School of Mathematics, Thapar Institute of Engineering & Technology, Patiala

UMA007 : Numerical Analysis

Assignment 9

Initial-Value Problems for Ordinary Differential Equations

1. Show that each of the following initial-value problems (IVP) has a unique solution, and find the solution.
   (a) $y' = y\cos t,\ 0 \le t \le 1,\ y(0) = 1.$
   (b) $y' = \dfrac{2}{t}y + t^2 e^t,\ 1 \le t \le 2,\ y(1) = 0.$

2. Apply Picard's method for solving the initial-value problem generate $y_0(t)$, $y_1(t)$, $y_2(t)$, and $y_3(t)$ for the initial-value problem
$$y' = -y + t + 1,\ 0 \le t \le 1,\ y(0) = 1.$$

3. Consider the following initial-value problem
$$x' = t(x + t) - 2,\ x(0) = 2.$$
Use the Euler method with stepsize $h = 0.2$ to compute $x(0.6)$.

4. Given the initial-value problem
$$y' = \frac{1}{t^2} - \frac{y}{t} - y^2,\quad 1 \le t \le 2,\ y(1) = -1,$$
with exact solution $y(t) = -\dfrac{1}{t}$:
   (a) Use Euler's method with $h = 0.05$ to approximate the solution, and compare it with the actual values of y.
   (b) Use the answers generated in part (a) and linear interpolation to approximate the following values of y, and compare them to the actual values.
   i. $y(1.052)$ ii. $y(1.555)$ iii. $y(1.978)$.

5. Solve the following IVP by second-order Runge-Kutta method
$$y' = -y + 2\cos t,\ y(0) = 1.$$
Compute $y(0.2)$, $y(0.4)$, and $y(0.6)$ with mesh length 0.2.

6. Compute solutions to the following problems with a second-order Taylor method. Use step size $h = 0.2$.
   (a) $y' = (\cos y)^2,\ 0 \le x \le 1,\ y(0) = 0.$
   (b) $y' = \dfrac{20}{1 + 19e^{-x/4}},\ 0 \le x \le 1,\ y(0) = 1.$

7. A projectile of mass $m = 0.11$ kg shot vertically upward with initial velocity $v(0) = 8\ m/s$ is slowed due to the force of gravity, $F_g = -mg$, and due to air resistance, $F_r = -kv|v|$, where $g = 9.8\ m/s^2$ and $k = 0.002\ kg/m$. The differential equation for the velocity v is given by
$$mv' = -mg - kv|v|.$$
   (a) Find the velocity after $0.1, 0.2, \cdots, 1.0$ s.
   (b) To the nearest tenth of a second, determine when the projectile reaches its maximum height and begins falling.

8. Using Runge-Kutta fourth-order method to solve the IVP at $x = 0.8$ for
$$\frac{dy}{dx} = \sqrt{x + y},\ y(0.4) = 0.41$$
with step length $h = 0.2$.

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9. Water flows from an inverted conical tank with circular orifice at the rate

$$\frac{dx}{dt} = -0.6\pi r^2\sqrt{2g}\frac{\sqrt{x}}{A(x)},$$

where r is the radius of the orifice, x is the height of the liquid level from the vertex of the cone, and $A(x)$ is the area of the cross section of the tank x units above the orifice. Suppose $r = 0.1$ ft, $g = 32.1$ ft/s^2, and the tank has an initial water level of 8 ft and initial volume of $512(\pi/3)$ ft^3 . Use the Runge-Kutta method of order four to find the following.

a. The water level after 10 min with $h = 20$ s.

b. When the tank will be empty, to within 1 min.

10. The following system represent a much simplified model of nerve cells

$$\begin{aligned}\frac{dx}{dt} &= x + y - x^3,\ x(0) = 0.5\\ \frac{dy}{dt} &= -\frac{x}{2},\ y(0) = 0.1\end{aligned}$$

where $x(t)$ represents voltage across the boundary of nerve cell and $y(t)$ is the permeability of the cell wall at time t. Solve this system using Runge-Kutta fourth-order method to generate the profile up to $t = 0.2$ with step size 0.1.

