NATIONAL UNIVERSITY OF SINGAPORE

SCHOOL OF COMPUTING

MID-TERM TEST
AY2019/20 Semester 1

CS1231/CS1231S — DISCRETE STRUCTURES

5 October 2019 Time Allowed: 1 hour 20 minutes

INSTRUCTIONS

1. This question paper contains **THIRTEEN (13)** questions and comprises **FIVE (5)** printed pages.
2. This is an **OPEN BOOK** assessment.
3. You are allowed to use NUS approved calculators. Other electronic devices are not allowed.
4. Answer Section A on the **MCQ form** and Section B on the **ANSWER SHEET** provided.
5. Write and shade your **Student Number** on the MCQ form with a 2B pencil. You do not need to fill in any other particulars. Check that you have written and shaded your student number correctly! (Wrong or incomplete student number will lead to no marks awarded for section A.)
6. Write your **Student Number** and **Tutorial Group Number** on the first page of the Answer Sheet using a **PEN**. Do NOT write your name on it.
7. You may use pen or pencil (at least 2B) to write your answers on the Answer Sheet, but please erase cleanly, and write legibly. Marks may be deducted for illegible handwriting.
8. Submit only the MCQ form and the Answer Sheet at the end of the test. You may keep the question paper.
9. Turn off your phone or set it to silent mode and put the phone away.
10. You are not allowed to leave the hall in the last 15 minutes of the assessment.

——— END OF INSTRUCTIONS ———

Section A: Each multiple-choice-question has only <u>one</u> correct answer. Shade your answers on the **MCQ form**. Two (2) marks are awarded for each correct answer and no penalty for wrong answer. [Total: 20 marks]

1. Which of the following are true?

   (I) $(p \to q) \equiv ({\sim}q \to {\sim}p)$ ✓

   (II) $(p \to q) \equiv ({\sim}p \to {\sim}q)$ ✗

   (III) $(p \text{ only if } q) \equiv (q \to p)$ ✗

   Then Only sufficient →

   A. (I) only
   B. (II) only
   C. (III) only
   D. (I) and (III) only
   E. All of (I), (II) and (III)

   A ✓

2. Given that $a \mid b$ means "a divides b", the predicate $P(x)$, where $x \in \mathbb{N}$, is defined as follows:

$$P(x) = \big((400 \mid x) \lor (4 \mid x \land 100 \nmid x) \big).$$

   Which of the following statements is/are true?

   (I) $P(2000)$ ✓ (II) $P(1900)$ ✗ (III) $P(1231)$ ✗ (IV) $P(604)$ ✓

   A. (I) only
   B. (II) only
   C. (II) and (III) only
   D. (I) and (IV) only
   E. None of (I), (II), (III) and (IV)

   D ✓

   $p \to q$
   $q \to p$
   ${\sim}p \to {\sim}q$

3. "If it rains tomorrow, we will have a mid-term test," announced the Evil Prof™. On the very next day, to the joy of the students, it doesn't rain, and therefore there would be no mid-term test. What can we say about the students' joy?

   p $p \to q$ q ${\sim}p$ ${\sim}q$

   See notes.

   A. It is justifiable, since by ~~modus~~ tollens there will not be a mid-term test if it does not rain. See notes.
   B. They are mistaken and have committed a converse error.
   C. It is justifiable, since by contrapositive there will not be a mid-term test if it does not rain.
   D. They are mistaken and have committed an inverse error. ✓
   E. They are mistaken and have committed a contrapositive error.

   A
   ✗ D

4. Which of the following are true, given that the domain of x is non-empty?

(I) $\forall x\, P(x) \rightarrow \exists x\, P(x)$

(II) $\forall x\big(P(x) \wedge Q(x)\big) \equiv \forall x\, P(x) \wedge \forall x\, Q(x)$

(III) $\exists x\,\big(P(x) \wedge Q(x)\big) \rightarrow \exists x\, P(x) \wedge \exists x\, Q(x)$

(IV) $\exists x\,\big(P(x) \wedge Q(x)\big) \equiv \exists x\, P(x) \wedge \exists x\, Q(x)$

A. (I) and (II) only

B. (II) and (IV) only

C. (I), (II) and (III) only

D. (I), (II), and (IV) only

E. All of (I), (II), (III) and (IV)

A ✗ C

5. Given the following predicate:

$$P(x) = (x \neq 1) \wedge \forall y, z\,\big(x = yz \rightarrow ((y = x) \vee (y = 1))\big)$$

and that the domain of x, y and z is $\mathbb{Z}^+$, what is $P(x)$?

A. $P(x)$ is true if and only if x is a prime number.

B. $P(x)$ is true if and only if x is a number other than 1.

C. $P(x)$ is always true irrespective of the value of x.

D. $P(x)$ is true if and only if x has exactly two factors other than 1 and x.

E. None of the above.

A

6. Given $A \subseteq \mathbb{R}$, $A \neq \emptyset$, $x \in \mathbb{R}$ and the predicate $P(x)$ to mean "x is an upper bound for A". Which of the following defines α to be the least upper bound for A?

