Math 340: Lec 14 Big Ideas Journal (Discrete joint, marginal, and conditional distributions)

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Definition of distributions

Definition 1 (joint distribution)**.** Let X and Y be two discrete random variables. Their joint distribution is

$$\mathbb{P}(X = k, Y = j) \text{ for } k \in R(X), j \in R(Y)$$

Remark. The joint distribution can be considered a mass function with domain in the 2D-plane.

Remark. If X, Y are independent, $\mathbb{P}(X = k, Y = j) = \mathbb{P}(X = k)\mathbb{P}(Y = j)$, and we say that the joint probability distribution has a product structure.

Definition 2 (marginal distribution)**.** Let X be a discrete random variable. The marginal distribution of X is $\mathbb{P}(X = k)$.

Remark. Even if a set of three random variables have the same marginal distributions, they may have different joint distributions if one of the random variables is dependent on another.

Definition 3 (Conditional distribution)**.** Let X and Y be discrete random variables. A conditional distribution looks like $\mathbb{P}(X = k|Y = j)$. The conditional distribution is given by:

$$\mathbb{P}(X = k|Y = j) = \frac{\mathbb{P}(X = k, Y = j}{\mathbb{P}(Y = j)}$$

Moving between distributions

Remark. We can find the marginal distribution $\mathbb{P}(X = k|Y = j)$ given the joint probability distribution $\mathbb{P}(X = k, Y = j)$ by normalizing the joint probabilities of X and Y for a given $Y = j$. We can use the same process to find $\mathbb{P}(Y = j|X = k)$.

Remark. We can relate the joint and marginal distributions using the partition rule. In particular, if we know the joint distributions of X and some some other r.v. Y, we can recover the marginal distribution of X (and/or Y). But if we know the marginal distribution of X, we cannot recover the joint distribution of X and Y without add'l info.

Example (Calculating expected value of the marginal distribution using the conditional)**.** By the partition rule,

$$\mathbb{E}[X] = \sum_{j \in R(Y)} \mathbb{E}[X|Y = j]\mathbb{P}(Y = j)$$

This lets us calculate $\mathbb{E}[X]$ when it's difficult to think directly about X but easier to think about $X|Y$. And note

$$\mathbb{E}[X|Y = j] = \sum_{k \in R(X)} k\mathbb{P}(X = k|Y = j)$$