11. Use Runge-Kutta method of order four to solve

$$y'' - 3y' + 2y = 6e^{-t},\ 0 \le t \le 1,\ y(0) = y'(0) = 2$$

for $t = 0.2$ with stepsize 0.2.

Q.1
(a) $y' = y\cos t \quad ; 0 \leq t \leq 1, \quad y(0) = 1$
$= f(t, y)$

(i) $f(t,y) = y\cos t$ is continous

(ii) for (t, y_1) & (t, y_2)

$$|f(t,y_1) - f(t,y_2)| = |y_1\cos t - y_2\cos t|$$
$$= |\cos t||y_1 - y_2|$$
$$\leq 1 \times |y_1 - y_2| \quad (\text{since } |\cos t| \leq 1 \ \forall t \in \mathbb{R}$$

Hence using uniqueness thm., the IVP has a unique soln.

$$y_0(t) = y(0) = 1$$

$$y_1(t) = 1 + \int_0^t f(s, y_0(s))\,ds = 1 + \int_0^t 1 \times \cos(s)\,ds$$
$$= 1 + [\sin(s)]_0^t = 1 + \sin t$$

$$y_2(t) = 1 + \int_0^t f(s, y_1(s))\,ds = 1 + \int_0^t [y_1(s)]\cos s\,ds$$
$$= 1 + \int_0^t (1 + \sin s)\cos s\,ds$$

$$= 1 + \int_0^t \left[\cos s + \frac{1}{2}\sin(2s)\right] ds$$

$$= 1 + \left[\sin s\right]_0^t + \frac{1}{2}\left[\frac{\cos(2s)}{2}\right]_0^t$$

$$= 1 + \sin t + \frac{1}{2}\left[\cos(2t) + 1\right]$$

$$y_2(t) = 1 + \frac{1}{2} + \sin t + \frac{1}{2}\cos(2t)$$

$$y_3(t) = 1 + \int_0^t f(s, y_2(s))\, ds$$

$$= 1 + \int_0^t \left[y_2(s)\cos s\right] ds$$

$$= 1 + \int_0^t \left[\frac{3}{2} + \sin s + \frac{1}{2}\cos(2s)\right] \cos(s)\, ds$$

$$= 1 + \int_0^t \left[\frac{3}{2}\cos(s) + \frac{1}{2}\sin(2s) + \frac{1}{2}\cos(s)\cos(2s)\right] ds$$

$$= 1 + \int_0^t \left[\frac{3}{2}\cos(s) + \frac{1}{2}\sin(2s) + \frac{1}{2}\left[\cos(3s) + \cos(s)\right]\right] ds$$

$$= 1 + \left[\frac{3}{2}\sin(s) + \frac{1}{4}\cos(2s) + \frac{1}{6}\sin(3s) + \frac{1}{2}\sin(s)\right]_0^t$$

$$y_3(t) = 1 + \frac{1}{4} + 2\sin t + \frac{1}{2^2}\cos(2t) + \frac{1}{6}\sin(3t)$$

and so-on.

(b) $y' = \frac{2}{t}y + t^2e^t$; $1 \leq t \leq 2$, $y(1) = 0$.

$= f(t,y)$

(i) Since $t \neq 0$, $f(t,y)$ is Continous.

(ii) for (t,y_1) + (t,y_2)

$$|f(t,y_1) - f(t,y_2)| = \left|\frac{2}{t}y_1 + \cancel{t^2e^t} - \frac{2}{t}y_2 - \cancel{t^2e^t}\right|$$

$$= \left|\frac{2}{t}\right| |y_1 - y_2|$$

$\frac{1}{t}$ is dec. function.

$$\Rightarrow |f(t,y_1) - f(t,y_2)| \leq 2|y_1 - y_2|$$

Hence using uniqueness thm., the IVP has a unique soln.

