NATIONAL UNIVERSITY OF SINGAPORE

SEMESTER 1, 2021/2022

MA1521 Calculus for Computing **Tutorial 10**

1. (a) Evaluate $\iint_R \frac{1}{xy} dA$ where R is the square $1 \leq x \leq 2, 1 \leq y \leq 2$.

   (Thomas' Calculus (14th edition), p. 855, Problem 25)

   (b) Evaluate $\int_0^2 \int_0^1 \frac{x}{1+xy}\, dxdy$.

   (Thomas' Calculus (14th edition), p. 855, Problem 37)

2. (a) Evaluate $\int_0^\pi \int_x^\pi \frac{\sin y}{y}\, dydx$.

   (Thomas' Calculus (14th edition), p. 863, Problem 47)

   (b) Evaluate $\int_0^3 \int_{\sqrt{x/3}}^1 e^{y^3}\, dydx$.

   (Thomas' Calculus (14th edition), p. 863, Problem 52)

3. (a) Find the volume of the region bounded above by the paraboloid $z = x^2 + y^2$ and below by the triangle enclosed by the lines $y = x, x = 0$ and $x + y = 2$.

   (Thomas' Calculus (14th edition), p. 863, Problem 57)

   (b) Find the volume of the solid that is bounded above by the cylinder $z = x^2$ and below by the region enclosed by the parabola $y = 2 - x^2$ and the line $y = x$ in the xy-plane.

   (Thomas' Calculus (14th edition), p. 863, Problem 58)

4. Without evaluating $\int \tan^{-1} x\, dx$, evaluate
$$\int_0^2 (\tan^{-1}(\pi x) - \tan^{-1} x)\, dx$$
using a double integral.

(Thomas' Calculus (14th edition), p. 864, Problem 78)

5. Change the Cartesian integral into an equivalent polar integral and then evaluate the polar integral.
   (a) Evaluate $\displaystyle\int_0^1 \int_0^{\sqrt{1-y^2}} (x^2 + y^2)\, dxdy.$

   (Thomas' Calculus (14th edition), p. 872, Problem 10)

   (b) Evaluate $\displaystyle\int_{-1}^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \ln(x^2 + y^2 + 1)\ dxdy.$

   (Thomas' Calculus (14th edition), p. 873, Problem 20)

6. Find the extreme values of $f(x, y) = xy$ subject to the constraint $g(x, y) = x^2 + y^2 - 10 = 0$.

(Thomas' Calculus (14th edition), p. 832, Problem 2)

7. (a) Find the extreme values of $f(x, y) = (xy)^2$ subject to the constraint that $g(x, y) = x^2 + y^2 - r^2 = 0$ and deduce that for positive real numbers a, b,
$$\sqrt{ab} \leq \frac{a+b}{2}.$$
   (b) (Optional) The Lagrange multiplier method works for functions of n variables. Find the extreme values of
$$f(x_1, x_2, \cdots, x_n) = (x_1 x_2 \cdots x_n)^n$$
subject to the constraint that
$$g(x_1, x_2, \cdots, x_n) = x_1^n + x_2^n + \cdots + x_n^n - r^n = 0$$
for some $r > 0$ and deduce that for positive real numbers $a_1, a_2, \cdots, a_n$,
$$(a_1 a_2 \cdots a_n)^{1/n} \leq \frac{a_1 + a_2 + \cdots + a_n}{n}.$$

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1, a) $\int_1^2 \int_1^2 \frac{1}{xy} \, dy \, dx$

$= \int_1^2 \frac{1}{x} dx \cdot \int_1^2 \frac{1}{y} dy.$

$= (\ln 2 - \ln 1)(\ln 2 - \ln 1.)$

$= \ln 2 \cdot \ln 2$

$= (\ln 2)^2.$

b) $\int_0^2 \int_0^1 \frac{x}{1+xy} \, dx \, dy$

$= \int_0^1 \int_0^2 \frac{x}{1+xy} \, dy \, dx$

$= \int_0^1 x \ln(1+2x) \, dx.$

$u = \ln(1+2x) \qquad dv = x$

$du = \frac{1}{1+2x} \qquad v = \frac{x^2}{2}$

$= \ln(1+2x) \cdot \frac{x^2}{2} - \int \frac{x^2}{2} \cdot \frac{1}{1+2x} \, dx.$

$= \frac{x^2}{2} \ln(1+2x) - \frac{1}{4}(1+2x) + \frac{1}{4}(\ln(1+2x)) \Big|_0^1$

$= \frac{1}{2} \ln 3 - \frac{3}{4} + \frac{1}{4} \ln 3 - \frac{1}{4}. \quad = \frac{3}{4} \ln 3 - 1$ //

$\frac{1}{2} \int \frac{x^2}{1+2x}$

$1+2x \,\overline{)\, x^2}$ quotient x

$-(x + x^2)$

$-x.$

$\frac{1}{2} \int \frac{1}{1+2x} - \frac{x}{1+2x}$

$\frac{1}{2}(\ln|1+2x|) - \frac{1}{2} \int \frac{x}{1+2x} \, dx$

$\int \frac{x}{1+2x}$

$u = 1+2x$

$x = \frac{u-1}{2}.$

$du = 2.$

$\frac{1}{2} \int \frac{\frac{u-1}{2}}{u}.$

$\frac{1}{4} \int \frac{u-1}{u} . du$

$= \frac{1}{4}\left(\int \frac{u}{u} - \int \frac{1}{u}\right)$

$= \frac{1}{4} u - \ln u$

$= \frac{1}{4}(1+2x) - \frac{1}{4}\ln(1+2x)$

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(Thomas' Calculus (14th edition), p. 863, Problem 52)

a) $$\int_0^\pi \int_x^\pi \frac{\sin y}{y} \, dy \, dx.$$

$$= \int_x^\pi \int_0^\pi \frac{\sin y}{y.} \, dx \, dy.$$

$$? \quad \int_x^\pi \left(\frac{\sin y}{y} \cdot x \right) \Big|_0^\pi \, dy.$$

$$= \int_x^\pi \frac{\sin y}{y} \cdot \pi \, dy.$$

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