Roll No. : ....

B.E. IV Semester
Time Limit: 02 Hours, Maximum Marks: 25

UCS410 : Probability & Statistics
Instructor : Dr. Jatinderdeep Kaur

Instructions: You are expected to answer all questions. Non-programmable calculators are permitted.

1. (a) The following data represent the length of life in years, measured to the nearest tenth, of 30 similar fuel pumps:

   2.0 3.0 0.3 3.3 1.3 0.4 0.2 6.0 5.5 6.5 0.2 2.3 1.0 6.0 5.6
   1.5 4.0 5.9 1.8 4.7 0.7 4.5 0.3 1.5 0.5 2.5 5.0 6.0 1.2 0.2

   (i) Construct a stem-and-leaf plot for the life in years of the fuel pumps.
   (ii) Compute the sample mean and sample range. [1+1.5 marks]

   (b) A producer of a certain type of electronic component ships to suppliers in lots of twenty. Suppose that 60% of all such lots contain no defective components, 30% contain one defective component, and 10% contain two defective components. A lot is picked, two components from the lot are randomly selected and tested, and neither is defective.
   (i) What is the probability that zero defective components exist in the lot?
   (ii) What is the probability that one defective exists in the lot? [3 marks]

   (c) If A and B are two events such that $P(A^c) = 0.6$, $P(B) = 0.7$, $P(A \cup B) = 0.6$, then find
   (i) $P(A \cap B)$ (ii) $P(B^c|A^c)$. [2.5 marks]

2. (a) The probability distribution function of a random variable X is given by

x	0	1	2
$p(x)$	$3k^2$	$4k - 10k^2$	$5k - 1$.

   Where $k > 0$. Find k and also find $P(X < 2)$. [2.5 marks]

   (b) If a random variable X is defined such that $E[(X-1)^2] = 10$ and $E[(X-2)^2] = 6$, find μ and σ^2. [2.5 marks]

   (c) A random variable X has a mean $\mu = 10$ and a variance $\sigma^2 = 4$. Using Chebyshev's theorem, find
   (i) $P(|X - 10| \geq 3)$;
   (ii) the value of the constant c such that $P(|X - 10| \geq c) \leq 0.04$. [3 marks]

3. (a) According to Chemical Engineering Progress, approximately 30% of all pipework failures in chemical plants are caused by operator error.
   (i) What is the probability that out of the next 20 pipework failures at least 10 are due to operator error?
   (ii) What is the probability that no less than 5 out of 20 such failures are due to operator error? (Given: $\sum_{x=0}^{4} B(x; n, p) = 0.2375$ and $\sum_{x=5}^{9} B(x; n, p) = 0.7145$) [3 marks]

   (b) If X is a Poisson variate such that $P(X = 2) = \frac{2}{3}P(X = 1)$; find variance and moment generating function of a random variable X. [3 marks]

   (c) In a normal distribution, 7% of the items are under 35 and 89% are under 63. Find mean and variance of the distribution. Given that $P(Z > 1.5) = .07$ and $P(Z < 1.3) = 0.89$. [3 marks]