A note about the aspect-ratio

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Abstract

In recent versions of CoMMA, the user can now choose how to compute the aspect-ratio (AR). We give in this document some insights about all these ways with graphical illustrations in order to help the user choose the one that suits their needs best.

1 General remarks

CoMMA being a *geometric* agglomerator, the aspect-ratio (AR) is a pivotal concept. There is no agreed-upon definition of AR, but, generally speaking, it is a quantity which translates into numbers the "quality" of a cell. Typically, it could be computed like the ratio of the maximum of the distances between the vertices of a cell over the minimum. The AR is then a positive quantity greater than or equal to 1: indeed, an AR equal to 1 is usually thought of as the best case scenario, a cell which is well formed and balanced; the more stretched the cell gets, the higher the AR.

The notion of AR plays two major roles in CoMMA, indeed, the AR is used to:

U1 identify anisotropic cells which are agglomerated with a special algorithm.

U2 choose a fine cell (FC) to be added to the current coarse cell (CC): the chosen FC is the one that should *minimize* the AR of the resulting CC.

In what follows, we discuss usage U2 only.

Notice that the definition employed above involving vertices is not viable in CoMMA since one does not have the necessary information (namely, the position of the vertices). In the past, CoMMA has always tried to use a definition which reminds of the one mentioned above. However, today, we use "AR" more freely (and indeed, we might use it in a non geometric sense). In most of the definitions that we define below, the AR will be compute as a ratio:

$$AR = \frac{Num}{Den} \tag{1}$$

We however fix some key concepts:

AR1 The AR is a positive quantity.

AR2 Whenever the ratio of two quantities is used to compute the AR, the result is a non-dimensional quantity.

AR3 A large value indicates an undesired cell.

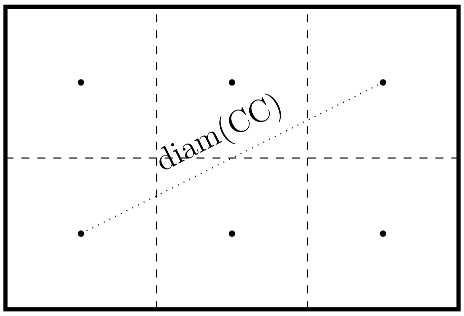


Figure 1: Example of a rectangular CC composed of 6 square FCs. The external faces of the CC are solid, the internal ones dashed, the dots denotes the barycenters of the FCs.

AR4 When agglomerating, CoMMA will always try to minimize the AR of a CC.

Notice that, in the context of usage U2, the AR does not have an absolute value: with this, we mean that the AR of CC is compared to those of other CC to find the optimal one. For this reason, practically, CoMMA may not use the definitions used below, but similar ones in which it avoids using computationally expensive operations such as roots. For instance, consider a 2D problem and suppose one wants to compare thethe perimeter and the area of a cell. In view of the non-dimensionality of the final results (remind AR2, instead of taking the square-root of the area, CoMMA rather takes the square of the perimeter.

2 Notation and definitions

We give here some definitions that will be serve us when describing the different ways of computing the AR. In what follows, we might mix a bit of terminology of 2- and 3D geometric concept in the hope that definitions are clearer.

Recall that CoMMA uses a graph-based description of the mesh. Each cell of the mesh has a weight, which corresponds to its measure, that is, its area (respectively, volume) in 2D (resp., 3D). Each cell is connected to other cells and the weights given to these connection are, in 2D (respectively, 3D) the length of the edge (resp., the area of the face) shared by the two cells. However, we also allow an *algebraic* description of the graph, where, hence, the all the weights are non-dimensional quantities.

We will denote the barycenter with $x^b_\cdot$, for instance for instance the barycenter of cell fc is x^b_{fc}. A CC is a set of several FC: $\mathrm{CC} = \{\mathrm{fc}_i\}_{i=1,2,\dots}$. The measure of a cell fc is denoted by vol(fc), hence the measure of a CC is simply:

$$\mathrm{vol}(\mathrm{CC}) \coloneqq \sum_{\mathrm{fc}\in\mathrm{CC}} \mathrm{vol}(\mathrm{fc}) \; . \tag{2}$$

For a CC, we use *external* faces in the usual sense of face; hence, the sum of the external weights ext(CC) corresponds to the "perimeter". On the other hand, an *internal* face is a face that is shared by two FCs belonging to the CC at hand; the internal weights int(CC) are thus the weights associated to internal faces. In the context of a CC, we denote *edge* the distance between the centers of any two FCs. The maximum edge is called *diameter*

$$\mathrm{diam}(\mathrm{CC}) \coloneqq \max_{\mathrm{fc}_i,\,\mathrm{fc}_j\in\mathrm{CC}} d(x^b_{\mathrm{fc}_i},\, x^b_{\mathrm{fc}_j}) \; . \tag{3}$$

