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Probabilistic electricity price forecasting with Bayesian stochastic volatility models

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ABSTRACT

The study is focused on probabilistic forecasts of day-ahead electricity prices. The Bayesian approach allows for conducting statistical inference about model parameters, latent volatility, jump times and their sizes. Moreover, the Bayesian forecasting takes into account uncertainty of parameter estimation. Using the PJM data sets we demonstrate that Bayesian stochastic volatility model with double exponential distribution of jumps and exogenous variables outperforms the non-Bayesian individual autoregressive models as well as three averaging schemes of spot price forecasts. We argue that the structure is a promising tool of modelling and forecasting electricity prices.

1. Introduction

In recent decades electricity markets have been subject to both deregulation and liberalization, enabling determination of competitive prices according to the market forces of supply and demand. The prices of electricity affect both economies and people's everyday lives. Modelling and forecasting electricity prices are of the utmost importance to manufacturing plants, as energy occupies a substantial cost position in their profit and loss accounts, and also to retailers, as they buy energy at volatile prices and sell it at fixed prices.

The difficulty in storing large quantities of electricity has a significant impact on the market and on price behaviour. The imbalance between supply and demand in real time, as well as the breakdown of power plants, political decisions, and weather conditions can, in consequence, lead to sharp price movements, called jumps (Weron, 2006; Janczura et al., 2013; Weron, 2014; Paraschiv et al., 2015) and periods of higher price fluctuations. As a result, modelling and forecasting prices, and risk management pose a considerable challenge, largely due to their jumps and time-variable volatility.

Although the term *jump* is commonly used in literature, its generally-agreed on definition does not exist. For example, while analysing a time series one can easily discern the values, or jumps, on account of their pronounced outlying position with respect to other observations; however, at the same time one does not know how to classify the remaining data points, that is, where to put the border line between observations that should or should not be classified as jumps. In this paper, it is assumed that a trajectory is generated by a stochastic process with jumps. The problem whether to classify a given data point as being (co-)generated by a jump component or not is solved by introducing latent variables under the Bayesian approach. In practice, inference about whether a jump has occurred over a given time interval is based on relevant posterior probability. A similar idea is applied in Johannes et al. (1999), Kostrzewski (2012), Brooks and Prokopczuk (2013), Kostrzewski (2015) and Kostrzewski (2016).

In the framework of electricity price modelling, the term *spike* is more often used than the term *jump*. Moreover, Weron (2006) criticizes using the terms interchangeably. The term *spike* can be informally described as follows. Within a 'very short' period of time, an electricity price can increase or decrease 'substantially' by a sequence of upward (or downward) jumps, and later it returns to its previous level by a sequence of downward (or upward) jumps.

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0140-9883/

In this paper, such periods of volatile prices are modelled by means of a stochastic volatility structure. Models are applied to modelling and forecasting electricity prices. The results are compared with the ones calculated under models or techniques commonly employed for analysing electricity prices. The models with jumps were applied to commodity markets by Brooks and Prokopczuk (2013).

In this study, a stochastic volatility process, precisely a log-volatility process, is a discrete time version of a mean-reverting diffusion process (Kloeden and Platen, 1999). Moreover, the relationship between electricity prices and volatility is determined by a leverage effect (or an inverse leverage effect) parameter.

In a formal manner Bayesian inference tackles uncertainty present in the estimation of parameters (e.g. Box and Tiao, 1973; Bernardo and Smith, 2009). It also allows one to formally incorporate expert knowledge into the model. Moreover, the Bayesian approach allows for conducting statistical inference about latent volatility, jump times and their sizes, and, what is most important in our study, facilitates forecasting by taking into account uncertainty about parameters.

Two Bayesian stochastic volatility models with jumps and exogenous variables and two models without exogenous variables are considered in the study. Extending a stochastic structure by additional variables, which carry information about prices or the differences in log-prices, is a common practice in commodity modelling (e.g. Weron and Misiorek, 2008; Kristiansen, 2012).

The main aim of our study is to present the Bayesian prediction intervals and to compare the results of electricity price prediction obtained using the Bayesian and non-Bayesian methods. In order to check the quality of the prediction intervals the unconditional and conditional coverage tests are applied (Christoffersen, 1998). The Berkowitz tests are employed for testing the goodness-of-fit of the predictive distributions (Berkowitz, 2001). Moreover, in order to compare forecasts, the Diebold-Mariano tests are conducted.

The study is focused on the application of the stochastic volatility models with jumps known from financial markets to the electricity prices. However, the dynamics of electricity prices is different from the share price dynamics. The electricity price models are often extended so as to incorporate seasonality or the relations between electricity prices and temperature. Adding exogenous variables allows for capturing these dependencies. The study presents the Bayesian approach and models which contribute to a better understanding of the nature of electricity price dynamics. To the best of our knowledge, this is the first study that applies the stochastic volatility models with double exponential distribution of jumps and exogenous variables for the probabilistic electricity price forecasting.

In the empirical part of the paper, we compare the results calculated under the Bayesian model and non-Bayesian techniques for two data sets. In the first case we analyse the same time series of day-ahead electricity prices as Nowotarski and Weron (2015), who consider 12 individual non-Bayesian models: autoregressive models, threshold models, mean-reverting jump diffusion models and semi-parametric autoregressive models, and calculate prediction intervals under the individual models and three averaging schemes. In the second case, we analyse the time series of hourly day-ahead locational marginal prices of electricity for the JCPL zone of the PJM Interconnection and compare the results of the Bayesian structure and non-Bayesian ARX model.

The rest of the paper is organized as follows. Firstly, we present the mathematical model which we apply in the empirical part, which is followed by a description of the Bayesian approach, and the definition of the Bayesian model. Next, we report the results of the comparison of the Bayesian models and present the results of the comparison of Bayesian and non-Bayesian forecasting of electricity prices. Conclusions and remarks end the paper.

2. The Bayesian SVDEJX

The jump-diffusion models are often applied in financial and commodity market analyses. One of the most known jump-diffusion models is the model of Merton (1976). Such models seem appealing because of a jump component in their structure which refers to the outliers observed in a data set. However, they are also criticized for the assumption of a constant value of a volatility parameter.

Time series of electricity prices and the difference in log-prices demonstrate sharp movements of values as well as periods of higher and lower volatility, and that is why a stochastic volatility model with jumps is employed in the study. The time-variant volatility is a hallmark of the model. It distinguishes the model from the jump-diffusion structures.

The electricity prices depend on many external factors which might be included into a model. Therefore, we consider the SV model with exogenous and dummy variables i.e. temperature and variables representing the trade on Saturdays, Sundays and Mondays.

The normal distribution of jump sizes is very often assumed. However, in our study the distributions of negative or positive jumps are governed by exponential distributions with different values of parameters. In other words, we assume the double exponential distribution of jumps. The same values of the parameters lead to symmetry. However, we decide not to assume a symmetric distribution of jumps.

Finally, our study is based on the discrete time version of the stochastic volatility model with a double exponential distribution of jumps, a leverage effect and exogenous variables (in short, the SVDEJX model). The SVDEJX model with specific exogenous variables is defined as follows:

$$
\begin{aligned}
y_{t_{i+1}} =& y_{t_i} + \mu + \overbrace{\psi X_{t_{i+1}}}^{\text{log(t...)}} + \overbrace{d_{Sat} D_{Sat,i+1} + d_{Sun} D_{Sun,i+1} + d_{Mon} D_{Mon,i+1}}^{\text{seasonality}} \\
\ln(S_{t_{i+1}}) \quad & + \sqrt{\exp(h_{t_i})}\varepsilon^{(1)}_{t_{i+1}} + J_{t_{i+1}}, \\
h_{t_{i+1}} =& h_{t_i} + \kappa_h(\theta_h - h_{t_i}) + \sigma_h\left(\rho\varepsilon^{(1)}_{t_{i+1}} + \sqrt{1-\rho^2}\varepsilon^{(2)}_{t_{i+1}}\right), \\
J_{t_{i+1}} =& -\xi^D_{t_{i+1}} \cdot \mathbb{I}(q_{t_{i+1}} = -1) + 0 \cdot \mathbb{I}(q_{t_{i+1}} = 0) + \xi^U_{t_{i+1}} \cdot \mathbb{I}(q_{t_{i+1}} = 1),
\end{aligned}
$$

