# k-NEAREST NEIGHBOURS

LECTURE 2
SECTION 2
JUNE 4TH

INSTITUTE FOR APPLIED
COMPUTATIONAL SCIENCE
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# MOTIVATION

# MODEL INTERPRETABILITY:

Interpreting our models help us evaluate them and extract new insights about our data.

$y$ = price ($)

$X_1$ = Size ($m^2$)

$X_2$ = Distance to city center (m)

**M1: Linear Regression**

$$y = 3.5X_1 - 0.1X_2 + 100{,}000$$

Train MSE: 1,200
Test MSE: 2,200

Is this a reasonable model?

What does it say about housing prices?

**M2: Polynomial Regression**

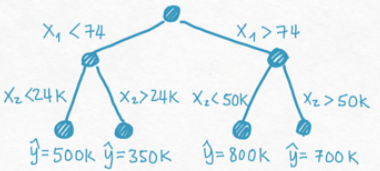
$$y = 0.01X_1^2 - 0.02X_2^2 + 2.1X_1X_2 + 0.1X_1 - 0.2X_2 + 10{,}000$$

Train MSE: 358
Test MSE: 657

Is this a reasonable model?

What does it say about housing prices?

**M3: Regression Tree**




Train MSE: 568
Test MSE: 1,000

Is this a reasonable model?

What does it say about housing prices?

Which model should we choose?

# EXPLANATION BY EXAMPLE:

Linear models and regression trees are interpretable in different ways. Another way to explain or interpret decisions is by looking at examples:

# k-NEAREST NEIGHBOURS

# K-NEAREST NEIGHBOURS:

The very human way of decision making by similar examples can be formalized as an algorithm:

**The k-Nearest Neighbor Algorithm:**

Given a data set $D = \{(x^{(1)}, y^{(1)}), \dots (x^{(N)}, y^{(N)})\}$. For every new $x$:

1. Find k-number of observations in $D$ most similar to $x$:
   $\{(x^{(n_1)}, y^{(n_1)}), \dots, (x^{(n_k)}, y^{(n_k)})\}$
   These are called the k-nearest neighbours of $x$.

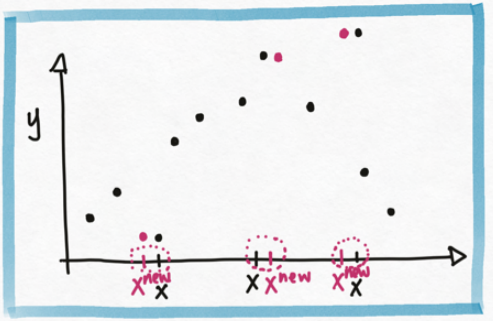
2. Average the output of the k-nearest neighbours of $x$:
   $$\hat{y} = \frac{1}{k} \sum_{k=1}^{K} y^{(n_k)}$$

3. Predict $\hat{y}$ for $x$.

# OVERFITTING AND UNDERFITTING:

- Training Data
- prediction for new data

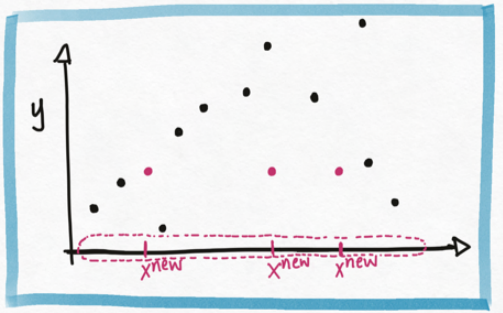
K=1




predictions on new data is influenced by noise in the data.

test error will be high.

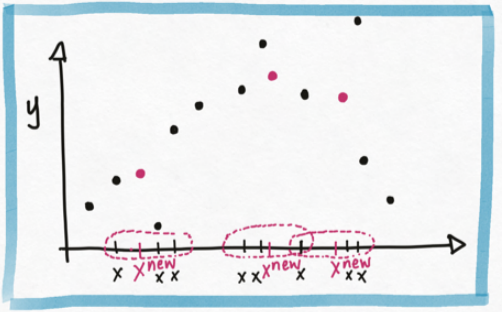
K=10




predictions is unaffected by local variations in the data.

test error will be high.

K=3




predictions take into account local variations but averaging several neighbours cancels the effect of noise.

# COMPARING MODELS

# COMPARISON OF MODELS:

Choosing the right model isn't just about minimizing test error.
We want to extract insights from our models.

| | Probabilistic | Has a fixed form $f_w(x)$ (parametric) | Easy to interpret |
|---|---|---|---|
| Linear Regression | YES | YES | YES |
| Polynomial Regression | YES | YES | NO |
| Regression Tree | NO | YES | If the tree is not big |
| K-Nearest Neighbours | NO | NO | YES |
| | Explicitly modeling the noise as a RV help us diagnose & improve the model. | Having an explicit functional form $f_w(x)$ makes it easy to store and use the model. | Interpretation helps us evaluate our model and understand the data. |