CA - 105

* Discrete Mathematics & Statistics.

Syllabus :

Unit ① Set theory & logic.
② Relation & Function
③ Counting & Probability.
④ Data Representation & Aggregation] Statistics.
⑤ Correlation theory & Sampling.

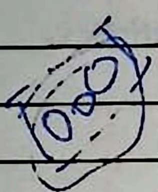
C-1 Set theory & logic.

- **Set :** Well defined collection of objects.

example →

① $N = \{1, 2, 3, 4, 5\}$

② x is a set of letters in the word 'COLLEGE'



$X = \{C, O, L, E, G\}$

→ Representation of Set.

1) Listing method ~~(Roaster)~~ (Roster)
2) Set - building form.

eg : 1) $A = \{x \mid x$ is a positive prime number less than $20\}$

$A = \{2, 3, 5, 7, 11, 13, 17, 19\}$

2) $A = \{1, 2, 3, 4, 5\}$

$A = \{x \mid x \geq 5, x \in N\}$

3) $B = \{x \mid x$ is +ve integer and $x^2 < 20\}$

$B = \{0, 1, 2, 3, 4\}$

NOTE : Equality of set : Two set A & B are called equal if both of them contain same elements, and in this case we write $A = B$

Examples 1) $A = \{1, 2, 3, 4\}$

$B = \{x \mid x$ is a positive integer and $x^2 < 20\}$

$C = \{x \mid x$ is integer & $x^2 < 20\}$

$D = \{0, 1, -1, 2, -2, 3, -3, 4, -4\}$

→ $B = \{1, 2, 3, 4\} = A$

set C can be written as $C = \{1, 2, 3, 4, -1, -2, -3, -4, 0\}$

$C = D$

2) $A = \{1, 2, 3\}$ $P(A) = \{\phi, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{2, 3\}, \{1, 3\}, \{1, 2, 3\}$

$|P(A)| = \cancel{8}\ 2^3 = 8$

3) $B = \{a, b, c, d\}$

$P(B) = \{\phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{b, c\}, \{c, d\}, \{a, c\}, \{a, b\}, \{a, d\}, \{a, b, c\}, \{b, d\}, \{a, b, d\}, \{b, c, d\}, \{a, c, d\}, \{a, b, c, d\}\}$

$|P(B)| = 2^4 = 16$

→ Operations on Set.

1) Union of sets : eg: $A = \{1, 2, 3\}$

$B = \{a, b\}$

$A \cup B = \{1, 2, 3, a, b\}$

2)

2) Intersection of sets : eg $A = \{1, 2, 3, 4\}$

$B = \{2, 3\}$

$A \cup B = \{2, 3\}$

eg 1) $A = \{1, 2\}$ $A \cup B = \phi$

$B = \{a, b\}$

Q1 Find union & intersection.

1) $A = \{1, 2, 3, 4, 5, 6\}$ $A \cap B = \{1, 2, 3, 4, 5, 6\}$

$B = \{3, 6\}$ $A \cup B = \{6\}$

2) $A = \{a, b, c, d\}$ $A \cap B = \{a, b, c, d, e, f\}$

$B = \{e, f\}$ $A \cup B = \phi$

- Subset :

~~B~~ if Every element of set A is also an element of set B then we say that A is subset of B

eg : $A = \{1, 2, 3\}$ & $B = \{1, 2, 3, 4, 5\}$

$\therefore$ $A \subseteq B$ & ~~B ⊄~~

$B \not\subseteq A$

- Empty Set : (Null set or Void set)

A set which contains no elements is called empty set. And is denoted by ϕ or $\{\}$

~~eg :~~

- ~~Equa~~ Power Set :

If A is set ~~an~~ then the set of all subsets of set A is known as Power set of A. and it is written as $P(A)$

eg $A = \{1, 2, 3\}$ $P(A) = \{\phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{2,3\}, \{1,3\}, \{1,2,3\}\}$

$|A| = 3$

$n(A) = 3$

- Cardinality of ~~a set~~ power set

$|P(A)| = 2^{|A|}$ where $|A|$ is cardinality of set A

$= 2^3 = 8$.

