NATIONAL UNIVERSITY OF SINGAPORE

SEMESTER 1, 2021/2022

MA1521 Calculus for Computing **Tutorial 6**

1. (a) Evaluate $\int x\sqrt{4-x}\,dx$.

(Thomas' Calculus (14^{th} edition), p. 313, Problem 44)

(b) Evaluate $\int \frac{1}{x^2}\sin\frac{1}{x}\cos\frac{1}{x}\,dx$.

(Thomas' Calculus (14^{th} edition), p. 313, Problem 35)

2. If f is a continuous function, find the value of the integral

$$I = \int_0^a \frac{f(x)}{f(x)+f(a-x)}\,dx$$

by making the substitution $u = a - x$ and adding the resulting integral to I.

(Thomas' Calculus (14^{th} edition), p. 324, Problem 87)

3. (a) Evaluate $\int x(\ln x)^2\,dx$.

(Thomas' Calculus (14^{th} edition), p. 476, Problem 33)

(b) Evaluate $\int \tan^{-1}(x)\,dx$.

(Thomas' Calculus (14^{th} edition), p. 475, Problem 11)

4. Evaluate $\int \frac{1}{x(x^4+1)}\,dx$.

Hint: Multiply by x^3/x^3.

(Thomas' Calculus (14^{th} edition), p. 313, Problem 44)

5. Evaluate $\int \sec x \, dx$ by
   (i) writing it as $\int \frac{1}{\cos x} \, dx$,
   (ii) followed by multiplying the function in the integral by $\frac{\cos x}{\cos x}$,
   (iii) followed by using a trigonometric identity,
   (iv) followed by a suitable substitution and
   (v) finally by using partial fraction.

   Remark: There is a shorter way of evaluating this integral.

   (Thomas' Calculus (14^{th} edition), p. 496, Problem 74(b))

6. Suppose y is a function of x and x and y are related by
$$x = \int_0^y \frac{1}{\sqrt{1+4t^2}} \, dt.$$
   Show that
$$\frac{1}{y} \frac{d^2y}{dx^2}$$
   is a constant C. Determine the constant C.

   (Thomas' Calculus (14^{th} edition), p. 328, Problem 4)

7. (a) Show that
$$-1 \leq \frac{2x}{1+x^2} \leq 1$$
   for all $x \in \mathbf{R}$.

   (b) Use Mean Value Theorem for Integral to deduce that
$$\left|\ln(1+b^2) - \ln(1+a^2)\right| \leq |b-a|$$
   for all real numbers a, b.

   (Thomas' Calculus (14^{th} edition), p. 263, Problem 8 (Modified))

8. (a) Evaluate $\int \frac{1}{x\sqrt{x+9}} \, dx$.

   (Thomas' Calculus (14^{th} edition), p. 496, Problem 48)

   (b) Evaluate $\int \frac{1}{x^6(x^5+4)} \, dx$,

   (Thomas' Calculus (14^{th} edition), p. 496, Problem 50)

9. (a) Evaluate $\displaystyle\int_2^\infty \frac{2}{t^2-1}\,dt$.

(Thomas' Calculus (14^{th} edition), p. 521, Problem 12)

(b) Evaluate $\displaystyle\int_0^1 (-\ln x)\,dx$,

(Thomas' Calculus (14^{th} edition), p. 521, Problem 26)

1. (a) Evaluate $\int x\sqrt{4-x}\,dx$.

(Thomas' Calculus (14th edition), p. 313, Problem 44)

(b) Evaluate $\int \frac{1}{x^2}\sin\frac{1}{x}\cos\frac{1}{x}\,dx$.

