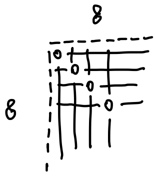
3. How many ways can you place 4 distinct rooks on an 8x8 chessboard such that no two rooks attack each other?

- 2822400
- 15249024
- 117600
- 635376



Total grid = 64.
Total rooks = 4.

if place one → 7R, 7C. free
if place 2 → 6R, 6C. free
if place 3 → 5R, 5C free.

$\therefore {}^{64}C_1 \cdot {}^{49}C_1 \cdot {}^{36}C_1 \cdot {}^{25}C_1 = 2822400.$

4. How many positive integers between 1 and 1231 inclusive are not divisible by 2, 3, or 5?

- 411
- 329
- 861
- 902

numbers divisible by:

2 – 615.
3 – 410
5 – 246

6 – 205
15 – 82
10 – 123

30 – 41

	2	3	5
2	1	3	5.
3	1		5
5	1	1	1

$1231 - x = 615 + 410 + 246 - 205 - 82 - 123 + 41.$

$= 329$

7. How many 3-digit even integers can you create if the digits are picked from $\{0, 2, 4, 5, 7, 8\}$ and no repetitions are allowed?

- 48
- 68
- 80
- 120

Even integers:

no 0.

$0 \to {}^3C_2 \times 2! \times 4. \quad - \quad {}^3C_1 \cdot {}^4C_1$

2 →
4 →
8 →.

no zero.

middle digit

number w zero infront

8. How many 3-digit odd integers can you create if the digits are picked from $\{0, 2, 4, 5, 7, 8\}$ and repetitions are allowed?

- ( ) 60
- (•) 72
- ( ) 120
- ( ) 216

odd integers: _ _ _

Case w/ 0 : _ 0 5 or 7 $\Rightarrow {}^5C_1 \cdot {}^2C_1$ ⇗ r-combi w/ repetitions

Case w/o 0: _ _ 5 or 7 $\Rightarrow +$ ${}^{5-1+2}C_2 \cdot {}^2C_1 \cdot 2! - \underline{5 \cdot 2}$.

repeats.

$= 60.$

9. A class consists of 30 students. 15 of them like to play soccer, 17 like to play basketball, and 17 like to play volleyball. The number of students who like to play soccer and basketball equals the number of students who like to play basketball and volleyball. It is also known that 8 of them like to play both soccer and volleyball, 4 of them do not like all three sports, and the number of students who like to play basketball and volleyball equals twice the number of students who like to play all three sports.

How many students like to play all three sports?

- ( ) 2
- ( ) 4
- ( ) 5
- (•) 10

15s
17b
17v

$s \cap b = b \cap v$
$s \cap v = 8$
$b \cap v = 2(s \cap b \cap v)$

$10 = 2(s \cap b \cap v)$
$s \cap b \cap v = 5.$ //

$30 - 4 = 26.$

$\therefore 26 = 15 + 17 + 17 - s \cap b - b \cap v - s \cap v + s \cap b \cap v.$
$-23 = - b \cap v - b \cap v - 8 + \frac{1}{2}(b \cap v).$
$-15 = -1.5\, b \cap v$
$b \cap v = 10.$

14. Covid-19 is finally over and the airline celebrates by welcoming the first 100 passengers onboard its 100-seater plane. Everyone has a ticket with an assigned seat number from 1 through 100.

The first passenger misplaces his ticket and takes a random seat. Every subsequent passenger will take his own seat if unoccupied, or take a random seat otherwise.

You happen to be the last passenger to board the plane. What is the probability that you will get to sit in your assigned seat?

- ( ) $\frac{1}{100}$
- (•) $\frac{1}{10}$
- ( ) $\frac{1}{2}$
- ( ) $\frac{99}{200}$
- ( ) $\frac{1}{3}$

_ _ _ _ . . . _

n= 1 2 3 4 100

First passenger → own seat
→ another seat × Second p → own seat
→ another seat . . . .

start.
↓
$(\frac{1}{2} \times \frac{1}{2}) + (\frac{1}{2})^3 + (\frac{1}{2})^4 + \dots (\frac{1}{2})^{100}.$

2:
3.
4. . . . . .
5. . . . . . .