- Cartesian product :

The cartesian product of a set A & B is the set $A \times B$ of all ordered pairs (x, y) where $x \in A$ & $y \in B$ i.e. ~~$A \times B =$~~

$$A \times B = \{(x, y) \mid x \in A, y \in B\}$$

exa : $A = \{a, b, c\}$, $B = \{1, 2\}$

The cartesian product of set A & B is,

$A \times B = \{(a,1), (a,2), (b,1), (b,2), (c,1), (c,2)\}$

$|A \times B| = 6$

exa : $A = \{\cancel{1, 2, 3}\}$ 2, 4, 5 & $B = \{a, b, c\}$

$A \times B = \{(2,a), (2,b), (2,c),$
$(4,a), (4,b), (4,c),$
$(5,a), (5,b), (5,c)\}$

$|A \times B| = 9$

NOTE : if $|A| = n$ & $|B| = m$ then

$$|A \times B| = |A| \cdot |B| = m \cdot n$$

eg exa : ① $A = \{1, 2, 3, 4\}$ & $B = \{3, 4, 5, 6, 7\}$

$$A \cup B = \cancel{\{3, 4\}} \{1, 2, 3, 4, 5, 6, 7\}$$

$$A \cap B = \{3, 4\}$$

$$A - B = \cancel{\{5, 6, 7\}} \{1, 2\}$$

② $\cancel{A}B - A = \{5, 6, 7\}$

② $A = \{1, 2, 3, 4, 5, 6\}$ $B = \{4, 5, 6, 7, 8, 9\}$

Verify $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$.

(LHS): $A - B = \{1, 2, 3\}$

$B - A = \{7, 8, 9\}$

$$(A - B) \cup (B - A) = \{1, 2, 3, 7, 8, 9\} \quad - ①$$

(RHS): $(A \cup B) = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$$(A \cap B) = \{4, 5, 6\}$$

$$(A \cup B) - (A \cap B) = \{1, 2, 3, 7, 8, 9\} \quad - ②$$

from ① & ② we verify that

$$(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$$

- Symmetric Difference.

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$A \oplus B = (A-B) \cup (B-A)$

$A = \{1, 2, 3, a, b, p\}$

$B = \{2, p, q, 8, m\}$

$(A-B) = \{1, 3, a, b\}$

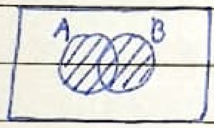
$(B-A) = \{q, 8, m\}$

$A \oplus B = \{1, 3, a, b\} \cup \{q, 8, m\}$

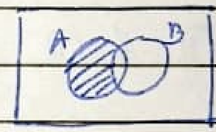
$= \{1, 3, a, b, 8, q, m\}$

- Venn Diagram.

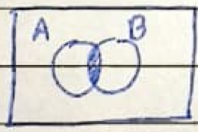
$A \cup B$



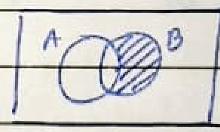
$A - B$



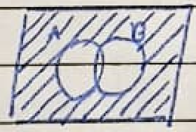
$A \cap B$



$B - A$



$(A \cup B)^c$



★ • De-Morgan's law

1) $(A \cap B)^c = A^c \cup B^c$

2) $(A \cup B)^c = A^c \cap B^c$

$\Rightarrow$ let $x \in (A \cap B)^c$

$\Rightarrow x \notin (A \cap B)$

$x \notin A$ and $x \notin B$

Homework

$x \in A^c$ and $x \in B^c$

$x \in A^c \cup B^c$

$\Rightarrow$ This implies that

~~$A \oplus$~~ $(A \cap B)^c \subseteq A^c \cup B^c$ —— (1)

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$x \in A^c \cup B^c$

$x \in A^c$ or $x \in B^c$

$x \notin A$ or $x \notin B$

$x \notin A \cap B$

$x \in (A \cap B)^c$

$\Rightarrow$ This implies that

$$A^c \cup B^c \subseteq (A \cap B)^c \quad \text{— (2)}$$

From (1) & (2) we get that

$$(A \cap B)^c = A^c \cup B^c$$

~~Types of Sets~~. Logic :

~~Propositional Law~~

1. Propositional Law :

a) Proposition : A proposition is a declaritive sentence. that is either true or false. but not both.

ex : 1) New Delhi is capital of India — True

2) $2 + 3 = 5$ — True

3) $3 + 3 = 4$ — False.

4) Washington D.C. is capital of Canada — False.

b) Propositional Variables : The variables that represent proposition is called proposition-al Variable.

It is denoted by ~~variables~~ letters p, q, r.

★ 2) Logical Connectives :

a) Negation (NOT / $\sim$)

ex : p : " Today is Friday"

$\sim$p : " Today is not Friday"

p	$\sim$p
T	F
F	T

b) Conjuction (AND / $\wedge$) :

Let p & q be two propositions

the conjuction of p & q denoted by

~~p∧q~~ $p \wedge q$ is the proposition "p and q".

The Conjuction is true when both p and q are ~~ture~~ ~~tur~~ true otherwise false.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

c) Disjunction (OR / $\vee$) :

Let p ~~or q~~ and q be two propositions

the disjunction of p and q is ~~den~~ denoted by $p \vee q$. The dis junction is ~~true~~ false when any ~~one of~~ when both p and q are false and otherwise True.

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p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F