## NATIONAL UNIVERSITY OF SINGAPORE

SEMESTER 1, 2021/2022

## MA1521 Calculus for Computing

Tutorial 9

1. Find the limits:

(a) 
$$\lim_{(x,y)\to(2,-3)} \left(\frac{1}{x} + \frac{1}{y}\right)^2$$
 (Thomas' Calculus (14<sup>th</sup> edition), p. 773, Problem 4)

(b) 
$$\lim_{\substack{(x,y)\to(0,0)\\x\neq y}} \frac{x-y+2\sqrt{x}-2\sqrt{y}}{\sqrt{x}-\sqrt{y}}$$
 (Thomas' Calculus (14<sup>th</sup> edition), p. 774, Problem 17)

(c) 
$$\lim_{(x,y)\to(0,0)} \frac{1-\cos(xy)}{xy}$$
 (Thomas' Calculus (14<sup>th</sup> edition), p. 774, Problem 22)

2. Show that the following limits do not exist.

(a) 
$$\lim_{(x,y)\to(1,-1)} \frac{xy+1}{x^2-y^2}$$
 (Thomas' Calculus (14<sup>th</sup> edition), p. 774, Problem 50)

(b) 
$$\lim_{(x,y)\to(1,1)} \frac{\tan y - y \tan x}{y - x}$$
 (Thomas' Calculus (14<sup>th</sup> edition), p. 774, Problem 54)

3. The inequality

$$1 - \frac{x^2 y^2}{3} < \frac{\tan^{-1} xy}{xy} < 1$$

holds for (x, y) "close to (0, 0)". What can you say about  $\lim_{(x,y)\to(0,0)} \frac{\tan^{-1} xy}{xy}$ ?

(Thomas' Calculus (14th edition), p. 775, Problem 59)

- 4. Find  $\partial f/\partial x$  and  $\partial f/\partial y$  for the following functions f.
  - (a)  $f(x,y) = (xy 1)^2$

(Thomas' Calculus (14<sup>th</sup> edition), p. 785, Problem 5)

- (b)  $f(x,y) = e^{xy} \ln y$  (Thomas' Calculus (14<sup>th</sup> edition), p. 785, Problem 16)
- (c)  $f(x,y) = x^y$  (Thomas' Calculus (14<sup>th</sup> edition), p. 785, Problem 19)
- 5. For each of the following functions, determine  $f_{xy}$ .
  - (a)  $f(x,y)=y+x^2y+4y^3-\ln(y^2+1)$  (Thomas' Calculus (14<sup>th</sup> edition), p. 786, Problem 61(d))
  - (b)  $f(x,y) = x \ln(xy)$  (Thomas' Calculus (14<sup>th</sup> edition), p. 786, Problem 61(d))
- 6. Show that the function  $f(x,y) = \ln(x^2 + y^2)$  satisfies the two-dimensional Laplace equation

$$f_{xx} + f_{yy} = 0.$$

(Thomas' Calculus (14<sup>th</sup> edition), p. 787, Problem 86)

- 7. Find the absolute maxima and minima of the functions on the given domains.
  - (a)  $D(x,y) = x^2 xy + y^2 + 1$  on the closed triangular plate in the first quadrant bounded by the lines x = 0, y = 4 and y = x.

(Thomas' Calculus (14<sup>th</sup> edition), p. 823, Problem 32)

(b) Find two numbers a and b with  $a \leq b$  such that

$$\int_{a}^{b} (6-x-x^2)dx$$

has its largest values.

(Thomas' Calculus ( $14^{\rm th}$  edition), p. 823, Problem 39)

8. Let

$$f(x,y) = \begin{cases} \frac{xy^2}{x^2 + y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0). \end{cases}$$

Show that  $f_x(0,0)$  and  $f_y(0,0)$  exist, but f is not differentiable at (0,0).

(Thomas' Calculus (14<sup>th</sup> edition), p. 788, Problem 101)

9. Find the extreme values of f(x,y) = xy subject to the constraint  $g(x,y) = x^2 + y^2 - 10 = 0$ .

(Thomas' Calculus (14<sup>th</sup> edition), p. 832, Problem 2)

10. (a) Find the extreme values of  $f(x,y) = (xy)^2$  subject to the constraint that  $g(x,y) = x^2 + y^2 - r^2 = 0$  and deduce that for positive real numbers a, b,

$$\sqrt{ab} \le \frac{a+b}{2}$$
.