$\therefore GP: \frac{a(1-r^{99})}{1-r}$

$= \frac{(\frac{1}{2} \cdot \frac{1}{2})(1-(\frac{1}{2})^{99})}{1-\frac{1}{2}}.$

15. In a certain city with n towns ($n \geq 2$), there is a road between any two of these towns, so one can travel from any town to any other town. Aiken is visiting the city and wants to visit each town exactly once. But on arrival, Aiken realizes that one of the roads is blocked. How many ways can Aiken visit every town exactly once?

Hamiltonian paths for complete graph
$\Rightarrow n!$

- (selected) $(n-1)!$
- $n(n-1)!$
- $n! - n$
- $n! - (n-1)!$
- $n! - 2(n-1)!$

$n! \rightarrow$ o o o o ... n.

o o o o o o ... n

Total - together

$n! - (n-1)! \times 2!$

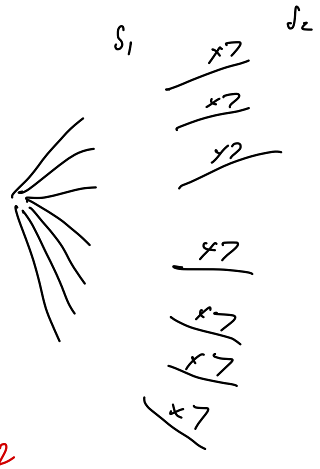
3. During a semester in NUS, Dueet will always choose the same number of ingredients at the yong tao foo stall (i.e if he chooses 3 ingredients on his first visit in sem 2, then he must stick to 3 ingredients everytime). In addition, every ingredient he chooses must be different. Let the number of ingredients Dueet chooses be N. What can N be given that the yong tao foo stall has 10 different types of ingredients and Dueet wishes to have more than 120 different combination of ingredients throughout the semester?

- [ ] 3
- [x] 4
- [x] 5
- [x] 6
- [x] ~~7~~
- [x] ~~8~~

$^{10}C_N > 120$

7. Potts has 3 pairs of socks for each color of the rainbow (i.e Potts has 42 socks in total). Left socks cannot be differentiated from the right socks. What is the probability of Potts finding at least 1 matching pair if she draws 5 socks out of her drawer at random?

Hint: ~~A probability tree might help.~~



- (selected) 5/7
- 495/594
- 8183/10127
- 79/97

Socks: C_1: 6, C_2: 6, C_3: 6, C_4: 6, C_5: 6, C_6: 6, C_7: 6 — 42

7 colour.

$1 - P(\text{no matches})$

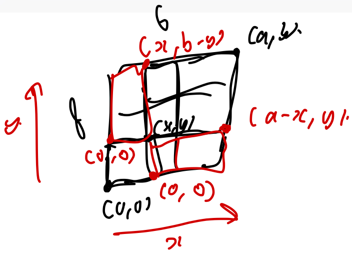
$= 1 - \dfrac{|\text{no matches}|}{|\text{all matches}|}$

$= 1 - \dfrac{^{42}C_1 \cdot {}^{36}C_1 \cdot {}^{30}C_1 \cdot {}^{24}C_1 \cdot {}^{18}C_1}{^{42}C_1 \cdot {}^{41}C_1 \cdot {}^{40}C_1 \cdot {}^{39}C_1 \cdot {}^{38}C_1} \approx 0.808$

9. Jack starts at (0, 0) on a grid. He wishes to get home which is at (6, 8) on the grid. Jack can only move rightwards or upwards. However, Jack needs to first visit Potts at (x, y) on the way. Which of the following positions should Potts stand on in order for Jack to have at least 1231 number of distinct routes to go back home?

Hint: ~~It might help to draw a small grid first. Each path is made up of a sequence of rightwards and/or upwards movement.~~

- [x] (4,3) 1227.
- [x] (2,2) 1270
- [ ] (5,6) 1271
- [x] (3,1) 1246.



pos on grid = minus subsequent R and C.

$\therefore {}^{6-1+8}C_8 = 1287.$

$- \dfrac{{}^{(a-x)-1+y}C_y}{} - {}^{x-1+(b-y)}C_{b-y}$

a = 6
b = 8.

