

# Math 221 Lec 16

## 4.4: Abstract linear transformations

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**Definition 1** (isomorphism).  $T : V \mapsto W$  is an **isomorphism** if  $T$  is bijective.

**Lemma 2.**  $T$  is an isomorphism iff  $\exists T^{-1} : W \mapsto V$  with  $T \circ T^{-1} = id_W \forall \mathbf{w} \in W$  and  $T^{-1} \circ T = id_V \forall \mathbf{v} \in V$

**Proposition 3.**  $T : V \mapsto W$  is an isomorphism iff  $T(\text{basis for } V)$  is a basis for  $W$ .

**Corollary 4.**  $\dim V = n$  iff  $V$  is isomorphic to  $\mathbb{R}^n$  (there's an isomorphism from  $V$  to  $\mathbb{R}^n$ . The isomorphism is that you choose a basis for  $V$  and send the basis vectors of  $\mathbb{R}^n$  to it.