Math 221 Lec 7 (2.1/2.2)

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1 Matrix operations

Proof tips: If proving something about a combined matrix AB, it may help to just focus on a single column of it, $(AB)_i$. For example, when proving $A + A')B = AB + A'B$, we can just consider column b of B. $(A + A')b$ is the b-th column of $(A + A')B$, so what we prove for it goes for every column of that matrix.

Remark. The jth column of AB is the product of A with the jth column vector of B.

Proposition 1. Let A and A' be $m \times n$ matrices, let B and B' be $n \times p$ matrices, let C be a $p \times q$ matrix, and let c be a scalar. Then

1. $AI_m = A = I_m A$
2. $(A + A')B = AB + A'B$ distributive property of matrix mult over matrix addition
3. $(cA)B = c(AB) = A(cB)$
4. (AB)C = A(BC) (associative property of matrix multiplication)

2 Linear transformations

Remark. Matrices can only represent a certain type of function: linear transformations.

Definition 2 (linear transformation)**.** A function $f : \mathbb{R}^n \mapsto \mathbb{R}^m$ is called a linear transformation if it satisfies

1. $f(c \cdot \mathbf{x}) = cf(\mathbf{x}$ for $x \in \mathbb{R}^n$ and $c \in \mathbb{R}$
2. $f(x + y) = f(x) + f(y) \forall x, y \in \mathbb{R}^n$

Remark. Geometrically, a linear transformation must keep the origin fixed in place and ensure that all lines remain lines.

Theorem 3 (matrices as linear transformations)**.** A function $f : \mathbb{R}^m \mapsto \mathbb{R}^n$ is represented by an $m \times n$ matrix A iff f is a linear transformation.

Remark. The theorem above means that every linear transformation can be represented by a matrix. And every matrix represents a linear transofmration.

Proof. Yeah, I'll type this up when I have more time. $\square$