Elastic behavior

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THAPAR INSTITUTE
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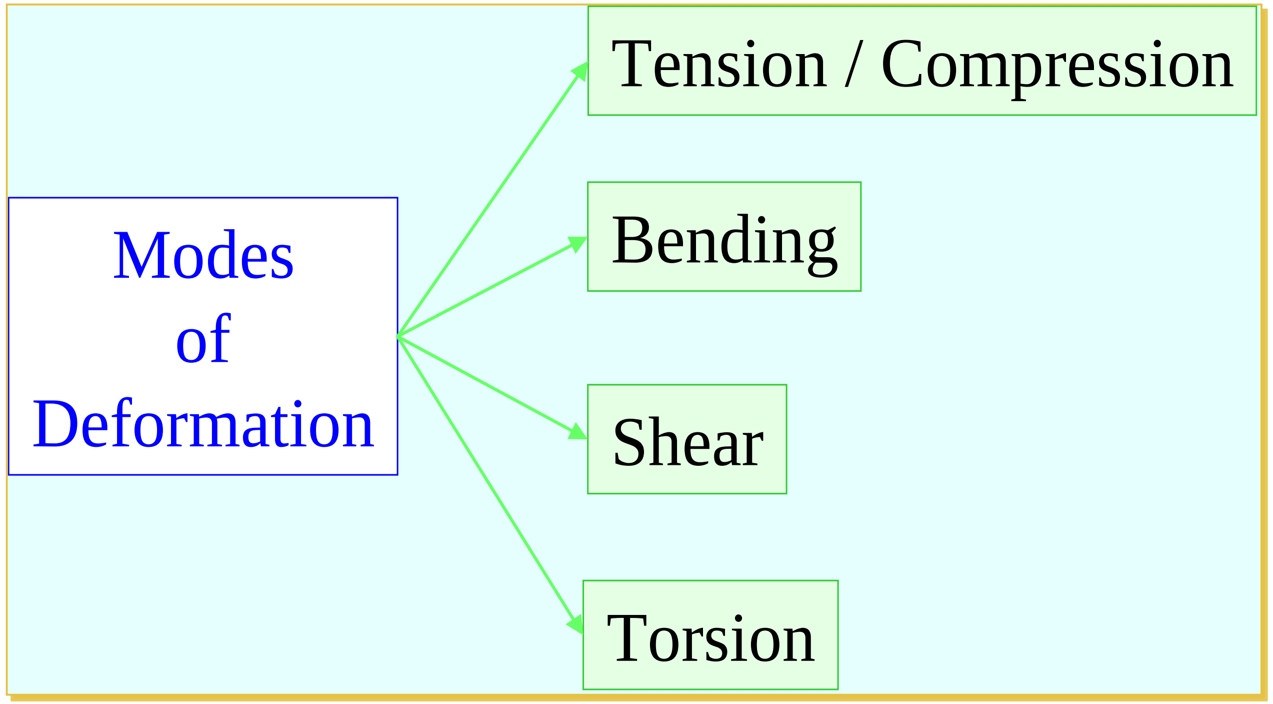
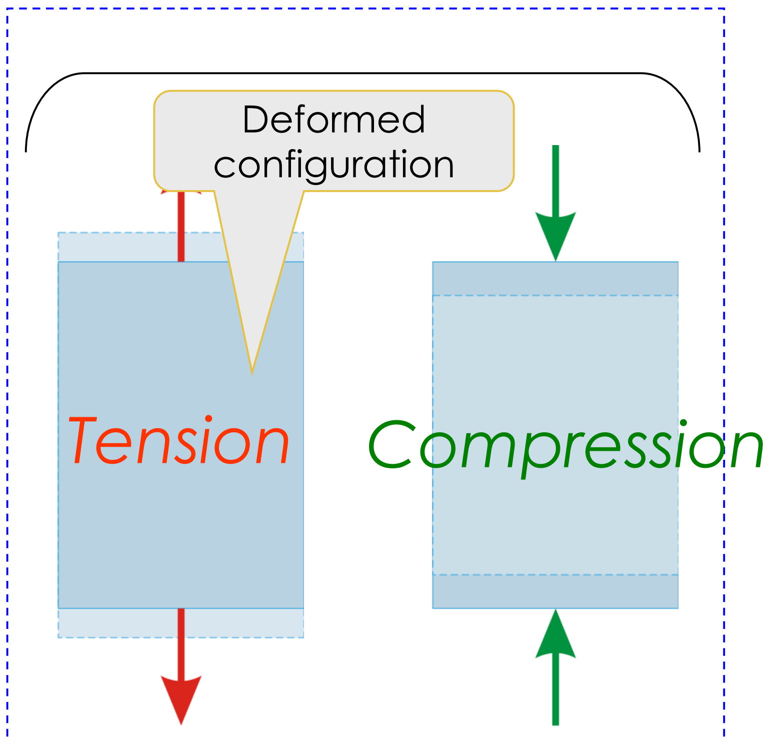
Syllabus

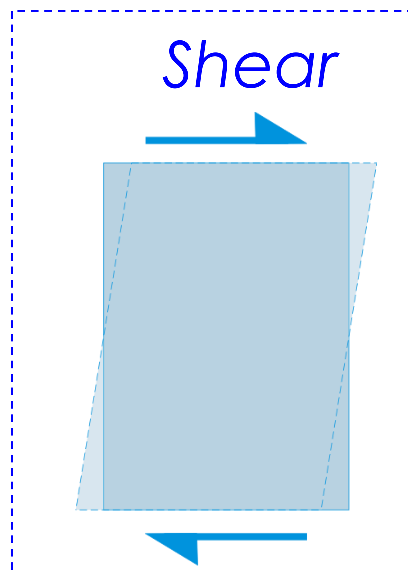
Mechanical properties of materials: Elastic, Anelastic and Viscoelastic behavior, Engineering stress and engineering strain relationship, True stress - true strain relationship, review of mechanical properties, Plastic deformation by twinning and slip, Movement of dislocations, Critical shear stress, Strengthening mechanism, and Creep.

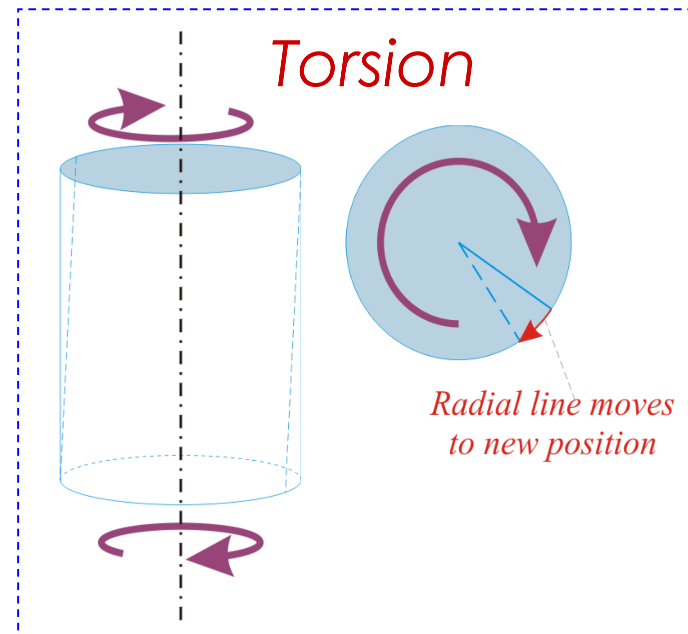
CLO: Distinguish between elastic and plastic behavior of materials.

