Joint probability distribution

Given two random variable X and Y defined on the same sample space S, the joint probability distribution is the corresponding probability distribution on all possible pair of outcomes.

If outcomes of $X = \{x_1, x_2, \ldots, x_m\}$
and outcomes of $Y = \{y_1, y_2, \ldots, y_n\}$
then joint probability distribution provides probability of all pairs (x_i, y_j), $P(x_i, y_j)$ for all i, j

Def^n The joint cummulative function for X and Y is $F_{X,Y}(x, y) = P(X \le x \text{ and } Y \le y)$

$\underline{\text{Def}^n}$ The joint probability mass function of X and Y is $p_{X,Y}(x,y) = P(X=x, Y=y)$ (discrete)

and the joint probability density function of X and Y is denoted by $f_{X,Y}(x,y)$.

Example

Consider tossing of two fair coins. Let $\underline{A}$ and $\underline{B}$ are associated random variables. If a coin flips "head" then associated r.v. takes the value 1, and it takes the value 0 otherwise.

* all possible outcomes are $\underline{(A=0, B=0)}$, $\underline{(A=0, B=1)}$, $\underline{(1,0)}$, $\underline{(1,1)}$

* the joint probability mass function:

$$P(A,B) = \frac{1}{4} \quad \text{for } A \in \{0,1\}, \; B \in \{0,1\}$$
$$= 0 \quad \text{otherwise}$$

Def^n

For two random variable X and Y, the marginal distribution function of X, denoted by $P(x)$, defined by

$$P(x) = \sum_{y} P(X = x, Y = y) \quad \left(\begin{array}{c}\text{discrete} \\ \text{case}\end{array}\right)$$

and $$P(x) = \int_{y=-\infty}^{\infty} P(x,y)\, dy \quad \left(\begin{array}{c}\text{continuous} \\ \text{case}\end{array}\right)$$

$P(y)$ = similar to $P(x)$

Def^n The conditional density function of X given Y, denoted by $P(X/Y)$ or $P_{X/Y}$, is defined by

$$P(X/Y) = \frac{P(x,y)}{P(y)} \quad ($$

Let $P(x)$ be the marginal distribution function of X, $\int_{-\infty}^{\infty} P(x)\, dx = 1 \Rightarrow \int_{-\infty}^{\infty} \left(\int_{-\infty}^{\infty} p(x,y)\, dy \right) dx = 1$

and $p(x,y) \geq 0$

and $p(x,y) = 0$

Example X and Y two random variable with joint distribution

$P(x=-1, y=0) = \frac{1}{3}$, $P(x=0, y=1) = \frac{1}{3}$, $P(x=1, y=1) = \frac{1}{3}$;

$P(x,y) = 0$ otherwise.

(a) find marginal destribution of X and Y?

(b) find conditional distribution of X given $y=1$?

X/Y	0	1	Total
-1	1/3	0	1/3
0	0	1/3	1/3
1	0	1/3	1/3
Total	1/3	2/3	

Ans (a) the marginal distribution function of X

$$P(x) = \sum_{y} P(x,y)$$

x	-1	0	1
$p(x)$	$1/3$	$\frac{1}{3}$	$\frac{1}{3}$

$$p(y) = \sum_{x} p(x, y)$$

y	0	1
$p(y)$	$1/3$	$\frac{2}{3}$

(b) $$P(X/Y=1) = \frac{P(x, 1)}{P(Y=1)}$$

$X/Y=1$	-1	0	1
$P(X/Y=1)$	$\frac{P(-1,1)}{2/3} = 0$	$\frac{P(0,1)}{2/3} = \frac{1/3}{2/3} = \frac{1}{2}$	$\frac{1}{2}$