One can think of linear algebra as an algebraic way to think about geometric concepts.

Definition 1 (vector)**.** A **vector** in $\mathbb{R}^N$ is a point $\mathbf{x} = (x_1, ..., x_n) \in \mathbb{R}^N$ or the line segment going from the origin to that point.

Definition 2 (vector magnitude/length)**.** The **magnitude** or **length** of a vector $\mathbf{x} = (x_1, ..., x_n)$ is $\sqrt{x_1^2 + x_2^2 + ...x_n^2}$

Definition 3 (unit vector)**.** A unit vector is a vector $\mathbf{v}$ s.t. $\|v\| = 1$

insert proof

Definition 4 (vector addition)**.** The **sum** of $\mathbf{x}$ and $\mathbf{y}$ in $\mathbb{R}^N$ is $\mathbf{x} + \mathbf{y} = (x_1 + y_1, ..., x_n + y_n)$

**Heading operations on vectors **

Definition 5 (scalar multiplication)**.** If $c \in \mathbb{R}, \mathbf{v} \in \mathbb{R}^n$,

$$c\mathbf{v} = (cx_1, cx_2, \ldots, cx_n) \tag{1}$$

$$\|c\mathbf{v}\| = c\sqrt{x_1^2 + x_2^2 + \ldots x_n^2} = |c| \cdot \|\mathbf{v}\| \tag{2}$$

Definition 6 (vector addition)**.** if $\mathbf{v} = (x_1, \ldots, x_n), \mathbf{w} = (y_1, \ldots y_n)$

$$\mathbf{v} + \mathbf{w} = (x_1 + y_1, x_2 + y_2, \ldots x_n + y_n) \tag{3}$$

Remark. Parallelogram rule: $0, \mathbf{v}, \mathbf{w}$, and $\mathbf{v}$+$\mathbf{w}$ form a parallelogram

Definition 7 (vector subtraction)**.** **v-w** is the vector that produces $\mathbf{v}$ when added to $\mathbf{w}$. For **v, w** $\in \mathbb{R}^n$,

$$\mathbf{v\text{-}w} = (x_1 - y_1, \ldots, x_n - y_n) \tag{4}$$

Remark. **v-w** starts at $\mathbf{w}$ and goes toward $\mathbf{v}$

Definition 8 (midpoint)**.** The **midpoint** of two points can be found by taking the average of their components. In two dimensions, the midpoint vecotr of points $\mathbf{v}$ and $\mathbf{w}$ is

$$\frac{\mathbf{v}+\mathbf{w}}{2} \tag{5}$$

We can also think of this as $\mathbf{w} + \frac{\mathbf{v\text{-}w}}{2} = \frac{\mathbf{v}+\mathbf{w}}{2}$. I.e. traveling along $\mathbf{w}$ then halfway along the vector connecting $\mathbf{v}$ and $\mathbf{w}$.

Theorem 9 (Diagonals of a parallelogram bisect each other)**.**

Proof. We can prove the diagonals bisect each other by showing their midpoints are equal. We showed above that the diagonal formed by $\mathbf{v\text{-}w}$ has midpoint $\frac{\mathbf{v}+\mathbf{w}}{2}$. The other diagonal runs from $\mathbf{0}$ to $\mathbf{v}+\mathbf{w}$. This is the vector $\mathbf{v}+\mathbf{w}$ which also clearly has midpoint $\frac{v+w}{2}$ Since the midpoints of the diagonals are equal, they bisect each other. □

Theorem 10 (Medians of a triangle intersect at a point 2/3 the way from each vertex to its opposite side)**.**

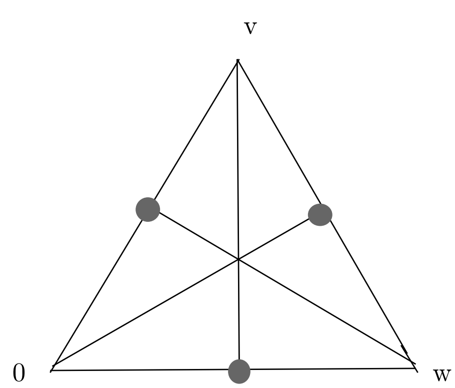

Figure 1: medians

Proof. For each point, we want to calculate the point 2/3 along its corresponding median.
One median is $\mathbf{w}/2 - \mathbf{v}$. 2/3 of that median is $2/3(\mathbf{w}/2 - \mathbf{v}) = \mathbf{v}/3 + \mathbf{w}/3$.
Another median is $\mathbf{v/2} - \mathbf{w}$. 2/3 of that median is $2/3(\mathbf{v}/2 - \mathbf{w}) = \mathbf{v}/3 + \mathbf{w}/3$.
The final median is $\mathbf{v}+\mathbf{w}/2$. 2/3 of that median is $2/3(\mathbf{v}+\mathbf{w}/2) = \mathbf{v}/3 + \mathbf{w}/3$
The same cooordinate lies 2/3 the way down each median.

□