Common types of deformation testing

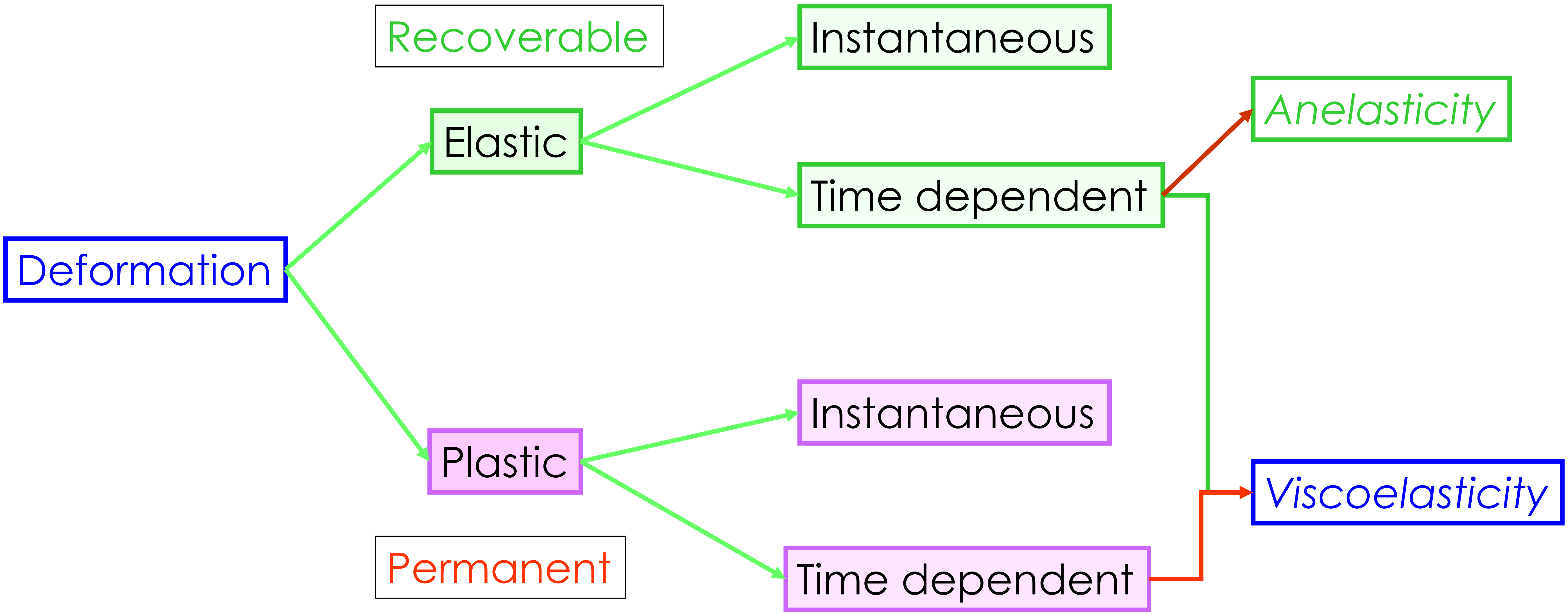




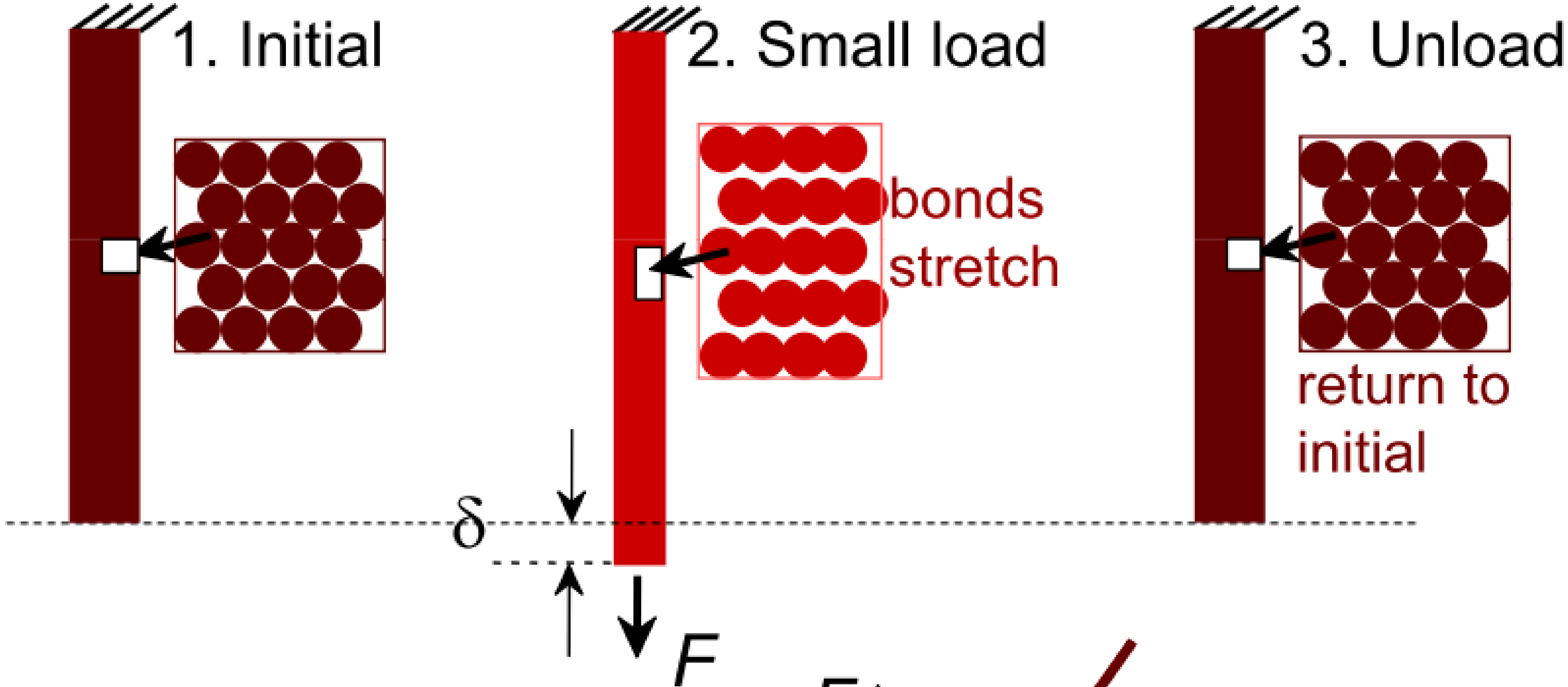


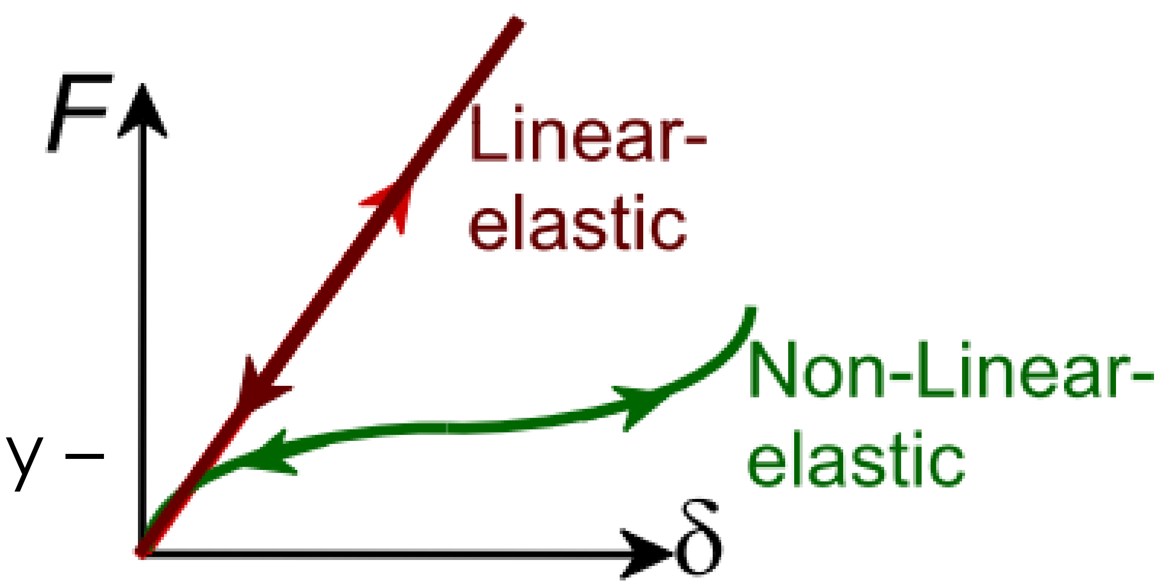


Types of deformation



Elastic Deformation