$$y_0(t) = y(1) = 0$$

$$y_1(t) = 0 + \int_1^t f(s, y_0(s))\,ds$$

$$= \int_1^t \left[s^2 e^s\right] ds$$

$$= \left[s^2 e^s\right]_1^t - \int_1^t 2s\, e^s\, ds$$

$$= t^2 e^t - e - 2\left[\left[s e^s\right]_1^t - \int_1^t e^s ds\right]$$

$$= -e + t^2 e^t - 2\left[t e^t - e\right] + 2\left[e^s\right]_1^t$$

$$= -e + t^2 e^t - 2te^t + \cancel{2e} + 2e^t - \cancel{2e}$$

$$= t^2 e^t + 2e^t - 2te^t - e$$

$$= e^t (t-1)^2 + (e^t - e)$$

$$y_2(t) = 0 + \int_1^t f(s, y_1(s))\,ds$$

$$= \int_1^t \left[\frac{2}{s}\left[e^s (s-1)^2 + (e^s - e)\right] + e^s s^2\right] ds$$

Solve for $y_2(t)$ and proceed like this.

Q2

$$y' = -y + t + 1, \quad 0 \le t \le 1, \quad y(0) = 1$$
$$= f(t, y)$$

$$y_0(t) = y(0) = 1$$

$$y_1(t) = y(0) + \int_0^t f(s, y_0(s))\,ds$$

$$= 1 + \int_0^t [\cancel{-1} + s + \cancel{1}]\,ds$$

$$= 1 + \int_0^t s\,ds = 1 + \frac{t^2}{2}$$

$$y_2(t) = y(0) + \int_0^t f(s, y_1(s))\,ds$$

$$= 1 + \int_0^t \left[\cancel{-1} - \frac{s^2}{2} + s + \cancel{1}\right] ds$$

$$= 1 + \int_0^t \left[s - \frac{s^2}{2}\right] ds$$

$$= 1 + \left[\frac{s^2}{2} - \frac{1}{2}\frac{s^3}{3}\right]_0^t = 1 + \frac{t^2}{2} - \frac{t^3}{3!}$$

$$y_3(t) = y(0) + \int_0^t f(s, y_2(s))\,ds$$

$$= 1 + \int_0^t \left[\cancel{-1} - \frac{s^2}{2} + \frac{s^3}{3!} + s + \cancel{1}\right] ds$$

$$= 1 + \left[\frac{s^2}{2} - \frac{s^3}{3!} + \frac{s^4}{4!} \right]_0^t$$

$$y_3(t) = 1 + \frac{t^2}{2!} - \frac{t^3}{3!} + \frac{t^4}{4!}$$

$$= 1 + \frac{(-t)^2}{2!} + \frac{(-t)^3}{3!} + \frac{(-t)^4}{4!}$$

In general, $y_k(t) = 1 + \frac{(-t)}{1!} + \frac{(-t)^2}{2!} + \frac{(-t)^3}{3!} + \cdots + \frac{(-t)^{k+1}}{(k+1)!} + t$

Hence,

$$\boxed{y(t) = \lim_{k \to \infty} y_k(t) = e^{-t} + t}$$

Q-3

$$x' = t(x+t) - 2 \quad , \quad x(0) = 2$$
$$= f(t, x)$$

$h = 0.2$, , $x(0.6) = ?$ using Euler's method.

$$x(0) = 2$$

$$x(0.2) = x(0 + 0.2) = x(0) + h f(0, x(0))$$

$$= 2 + 0.2 [0 (2+0) - 2]$$

$$= 2 - 0.4 = 1.6$$

$$x(0.4) = x(0.2 + 0.2) = x(0.2) + h f(0.2, x(0.2))$$
$$= 1.6 + 0.2 [0.2 (1.6 + 0.2) - 2]$$

$= 1.272$

$x(0.6) = x(0.4+0.2) = x(0.4) + 0.2\, f(0.4, x(0.4))$

$= 1.272 + 0.2\, f(0.4, 1.272)$

$= 1.272 + 0.2\left[0.4(1.272+0.4) - 2\right]$

$$\boxed{x(0.6) = 1.00576}$$

Q-4 $\quad y' = \dfrac{1}{t^2} - \dfrac{y}{t} - y^2 ; 1 \leq t \leq 2, y(1) = -1$

$= f(t, y)$

(a) Solve ~~fy~~ for y, using Euler's method with $h = 0.05$.