A. $\forall y \in \mathbb{R}\,\big(P(y) \rightarrow (\alpha \leq y)\big)$. not a bound

B. $\forall y \in \mathbb{R}\,\big(P(y) \wedge (\alpha \leq y)\big)$.

C. $P(\alpha) \wedge (\forall y \in \mathbb{R}\;\; (\alpha \leq y))$.

D. $P(\alpha) \wedge \Big(\forall y \in \mathbb{R}\,\big(P(y) \rightarrow (\alpha \leq y)\big)\Big)$.

E. None of the above.

bound.

P(x) α A

A ✗ D

7. Consider the following statements, where $\wp(X)$ denotes the power set of a set X:

write out

(I) There exists a set A such that $|\wp(A)| = 3$. ✗

(II) For all sets A and B, $\wp(A \cup B) = \wp(A) \cup \wp(B)$.

A. Both (I) and (II) are true.

B. (I) is true and (II) is false.

C. (I) is false and (II) is true.

D. Both (I) and (II) are false.

E. Insufficient information to determine whether (I) and/or (II) are true.

A = {a, b, c}. P(A) =

B = {b, c, d}. P(B) =

A ∪ B = {a, b, c, d}.

C ✗ D

$A = \{1,2\}$ $\quad$ $A \cup B = \{1,2,3\}$

$B = \{2,3\}$

$P(A) = \{ \{\emptyset\}, \{1\}, \{2\}, \{1,2\}\}$

$P(B) = \{\{\emptyset\}, \{2\}, \{3\}, \{2,3\}\}$

$P(A \cup B) = \{\{\emptyset\}, \{1\}, \{2\}, \{3\}, \{1,2\}, \{2,3\}, \underline{\{1,3\}}, \underline{\{1,2,3\}}\}$

$X = \{1,2\}$ $\quad$ $f(X) = \{f(1), f(2)\}$

$X' = \{2,3\}$ $\quad$ $f(X') = \{f(2), f(3)\}$

$X \cup X' = \{1,2,3\}$ $\quad$ $X \cap X' = \{2\}$

$f(X \cup X') = \{f(1), f(2), f(3)\}$

$f(X \cap X') = \{f(2)\}$

Special case. $\quad$ $f(1) = f(3)$.

$f(X) = \{f(1), f(2)\}$

$f(X') = \{f(2), f(3)\}$

$X \cap X' = \{1, 2, 3\}$

$f(X \cap X') = \{f(2)\}$ $\quad$ $\{f(1), f(2)\} \cap \{f(2), \underset{f(1)}{\overset{\shortparallel}{f(3)}}\}$

$\{f(2), f(1)\}$

$X: \{1, 2, 3\}$. $X': \{2, 3, 4\}$. $X \cup X': \{1, 2, 3, 4\}$

$f(X \cup X') = \{f(1), f(2), f(3), f(4)\}$

$f(X) = \{f(1), f(2), f(3)\}$.

$f(X') = \{f(2), f(3), f(4)\}$.

8. Consider the following statements:

   (I) For all functions $f: \mathbb{R} \to \mathbb{R}$ and for all subsets $X, X' \subseteq \mathbb{R}$, $f(X \cup X') = f(X) \cup f(X')$.

   (II) For all functions $f: \mathbb{R} \to \mathbb{R}$ and for all subsets $X, X' \subseteq \mathbb{R}$, $f(X \cap X') = f(X) \cap f(X')$.

   $f(X \cap X') = \{f(2), f(3)\}$.

   $f(X) \cap f(X')$

   = Same.

   for diff input → same output

   A. Both (I) and (II) are true.
   B. (I) is true and (II) is false.
   C. (I) is false and (II) is true.
   D. Both (I) and (II) are false.
   E. Insufficient information to determine whether (I) and/or (II) are true.

   A ✗ B.

9. Consider the following statements:

   we learn inverse?

   (I) For all functions $f: \mathbb{R} \to \mathbb{R}$, f is bijective if and only if $|f^{-1}(\{b\})| = 1$ for all $b \in \mathbb{R}$.

   domain

   y-value

   $\{b\}$.

   (II) For all functions $f: \mathbb{R} \to \mathbb{R}$, f is surjective if and only if $f^{-1}(X) \neq \emptyset$ for all $X \in \wp(\mathbb{R})$.

   $p(\mathbb{R}) = \{\emptyset\}, \{1\}, \{2\}$

   $\{\emptyset\}$.

   A. Both (I) and (II) are true.
   B. (I) is true and (II) is false.
   C. (I) is false and (II) is true.
   D. Both (I) and (II) are false.
   E. Insufficient information to determine whether (I) and/or (II) are true.

   B

10. Let A, B be finite sets and $f: A \to B$. Consider the following statements:

   $\forall f$

   (I) $\forall X \subseteq A\ |f(X)| \leq |X|$.

   (II) $\forall Y \subseteq B\ |f^{-1}(Y)| \geq |Y|$.

   $f(x)$

   B

   $x \in A$

   Are the statements above true? Choose the one from the following list that best describes your answer.

