Math 340 HW 4

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1. Meester 2.3.28
Prove that Markov's inequality follows from theorem 2.3.5

Proof. Theorem 2.3.25 states that for a positive-valued r.v. Y and $b > 0$,

$$\mathbb{P}(Y \geq b) \leq \frac{1}{b}\mathbb{E}[Y] \tag{1}$$

Assume $Y = |X|^k$ for a positive-valued r.v. X and $b = a^k$ □

Then

$$\mathbb{P}(|X|^k \geq a^k) \leq \frac{1}{a^k}\mathbb{E}[|X|^k]$$

And since $|X|^k \geq a^k \Leftrightarrow |X| \geq a$,

$$\mathbb{P}(|X| \geq a) = \frac{1}{a^k}\mathbb{E}[|X|^k] \tag{2}$$

Prove that Chebyshev's inequality follows from theorem 2.3.5

Proof. Theorem 2.3.25 states that for a positive-valued r.v. Y and $b > 0$,

$$\mathbb{P}(Y \geq b) \leq \frac{1}{b}\mathbb{E}[Y] \tag{3}$$

Assume $Y = \mathrm{Var}(X)$ for a positive-valued r.v. X and $b = a^2$ Then

$$\mathbb{P}(\mathrm{Var}(X) \geq a^2) \leq \frac{1}{a^2}\mathrm{Var}(X) \tag{4}$$

Integrating the definition of Var(X) and noting that $\forall m, m^2 = |m|^2$, we find

$$\mathbb{P}((X - \mathbb{E}[X])^2 \geq a^2) = \mathbb{P}(|X - \mathbb{E}[X]|^2 \geq a^2) \leq \frac{1}{a^2}\mathrm{Var}(X) \tag{5}$$

And once again, since for any event A, $\mathbb{P}(A)^2 \geq q^2 \Leftrightarrow \mathbb{P}(A) \geq q$

$$\mathbb{P}(|X - \mathbb{E}[X]| \geq a) \leq \frac{1}{a^2}\mathrm{Var}(X) \tag{6}$$

□