Tutorial Sheet-9 Solutions

1. The sum of degrees of vertices is is 6+6+6+5+3 = 26 = 2 ·13; there are 13 edges.

2. Yes, the graph is bipartite. V1 = {a,b,d,e} and V2 = {c,f}. Chromatic number = 2.

3. a) We construct a bipartite graph in which the vertex set consists of two subsets—one for the employees and one for the jobs. Let V_1 = {Zamora, Agraharam, Smith, Chou, Macintyre}, and let V_2 = {planning, publicity, sales, marketing, development, industry relations}. Then the vertex set for our graph is $V = V_1 \cup V_2$. Given the list of capabilities in the exercise, we must include precisely the following edges in our graph: {Zamora, planning}, {Zamora, sales}, {Zamora, marketing}, {Zamora, industry relations}, {Agraharam, planning}, {Agraharam, development}, {Smith, publicity}, {Smith, sales}, {Smith, industry relations}, {Chou, planning}, {Chou, sales}, {Chou, industry relations}, {Macintyre, planning}, {Macintyre, publicity}, {Macintyre, sales}, {Macintyre, industry relations}.

   b) Many assignments are possible. If we take it as an implicit assumption that there will be no more than one employee assigned to the same job, then we want a maximum matching for this graph. So we look for five edges in this graph that share no endpoints. A little trial and error gives us, for example, {Zamora, planning}, {Agraharam, development}, {Smith, publicity}, {Chou, sales}, {Macintyre, industry relations}. We assign the employees to the jobs given in this matching.

4. The given information tells us that $G \cup \overline{G}$ has 28 edges. However, $G \cup \overline{G}$ is the complete graph on the number of vertices n that G has. Since this graph has $n(n-1)/2$ edges, we want to solve $n(n-1)/2 = 28$. Thus $n = 8$.

5. i) These two graphs are isomorphic. Each consists of a K_4 with a fifth vertex adjacent to two of the vertices in the K_4 . Many isomorphisms are possible. One is $f(u_1) = v_1$, $f(u_2) = v_3$, $f(u_3) = v_2$, $f(u_4) = v_5$, and $f(u_5) = v_4$.

   ii) We claim that the digraphs are isomorphic. To discover an isomorphism, we first note that vertices u_1 , u_2 , and u_3 in the first digraph are independent (i.e., have no edges joining them), as are u_4 , u_5 , and u_6 . Therefore these two groups of vertices will have to correspond to similar groups in the second digraph, namely v_1 , v_3 , and v_5 , and v_2 , v_4 , and v_6 , in some order. Furthermore, u_3 is the only vertex among one of these groups of u's to be the only one in the group with out-degree 2, so it must correspond to v_6 , the vertex with the similar property in the other digraph; and in the same manner, u_4 must correspond to v_5 . Now it is an easy matter, by looking at where the edges lead, to see that the isomorphism (if there is one) must also pair up u_1 with v_2 ; u_2 with v_4 ; u_5 with v_1 ; and u_6 with v_3 . Finally, we easily verify that this indeed gives an isomorphism—each directed edge in the first digraph is present precisely when the corresponding directed edge is present in the second digraph.