- Actual soln. $y = \dfrac{-1}{t}$

$y(1) = -1$

$y(1.05) = y(1+0.05) = y(1) + 0.05\, f(1, y(1))$

$= -1 + 0.05\, f(1, -1)$

$= -1 + 0.05\left[\dfrac{1}{1} + \dfrac{1}{1} - 1\right]$

$= -1 + 0.05 = -0.95$

$y(1.10) = y(1.05 + 0.05) = y(1.05) + 0.05\, f(1.05, y(1.05))$

$$= -0.95 + 0.05\left[\frac{1}{(1.05)^2} - \frac{(-0.95)}{1.05} - (-0.95)^2\right]$$

$$= -0.95 + 0.05\left[0.9093\right]$$

$$= -0.9045$$

Similarly we can find rest of values, ie,

$y(1.15), y(1.20), \ldots, y(2.0)$

Actual result,

$$y(1.05) = \frac{-1}{1.05} = -0.9524$$

Error,

$$|-0.9524 + 0.95| = 0.0024$$

$$y(1.1) = \frac{-1}{1.1} = -0.9091$$

Error, $|-0.9091 + 0.9045| = 0.0046$

Similarly for rest of the terms.

(b) $y(1.052)$ can be find using linear interpolation using pts., 1.05 & 1.10.

$$P(x) = \frac{x-1.05}{1.10-1.05}\, y(1.1) + \frac{(x-1.1)}{1.05-1.1}\, y(1.05)$$

$$= \frac{x-1.05}{0.05}(-.9045) + \frac{(x-1.1)}{-0.05}(-0.95)$$

$$P(x) = \frac{-1}{0.05}\left[0.9045(x-1.05) + 0.95(x-1.1)\right]$$

$$P(1.052) = \frac{-1}{0.05}\left[0.9045(1.052-1.05) + 0.95(1.052-1.1)\right]$$

$$= -20\left[0.9045 \times 0.002 + 0.95 \times 0.048\right]$$

$$\boxed{P(1.052) = -0.9482}$$

Similarly for rest of the values.