(Thomas' Calculus (14th edition), p. 313, Problem 35)

a) $\int x\sqrt{4-x}\,dx$

Let $u = x$, $v' = \sqrt{4-x} = (4-x)^{\frac{1}{2}}$

$u' = 1$, $v = \frac{(4-x)^{\frac{1}{2}+1}}{\frac{1}{2}+1} = \frac{(4-x)^{\frac{3}{2}}}{\frac{3}{2}}$

$= -\frac{2}{3}(4-x)^{\frac{3}{2}}$

$\therefore = x\left(-\frac{2}{3}(4-x)^{\frac{3}{2}}\right) - \int -\frac{2}{3}(4-x)^{\frac{3}{2}}\cdot 1\,dx$

$+\frac{2}{3}\int (4-x)^{\frac{3}{2}}dx$

$= -\frac{(4-x)^{\frac{3}{2}+1}}{\frac{3}{2}+1}$

$= x\left(-\frac{2}{3}(4-x)^{\frac{3}{2}}\right) - \frac{2}{3}\cdot\frac{2}{5}(4-x)^{\frac{5}{2}}$

b) $\int \frac{1}{x^2}\sin\frac{1}{x}\cos\frac{1}{x}\,dx$

$u = -\sin\frac{1}{x}$, $v' = -\frac{1}{x^2}\cos\frac{1}{x}$,

$u' = +\frac{1}{x^2}\cos\frac{1}{x}$, $v = +\sin\frac{1}{x}$

$= (-\sin\frac{1}{x})(\sin\frac{1}{x}) - \int \sin\frac{1}{x}\cdot\frac{1}{x^2}\cos\frac{1}{x}\,dx$

$= \frac{-\sin^2\frac{1}{x}}{2}$

2. If f is a continuous function, find the value of the integral

$$I = \int_0^a \frac{f(x)}{f(x) + f(a-x)}\, dx$$

$dx = -du$

by making the substitution $u = a - x$ and adding the resulting integral to I.

(Thomas' Calculus (14th edition), p. 324, Problem 87)

2.

$$I = \int_0^a \frac{f(x)}{f(x)+f(a-x)}\, dx$$

let $u = a - x$, $\frac{du}{dx} = -1$

$x = a - u$ $\quad du = -dx$

$$I = -\int_0^a \frac{f(a-u)}{f(a-u)+f(u)}\, dx$$

$$\frac{f(a)-f(u)}{f(a)-\cancel{f(u)}+f(u)}$$

$$\frac{f(a)}{f(a)} - \frac{f(u)}{f(a)}$$

$$-\int_0^a 1 - \frac{f(u)}{f(a)}$$

$$= -a + \frac{1}{f(a)} \int_0^a f(u)\, du$$

$$= \frac{1}{f(a)} \left[\frac{f(u)^2}{2}\right]_0^a - a$$

$$= \frac{1}{f(a)} \left[\frac{f(a-x)^2}{2}\right]_0^a - a$$

$$= \frac{1}{f(a)} \left[\frac{f(a-a)^2}{2} - \frac{f(a-0)^2}{2}\right) - a$$

$$= \frac{1}{f(a)} \left[\frac{f(0)^2}{2} - \frac{f(a)^2}{2}\right) - a$$

$$= \frac{f(0)^2}{2f(a)} - \frac{f(a)}{2} - a + k.$$

$$I = -\int_a^0 \frac{f(a-u)}{f(a-u)+f(u)}\, du.$$

$u = a - x$ $\quad du = -dx$

$x = a - u$

when $x = 0$, $u = a$

$x = a$, $u = 0$.

$$= +\int_0^a \frac{f(a-u)}{f(a-u)+f(u)}\, du$$

$$2I = I + I$$

$$=$$

$$\text{U}\mathcal{H}\text{U}: \int_0^a \frac{f(t)}{f(t)+f(a-t)}\, dt + \int_0^a \frac{f(a-t)}{f(a-t)+f(t)}\, dt = \int_0^a \frac{f(t)+f(a-t)}{f(t)+f(a-t)} = \int_0^a 1\, dt$$

$2I = a.$

$I = \frac{a}{2}$ $\square$.

3. (a) Evaluate $\int x(\ln x)^2\,dx$.

(Thomas' Calculus (14th edition), p. 476, Problem 33)

(b) Evaluate $\int \tan^{-1}(x)\,dx$.