(b) (Optional) The Lagrange multiplier method works for functions of n variables. Find the extreme values of

$$f(x_1, x_2, \cdots, x_n) = (x_1 x_2 \cdots x_n)^n$$

subject to the constraint that

$$g(x_1, x_2, \dots, x_n) = x_1^n + x_2^n + \dots + x_n^n - r^n = 0$$

for some r > 0 and deduce that for positive real numbers  $a_1, a_2, \dots, a_n$ ,

$$(a_1 a_2 \cdots a_n)^{1/n} \le \frac{a_1 + a_2 + \cdots + a_n}{n}.$$

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$$\lim_{\substack{(x,y)\to(0,0)\\x\neq y}} \frac{x-y+2\sqrt{x}-2\sqrt{y}}{\sqrt{x}-\sqrt{y}}$$

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(c) 
$$\lim_{(x,y)\to(0,0)} \frac{1-\cos(xy)}{xy}$$

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		following limits do not exist.	
(8	$\lim_{(x,y)\to(1,-1)}\frac{1}{x}$	$\frac{xy+1}{x^2-y^2}$	
	$(x,y) \rightarrow (1,-1)$ a	(Thomas' Calculus (14 <sup>th</sup> edition), p. 774, Problem 50)	
(h	$\frac{\tan \frac{\tan x}{2}}{2}$	an y - y tan x	
	$\lim_{(x,y)\to(1,1)}\frac{\mathrm{ta}}{}$		
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3. The inequality	$x^2y^2 = \tan^{-1}xy$	_	
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holds for $(x, y)$ "cl	lose to $(0,0)$ ". What can you say ab	out $\lim_{(x,y)\to(0,0)} \frac{\tan^{-1}xy}{xy}$ ?	
	(Thomas' Calculus (14 <sup>th</sup> edition),		
	(Thomas Calculus (14 Carton),	p. 110, 110bichi 05)	

	$\partial f/\partial y$ for the following functions $f$ .	
(a) $f(x,y) = (xy)$	(Thomas' Calculus (14 <sup>th</sup> edition), p. 785, Problem 5)	
(b) $f(x,y) = e^{xy}$	$\ln y$ (Thomas' Calculus (14 <sup>th</sup> edition), p. 785, Problem 16)	
	(Thomas Calcinus (14 Edition), p. 166, 1166fem 16)	
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6. Show that the function $f(x, y) = \ln(x^2 + y^2)$ s Laplace equation	atisfies the two-dimensional		
Laplace equation $f_{xx} + f_{yy} = 0.$			
(Thomas' Calculus (14 <sup>th</sup> ed	ition), p. 787, Problem 86)		

7. Find the absolute maxima and minima of the functions on the given	
domains.	
(a) $D(x,y) = x^2 - xy + y^2 + 1$ on the closed triangular plate in the first	
quadrant bounded by the lines $x = 0$ , $y = 4$ and $y = x$ .	
(Thomas' Calculus (14 <sup>th</sup> edition), p. 823, Problem 32)	
(b) Find two numbers $a$ and $b$ with $a \leq b$ such that	
$\int_{a}^{b} (6 - x - x^2) dx$	
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(Thomas' Calculus (14 <sup>th</sup> edition), p. 823, Problem 39	
(Thomas Calculus (14 edition), p. 625, 1 foblem 55	

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$$f(x,y)=\left\{\begin{array}{ll} \frac{xy^2}{x^2+y^2}, & (x,y)\neq (0,0)\\ 0, & (x,y)=(0,0). \end{array}\right.$$
 Show that  $f_x(0,0)$  and  $f_y(0,0)$  exist, but  $f$  is not differentiable at  $(0,0)$ .

(Thomas' Calculus (14th edition), p. 788, Problem 101)

9. Find the extreme values of $f(x,y) = xy$ subject to the constraint $g(x,y) = x^2 + y^2 - 10 = 0$ .
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10. (	a)	Find the extreme values of $f(x,y)=(xy)^2$ subject to the constraint that $g(x,y)=x^2+y^2-r^2=0$ and deduce that for positive real numbers $a,b$ ,	
		$\sqrt{ab} \le \frac{a+b}{2}.$	
(1	b)	(Optional) The Lagrange multiplier method works for functions of $n$ variables. Find the extreme values of	
		$f(x_1, x_2, \cdots, x_n) = (x_1 x_2 \cdots x_n)^n$	
		subject to the constraint that $g(x_1,x_2,\cdots,x_n)=x_1^n+x_2^n+\cdots+x_n^n-r^n=0$	
		for some $r>0$ and deduce that for positive real numbers $a_1,a_2,\cdots,a_n,$	
		$(a_1 a_2 \cdots a_n)^{1/n} \le \frac{a_1 + a_2 + \cdots + a_n}{n}.$	