Q-5

$$y' = -y + 2\cos t = f(t,y)\ ;\ y(0) = 1$$

$y(0.2) = ?$, $y(0.4) = ?$, $y(0.6) = ?$ with $h = 0.2$

Using modified Euler's method,

$$y(0.2) = y(0 + 0.2)$$

$$\cancel{K_1} = K_1 = h f(0, y(0)) = 0.2 f(0, 1)$$

$$= 0.2\left[-1 + 2\cos(0)\right]$$

$$K_1 = 0.2$$

$$K_2 = h\, f(0 + 0.2,\ y(0) + K_1)$$

$$= 0.2\, f(0.2,\ \cancel{0.2}\,1.2)$$

$$= 0.2\left[-1.2 + 2\cos(0.2)\right]$$

$$K_2 = 0.152$$

$$y(0.2) = y(0) + \frac{K_1 + K_2}{2} = 1 + \frac{0.2 + 0.152}{2}$$

$$y(0.2) = 1.176$$

$$y(0.4) = y(0.2 + 0.2)$$

$$K_1 = h\, f(0.2,\ y(0.2))$$

$$= 0.2\, f(0.2,\ 1.176)$$

$$= 0.2\left[-1.176 + 2\cos(0.2)\right]$$

$$K_1 = 0.1568$$

$$K_2 = h\, f(0.2 + 0.2,\ y(0.2) + K_1) = 0.2\, f(0.4,\ 1.3328)$$

$$= 0.2\left[-1.3328 + 2\cos(0.4)\right]$$

$$= 0.1019$$

$$y(0.4) = y(0.2) + \frac{K_1+K_2}{2}$$

$$= 1.176 + \frac{0.1568 + 0.1019}{2}$$

$$y(0.4) = 1.3054$$

$$y(0.6) = y(0.4+0.2)$$

$$K_1 = hf(0.4, y(0.4)) = 0.2 f(0.4, 1.3054)$$

$$= 0.2[-1.3054 + 2\cos(0.4)]$$

$$= 0.1073$$

$$K_2 = hf(0.6, 1.3054 + 0.1073) = 0.2 f(0.6, 1.4127)$$

$$= 0.2[-1.4127 + 2\cos(0.6)]$$

$$= 0.048$$

$$y(0.6) = y(0.4) + \frac{K_1+K_2}{2} = 1.3054 + \frac{0.1073 + 0.048}{2}$$

$$y(0.6) = 1.383$$

Q-6 (a) $y' = (\cos y)^2 = f(x,y)$; $0 \le x \le 1$

$y(0) = 0$, $h = 0.2$

We need to find $y(0.2)$, $y(0.4)$, $y(0.6)$, $y(0.8)$, $y(1.0)$

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Second order Taylor series,

$$y(x_i + h) = y(x_i) + h\, y'(x_i) + \frac{h^2}{2} y''(x_i)$$

$$\frac{dy}{dx} = (\cos y)^2$$

$$y'' = \frac{d^2y}{dx^2} = 2\cos y \times (-\sin y)\frac{dy}{dx}$$

$$= -\sin(2y)(\cos y)^2$$

$$y'' = -2\sin y\,(\cos y)^3$$

$$y(0.2) = y(0 + 0.2) = y(0) + 0.2\,(\cos(y(0)))^2 +$$

$$h^2\left[-2\sin(y(0))\cos^3(y(0))\right]$$

$$= 0 + 0.2 \times \cos^2(0) = 0.2$$

Similarly,

$y(0.4) = y(0.2 + 0.2)$ and so-on.

(b) Similar to part (a).

Q-7 $\quad mv' = -mg - kv|v|$

$g = 9.8\ m/s^2$, $\quad k = 0.002\ kg/m$

$m = 0.11\ Kg$, $\quad v(0) = 8\ m/s$.

$$\frac{dv}{dt} = -g - \frac{k}{m} v|v|$$

$$= -9.8 - \frac{0.002}{0.11} v|v|$$

$$= -9.8 - 0.0182\, v|v|$$

$$= f(t, v).$$

Using Runge-Kutta Order-4, we find $\cancel{y}(0.1)$, $\cancel{y}(0.2)$, - - - - -, $\cancel{y}(1.0)$.

for $v(0.1) = v(0+0.1)$, $h = 0.1$

$$K_1 = h f(0, v(0)) = h f(0, 8)$$

$$= (-9.8 - 0.018 \times 8^2) \times 0.1$$

$$= \cancel{-10} \; -1.096.$$

$$K_2 = h f\left(0 + \frac{h}{2}, v(0) + \frac{K_1}{2}\right)$$

$$= 0.1 f\left(0.05 + 8 - 0.5482\right)$$

$$= 0.1 f\left(0.05, 7.4518\right)$$

$$= 0.1\left[-9.8 - 0.0182 \times (7.4518)^2\right]$$

$$= -1.0811$$

$$K_3 = \cancel{hf(0.05, 8-0.54)}\; h f\left(0 + \frac{h}{2}, v(0) + \frac{K_2}{2}\right)$$

$= 0.1 f(0.05, 8 - 0.54) = 0.1 f(0.05, 7.46)$

$= 0.1 \left[-9.8 - 0.0182 \times (7.46)^2\right].$

$K_3 = -1.0813$

$K_4 = h f(0 + h, v(0) + K_3)$

$= h f(0.1, 8 - 1.0813) = 0.1 f(0.1, 6.919)$

$= 0.1 \left[-9.8 - 0.0182 (6.919)^2\right]$

$= -1.0671$

$$y(0.1) = y(0) + \frac{K_1 + 2K_2 + 2K_3 + K_4}{6}$$

$$= 8 + \frac{(-1)(1.096 + 2 \times 1.0871 + 2 \times 1.0813 + 1.0671)}{6}$$

$y(0.1) = 8 - 1.08$

$y(0.1) = 6.9187$

Similarly, we can find the rest of the values.