(Thomas' Calculus (14th edition), p. 475, Problem 11)

a) $\int x\,(\ln x)^2\,dx$

By parts: $u = \ln x \quad dv = x$

$\frac{du}{dx} = \frac{1}{x} \quad v = \frac{x^2}{2}$.

$du = \frac{1}{x}\,dx$.

$\therefore \int u\,dv = uv - \int v\,du$.

$\int x(\ln x)^2\,dx = \frac{x^2}{2}\ln x - \int \frac{x^2}{2}\cdot\frac{1}{x}\,dx$

$= \frac{x^2}{2}\ln x - \int \frac{x}{2}\,dx$

$= \frac{x^2}{2}\ln x - \frac{1}{2}\int x\,dx$

$= \frac{x^2}{2}\ln x - \frac{1}{2}\cdot\frac{x^2}{2}$

$= \frac{x^2}{2}\ln x - \frac{x^2}{4} + C$

$\frac{(x\ln x)^2}{2} - \frac{x^2\ln x}{2} + \frac{x^2}{4} + C$?

$\int x(\ln x)^2\,dx$.

$u = (\ln x)^2 \quad x\,dx = dv$

$du = \frac{2\ln x}{x} \quad \frac{x^2}{2} = v$

$\int (\ln x)^2\,dv + \int v\,du = uv$

$\int (\ln x)^2 x\,dx = \frac{x^2}{2}(\ln x)^2 - \int \frac{x^2}{2}\cdot\frac{2\ln x}{x}\,dx$

$= \frac{x^2(\ln x)^2}{2} - \left(\frac{x^2}{2}\ln x - \int \frac{x^2}{2}\cdot\frac{1}{x}\,dx\right)$

$= \frac{x^2(\ln x)^2}{2} - \frac{x^2\ln x}{2} + \frac{x^2}{4} + C$.

b) $\int \tan^{-1}(x)\,dx$.

By parts:

$u = \tan^{-1}(x) \quad dv = 1$

$\frac{du}{dx} = \frac{1}{1+x^2} \quad v = x$

$du = \frac{1}{1+x^2}\,dx$

$\therefore \int \tan^{-1}(x)$

$= x\tan^{-1}(x) - \int x\,\frac{1}{1+x^2}\,dx$

$\int \frac{x}{1+x^2}\,dx$

$u = 1+x^2, \quad \frac{du}{dx} = 2x$.

$= \frac{1}{2}\int \frac{1}{u}\,du$.

$= \frac{1}{2}\ln u$

$= \frac{1}{2}\ln(1+x^2) + C$

$\therefore \int \tan^{-1}(x) = x\tan^{-1}(x) - \frac{1}{2}\ln(1+x^2) + C$

4. Evaluate $\int \frac{1}{x(x^4+1)}\,dx$.

Hint: Multiply by x^3/x^3.

(Thomas' Calculus (14th edition), p. 313, Problem 44)

(x^4+1)
$= (x^2+bx+c)(x^2+dx+c)$.

$$\int \frac{1}{x(x^4+1)} \cdot \frac{x^3}{x^3}\,dx$$

$$= \int \frac{x^3}{x^4(x^4+1)}\,dx$$

$u = x^4$ $\quad \frac{du}{dx} = 4x^3$

$du = 4x^3\,dx$

$$= \frac{1}{4}\int \frac{1}{u(u+1)}\,du$$

$$= \frac{1}{4}\int \frac{1}{(\frac{1}{u}+1)u^2}\,du.$$

$v = \frac{1}{u}+1$

$\frac{dv}{du} = -\frac{1}{u^2}$

$du = -u^2\,dv$

$$= -\frac{1}{4}\int \frac{1}{v}\,dv.$$

$$= -\frac{1}{4}\ln(v) + C$$

$$= -\frac{1}{4}\ln\left(\frac{1}{u}+1\right) + C$$

$$= -\frac{1}{4}\ln\left(\frac{1}{x^4}+1\right) + C.$$

or $=$

partial frac.

$$\int \frac{1}{u} - \frac{1}{u+1}\,du$$

$$= \int \frac{1}{u}\,du - \int \frac{1}{u+1}\,du.$$

$$= \ln u - \ln(u+1)$$

$$= \frac{1}{4}\left(\ln x^4 - \ln(x^4+1)\right) + C'$$

5. Evaluate $\int \sec x \, dx$ by

(i) writing it as $\int \frac{1}{\cos x} \, dx$,

(ii) followed by multiplying the function in the integral by $\frac{\cos x}{\cos x}$,

(iii) followed by using a trigonometric identity,

(iv) followed by a suitable substitution and

(v) finally by using partial fraction.

