HW3

Kevin Chang

October 13, 2025

1

Recall that for a vector $w \in \mathbb{R}^d$, $\mathcal{H}_w := \{z : <w, z> = 0\}$. Let $S = \{(x_i, y_i)\}$ be a set of linearly separable data in $\mathbb{R}^d$ (i.e., $x_i \in \mathbb{R}^d$ and $y_i \in \{-1, 1\}$). Define the set $\mathcal{M}_S$ to be the set of all vectors which separate the data with large dot product:

$$\mathcal{M}_S = \{w : y_i <w, x_i> \geq 1 \text{for} i = 1, ..., n\}.$$

- Let w^* denote the element of $\mathcal{M}_S$ with smallest norm. Show that for any other w that separates the data
$$\min dist_{1 \leq i \leq n}(x_i, \mathcal{H}_w) \leq \min_{1 \leq i \leq n} dist(x_i, \mathcal{H}_{w^*}).$$
- Show that there are real numbers α_i such that $w^* = \sum_{i=1}^n \alpha_i x_i$.
- Show that the α_i can be chosen so that $yi\alpha_i$ are all nonnegative.