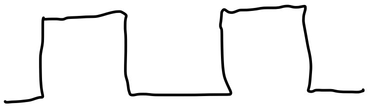
Boolean Algebra.

Digital Circuits

- 2 voltage levels
  - 1 (High)
  - 0 (Low)



note its squarish!

- Advantages over analog
  - More reliable – simpler circuits, less noise-prone
  - Specified accuracy – determinable.
  - Abstraction – using Boolean Algebra.
  - Ease design, analysis and simplification of digital circuit – Digital Logic Design

- 2 Types
  - Combinational
    - no memory
    - output depends solely on inputs
    - eg. Gates, Decoders, Multiplexers, Adders, Multipliers

    i.e. functions
  - Sequential
    - have memory
    - output depends on both input and current state → State machine.
    - eg. Counters, registers, memories.

Boolean Algebra.

- 1 = True
- 0 = False.
- Connectives

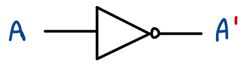
high priority

Precedence

low priority

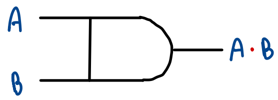
- NOT - Negation "inverter"

  A' / $\bar{A}$ / $\neg A$



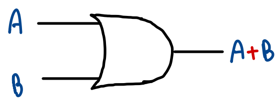
- AND - Conjunction

  $A \cdot B$ / $A \wedge B$



- OR - Disjunction

  $A + B$ / $A \vee B$



- Truth Table
  - Prove equations - identical columns.

- Laws (1 is True, 0 is False)

Identity laws	
A + 0 = 0 + A = A	A · 1 = 1 · A = A
Inverse/complement laws	
A + A' = A' + A = 1	A · A' = A' · A = 0
Commutative laws	
A + B = B + A	A · B = B · A
Associative laws *	optional
A + (B + C) = (A + B) + C	A · (B · C) = (A · B) · C
Distributive laws	compulsory
A · (B + C) = (A · B) + (A · C)	A + (B · C) = (A + B) · (A + C)

- Duality
  - If the AND / OR operators and identify elements 0 / 1 in a boolean equation are interchanged, it remains valid. (not expressions!)
  - i.e. If $a + (b \cdot c) = (a+b) \cdot (a+c)$ is true, then $a \cdot (b+c) = (a \cdot b) + (a \cdot c)$ is also true. "Shortcut"

- Theorems

Idempotency	Duality
$X + X = X$	$X \cdot X = X$
One element / Zero element	
$X + 1 = 1 + X = 1$	$X \cdot 0 = 0 \cdot X = 0$
Involution (Double negation law)	
$(X')' = X$	
Absorption 1	
$X + X \cdot Y = X$	$X \cdot (X + Y) = X$
Absorption 2	
$X + X' \cdot Y = X + Y$	$X \cdot (X' + Y) = X \cdot Y$
DeMorgans' (can be generalised to more than 2 variables)	
$(X + Y)' = X' \cdot Y'$	$(X \cdot Y)' = X' + Y'$
Consensus	
$X \cdot Y + X' \cdot Z + Y \cdot Z = X \cdot Y + X' \cdot Z$	$(X+Y) \cdot (X'+Z) \cdot (Y+Z) = (X+Y) \cdot (X'+Z)$

removed

- Standard Forms

  - Literals
    - A boolean variable on its own or in its complemented form
    - eg. x, x', y, y'

  - Product term
    - A single literal or a logical product (AND) of several literals
    - eg. x, $x \cdot y \cdot z'$, $A' \cdot B$, $A \cdot B$, $d \cdot g' \cdot v \cdot w$

  - Sum term
    - A single literal or a logical sum (OR) of several literals
    - eg. x, $x+y+z'$, $A'+B$, $A+B$, $c+d+h'+j$

Every boolean expression can be expressed in SOP or POS.

- Sum of products (SOP)
  - A product term or logical sum (OR) of several product terms
  - eg. x, $x + y \cdot z'$, $x \cdot y' + x' \cdot y \cdot z$, $A \cdot B + A' \cdot B'$, $A + B' \cdot C + A \cdot C' + C \cdot D$

- Product of sums (POS)
  - A sum term or logical product (AND) of several sum terms
  - eg x, $x \cdot (y+z')$, $(x+y') \cdot (x'+y+z)$, $(A+B) \cdot (A'+B')$, $(A+B+C) \cdot D' \cdot (B'+D+E')$

Each minterm is the complement of its corresponding maxterm. ie. $m2' = M2$.

- Minterm
  - product term that contains n literals from all the variables. → 2^n minterms.
  - eg. $x, y \rightarrow \underset{m0}{\overset{00}{x' \cdot y'}}, \underset{m1}{\overset{01}{x' \cdot y}}, \underset{m2}{\overset{10}{x \cdot y'}}, \underset{m3}{\overset{11}{x \cdot y}}$ (minterms)

- Maxterm
  - sum term that contains n literals from all the variables. → 2^n maxterms.
  - eg. $x, y \rightarrow \underset{M3}{\overset{11}{x'+y'}}, \underset{M2}{\overset{10}{x'+y}}, \underset{M1}{\overset{01}{x+y'}}, \underset{M0}{\overset{00}{x+y}}$ (maxterms)

- Canonical Forms
  - A unique form of representation.
  - Sum-of-minterms = Canonical sum-of-products.
    (impt. for logic circuits)
    - Gather the minterms of the function (output = 1)

x	y	z	F1	F2	F3
0	0	0	0	0	0
0	0	1	0	1	1
0	1	0	0	0	0
0	1	1	0	0	1
1	0	0	0	1	1
1	0	1	0	1	1
1	1	0	1	1	0
1	1	1	0	1	0

$F1 = x \cdot y \cdot z' = m6$

$F2 = x' \cdot y' \cdot z + x \cdot y' \cdot z' + x \cdot y' z + x \cdot y \cdot z' + x' \cdot y' \cdot z'$
$= m1 + m4 + m5 + m6 + m7$
$= \Sigma m(1,4,5,6,7)$

$F3 = x' \cdot y' \cdot z + x' \cdot y \cdot z + x \cdot y' \cdot z' + x \cdot y' \cdot z$
$= m1 + m3 + m4 + m5$
$= \Sigma m(1,3,4,5)$

  - Product-of-maxterms = Canonical product-of-sums.
    - Gather the maxterms of the function (output = 0)

x	y	z	F1	F2	F3
0	0	0	0	0	0
0	0	1	0	1	1
0	1	0	0	0	0
0	1	1	0	0	1
1	0	0	0	1	1
1	0	1	0	1	1
1	1	0	1	1	0
1	1	1	0	1	0

$F1 = \ldots$

$F2 = (x+y+z) \cdot (x+y'+z) \cdot (x+y'+z')$
$= M0 \cdot M2 \cdot M3$
$= \Pi M(0,2,3)$

$F3 = (x+y+z) \cdot (x+y'+z) \cdot (x'+y'+z) \cdot (x'+y'+z')$
$= M0 \cdot M2 \cdot M6 \cdot M7$
$= \Pi M(0,2,6,7)$

Conversion of standard forms

$F2 = \Sigma m(1,4,5,6,7)$
$= \Pi M(0,2,3)$

x	y	z	F2	F2'
0	0	0	0	1
0	0	1	1	0
0	1	0	0	1
0	1	1	0	1
1	0	0	1	0
1	0	1	1	0
1	1	0	1	0
1	1	1	1	0