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Name: ______________________ RollNo.: ______________________

THAPAR INSTITUTE OF ENGINEERING & TECHNOLOGY-PATIALA
Department of Computer Science and Engineering
Written Test (Oct. 19, 2021)

Probability and Statistics (UCS410) **M.M. 45 Time: 2 Hours**
Instructors: RJK, SHG, SWT, AMT

Note: Attempt any FIVE questions. Assume missing data suitably, if any.
Some useful data $P(Z > 1.67) = 0.04746$, $P(Z < -3) = 0.00135$, $P(Z < 3) = 0.99865$, $P(Z < 0) = 0.5$, $P(Z < -1.5) = 0.06681$, $P(-2 < Z < 2) = 0.9544$, $P(-1 < Z < 1) = 0.6826$ Here, Z is the standard Normal variable.

Q1 (a)	Two computers A and B are to be marketed. A salesman who is assigned the job of finding customers for them has 60% and 40% chances respectively of succeeding in case of computer A and B. The two computers can be sold independently. Given that he was able to sell at least one computer, what is the probability that computer A has been sold?	**3 Marks**
Q1 (b)	(i) State Bayes Theorem. (ii) Suppose that a product is produced in three factories X, Y, and Z. It is known that factory X produces thrice as many items as factory Y, and that factories Y and Z produce the same number of items. Assume that it is known that 3 per cent of the items produced by each factories X and Z are defective while 5 percent of those manufactured by factory Y are defective. All the items produced in the three factories are stocked, and an item of product is selected at random. • What is the probability that this item is defective? • If an item selected at random is found to defective, what is the probability that it was produced by factory X, Y and Z respectively?	**6 Marks**
Q2(a)	Define rectangular distribution (Continuous Uniform distribution) and derive its expectation and variance.	**6 Marks**
Q2 (b)	Consider random variable X which is distributed exponentially. Its pdf is given by $$f(x) = \begin{cases} e^{-x}, & x \geq 0 \\ 0\ , & elsewhere \end{cases}$$ Establish that Markov's inequality holds good for random variable X.	**3 Marks**
Q3 (a)	Find Moment Generating Function(MGF) for exponential distribution.	**3 Marks**
Q3 (b)	A component exhibits Normal Distribution for failure rate with mean of 3750 hrs. and standard deviation of 500 hrs. What percentage of parts will survive up to 4500 hrs.?	**4 Marks**
Q3(c)	What is the probability that at least two out of n people have the same birthday? Assume 365 days in a year and that all days are equally likely.	**2 Marks**

Q4 (a)	Fit the curve $y = ax^b$ for the following data <table><tr><td>x:</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr><tr><td>y:</td><td>1200</td><td>900</td><td>600</td><td>200</td><td>110</td><td>50</td></tr></table>	7 Marks
Q4 (b)	Does there exist a variate that satisfy following relationship $P[\mu - 2\sigma < X < \mu + 2\sigma] = 0.6$ To conclude use chebyshev's inequality.	2 Marks
Q5 (a)	Random variable X follows the continuous uniform distribution over the interval 0 to 1 i.e. [0,1]. Find the probability density function (pdf) of $Y = \frac{1}{X}$?	5 Marks
Q5(b)	The first four moments of a distribution about the value 4 of the variable are -1·5, 17, -30 and 108. Find the first four moments about the mean.	4 Marks
Q6	Let the joint probability density function for (X , Y) be $f(x, y) = \begin{cases} \frac{x+y}{2}, & 0 < x,\ 0 < y,\ and\ 3x + y < 3 \\ 0, & Otherwise \end{cases}$ i. Find the probability P(X < Y) . Draw the clear target region under consideration ii. Find the marginal probability density function of X. iii. Find the marginal probability density function of Y. iv. Are X and Y independent? If not, find Cov (X, Y).	9 Marks
Q7 (a)	Suppose that the marks of students in a course are normally distributed with mean 50 and standard deviation 15. We take a sample of size 30, and note that the marks of students are: 70, 45, 80, 49, 59, 43, 36, 76 ,72 ,48 ,64, 60, 38, 46, 68, 35, 61, 58, 76, 56, 62, 38, 63, 37 ,78, 37, 60, 47, 38, 14. Find the 95.44% confidence interval μ.	5 Marks
Q7(b)	Define a Sample and Sample Mean. Also, Show that Sample Mean is an unbiased estimate of population mean .	4 Marks