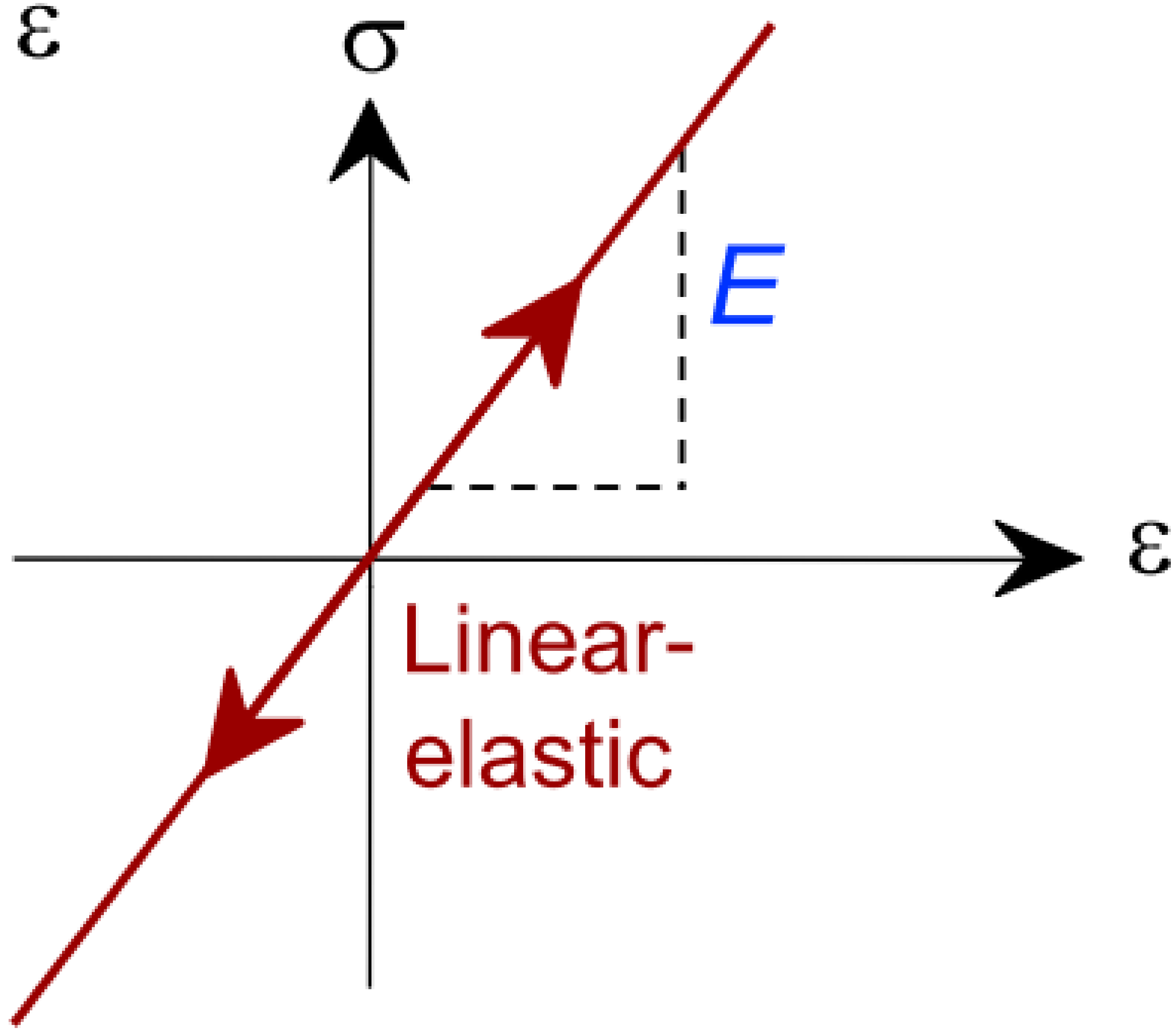
Elastic means reversible!

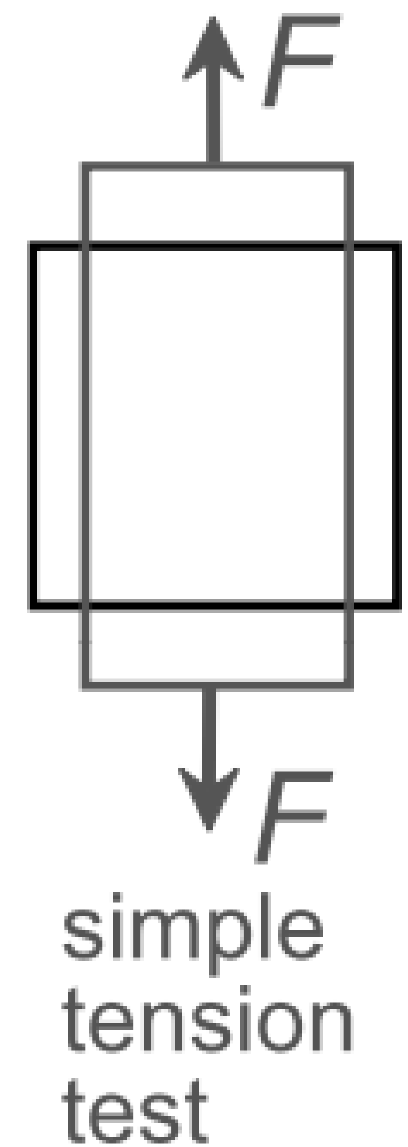
Material may deform temporarily – Elastic deformation

Youngs Modulus

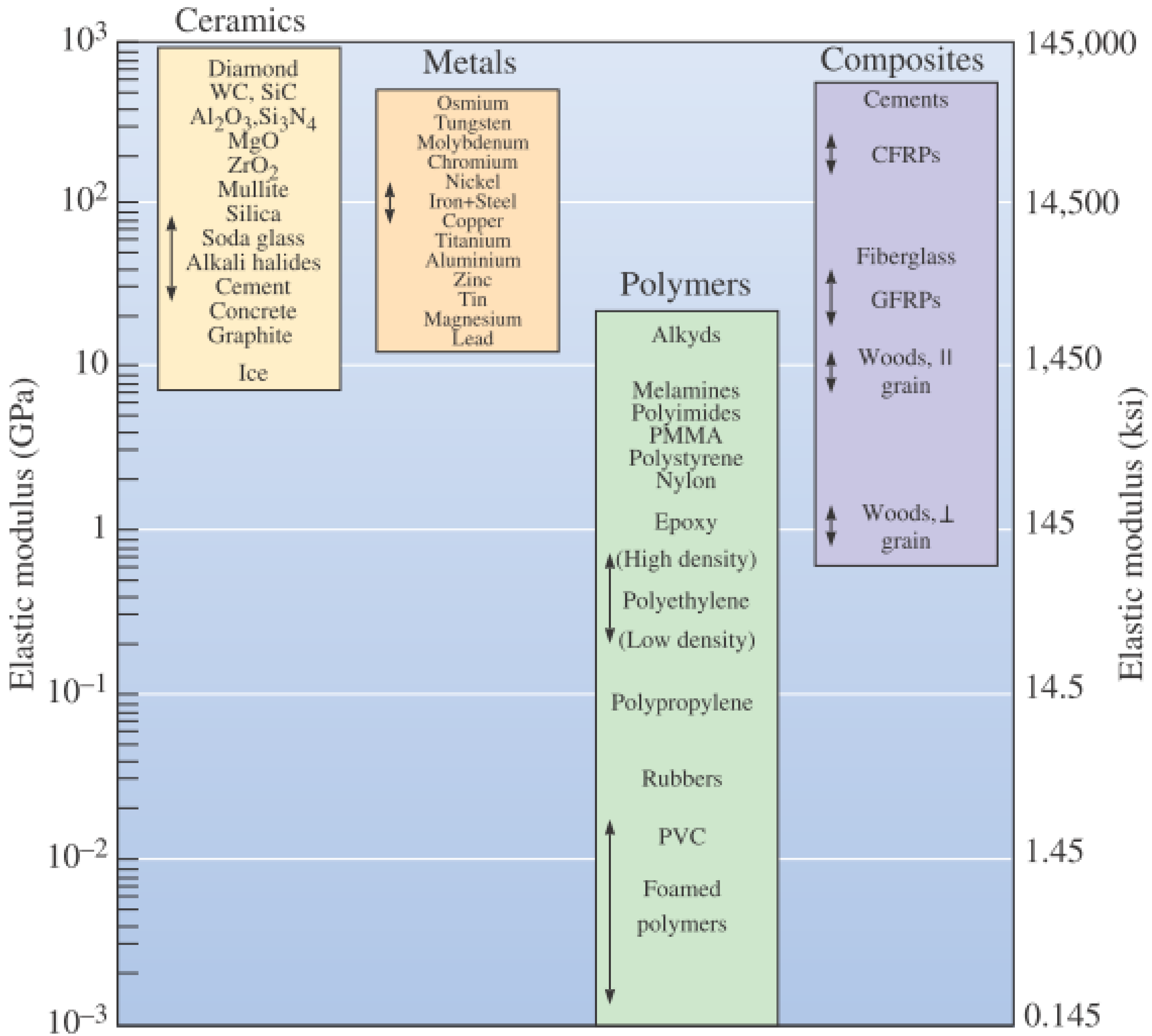
- **Hooke's Law**:

$$\sigma = E\,\varepsilon$$

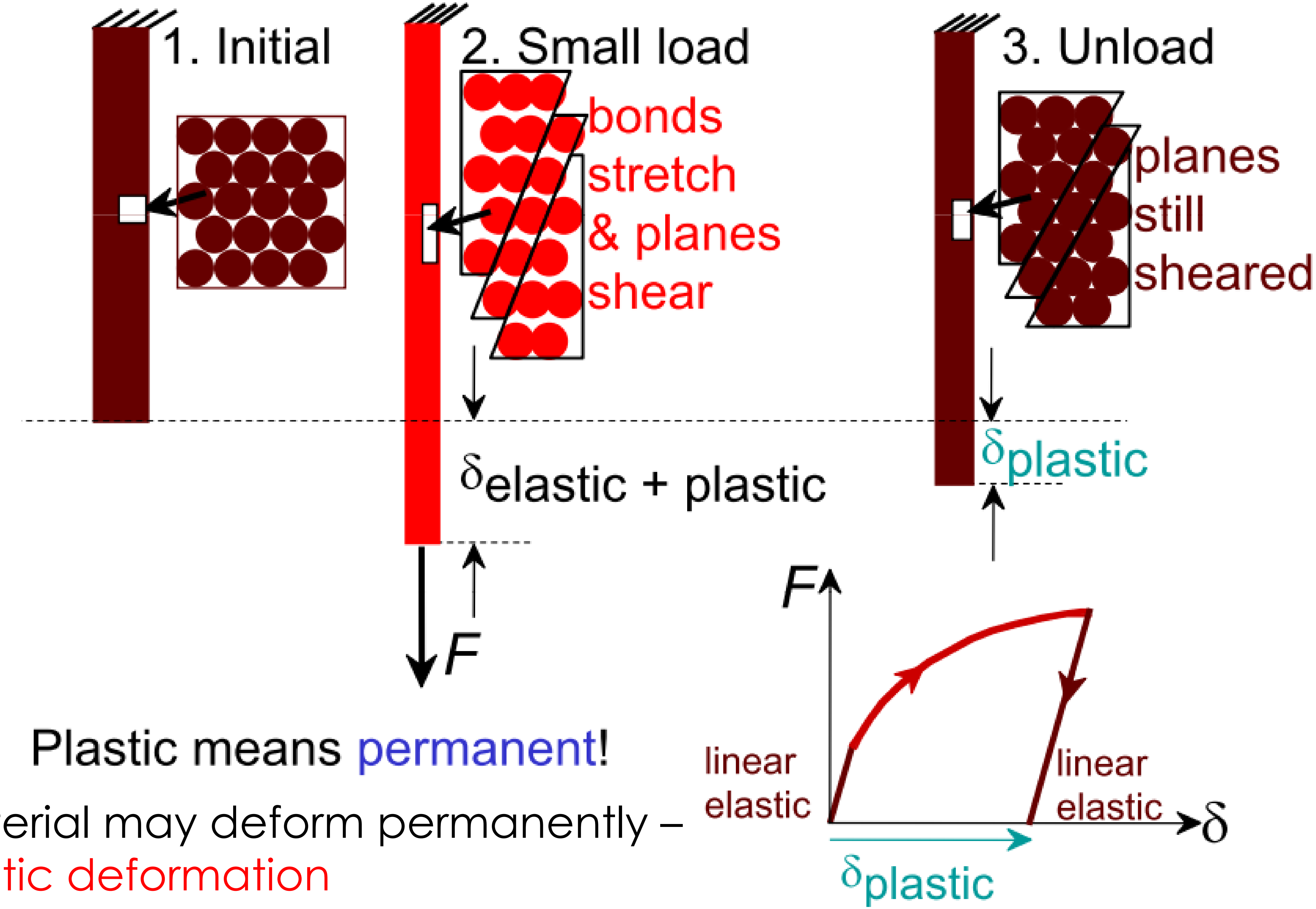




Young's Moduli of different materials



Plastic Deformation



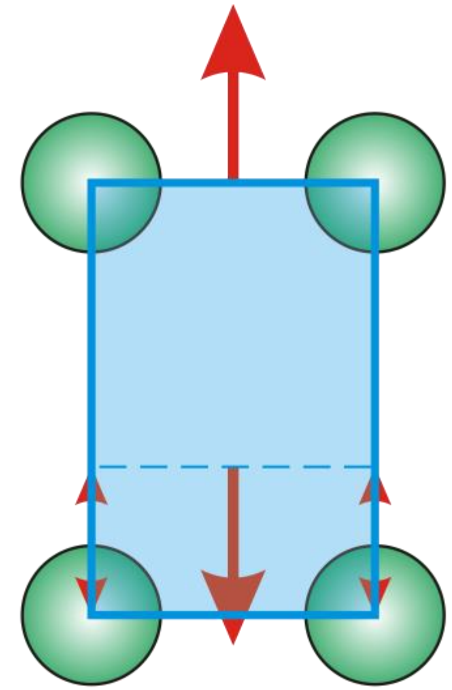
Plastic means permanent!