Remark: There is a shorter way of evaluating this integral.

(Thomas' Calculus (14th edition), p. 496, Problem 74(b))

i) $\int \sec x \, dx$

$= \int \frac{1}{\cos x} \, dx$

ii) $\int \frac{1}{\cos x} \cdot \frac{\cos x}{\cos x} \, dx$

$= \int \frac{\cos x}{\cos^2 x} \, dx$.

iii) $\sin^2 x + \cos^2 x = 1$.

$\cos^2 x = 1 - \sin^2 x$.

$\int \frac{\cos x}{\cos^2 x} \, dx \int \frac{\cos x}{1 - \sin^2 x} \, dx$

iv) $\int \frac{\cos x}{1 - \sin^2 x} \, dx$

$u = \sin x$

$du = \cos x \, dx$.

$\int \frac{\cos x}{1 - \sin^2 x} = \int \frac{1}{1 - u^2} \, du$.

v) $\int \frac{1}{1-u^2} \, du$

$\frac{1}{1-u^2} = \frac{1}{(1-u)(1+u)} = \frac{A}{1-u} + \frac{B}{1+u}$

$A = \frac{1}{2}$

$B = \frac{1}{2}$

$= \frac{1}{2(1-u)} + \frac{1}{2(1+u)}$

$\int \frac{1}{1-u^2} = \int \frac{1}{2(1-u)} \, du + \int \frac{1}{2(1+u)} \, du$.

$= -\frac{1}{2} \ln|1-u| + \frac{1}{2} \ln|1+u| + C$.

$= \frac{1}{2} \left(\ln \left| \frac{1+\sin x}{1-\sin x} \right| \right) + C$. //

Another way:

$\int \sec x \, dx = \int \sec x \frac{(\tan x + \sec x)}{(\tan x + \sec x)} \, dx$

$= \int \frac{\sec x \tan x + \sec^2 x}{\tan x + \sec x} \, dx$

$= \int \frac{d(\sec x + \tan x)}{\tan x + \sec x}$

$= \ln(\sec x + \tan x) + C$.

6. Suppose y is a function of x and x and y are related by

$$x = \int_0^y \frac{1}{\sqrt{1+4t^2}}\,dt.$$

Show that

$$\frac{1}{y}\frac{d^2y}{dx^2}$$

is a constant C. Determine the constant C.

(Thomas' Calculus (14th edition), p. 328, Problem 4)

$$\frac{dx}{dy} = \frac{1}{\sqrt{1+4y^2}}$$

$$x = \int_0^y \frac{1}{\underbrace{\sqrt{1+4t^2}}_{1+(2t)^2}}\,dt.$$

$u = 2t$

$du = 2\,dt$

$x = \frac{1}{2}\int_0^y \frac{1}{\sqrt{1+u^2}}\,du$

$$\int \frac{1}{\sqrt{1+x^2}}\,dx.$$

$u = \frac{1}{\sqrt{1+x^2}}$ $\quad dv = 1$

$du = \frac{2x}{2\sqrt{1+x^2}}\,dx$ $\quad v = x$.