(b) At maximum height, $v = 0$

ie we need to find t s.t

$v(t) = 0$

To do this,

$$\frac{dt}{dv} = \frac{-1}{9.8 + 0.0182\, v|v|} = g(v,t)$$

gives t as a function of 'v'

~~$v(0) = 8\,m/s$.~~ $t(8) = 0$ (given)

find $t(0) = ?$

→ use Runge-Kutta order four with $h = 1$.

→ Then round-off to ~~on~~ first decimal place (ie $1/10$ of a second).

Q8

$$\frac{dy}{dx} = \sqrt{x+y} = f(x,y)$$

$y(0.4) = 0.41$, $h = 0.2$,

we need to find $y(0.8) = ?$, using fourth-order Runge-Kutta.

first we find $y(0.6) = y(0.4 + 0.2)$

and then $y(0.8) = y(0.6 + 0.2)$.

Usage of Runge-Kutta fourth-order is as done in ~~previo~~ ~~prev~~ previous problem.

Q9

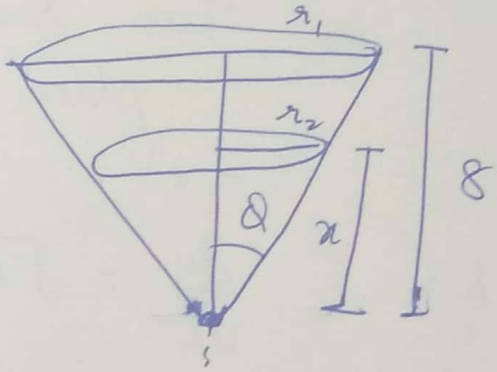
$$\frac{dx}{dt} = -0.6\,\pi r^2 \sqrt{2g}\,\frac{\sqrt{x}}{A(x)}$$

$r = 0.1$ ft, $g = 32.1$ ft/s^2, $x(0) = 8$ ft (initial ~~height~~ water level)

(a) we need to find ~~x(t)~~ x at $t = 10$ min $= 600$ sec

with $h = 20$ s.

first we need to find $A(x)$ as a function of x.



We ~~are~~ are given Volume when $x = 8$

$$V_0 = 512\left(\frac{\pi}{3}\right) ft^3$$

$$\Rightarrow \frac{\pi}{3} r_1^2\, 8 = 512\left(\frac{\pi}{3}\right)$$

$$\Rightarrow r_1 = 8$$

$$\tan\theta = \frac{r_1}{8} = 1 \Rightarrow \theta = 45^\circ$$

Also, $\tan\theta = \frac{r_2}{x} \Rightarrow \boxed{r_2 = x}$

Hence $A(x) = \pi r_2^2 = \pi x^2$

$$\Rightarrow \frac{dx}{dt} = -0.6 \cancel{\pi} (0.1)^2 \sqrt{2\times 32.1} \frac{\sqrt{x}}{\cancel{\pi} x^2}$$

$$\frac{dx}{dt} = -0.048\, x^{-3/2} = f(t, x)$$

$$x(0) = 8$$

Apply fourth-order Runge-Kutta to find $x(600)$ with $h = 20$.

(b) To find t when $x = 0$.

$$\frac{dt}{dx} = -\frac{1}{0.048} x^{3/2} = g(x, t)$$

$$t(8) = 0$$

find, $t(0) = ?$, ~~the~~

then round-off to nearest minute

Q-10 $$\frac{dx}{dt} = x + y - x^3 \; ; \; x(0) = 0.5$$
$$= f(t, x, y)$$

$$\frac{dy}{dt} = -\frac{x}{2} \; ; \; y(0) = 0.1$$
$$= g(t, x, y)$$

Given, $h = 0.1$, To find $x(0.1), y(0.1), x(0.2), y(0.2)$

$$K_1 = h f(0, x(0), y(0))$$
$$= 0.1 f(0, 0.5, 0.1)$$
$$= 0.1 (0.5 + 0.1 - (0.5)^3)$$
$$= 0.1 (0.6 - 0.125) = 0.0475$$

$$L_1 = h g(0, x(0), y(0))$$
$$= 0.1\, g(0, 0.5, 0.1) = 0.1 \left(-\frac{0.5}{2}\right)$$
$$= -0.025$$

$$K_2 = h f\left(0 + \frac{h}{2}, x(0) + \frac{K_1}{2}, y(0) + \frac{L_1}{2}\right)$$
$$= h f\left(0.05, 0.5 + 0.0475\}, 0.1 - \frac{0.025}{2}\right)$$
$$= h f(0.05, 0.52375, 0.0875)$$
$$= 0.1 (0.52375 + 0.0875 - (0.52375)^3)$$
$$= 0.04676$$

$$L_2 = h g\left(0 + \frac{h}{2}, x(0) + \frac{K_1}{2}, y(0) + \frac{L_1}{2}\right)$$