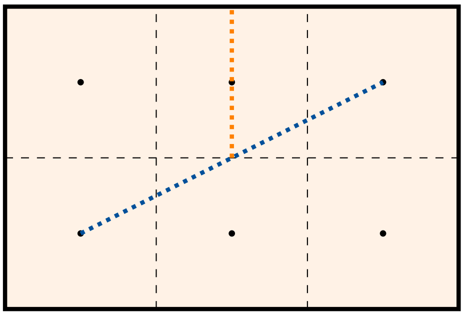
With *radius*, $\rho(\mathrm{CC})$ we define an approximation of the characteristic size of the cell computed from the measure:

$$\rho(\mathrm{CC}) \coloneqq \sqrt[d]{\mathrm{vol}(\mathrm{CC})}\,, \quad d = 2, 3\,. \tag{4}$$

3 AR definitions

We now give below the list of the types of AR available for usage U2 with graphical illustrations. Notice that in these pictures, we draw in blue (respectively, orange), the geometrical features that compose the numerator (resp., denominator) of the ratio used for the AR computation, see (1). The keys that the user should pass to CoMMA to choose a given AR computation are also added.

3.1 Diameter over radius

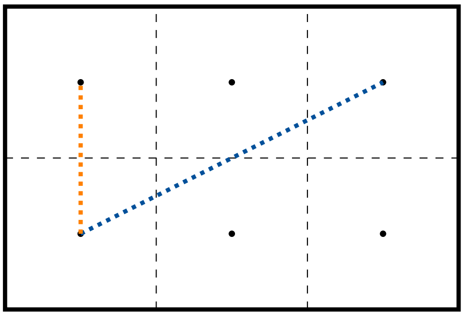


$$AR = \frac{\mathrm{diam}(\mathrm{CC})}{\rho(\mathrm{CC})} \tag{5}$$

This is the definition used by default in CoMMA from version 1.1 up to 1.3.2. It aims to see how a cell is stretched with respect to a sphere. It is a almost costless formula that approximate the typical AR definition. One drawback is that we divide by the radius, which is computed from the measure, hence larger FCs (higher measure) can be favored.

This AR can be selected with key `DIAMETER_OVER_RADIUS`.

3.2 Diameter over min edge

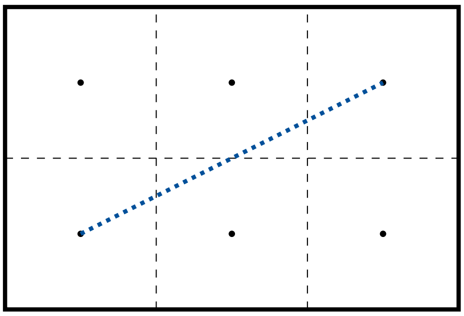


$$AR = \frac{\mathrm{diam}(\mathrm{CC})}{\min_{\mathrm{fc}_i,\,\mathrm{fc}_j \in \mathrm{CC}} d(x^b_{\mathrm{fc}_i},\, x^b_{\mathrm{fc}_j})} \tag{6}$$

Differently from the previous version, this version does not depend on the volume. However, often the minimum is not well approximated and does not change after that the second / third FC is agglomerated.

This AR can be selected with key `DIAMETER_OVER_MIN_EDGE`.

3.3 Diameter

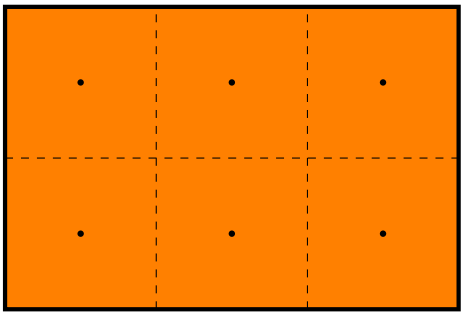


$$AR = \frac{\mathrm{diam}(\mathrm{CC})}{1} \tag{7}$$

We consider only the maximum edge, how much the cell is stretched, without taking into account its measure.

This AR can be selected with key `DIAMETER`.

3.4 Reciprocal of the measure

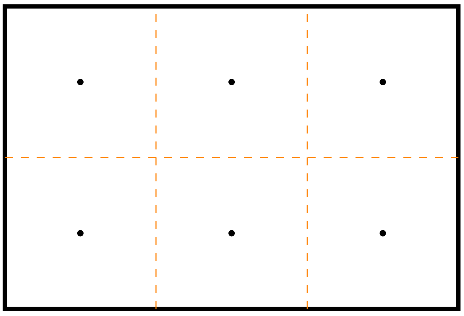


$$AR = \frac{1}{\mathrm{vol}(\mathrm{CC})} \tag{8}$$

Since we are considering the reciprocal, we prefer large CCs.

This AR can be selected with key `ONE_OVER_MEASURE`.

3.5 Reciprocal of the internal weights

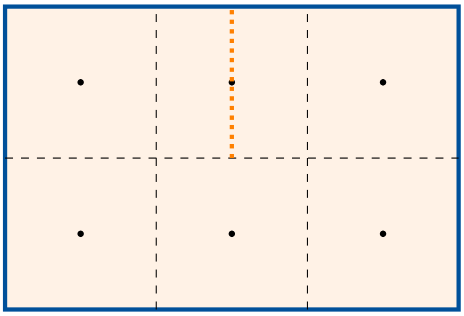


$$AR = \frac{1}{\mathrm{int}(\mathrm{CC})} \tag{9}$$

Since we are considering the reciprocal, we prefer CCs with high internal weights.