   $Y \subseteq B$

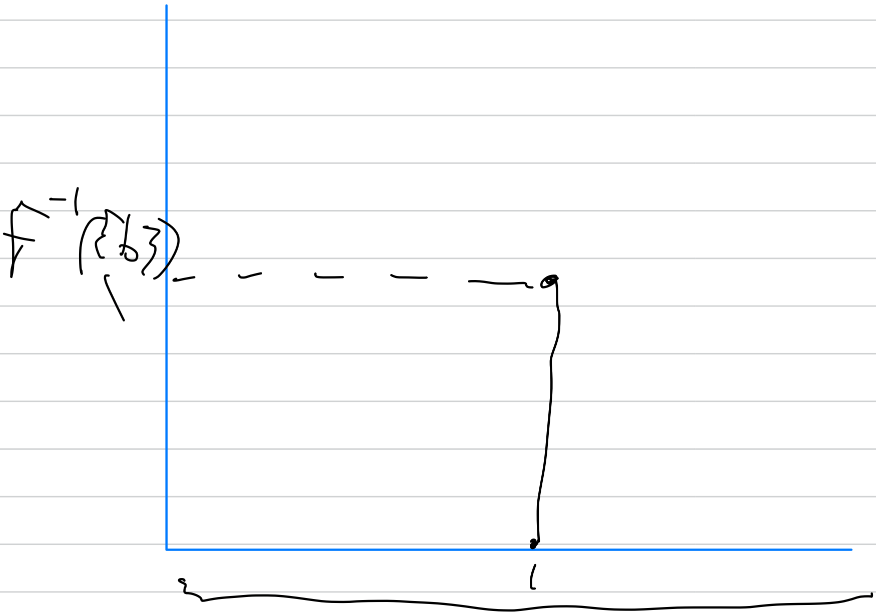
   A

   $y = f(x)$

   $x = f^{-1}(y)$.

   A. Both (I) and (II) are true.
   B. (I) is true and (II) is false.
   C. (I) is false and (II) is true.
   D. Both (I) and (II) are false.
   E. The truth of at least one of (I), (II) depends on the ~~choice of~~ A, B and f.

   D. ✗ E.



$\{b\}: \quad b = 1, 2, 3, \ldots$

not a function
is one to many

Section B: Write your answers on the **Answer Sheet.** You may write your answers in pencil. Remember to write your **Student Number** and **Tutorial Group Number** in **pen** on the first page of the Answer Sheet. [Total: 30 marks]

11. [12 marks]
For part (a), translate the given quantified statement into an English sentence, assuming that the universe of discourse (domain) is the set of all animals.

a. $\forall x\ (Bird(x) \wedge Fly(x))$

where $Bird(x)$ means "x is a bird" and $Fly(x)$ means "x can fly". [3 marks]

For all x, x is a bird and x can fly.

For parts (b) – (d), express the given English sentences using quantified statements. Do not use $\exists!$ and do not begin your answer with a negated quantifier (i.e $\sim\forall$, $\sim\exists$).

b. "There is no free lunch."

You may use the following predicates and no other predicates:

- $Free(x)$: "x is free"
- $Lunch(x)$: "x is lunch" [3 marks]

$\forall x\ (Lunch(x) \rightarrow \neg Free(x))$.

c. "Every child irritates his or her parent."

You may use the following predicates and no other predicates:

- $Child(x)$: "x is a child"
- $Parent(x, y)$: "x is the parent of y"
- $Irritate(x, y)$: "x irritates y" [3 marks]

$\forall x\ (Child(x) \rightarrow \exists y\ (Parent(\cancel{x,y}\ y,x) \wedge Irritate(x,y)))$

d. "For every odd integer there is a different integer such that the sum of these two numbers is even."

You may assume that the domain is the set of integers and you do not need to include it in your answer. An integer is either even or odd, but not both. You may use the following predicate and no other predicates:

- $Even(x)$: "x is even" [3 marks]

$\forall x \in \mathbb{Z}\ (\neg Even(x) \rightarrow \exists y\ (x \neq y \wedge Even(x+y)))$

12. Show that for all sets A, B, C,

$$A - (B - C) = (A - B) \cup (A \cap C).$$

You may assume our universal set is $U = A \cup B \cup C$ here if needed. [9 marks]

~~13~~. Prove by mathematical induction that, for all $n \in \mathbb{Z}^+$,

$$\sum_{i=1}^{n} i(i+1)(i+2) = \frac{1}{4}n(n+1)(n+2)(n+3).$$

[9 marks]

=== END OF PAPER ===

12 $A - (B - C) = (A - B) \cup (A \cap C)$

LHS. $= A \cap (\overline{B - C})$ ← by set difference

$= A \cap (\overline{B \cap \bar{C}})$

$= A \cap (\bar{B} \cup \bar{\bar{C}})$ by De morgan's.

$= A \cap (\bar{B} \cup C)$ by double complement

$= (A \cap \bar{B}) \cup (A \cap C)$ by distributive,

$= (A - B) \cup (A \cap C)$ by set difference.

$=$ RHS.