K_1

$$= h g(0.05, 0.52375, 0.0875)$$
$$= 0.1 \left(-\frac{0.52375}{2}\right) = -0.0261875$$

$$K_3 = hf\left(0+\frac{h}{2}, x(0)+\frac{K_2}{2}, y(0)+\frac{L_2}{2}\right)$$

$$= 0.1 f\left(0.05, 0.5+\frac{0.04676}{2}, 0.1-\frac{0.0262}{2}\right)$$

$$= 0.1 f\left(0.05, 0.52338, 0.0869\right)$$

$$= 0.1\left(0.52338 + 0.0869 - (0.52338)^3\right)$$

$$= 0.1\left(0.4669\right) = 0.04669$$

$$L_3 = hg\left(0+\frac{h}{2}, x(0)+\frac{K_2}{2}, y(0)+\frac{L_2}{2}\right)$$

$$= hg\left(0.05, 0.52338, 0.0869\right)$$

$$= 0.1\left(-\frac{0.52338}{2}\right) = -0.02617$$

$$K_4 = hf\left(0+h, x(0)+K_3, y(0)+L_3\right)$$

$$= 0.1 f\left(0.1, 0.5+0.04669, 0.1-0.02617\right)$$

$$= 0.1 f\left(0.1, 0.54669, 0.07383\right)$$

$$= 0.1\left(0.54669+0.07383-(0.54669)^3\right)$$

$$K_4 = 0.04571$$

$L_4 = hg(0+h, x(0)+K_3, y(0)+L_3)$

$= 0.1\, g(0.1, 0.54669, 0.07383)$

$= 0.1\left(-\frac{0.54669}{2}\right) = -0.02733$

$$x(0.1) = x(0) + \frac{K_1 + 2K_2 + 2K_3 + K_4}{6}$$

$$= 0.5 + \frac{0.0475 + 2\times 0.04676 + 2\times 0.04669 + 0.04571}{6}$$

$= 0.5 + 0.046685$

$= 0.546685$

$$y(0.1) = y(0.0) + \frac{L_1 + 2L_2 + 2L_3 + L_4}{6}$$

$= 0.1 - 0.02598$

$y(0.1) = 0.074$

Similarly, we can find $x(0.2) = x(0.1+0.1)$

$\quad \& \ y(0.2) = y(0.1+0.1)$

Q.11 ① $\leftarrow y'' - 3y' + 2y = 6e^{-t}, \quad 0 \leq t \leq 1,$

$y(0) = y'(0) = 2$

Solve at $t = 0.2$ with $h = 0.2$

$\not{y}$ Let $\dfrac{dy}{dt} = z = f(t, y, z)$; $y(0) = 2$

Using this in (1) we get,

$$\frac{d^2y}{dt^2} = \frac{d}{dt}\left(\frac{dy}{dt}\right) = \frac{dz}{dt} = 3y' - 2y + 6e^{-t}$$

$$= 3z - 2y + 6e^{-t}$$

$$= g(t, y, z) \; ; \; y'(0) = z(0) = 2$$

Hence we have,

$$\frac{dy}{dt} = z = f(t, y, z) \; ; \; y(0) = 2$$

$$\frac{dz}{dt} = 3z - 2y + 6e^{-2} = g(t, y, z) \; ;$$

$$z(0) = 0$$

find $y(0.2)$ $\cancel{z(0.2)}$ $= y(0 + \overset{h}{0.2})$

$z(0.2) = z(0 + 0.2)$

in the same way as in the last problem.