# Math 222 Lec 3

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### epsilon-delta proofs of limits

Steps for $\varepsilon - \delta$ proof:

1. Begin "given $\varepsilon > 0$, choose $\delta = \dots$"
2. Suppose $\|$**x-a**$\| < \delta$
3. Check that $\|\mathbf{f(x)} - L\| < \varepsilon$ **
   (a) In doing so, try to change the LHS to be some multiple of $\delta$. Then conculde $c\delta = \varepsilon$ satisfies our requirements for $\delta$
   (b) if in evaluating $\|$**f(x) - L**$\|$, we end up with some multiple of our $\delta$ bound times a factor involving $x$ (call it $\beta$), we need to take an add'l step
   (c) Note that $\delta = min(1, ValueWeSolveFor)$. Then $\|$**x - a**$\| < 1$ and $-1 < \|$**x-a**$\| < 1$. Manipulate this inequality so that the $x - a$ term resembles $\beta$. Then note that $\beta < 1$... Plug that back into **, thus concluding that $\delta = min(1, d\varepsilon$ for some $d$, which means $\delta \leq d\varepsilon$.

Tips for $\varepsilon - \delta$ proof: Remember:

1. Triangle inequality: $\|\mathbf{a}+\mathbf{b}\| \leq \|\mathbf{a}\| + \|\mathbf{b}\|$
2. Any term under a radical will alwasy be positive. Applyign this will often let us simplify inequalities by striking out radical terms.

### Computing limits

Tips:

1. l'hospital's.
   (a) $\lim_{x\to 0} = \frac{\sin x}{x} = 1$
2. Can replace complicated terms like $xy$ in $\lim_{x\to 0} \frac{\cos xy}{xy}$, yielding $\lim_{x\to 0} \frac{\cos t}{t}$
3. If don't see a way to simplify the limit, try checking whether it doesn't exist. Plug in approaches from various paths.

**Definition 1** (Differentiable). $f : U \subset \mathbb{R}^n \mapsto \mathbb{R}^m$ is **differentiable** at $\vec{x_0}$ if

1. All partials of $f$ exist at $\vec{x_0}$
2. The tangent plane at $x_0$ provides a linear approximation of $f(\vec{x_0})$

$$\lim_{x\to x0} \frac{\|f(\vec{x} - (f(\vec{x_0}) + Df(x_0)(x - x_0))\|}{x - x_0 = 0}$$

**Definition 2** ($C^1$). Let $f : U \subset \mathbb{R}^n \mapsto \mathbb{R}^m$. $f \in C^1$ iff the partials of $f$ exist and are **continuous** in a neighbordhoos of $\vec{x_0}$.

**Remark.** $f \in C^1$ (is continuous and has continuous partials) $\implies$ $f$ is differentiable $\implies$ $f \in C^0$ (is continuous)