Material may deform permanently – plastic deformation

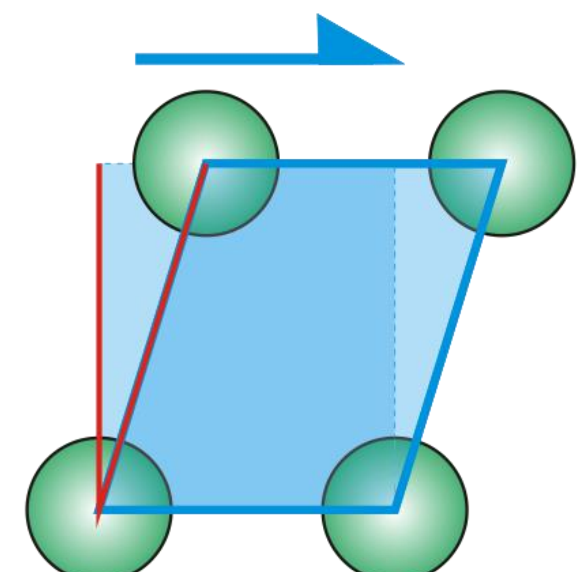
Deformation

At a more fundamental (material) level there are only two types of deformations

1. Tension/compression ➔ wherein bond length is increased/decreased
2. Shear ➔ where bond angle is distorted



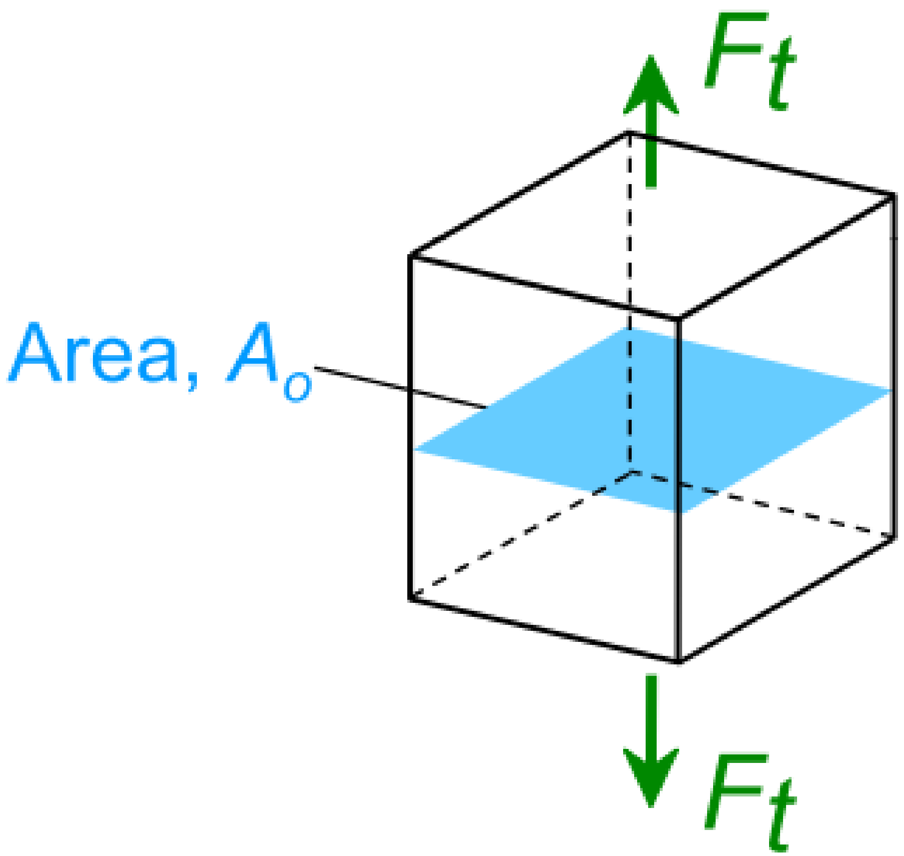
Change in bond length



Change in bond angle

Some Definitions

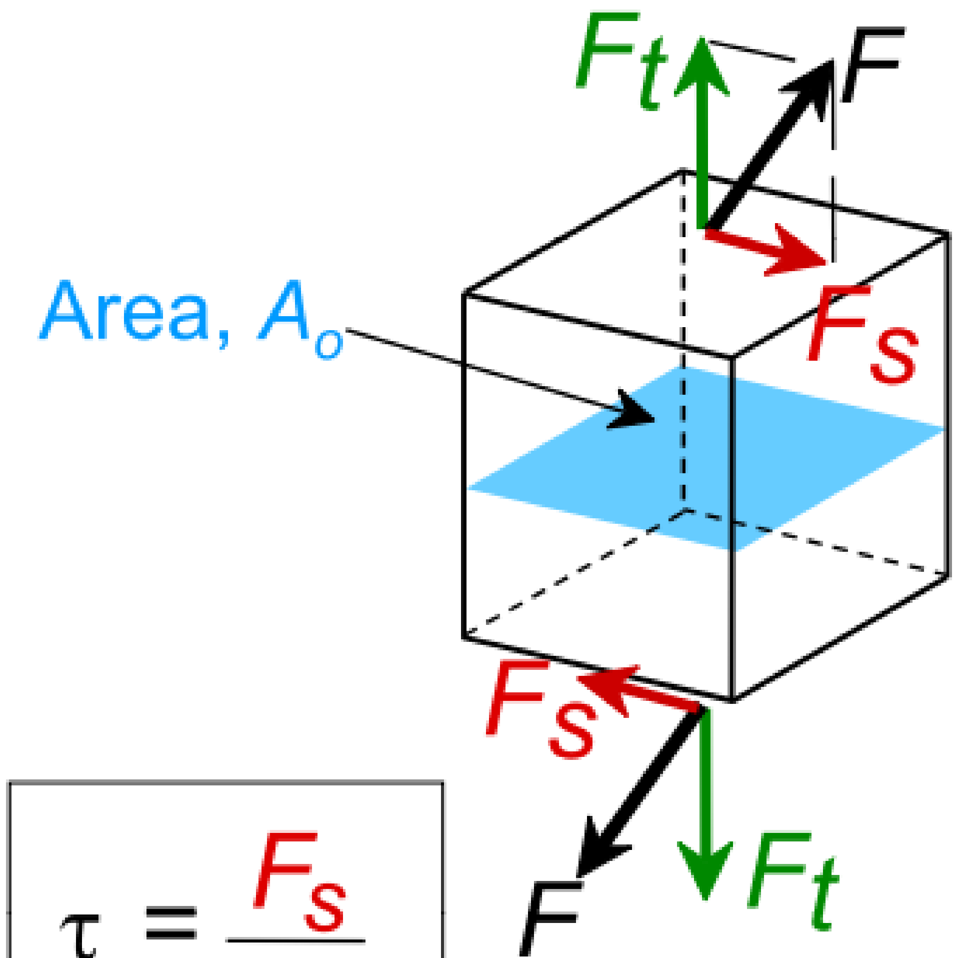
- Tensile stress, σ:



$$\sigma = \frac{F_t}{A_o} = \frac{lb_f}{in^2} \text{ or } \frac{N}{m^2}$$

original area before loading

- Shear stress, τ:



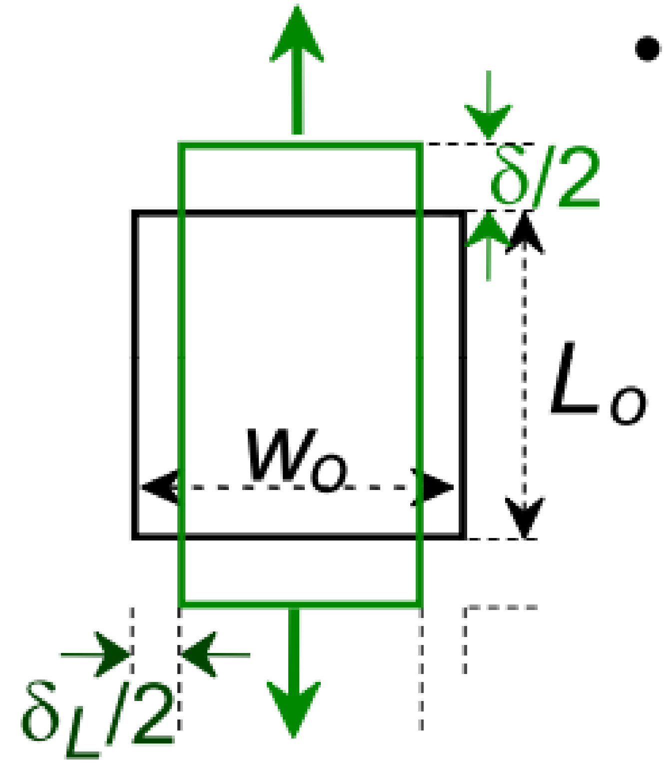
$$\tau = \frac{F_s}{A_o}$$

$\therefore$ Stress has units: N/m^2 or lb_f/in^2

Some Definitions

- **Tensile strain:**

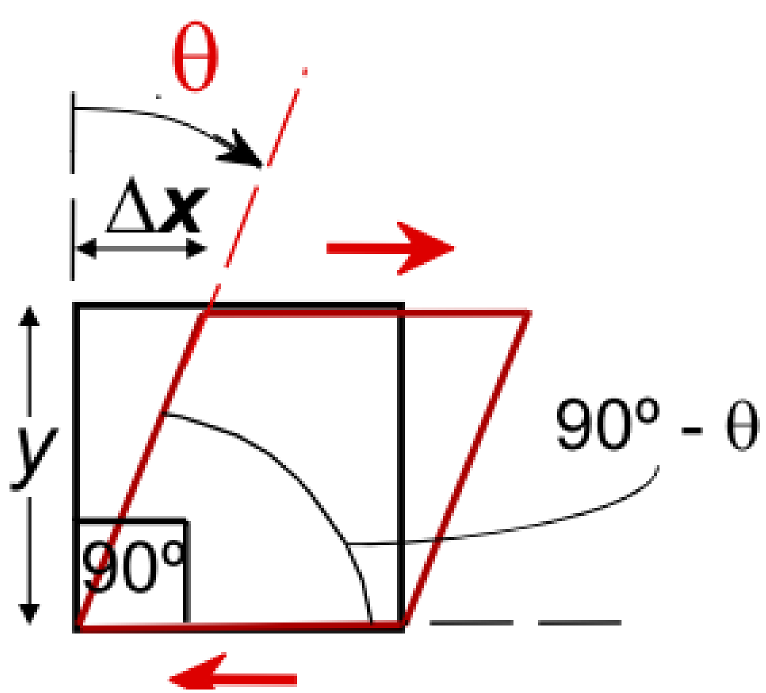
$$\varepsilon = \frac{\delta}{L_o}$$



- **Lateral strain:**

$$\varepsilon_L = \frac{-\delta_L}{w_o}$$

- **Shear strain:**



$$\gamma = \Delta x / y = \tan\theta$$

Strain is always dimensionless.

