Homework 7

Reading: Meester: Intermezzo (I), Chapter 5.

Idea Journal: Remember to submit the idea journal entries after each class.

Problems/Exercises: All of these are due **Friday, March 22 by 5:00pm**.

1. Let S_n be a simple random walk on the integers, starting from $S_0 = 0$. For which value of $\alpha > 0$ are both of the following true?

$$\lim_{n\to\infty} \mathbb{P}\left(|S_n| < n^r\right) = 0, \quad \text{if } r < \alpha,$$

   and

$$\lim_{n\to\infty} \mathbb{P}\left(|S_n| < n^r\right) = 1, \quad \text{if } r > \alpha.$$

   Justify your answer. Hint: think about the central limit theorem.

2. The Postal Service must estimate the weight that will be loaded into a truck containing flat-rate boxes. Each truck has the volume to hold 2500 flat-rate boxes (all boxes have the same dimensions). Let us assume that the weights of the flat-rate boxes (which depend on what the customers are mailing) are independent and identically distributed random variables, each having a mean of 2 lbs and standard deviation of 2 lbs. Estimate the probability that the total weight of boxes in a fully-loaded truck will be between 4850 and 5150 lbs. Express your answer in terms of the function $\Phi(x)$:

$$\Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}}\, dy$$

   .

3. Suppose $X \sim \text{Exponential}(\lambda)$ for some $\lambda > 0$. What is the distribution of $Y = cX$, where $c > 0$ is some constant?

4. Is the following statement True or False? *If $f(x)$ is the density for a continuously distributed random variable, then $0 \le f(x) \le 1$ must hold for all x.* Explain your answer.

5. Suppose that X is uniformly distributed on the interval $[2, 4]$.
   (i) What is the density for X?
   (ii) What is the CDF for X?
   (iii) What are the CDF and density for the random variable $Y = X^2 + 1$?
   (iv) What are the mean and variance of Y?

6. Suppose X is continuously distributed on $\mathbb{R}$ with density

$$f(x) = \begin{cases} cx^2, & x \in [-1, 1] \\ 0, & |x| > 1 \end{cases} \tag{0.1}$$

for some $c > 0$. What is c? Using this, compute $\mathbb{P}(X > \frac{1}{2})$.

7. Suppose X is a random variable that is continuously distributed on $\mathbb{R}$, having CDF $F(x) = \mathbb{P}(X \leq x)$. Assume F is increasing, so that the inverse F^{-1} is well-defined in the usual way. What is the distribution of the random variable $Y = F(X)$? Justify your answer.