log-volatility next $ vol. of h

Jump occurence

where S_{t_i} stands for a price, $y_{t_i} = \ln(S_{t_i})$, $\varepsilon^{(1)}_{t_i}, \varepsilon^{(2)}_{t_i} \sim iid\, N(0,1)$, a discrete time scale $t_i = 0, 1, \ldots, n$. Moreover, $\mathbb{I}$ is an indicator function, $\left\{\xi^D_{t_i}\right\} \sim iid \exp(\eta_D)$, $\left\{\xi^U_{t_i}\right\} \sim iid \exp(\eta_U)$ and q_{t_i} is a discrete variable which corresponds to a jump occurrence at t_i. The variables $\left\{\xi^D_{t_i}\right\}$, $\left\{\xi^U_{t_i}\right\}$ and $\{q_{t_i}\}$ are independent. The variable X_{t_i} refers to the logarithm of the hourly temperature at t_i. The three dummy variables – $D_{Sat,i} = \mathbb{I}_{\{t_i = Saturday\}}$, $D_{Sun,i} = \mathbb{I}_{\{t_i = Sunday\}}$, $D_{Mon,i} = \mathbb{I}_{\{t_i = Monday\}}$ (for Saturdays, Sundays and Mondays, respectively) – account for weekly seasonality. The value ρ is called a leverage effect parameter (if $\rho < 0$) or an inverse leverage parameter (if $\rho > 0$). If higher log-prices correspond to higher volatility, we expect a positive value of ρ. It is assumed that the log-volatility value h_{t_i} (but not $h_{t_{i+1}}$ as in Jacquier et al., 2004) appears in the formula for the difference in log-prices $y_{t_{i+1}} - y_{t_i}$, which is in line with the idea presented by Yu (2005), Omori et al. (2007), Li et al. (2008), and Johannes and Polson (2010).

The variables $\{q_{t_i}\}$, $\left\{\xi^D_{t_i}\right\}$, $\left\{\xi^U_{t_i}\right\}$ and $\{h_{t_i}\}$ are not observed and called latent variables. Under the Bayesian approach they are estimated as unknown parameters. On the other hand, latent variables make it possible to define and detect jumps, as well as incorporate stochastic volatility into the model. Formally, the occurrence of a jump at the i-th moment is equivalent to $q_{t_i} \neq 0$,

$$
\text{where } q_{t_i} = \begin{cases} -1 & \text{with probability} p_D, \\ 0 & \text{with probability} p_0, \\ 1 & \text{with probability} p_U. \end{cases}
$$

The values of the latent variables $\{q_{t_i}\}$ are not observed, but we can estimate the probability of a jump.

3. The Bayesian approach

Fuck this part

The Bayesian inference about a vector $\theta \in \Theta$ of unknown quantities (parameters and latent variables) is based on a posterior distribution with density $p(\theta|y) = \frac{p(y|\theta)p(\theta)}{\int_\Theta p(y|\theta)p(\theta)d\theta}$, where $y = (y_{t_1}, \ldots, y_{t_n})$ is a vector of observations, $p(y|\theta)$ is a likelihood function, $p(\theta)$ is a prior density. The Bayesian model is defined according to the formula $p(y,\theta) = p(y|\theta)p(\theta)$. Bayesian forecasting for a moment $t_n + k$, under a model M, is based on a predictive distribution with a predictive density $p(y_{t_n+k}|y, M) = \int_\Theta p(y_{t_n+k}|\theta, y, M)\, p(\theta|y, M)\, d\theta$, where $k > 0$. The predictive density integrates uncertainty about the future value y_{t_n+k} and uncertainty about parameters and latent variables θ both conditional on the previous observations $y_{t_1}, \ldots, y_{t_n}$ and the assumptions of the model M.

The Bayesian SVDEJX model or, for the sake of simplicity, the SVDEJX model is defined with an extended parameter space including also latent variables. The vector of unknown quantities is as follows:

SBER

$$\underbrace{\mu, \kappa_h, \theta_h, \sigma_h, \rho, \eta_D, \eta_U, p_D, p_0, p_U, \psi, d_{Sat}, d_{Sun}, d_{Mon}}_{=\theta},$$
$$\underbrace{h_{t_0}, \ldots, h_{t_{n-1}}}_{=h}, \underbrace{q_{t_1}, \ldots, q_{t_n}}_{=q}, \underbrace{\xi^D_{t_1}, \xi^U_{t_1} \ldots, \xi^D_{t_n}, \xi^U_{t_n}}_{=\xi}).$$

The Bayesian model is defined by a joint distribution:

$$p(y, \theta, h, q, \xi) = p(y|\theta, h, q, \xi)\, p(\theta, h, q, \xi).$$

Following Jacquier et al. (2004) we apply the reparametrisation $(\sigma_h, \rho) \to (\phi_h, \omega_h)$; $\phi_h = \sigma_h \rho$, $\omega_h = \sigma_h^2(1-\rho^2)$. The prior structure is defined as follows:

$$\mu \sim N(0,10), \kappa_h \sim N(1,6)\mathbb{I}_{(0,2)}, \theta_h \sim N(0,10), \omega_h \sim IG\left(3, \tfrac{1}{20}\right),$$
$$\phi_h|\omega_h \sim N\left(0, \tfrac{1}{2}\omega_h\right), p(\eta_D) \sim IG(1.86, 0.43),$$
$$p(\eta_U) \sim IG(1.86, 0.43),$$

Inv. Gaussian

Fucking bullshit

$$(p_D, p_0, p_U) \sim Dirichlet(1,1,1), \psi \sim N(0,10),$$
$$d_{Sat} \sim N(0,10), d_{Sun} \sim N(0,10), d_{Mon} \sim N(0,10),$$
$$p(q_{t_i}|\theta) = \begin{cases} p_D & \text{on} \quad q_{t_i} = -1 \\ p_0 & \text{on} \quad q_{t_i} = 0 \\ p_U & \text{on} \quad q_{t_i} = 1 \end{cases},$$
$$\xi^D_{t_i}|\theta \sim \exp(\eta_D), \xi^U_{t_i}|\theta \sim \exp(\eta_U).$$

The values of the hyperparameters are assumed according to Szerszen (2009) and Kostrzewski (2016). We do not favour (by prior assumptions) any of the states: no jump, a negative jump and a positive jump under SVDEJX. Moreover, fairly diffuse prior independent normal distributions with means equal zero and standard deviations equal ten are chosen for the parameters of the exogenous variables. The means equal zero express the lack of prior knowledge about a direction (a sign) of a relationship between the variables and the price. The prior distributions are primarily conjugate. It means, that the posterior and prior distributions belong to the same class of distributions. It facilitates direct sampling from conditional posterior distributions under the Gibbs sampler. In other cases, the marginal posterior distribution is not known. In order to generate values from the non-standard distribution the Metropolis-Hastings algorithm is additionally embedded in the Gibbs sampler. An application of the algorithm takes extra time of calculations. The parameter η_U (and η_D) represents a mean of a jump size ($-\eta_D$ for negative jumps). The inverse gamma distribution is a conjugate prior distribution for η_U and η_D. The distribution has a positive support and a long right tail. The latter hallmark is useful to shift a mass of prior probability of a jump size 'away' from zero. The study assumes the inverse gamma distribution $IG(1.86, 0.43)$ for η_U (and η_D) with the prior mean and mode equal 0.5 and 0.15, respectively. The variance does not exist. Following our expectation and prior assumptions, 'small' changes of time series should be modelled by a (pure) SV part rather than by a jump component. The prior distribution supports (by a 'higher' probability) 'higher' values of η_U (and η_D). It expresses a prior expectation of 'larger' jumps.

4. Empirical study

Most important!

The following subsections discuss the results of the Bayesian and non-Bayesian forecasting.