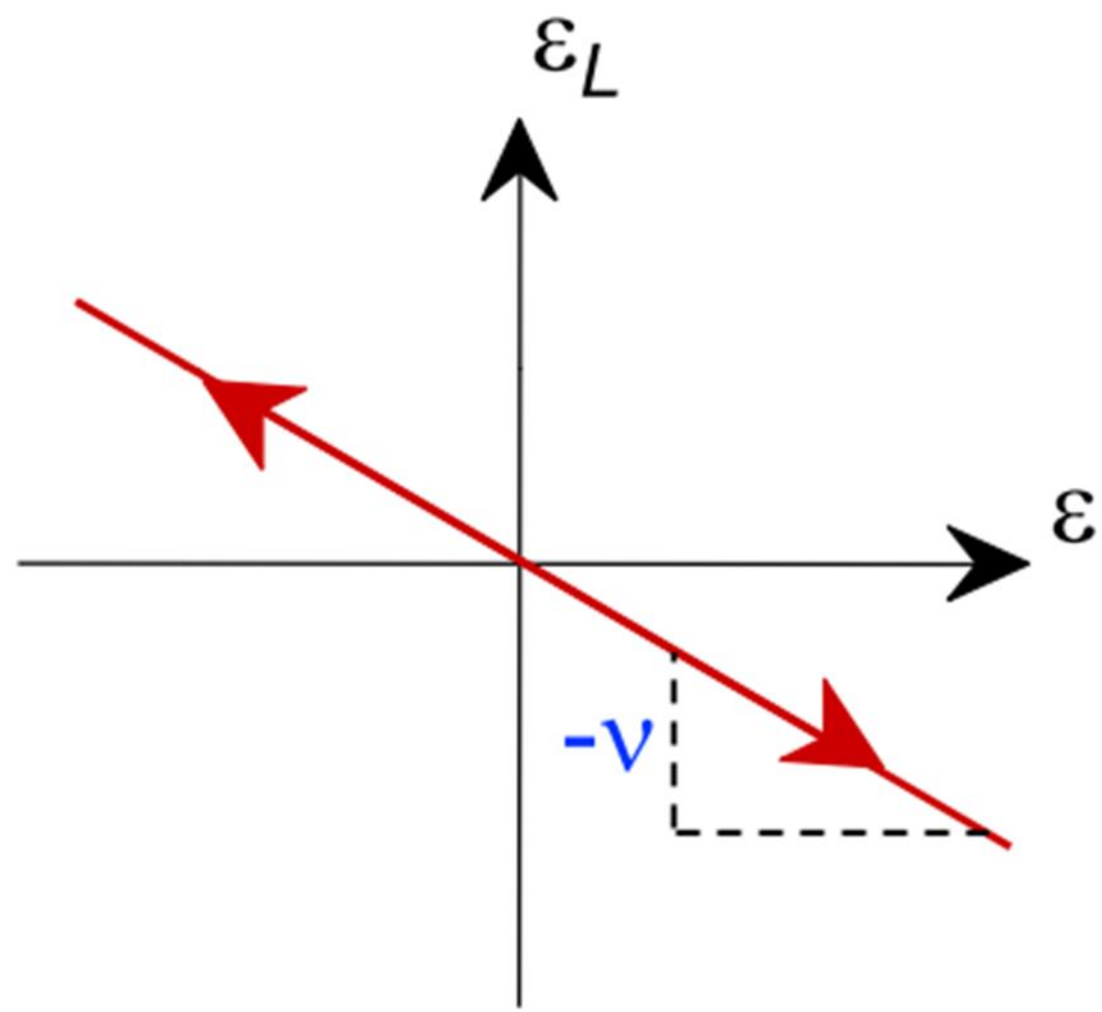
Some Definitions

Shear Modulus

$$\mu = \frac{\textit{shear stress}}{\textit{shear strain}} = \frac{\tau}{\gamma}$$

Poisson's ratio

$$\nu = -\frac{\varepsilon_L}{\varepsilon}$$



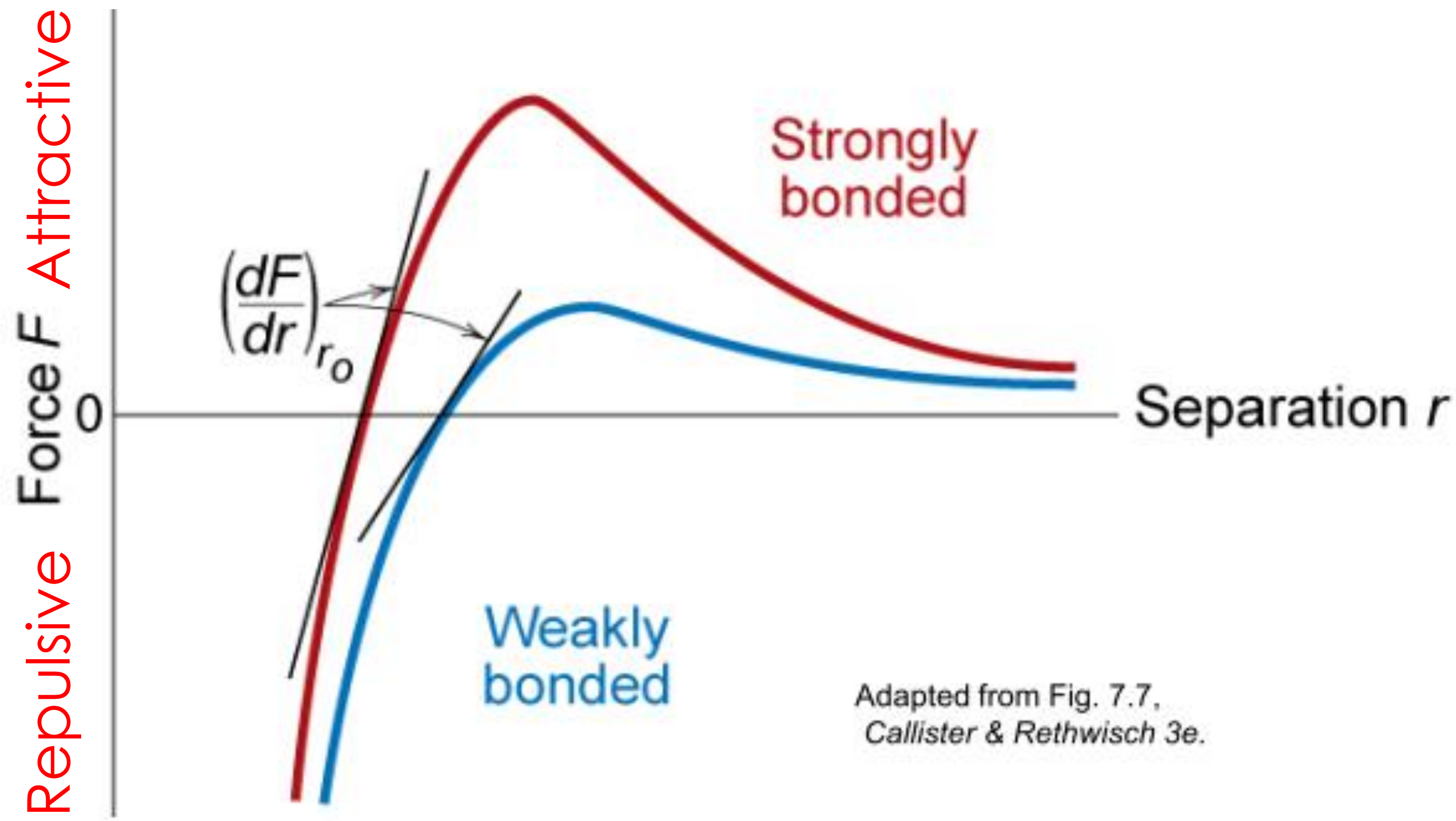
metals: $\nu \sim 0.33$
ceramics: $\nu \sim 0.25$
polymers: $\nu \sim 0.40$

Units:
E: [GPa] or [psi]
ν: dimensionless

$\nu > 0.50$ density increases

$\nu < 0.50$ density decreases (voids form)

