Math 340: Lec 2 Big Ideas Journal

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January 16, 2024

The probability mass function

Assuming Ω is finite, we can define a probability mass function $\rho : \Omega \mapsto [0, 1]$ that assigns probabilities to the events containing each $\omega \in \Omega$. $\mathbb{P}(A) \subset \Omega$, then, is $\sum_{\omega \in A} \rho(\omega)$ since $A = \bigcup_{k=1}^{\infty} \{\omega_k\}$

Equally likely outcomes

Asssuming Ω is finite and the probability measure / probabililty mass function are defined such that all outcomes are equally likely, $\mathbb{P}(A) = \frac{|A|}{|\Omega|}$ for any $A \subset \Omega$.

Combinatorics experiments

Note: for combinatorics problems, we can define Ω in a way that best suits the problem at hand. For example, if we run an experiment where we toss a coin n times, but we only care about the the first toss in each outcome, we could define $\Omega = \{H, T\}$ instead of $\Omega = \{H, T\}^N$.
When setting up problems, remember that we should embed our assumptions within the probability measure. See example below

Example (Tossing a fair coin)**.** Imagine we toss a fair coin n times. Then let $\Omega = \{H, T\}^N$. Since we have assumed we have a fair coin, all outcomes are equally likely. Thus $\rho(\omega) = 1/|\Omega|$. We rely on this assumption to define $\mathbb{P}(A)$. $A = \bigcup_{\omega \in A} \omega$, so $\mathbb{P}(A) = \sum_{i=1}^{|A|} \rho(\omega_i) = |A| * \rho(\omega_i) = \frac{|A|}{|\Omega|}$

n choose k

In combinatorics problems, we often want to find the number of ways to choose k things from a collection of n things. We do so by calculating $\binom{n}{k}$.

$$\binom{n}{k} = \frac{n!}{(n-k)!k!}$$

This quantity represents the number of ways to permute n things ($n!$) divided by the number of ways to arrange the $n - k$ items not selected (since we don't care about them), divided again by the number of ways to arrange the k items we did select (since we're finding the number of unordered subsets).

Example (Chance of drawing 5 cards and getting a full house hand)**.** A full house consists of 3 cards of one rank and 2 cards of a second rank. To find the number of full house combinations, we note that we can initiate our full house hand with a card from any of the 13 ranks for the trio. Given a rank, there are $\binom{4}{3}$ ways to select a trio of cards from that rank. There are then 12 choices for the second rank, and $\binom{4}{2}$ ways to select a pair of cards of that rank. The total number of combinations of full houses is thus $13 * 12 * \binom{4}{3} * \binom{4}{2}$.

To find the probability of drawing a full house, we calculate $\mathbb{P}(FullHouse)/|\Omega|$. Ω is the set of all combinatgions of 5 cards. There are $\binom{52}{5}$ ways to select 5 cards from a full deck, so

$$\mathbb{P}(FullHouse) = \frac{\binom{13}{1} * \binom{4}{3} * \binom{12}{1} * \binom{4}{2}}{\binom{52}{5}} \approx 0.00144$$