Firstly, we analyse the time series previously considered by Nowotarski and Weron (2015), i.e. the series of electricity spot prices (USD/MWh) downloaded from the GDF Suez website (http://www.gdfsuez-energy-resources.com) and containing "hourly day-ahead locational marginal prices (LMPs) for the Jersey Central Power and Light Company (JCPL) of the Pennsylvania–New Jersey–Maryland (PJM) Interconnection (U.S.)" (Nowotarski and Weron, 2015). This data set covers the period from August 22, 2010 to January 14, 2012.

The second analysis covers a data set of hourly day-ahead locational marginal prices (LMPs) of electricity for the JCPL zone of the PJM Interconnection downloaded from the PJM website (http://dataminer2.pjm.com). The data span the period from January 4, 2015 to October 28, 2017. The prices are called zonal day-ahead LMPs. We consider the temperature in New York again which is a good enough representative of the JCPL zone temperature. New York is not a part of JCPL, but the geographical distance to the zone is important. The data set was downloaded from http://www.wunderground.com.

The PJM Interconnection coordinates the movement of electricity through all or parts of Delaware, Illinois, Indiana, Kentucky, Maryland, Michigan, New Jersey, North Carolina, Ohio, Pennsylvania, Tennessee, Virginia, West Virginia and the District of Columbia. The JCPL zone is one of 20 control zones within PJM and covers central and northern New Jersey.

Participants of the day-ahead PJM energy market participants may submit offers to sell and bids to buy energy for each hour of the next day. Loads and prices are posted daily by 1:30 p.m. and are financially binding. The day-ahead energy market is based on the locational marginal prices.

The locational marginal price (in USD/MWh) is defined as the marginal price for energy at the location where the energy is delivered or received. The day-ahead LMP is calculated at every location (bus, node) for each hour from the day-ahead dispatch required to meet estimated nodal loads and supply. Zonal day-ahead LMP, i.e. zonal day-ahead, load-weighted LMP, is calculated from the nodal day-ahead LMPs using zonal distribution factors – calculated from historical real-time, bus-level load distributions which were in operation at 8 a.m. a week prior – as the load weights.

The LMP is a pricing approach that takes into account transmission system congestion and loss costs. The LMP at a particular location consists of three components: congestion cost, marginal loss cost and system marginal price. The marginal loss cost represents the power lost when power moves across the transmission system. The congestion cost represents the inability to use the least expensive generation to meet the electricity demand due to transmission limitations. The system marginal price is calculated at the distributed load reference location, where the loss and congestion contribution to LMP are zero.

4.1. Data set I

In the first part of the empirical study we analyse the time series of hourly day-ahead prices of electricity (USD/MWh), taken from the Jersey Central Power and Light Company of the Pennsylvania–New Jersey–Maryland Interconnection. We analyse two data spans in the same way as Nowotarski and Weron (2015), who consider the same data. The in-sample data span from August 22, 2010 to September 22, 2011 (hourly observations for 397 days). The out-of-sample data

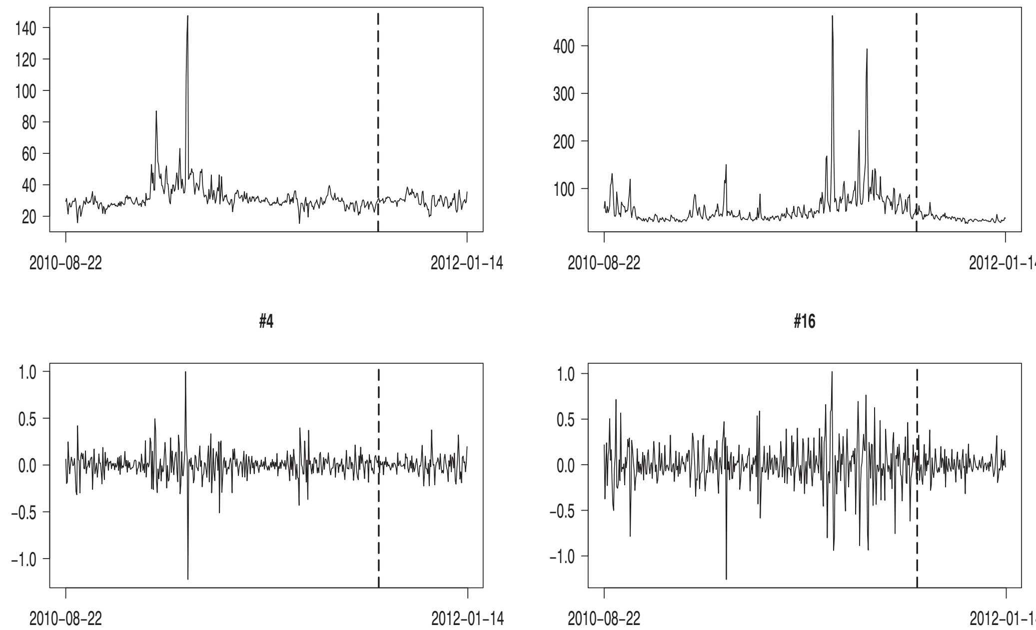

Fig. 1. Day-ahead/spot electricity prices (USD/MWh) for the JCPL zone of the PJM Interconnection (at the top) and differences in log-prices (at the bottom) at #4 hour (left panels) and #16 hour of a day (right panels). Dashed vertical lines split into in-sample (August 22, 2010–September 22, 2011) and out-of-sample (September 23, 2011–January 14, 2012) periods.

Table 1
Descriptive statistics of the electricity prices for #4 and #16 hours of a day calculated over the in-sample data span.

	#4 hour	#16 hour
Mean	32.305	60.967
Sd	11.405	44.415
Skewness	5.835	5.151
Kurtosis	50.695	39.084
Min	15.390	29.430
Max	147.600	463.400

span from September 23, 2011 to January 14, 2012 (hourly observations for 114 days). The logarithm of hourly air temperatures (in Fahrenheit degrees) in New York and three dummy variables $D_{Sat,i}$, $D_{Sun,i}$, $D_{Mon,i}$ are exogenous variables within the SVDEJX model.

First, we analyse two series of prices for #4 hour of a day (an example of an off-peak hour) and #16 hour of a day (an example of a peak hour). Prices at #4 hour and #16 hour are presented in the graphs on the left side of Fig. 1, and the differences in log-prices are shown on the right. One of the main characteristics of such time series are sharp upward or downward movements. Therefore, the jumps of prices or the jumps of the differences in log-prices should be taken into account in modelling such data sets. Moreover, it is easy to notice periods of high variability, which justifies the employment of the stochastic volatility component.

Table 1 presents descriptive statistics for the two time series. The mean price for #16 hour is higher than for #4 hour. It corresponds to the expectation that electricity at the peak hour is more expensive. The higher value of a standard deviation, and the wider span between minimum and maximum values suggest that the prices at the peak hour are much more volatile. The values of skewness and kurtosis suggest a non-normal distribution of prices.

The real temperature is used in the study as a one day-ahead prediction. The same data set is applied in Nowotarski and Weron (2015). Fig. 2 presents hourly air temperatures (in Fahrenheit degrees) in New York.

The time series of prices for #4 and #16 hours are analysed under the SVDEJX model and three additional Bayesian specifications. We apply the SVNJX model which is similar to the SVDEJX model. The only difference is the assumption about the distribution of jumps. In this case a normal distribution is assumed. Additionally, we apply two similar models to SVDEJX and SVNJX but without exogenous variables. The Bayesian version of the stochastic volatility model with normal distribution of jumps and without exogenous variables is proposed by Szerszen (2009). The Bayesian version of the stochastic volatility model with double exponential distribution of jumps but without exogenous variables is proposed by Kostrzewski (2016).