Youngs Modulus and bonding

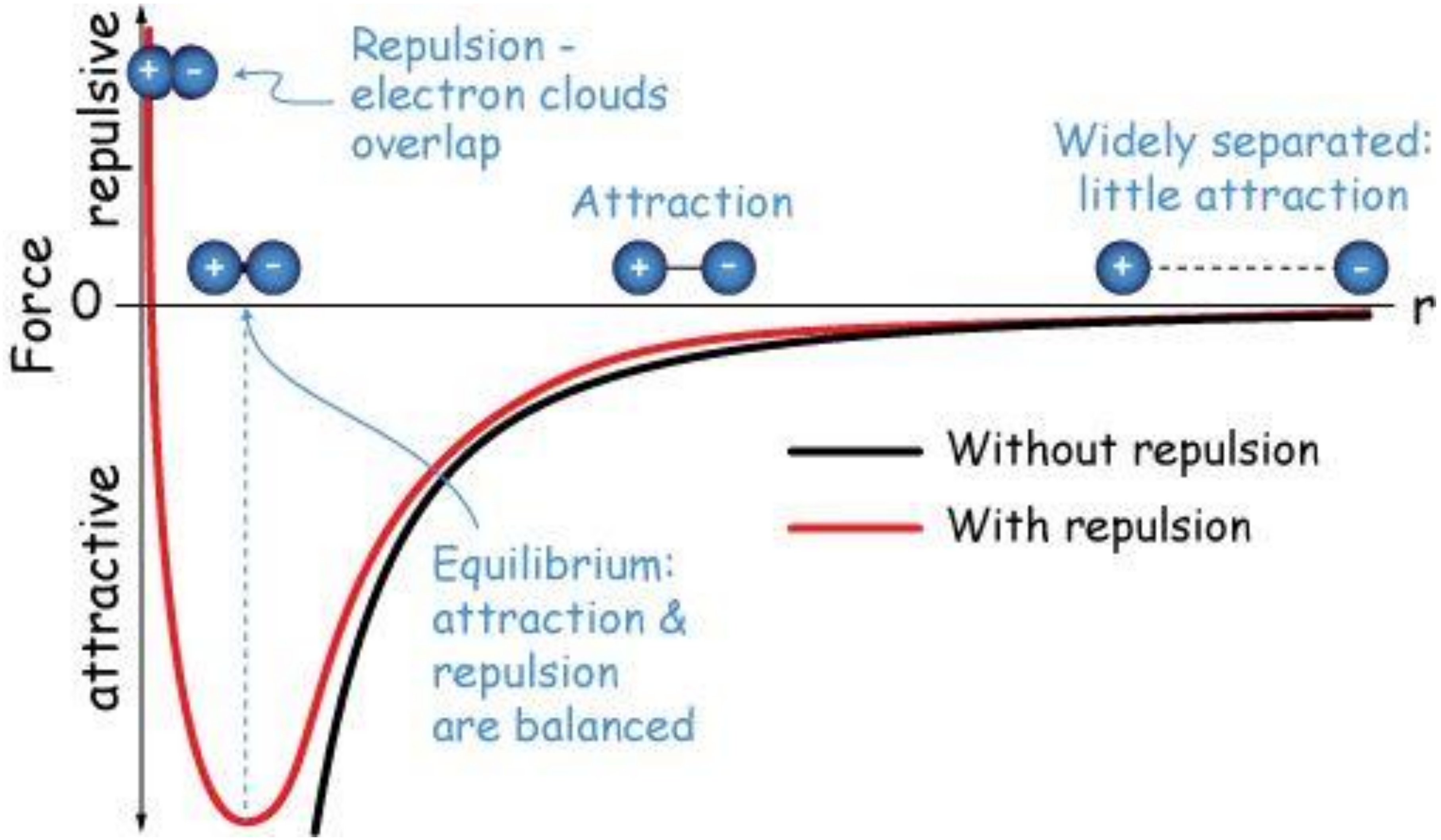


Elastic deformation is due to elongation of bonds.

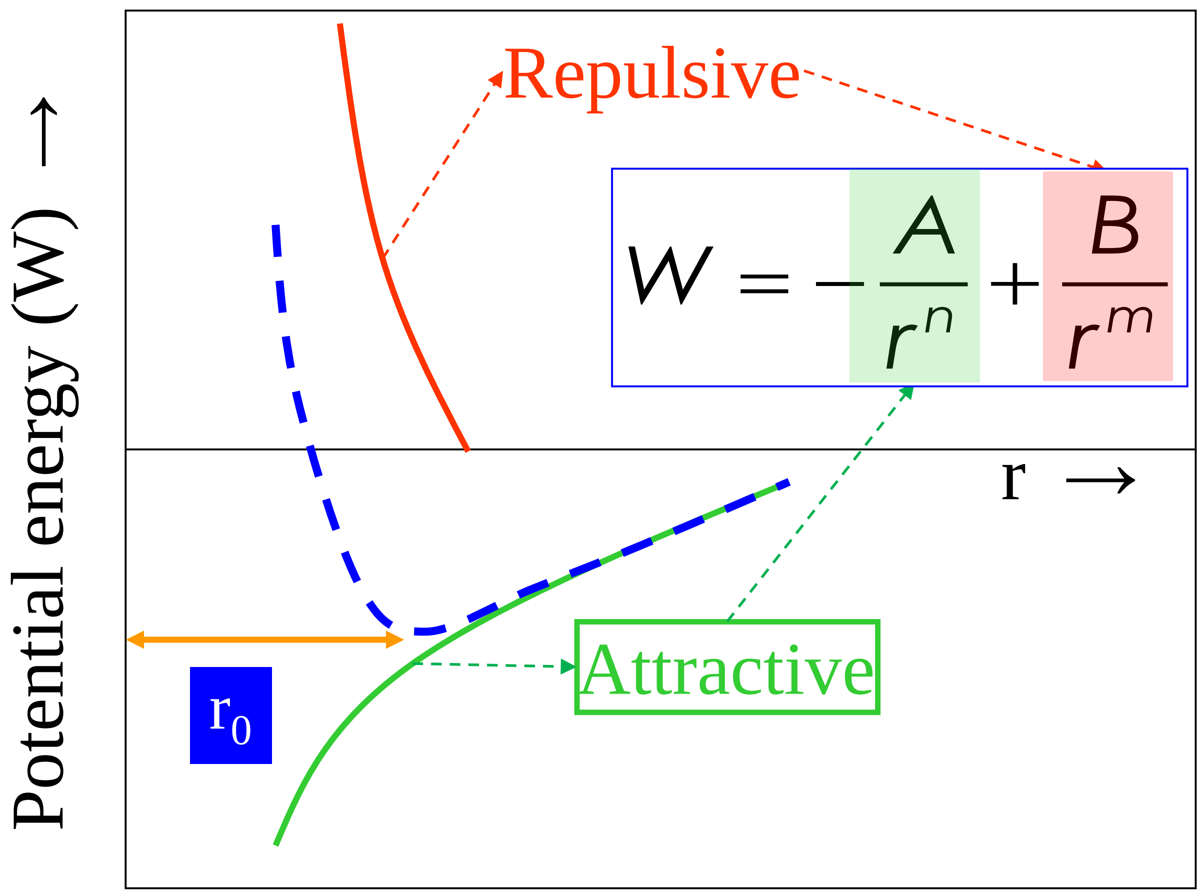
- Instantaneous (negligible time delay)
- Stronger the interatomic bond, higher the Young's modulus

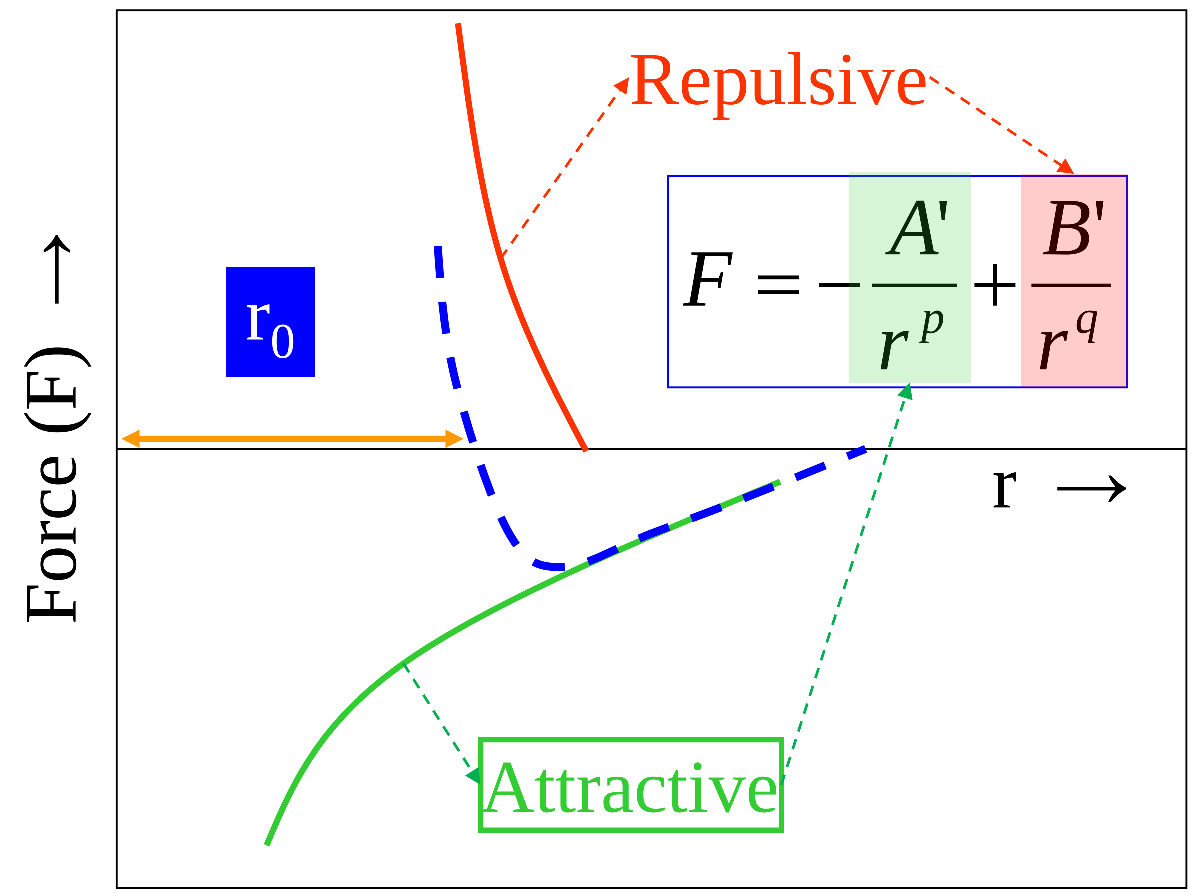
Can we have a model to describe this?