$= \frac{x}{\sqrt{1+x^2}} - \int x \frac{2x}{2\sqrt{1+x^2}}\,dx.$

$\frac{x}{\sqrt{1+x^2}} - \int \frac{x^2}{\sqrt{1+x^2}}\,dx.$

$$\Rightarrow \frac{dy}{dx} = \sqrt{1+4y^2}. \quad (f(f^{-1}(x)))' = 1.$$

implicit diff

$$\frac{d^2y}{dx^2} = \frac{1}{2\sqrt{1+4y^2}}(8y)\frac{dy}{dx}.$$

$$= \frac{1}{\sqrt{1+4y^2}}\cdot 4y \cdot \sqrt{1+4y^2}$$

$$= 4y.$$

$$\frac{1}{y}\cdot\frac{d^2y}{dx^2} = 4. \quad \therefore C = 4$$

7. (a) Show that
$$-1 \le \frac{2x}{1+x^2} \le 1 \quad \Rightarrow \quad \left|\frac{2x}{1+x^2}\right| \le 1 .$$
for all $x \in \mathbf{R}$.

(b) Use Mean Value Theorem for Integral to deduce that
$$\left|\ln(1+b^2) - \ln(1+a^2)\right| \le |b-a|$$
for all real numbers a, b.

(Thomas' Calculus (14th edition), p. 263, Problem 8 (Modified))

a) $\frac{d}{dx}\left(\frac{2x}{1+x^2}\right)$

$= \frac{(1+x^2)(2) - (2x)(2x)}{(1+x^2)^2}$ ✓

$= \frac{2(1+x^2) - (2x)^2}{(1+x^2)^2} = 0.$ or $\frac{2(1-x)(1+x)}{(1+x^2)^2}.$

$2(1+x^2) - (2x)^2 = 0$ ✓

$2 + 2x^2 - 4x^2 = 0.$

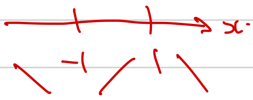
$2 - 2x^2 = 0$

$-2x^2 = -2.$

$x^2 = 1$

$x = \pm 1.$ ✓

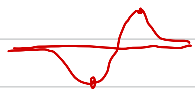
first deriv test :



Since the absolute maxima and minima is either $x=1$ or $x=-1$, then the function $\frac{2x}{1+x^2}$ lies between $[-1, 1]$.

$x=-1$ is local min,
$x=1$ is local max.

$\frac{2x}{1+x^2} \to 0$ as $x \to \infty$. $\Rightarrow$



$\to 0$ as $x \to -\infty$.

$x=-1$ & 1 are abs. min & max.

$-1 \le \frac{2x}{1+x^2} \le 1$ //

b) $\left|\ln(1+b^2) - \ln(1+a^2)\right| \le |b-a|$

$= \ln\left|\frac{1+b^2}{1+a^2}\right|$

let f be $\ln\left|\frac{1+b^2}{1+a^2}\right|$ and f is continuous on the closed interval $[a-b, b-a]$ and differentiable on the open interval $(a-b, b-a)$,

then by mean value theorem, there is a number $a-b < c < b-a$ such that $f'(c) = \frac{f(b-a) - f(a-b)}{b-a-(a-b)}$

$= \frac{2f(b-a)}{2b-2a}$

by sub.

$\int \frac{2x}{1+x^2}\,dx = \int \frac{d(x^2)}{(1+x^2)} = \ln(1+x^2) + C.$

MVT for integrals : $\ln(1+b^2)$

$b>a$, $\frac{\ln(1+b^2) - \ln(1+a^2)}{b-a} = \frac{1}{b-a}\int_a^b \frac{2x}{1+x^2}\,dx$

$= \frac{2c}{1+c^2}$ for some $c \in (a,b)$

$$\left|\frac{\ln(1+b^2)-\ln(1+a^2)}{b-a}\right| = \left|\frac{2c}{1+c^2}\right| \leq 1 \quad \text{by (a)}$$

$$\therefore \left|\ln(1+b^2)-\ln(1+a^2)\right| \leq |b-a|.$$

$\underline{a>b}$:

$$\frac{\ln(1+a^2)-\ln(1+b^2)}{a-b}$$

$$= \left|\frac{1}{a-b}\int_b^a \frac{2x}{1+x^2}\,dx\right|$$

$$= \left|\frac{2c}{1+c^2}\right| \text{ for some } c \in (b,a) \leq 1$$

Another way of doing pt. (a).