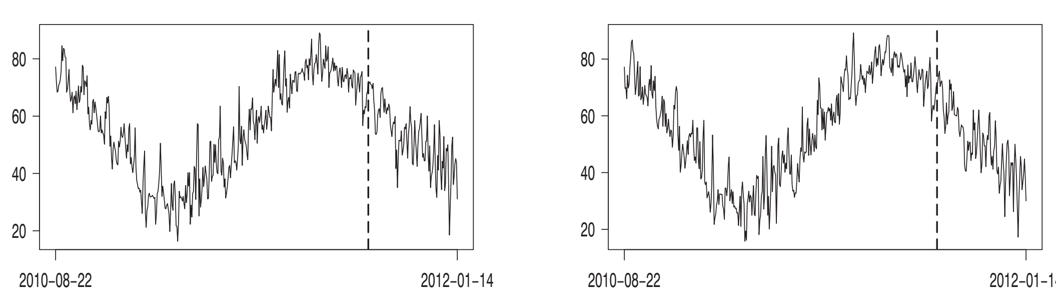

Fig. 2. Hourly air temperatures (in Fahrenheit degrees) in New York at #4 hour (left panel) and #16 hour of a day (right panel). Dashed vertical lines split into in-sample (August 22, 2010–September 22, 2011) and out-of-sample (September 23, 2011–January 14, 2012) periods.

Kou et al. (2017) consider a similar model but with a different h, i.e. they assume that h is defined as a square-root process (Cox et al., 1985).

Bayesian selection of the best model M is based on the choice of the model with the highest posterior probability $p\left(M|\, y\right)$. In order to choose between two models M_1 and M_2, the Bayes factors $\frac{p\left(M_1|y\right)}{p\left(M_2|y\right)}$ are calculated. The study assumes that prior probabilities of the models are the same i.e. $P\left(M_1\right) = P\left(M_2\right)$. In other words, neither of them is prior favoured. Therefore, the Bayes factors $\frac{p\left(M_1|y\right)}{p\left(M_2|y\right)} = \frac{p\left(y|M_1\right)}{p\left(y|M_2\right)}$, where $p\left(y|\, M_i\right)$ is a marginal data density of a model M_i. The harmonic mean estimator (Newton and Raftery, 1994) is applied to estimate the marginal data density. Due to space constraints, we report only the conclusions of the model comparison pertinent to our study. The models with the exogenous variables turn out to be superior to those without such variables. Moreover, the winner of the comparison is the stochastic volatility model with a double exponential distribution of jumps, a leverage effect and exogenous variables, and this is why the study is limited to the SVDEJX model.

Numerical methods of multidimensional integration – the Gibbs sampler and the Metropolis-Hastings algorithm – allow for conducting the Bayesian inference under the models. The numerical algorithms belong to the Markov Chain Monte Carlo (MCMC) methods (Gamerman and Lopes, 2006; Johannes and Polson, 2010) and they are applied in order to approximate non-standard posterior distributions and their characteristics. A convergence of the MCMC algorithms is verified by the visual inspection of ergodic means, standard deviations and CUMSUM statistics plots (Yu and Mykland, 1998). When jumps are normally distributed, the multimodal posterior distributions and problems with numerical convergence are observed. All calculations are performed in R.

4.1.1. *Bayesian and non-Bayesian forecasting*

In this part of the empirical study, the forecasts of day-ahead electricity prices independently for each of the 24 h for consecutive 114 days between September 23, 2011 and January 14, 2012 are calculated under the SVDEJX model. The results are compared with the outcomes published in Nowotarski and Weron (2015). Forecasting is carried out following the procedure described below. Firstly, we estimate the Bayesian model and calculate the first day-ahead forecast (a predictive distribution) over the in-sample data set. Secondly, we extend the in-sample data set by one observation, estimate the model and calculate the second day-ahead forecast. Thirdly, we extend the in-sample data set by one observation, estimate the model and calculate the third day-ahead forecast, and so on.

In the first step, many MCMC draws are applied. The estimation of the model is based on 200,000 MCMC draws, preceded by 1,000,000 burn-in cycles. We observe the convergence of the Gibbs sampler. Subsequent re-estimations and forecasts are based on a data window expanded by a consecutive observation. In order to estimate parameters, the Gibbs sampler is applied again using the results obtained in the previous step. The starting points of the numerical algorithm are set to means of the MCMC draws from the previous step, i.e. the starting points are equal to the estimators of posterior means of

Table 2
The unconditional percentage coverage of the PIs as well as means, standard deviations, medians and inter-quartile ranges of the PIs width calculated for two non-Bayesian individual models (ARX, SNARX), for three averaging schemes (SIMPLE, LAD and QRA), and for Bayesian PIs (B_Q, B_HPD). The results collected for all 24 h.
Source: Nowotarski and Weron (2015) and own calculations.

PI(%)	ARX	SNARX	SIMPLE	LAD	QRA	B_Q	B_HPD
Unconditional coverage							
50	69.74	56.51	58.63	56.36	53.55	**53.33**	**53.22**
90	96.13	94.23	94.44	93.64	92.07	**90.28**	**90.72**
Mean (standard deviation) of the PI width							
50	8.63	6.09	6.32	6.73	6.4	**5.6**	**5.52**
	(3.33)	(2.64)	(2.89)	(3.66)	(3.78)	(4.02)	(3.91)
90	21.28	20.73	25.73	26.2	21.1	**16.08**	**15.79**
	(8.29)	(8.78)	(15.74)	(17.21)	(12.09)	(11.15)	(10.71)
Median (inter-quartile range) of the PI width							
50	8.66	5.94	5.89	5.79	5.62	**4.37**	**4.29**
	(5.25)	(4.21)	(5.77)	(6.93)	(5.19)	(3.86)	(3.79)
90	21.34	20.64	23.22	21.87	19.51	**12.88**	**12.70**
	(13.02)	(15.28)	(25.86)	(26.33)	(18.51)	(10.63)	(10.43)

Table 3
The unconditional coverage of the 50% PIs for each of the 24 h separately calculated for two individual models (ARX, SNARX), for three averaging schemes (SIMPLE, LAD and QRA), and for Bayesian PIs (B_Q, B_HPD).
Source: Nowotarski and Weron (2015) and own calculations.

Hour	ARX	SNARX	SIMPLE	LAD	QRA	B_Q	B_HPD
#1	73.68	56.14	53.51	56.14	59.65	**51.75**	52.63
#2	72.81	57.02	58.77	55.26	61.4	52.63	**49.12**
#3	70.18	59.65	58.77	**53.51**	58.77	56.14	57.02
#4	72.81	62.28	56.14	**53.51**	57.89	55.26	54.39
#5	68.42	58.77	54.39	**49.12**	52.63	56.14	53.51
#6	**49.12**	37.72	38.6	34.21	45.61	**50.88**	53.51
#7	52.63	42.98	32.46	35.96	44.74	**49.12**	45.61
#8	64.91	**52.63**	43.86	44.74	44.74	41.23	37.72
#9	73.68	57.89	54.39	57.89	54.39	46.49	**48.25**
#10	71.05	61.4	60.53	60.53	**54.39**	55.26	**54.39**
#11	78.07	63.16	73.68	71.93	69.3	**57.02**	59.65
#12	78.07	57.02	76.32	70.18	**50.88**	62.28	57.02
#13	77.19	65.79	78.07	64.91	**50.88**	57.02	57.02
#14	76.32	58.77	82.46	66.67	**49.12**	52.63	51.75
#15	81.58	71.05	79.82	58.77	**48.25**	57.02	55.26
#16	78.95	64.04	74.56	74.56	62.28	56.14	**51.75**
#17	62.28	**49.12**	**49.12**	46.49	42.98	55.26	57.89
#18	59.65	57.89	**50**	**50**	51.75	55.26	55.26
#19	53.51	42.98	45.61	55.26	**50.88**	**50.88**	51.75
#20	65.79	52.63	57.89	57.02	55.26	**50.00**	51.75
#21	72.81	58.77	63.16	67.54	60.53	**54.39**	60.53
#22	75.44	53.51	55.26	63.16	57.02	**52.63**	56.14
#23	72.81	53.51	52.63	**50**	48.25	57.02	57.89
#24	71.93	61.4	57.02	55.26	53.51	**47.37**	**47.37**

Table 4
The unconditional coverage of the 90% PIs for each of 24 h separately calculated for two individual models (ARX, SNARX), for three averaging schemes (SIMPLE, LAD and QRA), and for Bayesian PIs (B_Q, B_HPD).
Source: Nowotarski and Weron (2015) and own calculations.