Some common definitions



Atomic model for elasticity





r_0 → Equilibrium separation between atoms

Atomic model for elasticity

$$W = -\frac{A}{r^n} + \frac{B}{r^m}$$

$$F = -\frac{dW}{dr} = 0 \text{ at } r_0$$

$$F_{app} = -F = \frac{dW}{dr}$$

Tensile or compressive

$$\boldsymbol{Y \propto -\frac{dF}{dr} = \frac{d^2W}{d^2r}}$$

Curvature of force – distance curve

$$Y = \frac{\text{Force/area}}{\text{change in length/length}} \approx \frac{dF/r_0^2}{dr/r_0} = \frac{1}{r_0}\frac{d^2W}{d^2r}$$

Youngs modulus and Bonding

1. Materials with strong bonds have a deep potential energy well with a high curvature ➔ high elastic modulus.
2. Along the period of a periodic table the covalent character of the bond and its strength increase ➔ systematic increase in elastic modulus.
3. Down a period the covalent character of the bonding ↓ ➔ ↓ in Y.

Along the period →	Li	Be	B	$C_{diamond}$	$C_{graphite}$
Atomic number (Z)	3	4	5	6	6
Young's Modulus (GN / m^2)	11.5	289	440	1140	8

Down the row →	$C_{diamond}$	Si	Ge	Sn	Pb
Atomic number (Z)	6	14	32	50	82
Young's Modulus (GN / m^2)	1140	103	99	52	16

Modulus as a design parameter

What is the criteria ??

In many applications high modulus in conjunction with good ductility should be chosen (good ductility avoids catastrophic failure in case of accidental overloading).

What types of materials we can choose

METALS

POLYMERS

CERAMICS

COVALENT MATERIALS

Modulus as a design parameter

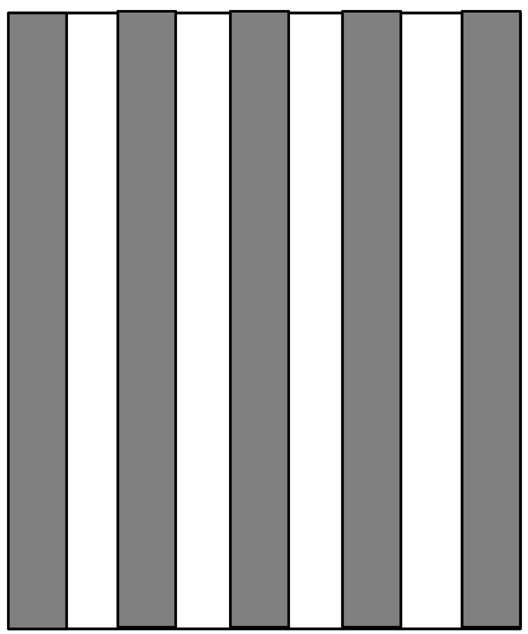
1. Metals have(i) reasonable elastic modulus, (ii) good ductility, (iii) ability to be alloyed to give good combination of properties, (iv) amenable to many types of fabrication techniques (casting, forging, extrusion, etc.).Metals of the First transition series posses a good combination of ductility & modulus (200 GPa). Second & third transition series have an even higher modulus, but their higher density is a shortcoming.
2. Polymers are light weight but they have a low modulus dependent and cannot withstand high temperatures. They have a poor wear resistance as well.
3. Covalently bonded materials (e.g. diamond has a high E (1140 GPa)) in spite of their high modulus are rarely used in engineering applications as they are brittle (poor tolerance to cracks).
4. Ionic solids have high Youngs modulus but are very brittle.

Modulus as a design parameter

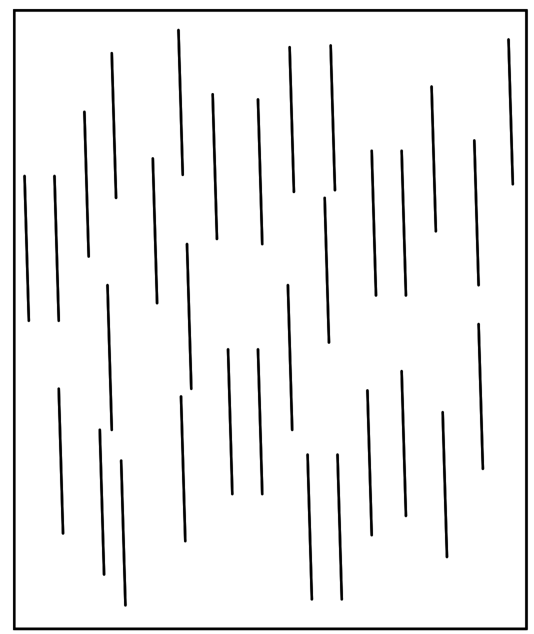
- The modulus of a metal can be increased by suitable alloying.
- E is a structure *(microstructure)* insensitive property, and this implies that grain size, dislocation density, etc. do not play an important role in determining the elastic modulus of a material.
- One of the important strategies to increase the modulus of a material is by forming a hyrbrid/composite with an elastically 'harder' (stiffer) material. E.g. TiB_2 is added to Fe to increase the modulus of Fe.

Composites (Not solid solutions)

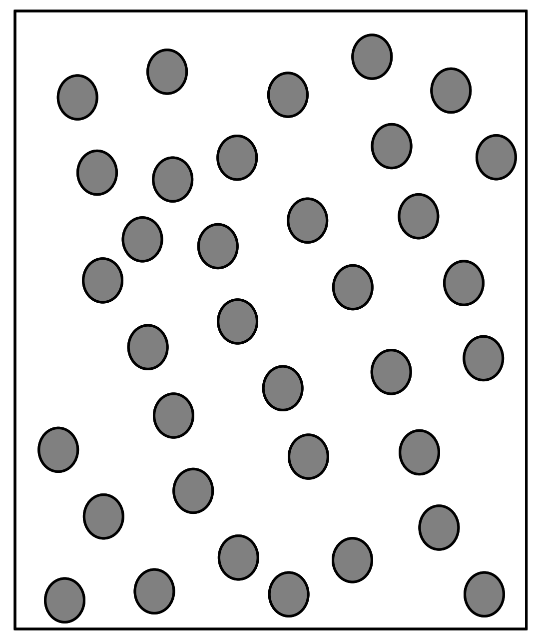
- A hard second phase *(termed as reinforcement)* can be added to a low E material to increase the modulus of the base material. The second phase can be in the form of *particles, fibres, laminates, etc*.
- Typically the second phase though harder is brittle and the ductility is provided by the matrix. If the reinforcement cracks the propagation of the crack is arrested by the matrix.



Laminate composite



Aligned fiber composite



Particulate composite

Composite

Suppose we want to design a light weight but strong material.

What are our choices?

Al is light weight (Y = 71 Gpa), but ductile (can withstand accidental overloading)

B is strong (Y = 440 GPa) but brittle (can't withstand accidental overloading)

By making a composite of Al with B fibres, a high strength light weight material can be designed.

$$Y_c = V_f Y_f + V_m Y_m$$

Summary

1. The Youngs modulus is depends upon the bonding nature. More the strong bond, higher the Youngs modulus.
2. The Youngs modulus increase along the period of a periodic table as the covalent character of the bond and its strength increase.
3. The Youngs modulus decreases down a period as the covalent character of the bonding decreases.
4. Design of a material for is important for lightweight, high performance applications.
5. Generally composites are used due to strength and are lightweight.