$$1+x^2-2x = (1-x)^2 \geq 0$$

$$\Rightarrow \frac{2x}{1+x^2} \leq 1 \qquad \text{“} 1+x^2 > 0 \text{”}$$

$$1+x^2+2x = (1+x)^2 \geq 0$$

$$2x \geq -(1+x^2) \Rightarrow \frac{2x}{1+x^2} \geq -1 \quad \forall x$$

no need $\underline{\text{calculus}}$

$$\therefore -1 \leq \frac{2x}{1+x^2} \leq 1$$

$$\left|\frac{2x}{1+x^2}\right| \leq 1$$

8. (a) Evaluate $\int \frac{1}{x\sqrt{x+9}}\,dx$.

(Thomas' Calculus (14th edition), p. 496, Problem 48)

x^{-6} is deriv of x^{-5}.

$\to u = x^{-5}$

(b) Evaluate $\int \frac{1}{x^6(x^5+4)}\,dx$, $\quad -\frac{1}{20x^5} + \frac{1}{80}\ln\left(1+\frac{4}{x^5}\right)+C$

$x^{-6} = \frac{\partial(x^{-5})}{\partial x}\cdot\left(\frac{1}{5}\right)$

(Thomas' Calculus (14th edition), p. 496, Problem 50)

a) $\int \frac{1}{x\sqrt{x+9}}\,dx$

$u = \sqrt{x+9}$. $\quad x = u^2 - 9$.

$\frac{du}{dx} = \frac{1}{2\sqrt{x+9}}$.

$\therefore\ 2\int \frac{1}{2\sqrt{x+9}\,x}\,dx$

$= 2\int \frac{1}{u^2-9}\,du$

$= 2\int \frac{1}{(u-3)(u+3)}\,du$.

$= 2\int \frac{1}{6(u-3)} - \frac{1}{6(u+3)}\,du$.

$= \frac{1}{3}\int \frac{1}{(u-3)} - \frac{1}{(u+3)}\,du$

$= \frac{1}{3}\ln(\sqrt{x+9}-3) - \frac{1}{3}\ln(\sqrt{x+9}+3) + C$. //

$= \frac{1}{3}\ln\left(\frac{\sqrt{x+9}-3}{\sqrt{x+9}+3}\right) + C$

b) $\int \frac{1}{x^6(x^5+4)}\,dx$.

$u = x^5+4 \qquad u-4 = x^5.$

$du = \frac{5x^4}{x^5+4} \qquad x^{10} = (u-4)^2$

$= \frac{1}{5}\int \frac{5x^4}{x^5+4\ \ x^6\cdot x^4}\,dx$

$= \frac{1}{5}\int \frac{1}{(u-4)^2}\,du$

$= \frac{1}{5}\cdot\frac{1}{4-u}$

$= \frac{1}{5}\cdot\frac{1}{4-(x^5+4)}$

$= -\frac{1}{5x^5} + C$ //

$u = x^{-5} = \frac{1}{x^5}$.

$du = -5x^{-6}$

$= -\frac{5}{x^6}$

$\therefore x^5 = \frac{1}{u}$.

$-\frac{1}{5}\int -\frac{5}{x^6(x^5+4)}\,du$

$= -\frac{1}{5}\int \frac{1}{\left(\frac{1}{u}+4\right)}\,du.$

$$-\frac{1}{5}\int \frac{1}{\left(\frac{1}{u}+4\right)}\,du$$

$$= -\frac{1}{5}\int \frac{1}{\frac{1+4u}{u}}\,du$$

$$\int \frac{u}{1+4u}\,du$$

$v = 1+4u$

$dv = 4$

$\frac{v-1}{4} = u$

$$\frac{1}{4}\int \frac{\frac{v-1}{4}}{v}\,dv$$

$$= \frac{1}{16}\int \frac{v-1}{v}\,dv$$

$$= \frac{1}{16}\int \frac{v}{v} - \frac{1}{v}\,dv$$

$$= \frac{1}{16}\left(v - \ln v\right)$$

$$= -\frac{1}{5}\cdot\frac{1}{16}\cdot\left(1+4u - \ln\left(1+4u\right)\right)$$

$$= -\frac{1}{80}\left(1+\frac{4}{x^5} - \ln\left(1+\frac{4}{x^5}\right)\right)$$