Hour	ARX	SNARX	SIMPLE	LAD	QRA	B_Q	B_HPD
#1	94.74	92.11	92.11	**90.35**	89.47	89.47	89.47
#2	93.86	92.11	91.23	**90.35**	92.11	**90.35**	89.47
#3	96.49	93.86	92.98	92.98	92.98	88.6	**89.47**
#4	98.25	96.49	92.11	**89.47**	91.23	**89.47**	**89.47**
#5	95.61	92.11	86.84	85.96	92.11	88.6	**89.47**
#6	89.47	83.33	75.44	80.7	88.6	89.47	**90.35**
#7	**91.23**	92.11	86.84	85.09	85.09	86.84	88.6
#8	95.61	94.74	91.23	88.6	86.84	89.47	**90.35**
#9	96.49	93.86	92.11	92.98	92.11	88.6	**89.47**
#10	96.49	94.74	97.37	97.37	**91.23**	92.98	92.98
#11	99.12	96.49	100	100	97.37	**92.11**	92.98
#12	100	97.37	100	96.49	87.72	**92.11**	92.98
#13	99.12	97.37	100	97.37	93.86	**92.11**	**92.11**
#14	98.25	98.25	100	99.12	93.86	**92.11**	**92.11**
#15	99.12	98.25	100	99.12	93.86	93.86	**92.11**
#16	98.25	97.37	100	100	99.12	**92.98**	93.86
#17	**92.98**	**92.98**	97.37	97.37	93.86	93.86	94.74
#18	93.86	93.86	93.86	92.98	92.98	87.72	**88.6**
#19	92.98	**91.23**	93.86	94.74	**91.23**	**91.23**	**91.23**
#20	96.49	95.61	97.37	95.61	92.98	85.09	**88.60**
#21	98.25	96.49	98.25	97.37	93.86	**90.35**	**90.35**
#22	98.25	95.61	96.49	96.49	95.61	**88.60**	**88.60**
#23	96.49	93.86	97.37	93.86	**89.47**	91.23	**89.47**
#24	95.61	91.23	93.86	92.98	92.11	89.47	**90.35**

Table 5
The summary of unconditional coverage. The number of times the method turned out to be the best in the group of: non-Bayesian PIs (*a*), non-Bayesian and B_Q PIs (*b*), non-Bayesian and B_HPD PIs (*c*) and all PIs (*d*).

	ARX	SNARX	SIMPLE	LAD	QRA	B_Q	B_HPD
50% PI							
(*a*)	2	6	4	6	**9**	–	–
(*b*)	1	2	2	5	6	**12**	–
(*c*)	2	4	2	5	6	–	**8**
(*d*)	1	2	2	5	6	**9**	**5**
90% PI							
(*a*)	3	7	3	5	**14**	–	
(*b*)	3	2	0	3	5	**18**	–
(*c*)	2	2	0	3	4	–	**18**
(*d*)	2	2	0	3	3	10	**16**

the parameters. Because the impact of one observation on the posterior distribution should not be large and because of the choice of the starting points, a lower number of MCMC draws is needed. The re-estimations of the model are based on 20,000 MCMC draws, preceded by 10,000 burn-in cycles. Similar results are obtained for a twice increased number of MCMC draws.

Generally, after a large number of forecast steps which cover a long time horizon or in case of a structural change in the market, it may be necessary to re-estimate the model for a large number of MCMC draws. However, such decision should be made individually for each specific situation.

We extend the set of exogenous variables by including a variable Z_t, which stands for the minimum of the previous day's 24 hourly log-prices, just as in Nowotarski and Weron (2015). The variable reflects the price signals from the entire previous day. Moreover, Nowotarski and Weron make the log-prices dependent on the log-prices of previous two days and the previous week. In our study such variables are not included, however, we improve the model in question by the SV structure. The SVDEJX model is considered in the new form:

$$\begin{aligned} y_{t+1} =& y_t + \mu + \psi_1 X_{t+1} + \psi_2 Z_{t+1} + \\ &+ d_{Sat} D_{Sat,t+1} + d_{Sun} D_{Sun,t+1} + d_{Mon} D_{Mon,t+1} + \\ &+ \sqrt{\exp(h_t)}\varepsilon^{(1)}_{t+1} + J_{t+1}, \\ h_{t+1} =& h_t + \kappa_h(\theta_h - h_t) + \sigma_h\left(\rho\varepsilon^{(1)}_{t+1} + \sqrt{1-\rho^2}\varepsilon^{(2)}_{t+1}\right), \end{aligned}$$

where $\varepsilon^{(1)}_t, \quad \varepsilon^{(2)}_t \sim iid\, N(0,1), \quad J_t \sim iid \quad DE,$
$D_{Sat,t} = \mathbb{I}_{\{t=Saturday\}}, D_{Sun,t} = \mathbb{I}_{\{t=Sunday\}}, D_{Mon,t} = \mathbb{I}_{\{t=Monday\}},$

X_t – the logarithm of the hourly temperature,
Z_t – the minimum of the previous day's 24 hourly log-prices.

The concept of prediction intervals, PIs in short, is becoming more and more popular. PIs allow for a better assessment of future uncertainty and the planning of different strategies for a wide range of possible outcomes (Nowotarski and Weron, 2015). The Bayesian approach provides a predictive distribution. Consequently, the prediction intervals are easy to calculate as by-products of the inference.

Two types of Bayesian prediction intervals are calculated. The first type is determined by quantiles of a predictive distribution, e.g. $(Q_{0.25}, Q_{0.75})$, where $Q_{0.25}$ and $Q_{0.75}$ are the first and the third quartiles, respectively. We call it the Bayesian quantile interval or B_Q, in short. The second type is calculated as the highest predictive density interval (Bernardo and Smith, 2009). We call it the Bayesian HPD interval or B_HPD, in short. An $(1-\alpha)$ HPD interval $C = \{S : p(S) \geq l_\alpha\}$, where p is a predictive density of S and l_α is a constant determined by the constraint $P(S \in C) = 1 - \alpha$, where P is a predictive probability of S.

Nowotarski and Weron consider twelve individual models: the autoregressive models (AR, ARX, p-AR (spike preprocessed AR), p-ARX), the threshold AR models (TAR, TARX), the mean-reverting jump diffusion models (MRJD, MRJDX) and the semiparametric autoregressive models (IHMAR, IHMARX, SNAR, SNARX). Prediction intervals calculated by Nowotarski and Weron are derived from individual specifications and also from combined spot price forecasts using three averaging schemes: the simple average method (SIMPLE), the least absolute deviation method (LAD), and the quantile regression averaging method (QRA). Nowotarski and Weron find that the SNARX based PIs are the most precise among the individual models, but they are outperformed by the QRA-based PIs.

The unconditional coverage of the day-ahead PIs by the spot electricity price jointly for all 24 h is considered. The idea of the unconditional coverage test proceeds by comparing a percentage of coverage to a nominal value, i.e. a true coverage probability (Christoffersen, 1998). Table 2 presents the unconditional coverage of the 50% and 90% prediction intervals by the spot price for ARX and SNARX, three averaging schemes calculated under the non-Bayesian models, and two Bayesian PIs. The method of calculating PIs is the more accurate, the closer to the nominal value 50% or 90% the percentage of coverage is. The best results – in each row – are in bold. The study focuses on a comparison between the Bayesian results and non-Bayesian ones. Each Bayesian result is compared, in each row, with non-Bayesian outcomes. Therefore, sometimes two Bayesian results are in bold, and, despite the fact that one is better than the other, both are still better than the non-Bayesian results. The best results are achieved by the Bayesian techniques for both analysed levels.

Tables 3 and 4 present the unconditional coverage of the 50% and 90% two-sided prediction intervals by the electricity price for each of the 24 h separately. The numbers written in bold indicate the best results in each row. In 13 cases (hours) the 50% Bayesian PIs bring the same or better results than the non-Bayesian ones (see Table 3). The intervals are at least as good as the non-Bayesian ones in 7 out of 12 off-peak hours and in 6 out of 12 peak hours[1]. Moreover, the 90% PIs bring the same or better results than the non-Bayesian ones in 20 cases (see Table 4). The intervals are at least as good as the non-Bayesian ones in 10 out of 12 off-peak hours as well as peak hours.

