})(v_2 - b)rac{c_{v+R}}{c_v}, \ T_1(v_1 - b)^{R/c_v} = T_2(v_2 - b)^{R/c_v}$
p, ν	$p = (p + \frac{a}{v^2}) \frac{v_u}{v - b} - \frac{a}{v^2}$	$p = p_1$	$\nu = \nu_1$	$p = -\frac{a}{v^2} + \left(p_1 + \frac{a}{v^2}\right) \left(\frac{v_1 - b}{v_m}\right)^{\frac{v_v + R}{R}}$
p,T	$T=T_1$	$p = p_1$	$p = \frac{T}{T_1} (p_1 + \frac{a}{v^2}) - \frac{a}{v_1^2}$	$p = -\frac{a}{v^2} + \left(p_1 + \frac{a}{v^2}\right) \left(\frac{T}{T_1}\right)^{\frac{c_V + R}{R}}$
ν, T	$T=T_1$	$T = T_1 \frac{v-b}{v_1-b} + \frac{a}{R}(v-b) \left(\frac{1}{v^2} - \frac{1}{v_1^2}\right)$	$\nu = \nu_1$	$T=T_1\left(rac{ u_1-b}{ u-b} ight)^{rac{R}{c_V}}$
<i>q</i> 12	$=RT_1\ln\left(\frac{v_2-b}{v_1-b}\right)$	$= rac{a}{v_1} - rac{a}{v_2} + c_{ u}(T_2 - T_1) + p_1(v_2 - v_1) \ = c_{ u}(T_2 - T_1)$		0 =
WV,12	$=-RT_1\ln\left(\frac{v_2-b}{v_1-b}\right)+\frac{a}{v_1}-\frac{a}{v_2}$	$=-p_1(v_2-v_1)$	= 0	$=rac{a}{v_1}-rac{a}{v_2}+c_{ u}(T_2-T_1)$
$s_2 - s_1$	$\left s_2 - s_1 \right = R \ln \left(\frac{v_2 - b}{v_1 - b} \right)$	$=c_{ u}\ln\left(rac{T_{2}}{T_{1}} ight)+R\ln\left(rac{ u_{2}-b}{ u_{1}-b} ight)$	$=c_ u \ln \left(rac{T_2}{T_1} ight)$	0 =