Stochastic Processes

Kevin Chang

January 13, 2022

1 Moment Generating Function

- Moment Generating Function: $\mathbb{E}[e^{tX}]$
  - Property:
    * $\mathbb{E}[e^{tx}] = \int_{-\infty}^{\infty} e^{tx} f_X(x)dx$
    * $\mathbb{E}[e^{tx}] = \sum_{k=0}^{\infty} E[X^k]\frac{t^k}{k!}$
      · $e^{tx} = \sum_{k=0}^{\infty} \frac{(tx)^k}{k!}$
      · $E[e^{tX}] = E[\sum_{k=0}^{\infty} \frac{(tX)^k}{k!}] = \sum_{k=0}^{\infty} E[X^k]\frac{t^k}{k!}$
    * $\frac{d\mathbb{E}[e^{tX}]}{dt} = \mathbb{E}[X]$
    * Not all random variables have Moment generating function
- Characteristic Function: $\mathbb{E}[e^{itX}]$
  - Property:
    * All random variables have Moment generating function
- Joint Moment Generating Function: $G(x, y) = \mathbb{E}[e^{xX}e^{yY}]$
- Property:
  - (Joint) moment generating funciton uniquely determines the (joint) CDF
- Example
  - Trapped miner's random walk
    * Miner has probability of $\frac{1}{3}$ to waste 3 hours in vain, $\frac{1}{3}$ to waste 5 hours in vain, and $\frac{1}{3}$ to spend 2 hours to go out of the mine.
    * X is the random variables of the hours to go out of the mine
    * Y_i is the random variables of the hours for the i-th action.
    * $\mathbb{E}[e^{tX}] = \mathbb{E}[e^{tX}|Y_1 = 2] + \mathbb{E}[e^{tX}|Y_1 = 3] + \mathbb{E}[e^{tX}|Y_1 = 5]$
      $= \mathbb{E}[e^{2t}] + \mathbb{E}[e^{t(X+3)}] + \mathbb{E}[e^{t(X+5)}]$
    * Find expectation and variance by joint moment generating function