Table 5 reports the number of times a particular method is the best. It sums up the results presented in Tables 3 and 4, in which more than one value in each row may be marked in bold. This means that two or more models give equally good results, and it happens in the case of 6th, 17th, 19th and 24th hours in Table 3, and 2nd, 4th, 13th, 14th, 17th, 19th, 21st, 22nd, 23rd hours in Table 4. Consequently, the numbers in the rows in Table 5 add up to more than 24. The first rows of Table 5 present the comparison between the non-Bayesian techniques for both levels 50% and 90%. The quantile regression averaging method (QRA) is the best non-Bayesian method. The second rows present the comparison between the non-Bayesian results and the Bayesian quantile intervals (B_Q). In the third rows non-Bayesian results are compared with the Bayesian HPD intervals (B_HPD). The last rows compare the results of all techniques.

The first conclusion is that the Bayesian results are better than the non-Bayesian ones. The B_Q method is better than B_HPD for the 50% PIs, while the B_HPD method is better than B_Q for the 90% PIs.

[1] The peak hours range from 9th hour to 20th hour i.e. they cover the interval between 8 a.m. to 8 p.m. The remaining hours are called the off-peak hours.

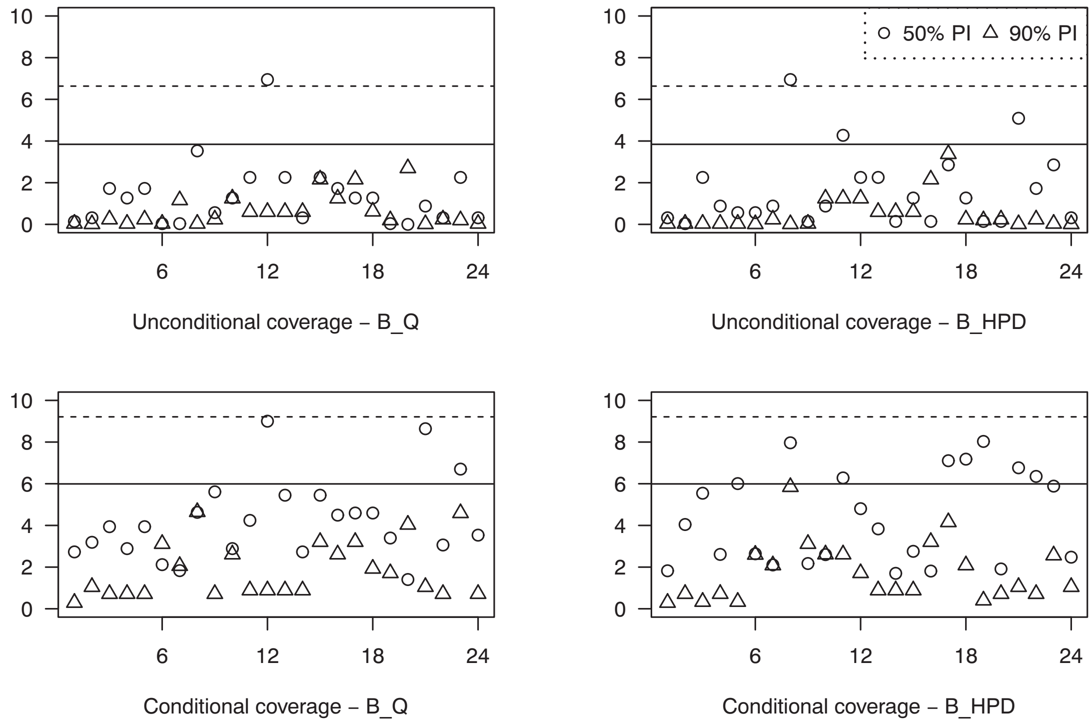

Fig. 3. The LR statistics of the unconditional and conditional coverage tests (the vertical axis) for the Bayesian B_Q PIs (left panels) and B_HPD PIs (right panels) for each of the 24 h (the horizontal axis) in the out-of-sample period September 23, 2011–January 14, 2012. The dashed and solid horizontal lines represent the 1% and 5% significance levels, respectively.

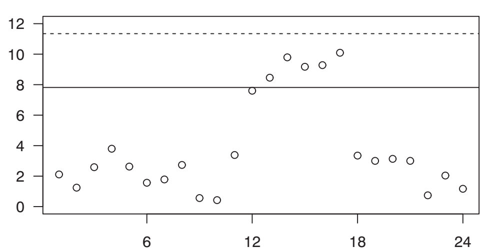

Fig. 4. Values of the Berkowitz test statistics (the vertical axis) calculated independently for each of the 24 h of a day (the horizontal axis) in the out-of-sample period September 23, 2011–January 14, 2012. The dashed and solid horizontal lines represent the 1% and 5% significance levels, respectively.

Therefore, the second conclusion is that there is no definitive winner between the Bayesian techniques.

Christoffersen's (1998) approach is applied in the study to test the unconditional and conditional coverage. This model independent approach is designed to overcome the clustering effect. The tests are carried out in the likelihood ratio (LR) framework. The LR statistics are calculated for the unconditional coverage and the conditional coverage, i.e. the joint test of the unconditional coverage and independence. The tests are conducted separately for each of the 24 h. Two significance levels, 1% and 5%, are considered. The results are presented in Fig. 3. For the majority of hours the null hypothesis is not rejected at both levels, which sounds promising and supports the idea of the applicability of the Bayesian techniques in modelling and forecasting electricity prices.

The Berkowitz test with the null hypothesis of independence and normality, is applied to test the goodness-of-fit of the predictive distributions (Berkowitz, 2001; Nowotarski and Weron, 2018). In order

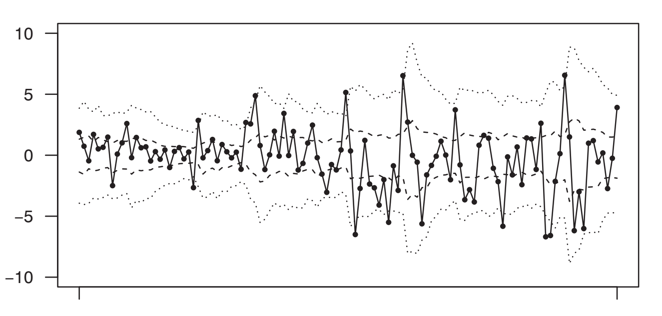

Fig. 5. The Bayesian 50% HPD PIs (dashed lines) and 90% HPD PIs (dotted lines) and the actual spot electricity prices at #4 hour of a day (dots) centred around the medians of predictive distributions in the out-of-sample period September 23, 2011–January 14, 2012.

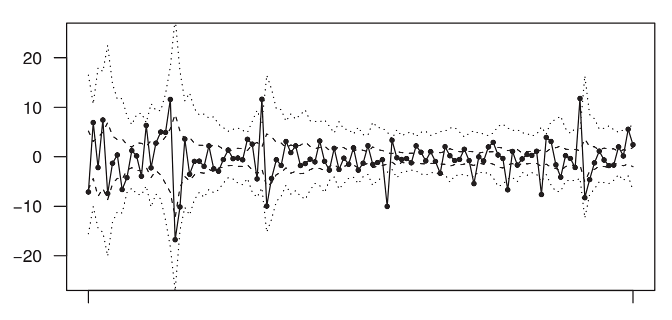

Fig. 6. The Bayesian 50% HPD PIs (dashed lines) and 90% HPD PIs (dotted lines) and the actual spot electricity prices at #16 hour of a day (dots) centred around the medians of predictive distributions in the out-of-sample period September 23, 2011–January 14, 2012.

to perform the test, the function BerkowitzTest from the RuGARCH package (R Environment) is employed. The results are presented in Fig. 4. The null hypothesis is rejected for only several hours at 5% significance level, and is not rejected for any of 24 h at 1% significance level. Thus, the Bayesian approach once again yields encouraging results.