=

//

10. Potts tosses a fair coin n times, where n is an odd integer greater than 1. What is the probability of Potts obtaining the same outcome every single toss (all heads or all tails)?

n is odd.

Hint: ~~Check out Lecture12 Counting 2, slide 32.~~

- (selected) $\frac{1}{2^n}$
- $\dfrac{1}{\binom{n}{0}+\binom{n}{2}+\binom{n}{4}+\ldots+\binom{n}{n-1}}$
- $\frac{1}{2^{2n}}$
- $\dfrac{\binom{n}{0}+\binom{n}{2}+\binom{n}{4}+\ldots}{2^n}$

H T H H T ...

↑ ↑ ↑ ↑ ↑.

$\dfrac{2}{{}^nC_1 + {}^nC_2 + {}^nC_3 + {}^nC_4 \ldots + {}^nC_n}$ } all outcomes.

$\dfrac{\ldots}{}$ n times. outcome

$= \dfrac{\cancel{2}\ 1}{\cancel{2}({}^nC_0 + {}^nC_2 + {}^nC_4 \ldots)}.$

11. Jack and Potts have 1 and 2 dice respectively. What is the probability of Jack rolling a higher number than the sum rolled by Potts?

- 1/128
- (selected) 5/54
- 2/23
- 3/32

J: ◇ P: ◇ ◇

6   3 2 + 2 3 + 2 2 + 1 2 + 2 1 + 1 1 + 4 1 + 1 4 → $(\frac{1}{6})^2 \cdot 10 \cdot \frac{1}{6}$

5   2 2 + 1 2 + 2 1 + 1 1 + 3 1 + 3 1 → $(\frac{1}{6})^2 \cdot 6 \cdot \frac{1}{6}$.   3 1 + 1 3.

4   1 2 + 2 1 + 1 1 → $(\frac{1}{6})^2 \cdot 3 \cdot \frac{1}{6}$

3   1 1 → $(\frac{1}{6})^2 \cdot \frac{1}{6}$.

$\frac{5}{54}$

all cases!

16. Polygraph tests are often routinely administered to employees. Let + be the event that the polygraph reading is positive, suggesting that the subject is lying, and let − be the event that the polygraph reading is negative, suggesting that the subject is telling the truth.

Let T be the event that the subject is telling the truth, and L be the event that the subject is lying.

According to polygraph reliability studies, P(+|L) = 0.88 and P(−|T) = 0.86. Suppose that on a particular question the vast majority of subjects have no reason to lie: P(T) = 0.99 and P(L) = 0.01.

If a subject produces a positive response on the polygraph, what is the probability that the polygraph is incorrect and that he/she is in fact telling the truth?

- (●) 0.08
- ( ) 0.47
- ( ) 0.92
- ( ) 0.94

independent

	T	L
−	0.08514	y
+	x	0.088
	↑ 0.99	↑ 0.01

independent

Given positive

$P(+|L) = 0.88$

$P(-|T) = 0.86$

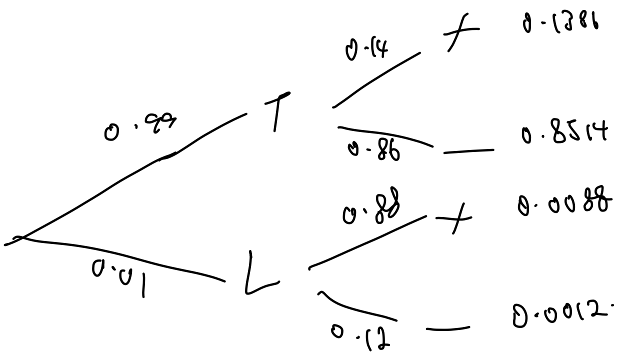
$P(T) = 0.99$

$P(L) = 0.01$

$P(+\cap L) = 0.088$

$P(-\cap T) = 0.86 \cdot 0.99 = 0.8514$

$P(T\cap +) = 0.99x$



$$P(T|+) = \frac{P(T\cap +)}{P(+)}$$

$$= \frac{0.1386}{0.0088 + 0.1386}$$

$$\approx 0.9402\ldots$$