Homework 3

Idea Journal: Remember to submit the idea journal entries for each class. Focus on the essential ideas from class and the associated reading.

Reading: Meester: Section 1.6, Section 2.1..

Problems/Exercises: All of these are due **Friday, February 2 by 5:00pm**.

1. At a certain 911 dispatch center, calls arrive at an average rate of 6 calls per hour, but the exact arrival time is random.

   (i) Suppose we model this as follows: during each of the 30-second intervals (dividing the hour into 120 intervals), a single call will arrive with probability p, or with probabililty $(1-p)$ no call will arrive during that interval. We assume that the arrival or non-arrival of the calls in different intervals are independent. Assume $p = 0.05$. Using this model, what is the probability that there are less than 3 calls in a given one hour period? Write an exact expression for this.

   (ii) Alternatively, suppose we model the number of arrivals in a single hour in this way: we assume the outcome space is $\Omega = \{0, 1, 2, 3, \dots\}$, where an outcome ω represents the number of calls that arrive in a single hour. We then use the probability mass function for the Poisson distribution (e.g. Example 1.5.13 in Meester) with approriate parameter λ. What should you choose for λ, to be consistent with the model in part (i)? Using this model based on Poisson approximation, what is the probability that there are less than 3 calls in a given one hour period?

2. This one is relatively easy, but it is essential that you have a good understanding of it. In your answer, explain precisely what is the the outcome space, and what are the events described (in terms of this outcome space)...Toss a p-coin repeatedly (independent tosses).

   (i) What is the probability that after n tosses you have not seen heads (i.e. you get n tails in a row)?

   (ii) What is the probability that you toss $n-1$ tails and then heads occurs for the first time on the n^{th} toss?

3. Suppose there are M coins, which are biased in different ways. The j^{th} coin lands heads with probability p_j. Let us assume $0 < p_1 < p_2 < \cdots < p_M < 1$. The coins are otherwise indistinguishable. Pick one of the M coins randomly from a box (you don't know how it is biased) and then toss it n times.

   (i) Let B_1 and B_2 be the events that your chosen coin lands heads on the 1^{st} and 2^{nd} tosses, respectively. Are these events independent? Justify.

(ii) Suppose your coin lands heads k times (out of the n tosses). What is the probability that your coin is the j^{th} coin (the one that lands heads with probability p_j)?

(iii) Suppose your coin lands heads n times in a row (no tails). You suspect it is the most biased coin (i.e. with largest $p = p_M$). How large would n have to be in order that the probability of it being the most biased coin is at least ten times as large as the probability of it being the next most biased (i.e. the one with $p = p_{M-1}$)?

4. Meester 1.7.36.

5. Meester 1.7.38

6. Meester 1.7.23

7. Meester 2.1.15: prove Lemma 2.1.14(b).

8. Suppose you are going to toss a fair coin 1000 times. Considering the estimate (1.5) in the proof of Theorem 1.6.1 (and in the notes on law of large numbers), what is the smallest value of m (as guaranteed by the estimate) such that the probability of tossing less than m heads is at least 0.99?