Finally, we again consider the off-peak hour #4 and the peak hour #16. Figs. 5 and 6 depict the Bayesian HPD PIs and the actual spot prices for the hours centred around the medians of predictive distributions. The range of intervals changes over time, and in most cases the intervals cover the observed prices. The periods with wider PIs correspond with the periods of more volatile prices. It should be noted here that the 90% PIs are significantly wider than the 50% PIs.

Table 6
The unconditional percentage coverage of the PIs as well as means, standard deviations, medians and inter-quartile ranges of the PIs width calculated for the non-Bayesian ARX model and Bayesian PIs (B_Q, B_HPD) for the SVDEJX model. Results collected for all 24 h.

PI(%)	ARX	B_Q	B_HPD
Unconditional coverage			
50	56.45	49.90	49.45
90	90.08	89.38	89.38
Mean (standard deviation) of the PI width			
50	6.56	6.08	5.93
	(3.23)	(6.73)	(6.13)
90	16.23	17.42	16.80
	(8.04)	(20.88)	(18)
Median (inter-quartile range) of the PI width			
50	5.86	4.51	4.44
	(3.52)	(3.82)	(3.71)
90	14.45	12.93	12.68
	(8.78)	(10.57)	(10.18)

4.2. Data set II

In the second part of the empirical study, we analyse the data set of hourly day-ahead locational marginal prices (LMPs) of electricity for the JCPL zone of the PJM Interconnection. The data span from January 4, 2015 to October 28, 2017. We divide the data into in-sample (January 4, 2015–August 5, 2017) and out-of-sample periods (August 6, 2017–October 28, 2017). The day-ahead forecasts of electricity prices for each of the 24 h for consecutive 84 days (12 weeks) are calculated under the SVDEJX model and the benchmark non-Bayesian model ARX.

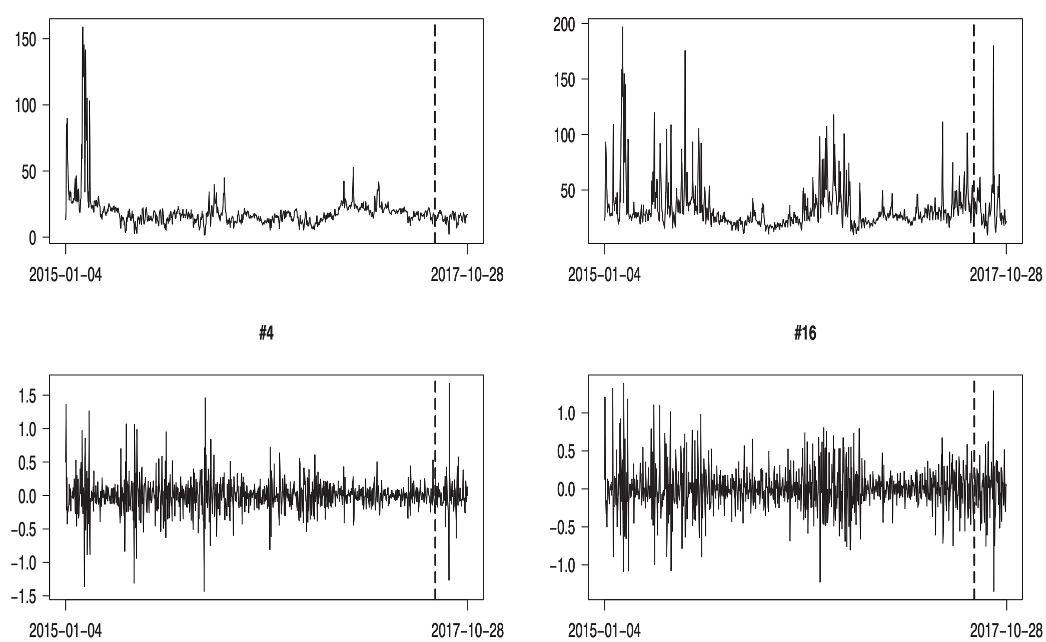

Fig. 7. Day-ahead electricity prices (USD/MWh) for the JCPL zone of the PJM Interconnection (at the top) and differences in log-prices (at the bottom) at #4 hour (left panels) and #16 hour of a day (right panels). Dashed vertical lines split into in-sample (January 4, 2015–August 5, 2017) and out-of-sample (August 6, 2017–October 28, 2017) periods.

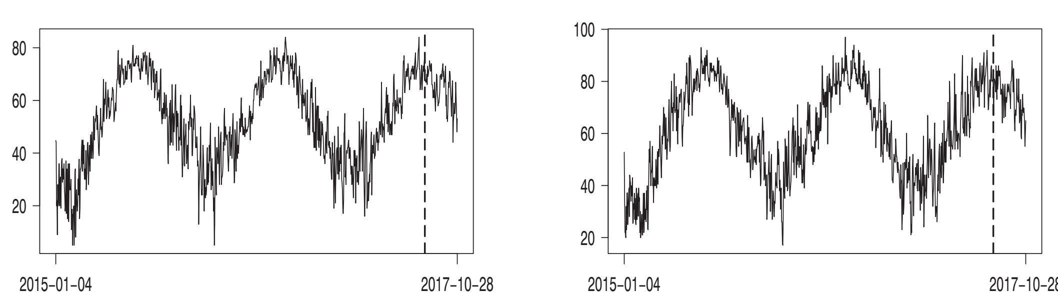

Fig. 8. Hourly air temperatures (in Fahrenheit degrees) in New York at #4 hour (left panel) and #16 hour of a day (right panel). The dashed vertical line splits into in-sample (January 4, 2015–August 5, 2017) and out-of-sample (August 6, 2017–October 28, 2017) periods.

Table 7
The number of hours for which the null hypotheses under the unconditional and conditional coverage Christoffersen's tests are not rejected at a significance level 1%.

PI(%)	ARX	B_Q	B_HPD
Unconditional coverage (0.01)			
50	22	24	24
90	24	24	24
Conditional coverage (0.01)			
50	19	24	23
90	20	21	22

Table 8
The number of hours for which the null hypotheses under the unconditional and conditional coverage Christoffersen's tests are not rejected at a significance level 5%.

PI(%)	ARX	B_Q	B_HPD
Unconditional coverage (0.05)			
50	19	24	24
90	24	24	24
Conditional coverage (0.05)			
50	13	24	22
90	16	18	17

Fig. 7 presents the electricity prices and the differences in log-prices for #4 hour (an off-peak hour) and #16 hour of a day (a peak hour). Fig. 8 presents hourly air temperatures in New York.

Firstly, the unconditional coverage of the PIs by the day-ahead electricity prices jointly for all 24 h is considered. Table 6 presents the unconditional coverage of the 50% and 90% prediction intervals by the price for ARX and two Bayesian PIs. The best results are achieved by the Bayesian techniques for the 50% PIs. However, the results for the 90% PIs are close to the nominal value for each method.

Christoffersen's (1998) approach is applied to test the unconditional and conditional coverage. The tests are conducted separately for each of the 24 h. Two significance levels, 1% and 5%, are considered. Tables 7–8 and Fig. 9 present the results. For the majority of hours and all models the null hypotheses are not rejected at both levels. However, the results of Christoffersen's tests favour the Bayesian approach.

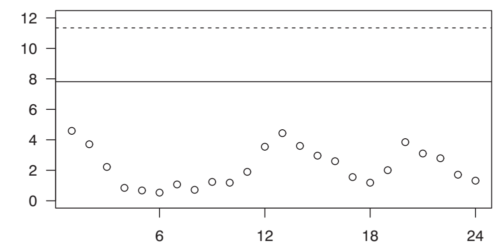

Fig. 10. Values of the Berkowitz test statistics (the vertical axis) calculated independently for each of the 24 h of a day (the horizontal axis) in the out-of-sample period August 6, 2017–October 28, 2017. The dashed and solid horizontal lines represent the 1% and 5% significance levels, respectively.

The Berkowitz test is applied to test the goodness-of-fit of the predictive distributions. For all hours the null hypothesis is not rejected at significance levels 1% and 5% under the SVDEJX model, see Fig. 10. However, the null hypothesis is rejected for all hours under the ARX model.