$$= -\frac{1}{80} - \frac{1}{20x^5} + \frac{1}{80}\ln\left(1+\frac{4}{x^5}\right)+C$$

$$= -\frac{1}{20x^5} + \frac{1}{80}\ln\left(1+\frac{4}{x^5}\right) + C$$

9. (a) Evaluate $\displaystyle\int_2^\infty \frac{2}{t^2-1}\,dt.$

(Thomas' Calculus (14$^{\text{th}}$ edition), p. 521, Problem 12)

(b) Evaluate $\displaystyle\int_0^1 (-\ln x)\,dx,$

(Thomas' Calculus (14$^{\text{th}}$ edition), p. 521, Problem 26)

a) $\displaystyle\int_2^\infty \frac{2}{t^2-1}\,dt$

$= \displaystyle\lim_{b\to\infty} \underbrace{\int_2^b \frac{2}{t^2-1}\,dt}$

$2\displaystyle\int_2^b \frac{1}{t^2-1^2}\,dt$

$= 2\displaystyle\int_2^b \frac{1}{(t-1)(t+1)}\,dt$

$= 2\displaystyle\int_2^b \frac{1}{2(t-1)} - \frac{1}{2(t+1)}\,dt$

$= 2\left(\frac{1}{2}\ln|t-1| - \frac{1}{2}\ln|t+1|\right)\Big|_2^b$

$= \ln\left|\frac{t-1}{t+1}\right|\Big|_2^b$ ✓

$= \ln\left|\frac{b-1}{b+1}\right| - \ln\left|\frac{1}{3}\right|$

$= \displaystyle\lim_{b\to\infty}\left(\ln\left|\frac{1-\frac{1}{b}}{1+\frac{1}{b}}\right| - \ln\frac{1}{3}\right)$

$= \ln 1 - \ln\frac{1}{3}$

$= \ln 3$. // ✓

b) $\displaystyle\int_0^1 (-\ln x)\,dx$

$= -\displaystyle\int_0^1 \ln x\,dx$

$u = \ln x$, $dv = 1$

$du = \frac{1}{x}$ $v = x$

$= -\left(x\ln x - \displaystyle\int x\cdot\frac{1}{x}\,dx\right)$

$= \displaystyle\int 1\,dx - x\ln x$

$= x - x\ln x$ ✓

$= x(1-\ln x)\Big|_{0\ \varepsilon}^{1}$

$= 1(1-\ln 1) - 0$

$= 1 - \ln 1$ //

$= 1$. ✓

$\varepsilon(1-\ln\varepsilon)$

$\displaystyle\lim_{\varepsilon\to 0^+}\int_\varepsilon^1 \ln x\,dx$

$= -1 - \displaystyle\lim_{\varepsilon\to 0}(\varepsilon\ln\varepsilon)$

$\displaystyle\lim_{\varepsilon\to 0^+}\frac{(-\ln\varepsilon)}{\frac{1}{\varepsilon}}$ l'hôpital

$= -\displaystyle\lim_{\varepsilon\to 0^+}\frac{-\frac{1}{\varepsilon}}{-\frac{1}{\varepsilon^2}}$

$= -\displaystyle\lim_{\varepsilon\to 0^+}\varepsilon = 0.$

~~7 (a)~~ 6 on

6 qns.

1 – 3 pts

2 – 3 pts.

6m – 2 pts.

differentiation

Topics l'hospital 15 m

Techniques of Integration 15 m

MVT + MVT for integrals 18 m.

differentiation → optimization 12 m.

⇒ a lot of hints

⇓

60 marks.

1h 45 mins

% of final — same weightage :C.

22%.

not as easy as first test.

test 3, app. of integrals, series & sequences.