This AR can be selected with key `ONE_OVER_INTERNAL_WEIGHTS`.

3.6 Perimeter over radius

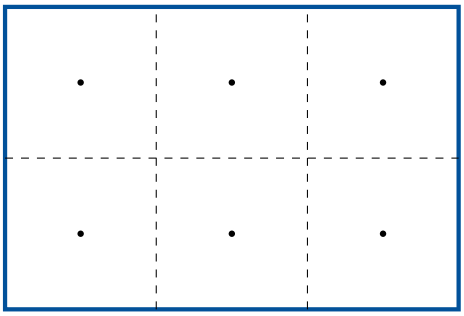


$$AR = \frac{\mathrm{ext}(\mathrm{CC})}{\rho(\mathrm{CC})} \tag{10}$$

The AR is the ratio between the perimeter and the radius (and not the surface in order to be non-dimensional). It is the definition used by CoMMA up to version 1.0.

This AR can be selected with key `PERIMETER_OVER_RADIUS`.

3.7 External weights, or perimeter, or min cut

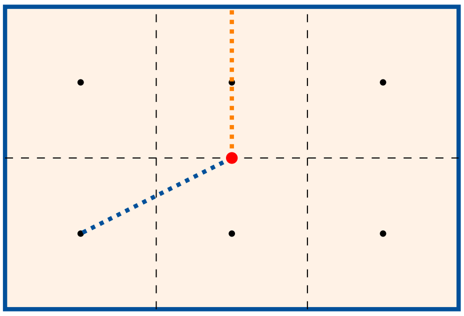


$$AR = \frac{\mathrm{ext}(\mathrm{CC})}{1} \tag{11}$$

With the current definition, CoMMA will try to minimize the perimeter of the CC: from a graph point of view, this corresponds to the so-called min-cut.

This AR can be selected with key `EXTERNAL_WEIGHTS`.

3.8 Max barycenter-based distance over radius

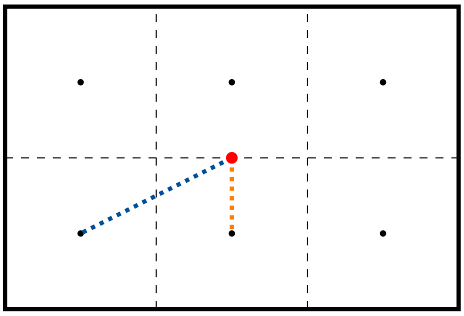


$$AR = \frac{\max_{\mathrm{fc}\in\mathrm{CC}} d(x^b_{\mathrm{CC}}, x^b_{\mathrm{fc}})}{\rho(\mathrm{CC})} \tag{12}$$

The barycenter of CC is computed (after each addition of a FC) as the weighted average of the centers of the FCs. Then, the distances between the CC barycenter and each FC barycenters are computed and maximum is selected. This can provide a good approximation of the classically-understood AR, however, it requires more computations than the other definitions.

This AR can be selected with key `MAX_BARY_DIST_OVER_RADIUS`.

3.9 Max over min barycenter-based distance

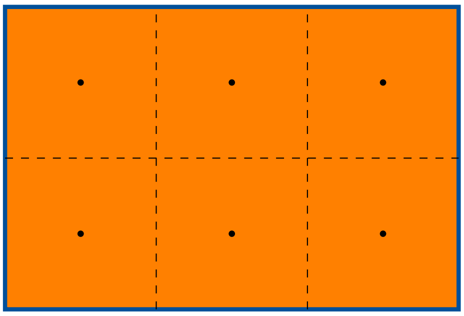


$$AR = \frac{\max_{\mathrm{fc}\in\mathrm{CC}} d(x^b_{\mathrm{CC}}, x^b_{\mathrm{fc}})}{\min_{\mathrm{fc}\in\mathrm{CC}} d(x^b_{\mathrm{CC}}, x^b_{\mathrm{fc}})} \tag{13}$$

The barycenter of CC is computed (after each addition of a FC) as the weighted average of the centers of the FCs. Then, the distances between the CC barycenter and each FC barycenters are computed and the minimum and maximum are selected. If the CC barycenter coincide with a FC one, this latter is not used in the computation of the min and max. This can provide a good approximation of the classically-understood AR, however, it requires more computations than the other definitions and it uses a minimum again which could be quite an awkward quantity to compute since, if there is a totally internal cell this will most likely be the argmin.

This AR can be selected with key `MAX_OVER_MIN_BARY_DIST`.

3.10 Algebraic perimeter-over-measure-like



$$AR = \frac{\mathrm{ext}(\mathrm{CC})}{\mathrm{vol}(\mathrm{CC})} \tag{14}$$

It is very similar to 3.6 above. However, here we suppose that it is not the we are using an algebraic definition of the graph, that this, the graph represent a matrix, rather than a mesh (in this case, CoMMA acts as an algebraic agglomerator rather than a geometric one).

This AR can be selected with key `ALGEBRAIC_PERIMETER_OVER_MEASURE`.