Figs. 11 and 12 present 50% and 90% HPD PIs and the actual day-ahead electricity prices at #4 and #16 hour of a day, respectively, centred around the medians of predictive distributions. In both cases, for many days the intervals cover the actual prices. The PIs for the peak hour are wider than those for the off-peak hour. Moreover, higher price volatility is accompanied by the PIs with a wider span.

The Diebold-Mariano tests (Diebold and Mariano, 1995; Diebold, 2015) are conducted in order to compare the forecasts yielded by two

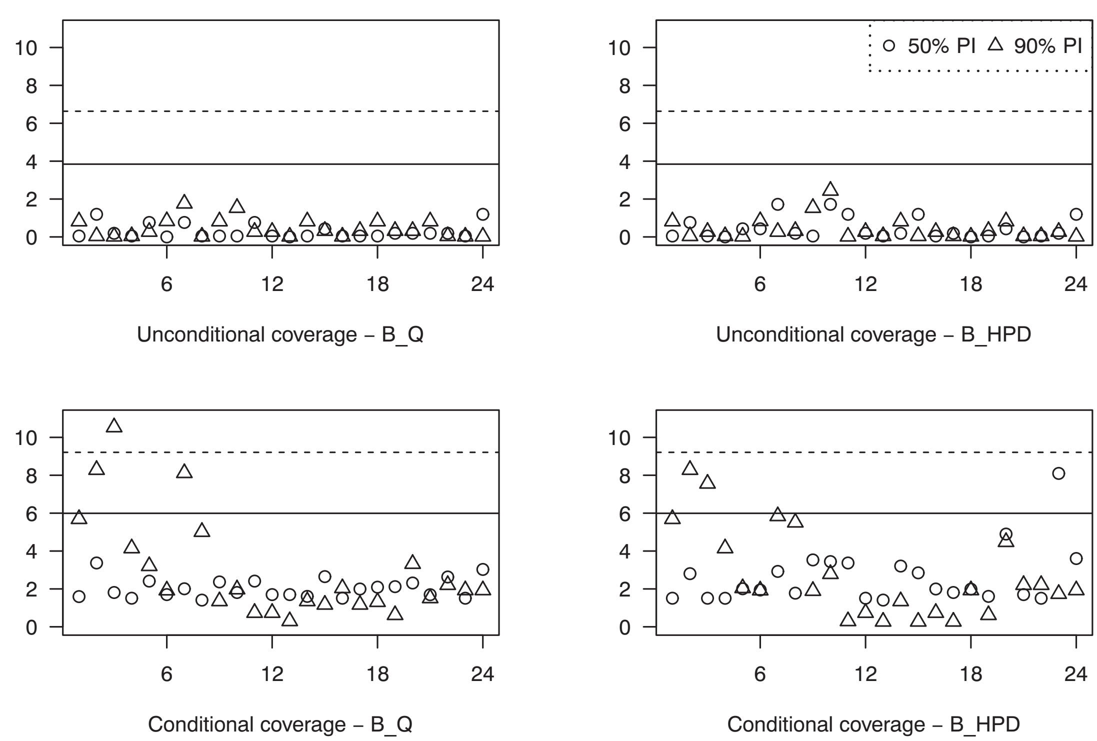

Fig. 9. The LR statistics of the unconditional and conditional coverage (the vertical axis) for the Bayesian B_Q PIs (left panels) and B_HPD PIs (right panels) for each of the 24 h (the horizontal axis) in the out-of-sample period August 6, 2017–October 28, 2017. The dashed and solid horizontal lines represent the 1% and 5% significance levels, respectively.

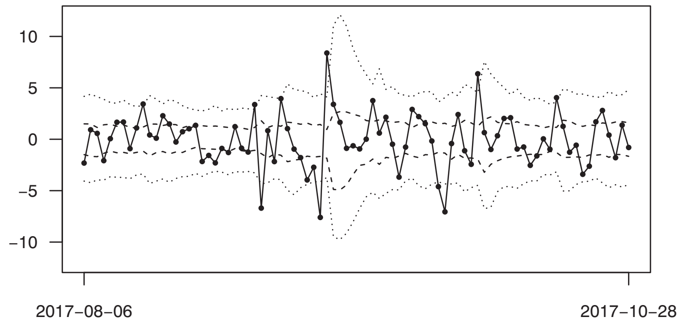

Fig. 11. The Bayesian 50% HPD PIs (dashed lines) and 90% HPD PIs (dotted lines) and the actual day-ahead electricity prices at #4 hour of a day (dots) centred around the medians of predictive distributions in the out-of-sample period August 6, 2017–October 28, 2017.

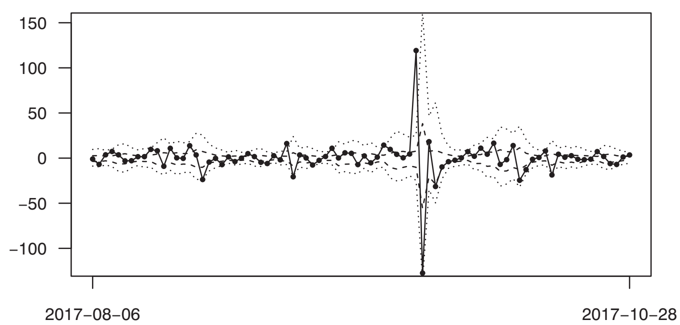

Fig. 12. The Bayesian 50% HPD PIs (dashed lines) and 90% HPD PIs (dotted lines) and the actual day-ahead electricity prices at #16 hour of a day (dots) centred around the medians of predictive distributions in the out-of-sample period August 6, 2017–October 28, 2017.

models. We employ the pinball loss (Maciejowska and Nowotarski, 2016) as a scoring rule for the test and a significance level 5%. The forecasts calculated under the SVDEJX model are significantly better than under the ARX model for 14 out of 24 h, and the forecasts calculated under the ARX model are significantly better than those under the SVDEJX for 5 out of 24 h. The results obtained in this part of our study indicate that in most cases the forecasts of electricity prices calculated by means of the Bayesian SVDEJX model are superior to the forecasts derived from the benchmark ARX model.

5. Conclusions and remarks

The study applies the Bayesian approach to forecast day-ahead electricity prices for the JCPL zone of the PJM Interconnection, however, the method can be applied to calculate probabilistic forecasts for any forecast horizon. The results of Bayesian and non-Bayesian interval predictions are compared, and in many cases the Bayesian predictions turn out superior. The results indicate that practitioners who address the problems of forecasting and risk management should turn to the Bayesian stochastic volatility model with double exponential jumps and exogenous variables. The superiority of the SV models with double exponential jumps over other SV specifications with jumps, in particular the models with Levy-type small jumps, is also confirmed by Kou et al. (2017), who analyse the S&P 500 and NASDAQ 100 indexes. The superiority of Bayesian day-ahead electricity forecasts over non-Bayesian ones is also confirmed by Gianfreda et al. (2018), who consider ARX and VARX models.

A drawback of the Bayesian methodology supported by the Gibbs sampler is connected with time consuming numerical calculations. However, under the Bayesian approach an unknown parameter is a random variable with an unknown distribution. The distribution (the posterior distribution) is estimated by means of the MCMC methods and a point estimator is only a by-product. The distribution is useful in handling uncertainty about unknown parameters. As in the case of estimation, in Bayesian forecasting the distribution (the predictive distribution) of a future (unknown) value is estimated and a point forecast is a by-product as before. Furthermore, this approach allows for determining predictive intervals (PIs).

The study demonstrates strong points of the Bayesian approach. It facilitates day-ahead electricity price forecasting by taking into account uncertainty about parameters. The results of Bayesian forecasting obtained in this study seem promising, although the methodology requires further empirical studies and should be tested on other commodity time series. We hope our analyses will encourage practitioners to employ this methods of electricity price forecasting.

Acknowledgments

We would like to thank Jakub Nowotarski and Rafał Weron for allowing us to use their data set and for inspiring us to tackle the problem of forecasting under Bayesian SV processes with jumps. This work was supported by the National Science Centre, Poland [grant number 2016/23/B/HS4/03018].

Appendix A. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.eneco.2019.02.004.

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