NATIONAL UNIVERSITY OF SINGAPORE

CS1231/CS1231S – Discrete Structures

(AY2019/20 Semester 1)

ANSWER SHEET

Time Allowed: 1 hour 20 minutes

FOR EXAMINERS' USE ONLY		
Questions	Marks	Obtained
Q11	12	
Q12	9	
Q13	9	
Total	30	

INSTRUCTIONS TO CANDIDATES

1. This Answer Sheet consists of **TWO (2)** printed pages.
2. Fill in your Tutorial Group Number (eg: T12) and Student Number below with a pen. Do not write your name.
3. You may write your answers in pencil.
4. Write within the boxes provided. Illegible handwriting and unnecessarily long answers will be penalized.

TUTORIAL GROUP: [ ]

STUDENT NUMBER: | A | | | | | | | | |

(Write your Student Number legibly with **a pen** to prevent accidental erasure.)

MCQ answers:

1. A 2. D 3. D 4. C 5. A 6. D 7. D 8. B 9. B 10. E

Q11. Logic [12 marks]

(a) $\forall x \left(Bird(x) \wedge Fly(x)\right)$ [3 marks]

All animals are birds and (all animals) can fly.

(b) "There is no free lunch." [3 marks]

$\forall x \left(\sim Free(x) \vee \sim Lunch(x)\right)$ or $\forall x \left(Lunch(x) \rightarrow \sim Free(x)\right)$

(c) "Every child irritates his or her parent." [3 marks]

or $\forall x \left(Child(x) \rightarrow \exists y \left(Parent(y,x) \wedge Irritate(x,y)\right)\right)$

$\forall x \left(Child(x) \rightarrow \forall y \left(Parent(y,x) \rightarrow Irritate(x,y)\right)\right)$

(d) "For every odd integer there is a different integer such that the sum of these two numbers is even." [3 marks]

$\forall x \left(\sim Even(x) \rightarrow \exists y \left(x \neq y \wedge Even(x+y)\right)\right)$

Q12. $A - (B - C) = (A - B) \cup (A \cap C)$. [9 marks]

$$
\begin{aligned}
A - (B - C) &= A - (B \cap \bar{C}) && \text{by the Set Difference Law;} \\
&= A \cap \overline{(B \cap \bar{C})} && \text{by the Set Difference Law;} \\
&= A \cap (\bar{B} \cup \bar{\bar{C}}) && \text{by the De Morgan's Law;} \\
&= A \cap (\bar{B} \cup C) && \text{by the Double Complement Law;} \\
&= (A \cap \bar{B}) \cup (A \cap C) && \text{by the Distributive Law;} \\
&= (A - B) \cup (A \cap C) && \text{by the Set Difference Law;}
\end{aligned}
$$

Q13. $\sum_{i=1}^{n} i(i+1)(i+2) = \frac{1}{4}n(n+1)(n+2)(n+3)$. [9 marks]

1. For each $n \in \mathbb{Z}^+$,
   let $P(n) = \left(\sum_{i=1}^{n} i(i+1)(i+2) = \frac{1}{4}n(n+1)(n+2)(n+3)\right)$.
2. **Base step:** $P(1) = \left((1)(2)(3) = 6 = \frac{1}{4}(1)(2)(3)(4)\right)$, which is true.
3. **Inductive step:**
   Assume $P(k)$, i.e. $\sum_{i=1}^{k} i(i+1)(i+2) = \frac{1}{4}k(k+1)(k+2)(k+3)$.
4. Then $P(k+1)$:

$$
\begin{aligned}
&\sum_{i=1}^{k+1} i(i+1)(i+2) \\
&= \sum_{i=1}^{k} i(i+1)(i+2) + (k+1)(k+2)(k+3) \\
&= \frac{1}{4}k(k+1)(k+2)(k+3) + (k+1)(k+2)(k+3) && \text{(applying } P(k)\text{)} \\
&= (\frac{1}{4}k + 1)(k+1)(k+2)(k+3) && \text{(basic algebra)} \\
&= \frac{1}{4}(k+4)(k+1)(k+2)(k+3) && \text{(basic algebra)}
\end{aligned}
$$

5. Therefore, $P(k+1)$ is true.
6. By MI, $P(n)$ is true for all $n \in \mathbb{Z}^+$.

=== END OF PAPER ===