HW3

Kevin Chang

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Recall that for a vector $w \in \mathbb{R}^d$, $\mathcal{H}_w := \{z : <w, z> = 0\}$. Let $S = \{(xi, yi)\}$ be a set of linearly separable data in $\mathbb{R}^d$ (i.e., $x_i \in \mathbb{R}^d$ and $y_i \in \{-1, 1\}$). Define the set $\mathcal{M}_S$ to be the set of all vectors which separate the data with large dot product: $\mathcal{M}_S = \{w : yi<w, xi> \geq 1 \text{for} i = 1, \ldots, n\}$.

- Let w_* denote the element of $\mathcal{M}_S$ with smallest norm. Show that for any other w that separates the data
$$\min_{1 \leq i \leq n} dist(x_i, \mathcal{H}_w) \leq \min dist(x_i, \mathcal{H}_{w_*}).$$
- Show that there are real numbers α_i such that $w_* = \sum_{i=1}^{n} \alpha_i x_i$.
- Show that the α_i can be chosen so that $y_i \alpha_i$ are all nonnegative.