Homework 2

Reading: Meester, Sections 1.4, 1.5.

Idea Journal: Remember to submit the idea journal entries for each class. Focus on the essential ideas from class and the associated reading.

Problems/Exercises: All of these are due **Friday, January 26 by 5:00pm**.

1. Meester Exercise 1.7.10

2. Meester Exercise 1.7.21

3. A box contains five tickets labeled 1,2,3,4,5, which are otherwise identical. Ten people take turns drawing a ticket randomly from the box, replacing and mixing the tickets after each draw so that the draws are independent.

    (i) For $k = 1, \ldots, 5$, let A_k be the event that ticket k was not chosen by anyone. What is $\mathbb{P}(A_k)$?

    (ii) Are the events A_k and A_j independent if $k \neq j$? Justify your answer with a computation.

    (iii) Let B be the event that each of the 5 tickets is chosen by at least one person. Compute $\mathbb{P}(B)$. Hint: consider how B is related to the A_k events, and consider using Lemma 1.3.2.

4. There are 20 dice. 17 of them are fair dice, and 3 of them are trick dice. The "trick dice" are just like the fair dice except that the faces on the trick dice are labeled $1, 1, 2, 3, 4, 5$ (while the faces on the fair dice are $1, 2, 3, 4, 5, 6$, as usual). Imagine that you draw a die randomly from the box, and then you roll it. Let T be the event that you drew a trick die. Let R_k be the event that your rolled the number k. Compute and compare the quantities $P(T)$, $\mathbb{P}(T \mid R_5)$, and $\mathbb{P}(T \mid R_1)$.

5. A group of 20 people, 5 people are chosen randomly to receive a prize, and the rest get nothing. Let A be the event that Alonzo gets a prize. Let B be the event that Biji gets a prize. Are the events A and B independent? Justify your answer with a calculation.

6. A box contains 10 cards: 5 cards are red, 5 cards are blue. The red cards are labeled 1,2,3,4,5. The blue cards are also labeled 1,2,3,4,5. Consider the following game:

    Joe draws a card at random, **without** replacement. Then Kamala chooses randomly from among the remaining cards. Kamala wins if either (a) her card has a higher numerical value than Joe's or (b) the color of her card is different from Joe's card. If neither (a) nor (b) happen, then Joe wins.

    (i) What is the probability that Kamala wins?

    (ii) Suppose we are told that Kamala won and that she drew a Red 4. Given this information, what is the probability that Joe's card was blue?

1

7. Suppose you toss a fair coin 20 times. Write an expression for the probability that less than 4 heads were tossed.

8. Toss 2 fair coins, and consider the following events: $A =$ the event that the first coin landed heads. $B =$ the event that the second coin landed heads. $C =$ the event that the two coins landed the same way (either both heads or both tails). Are these events pairwise independent? Are they independent in the sense of Definition 1.5.1?

Extra, not required: There are lots of good problems in Section 1.7 of Meester. Choose some to practice! Often the hardest part is getting started; for each problem start by asking: What is this problem about? What concepts are important here? How should I go about solving this problem?