Lecture-6

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Chapter 2: Solution of root-finding problem

Lecture 6: Numerical Analysis (UMA011)

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chapter 1 :-

1. Max error bound in Rounding, chopping.
2. Nested Arithmetic. → poly evaluation → open the series exp & use nested.
3. Roots of quadratic equations.
4. stability → write Algorithm, find C.N. at each step, modification.

Chapter 2: Solution of root-finding problem

$f(x)=0$ $x=?$

$x-3=0$, $x=3$

$x^2-2x+1=0$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x^{100}+9x^{99}+2x+1=0$

$\sin x+\cos x-e^x=0$

$x^2-2x+1=0$, $x=1,1$

Root-finding problem

Methods for root-finding problem:

To find a solution of an equation $f(x)=0$, we discuss the following four methods:

1. Bisection method ✓
2. Fixed point Iteration ✓
3. Newton method ✓
4. Secant method ✓

Iterative Methods $f(x)=0$

$x_0, x_1, x_2, x_3 \ldots x_n$ (app) → p → exact soln.

$f(x_0)\approx 0$, x_0 → guess, x_1 →, x_2 →, x_3 →

A.E. $|x_n-p|$

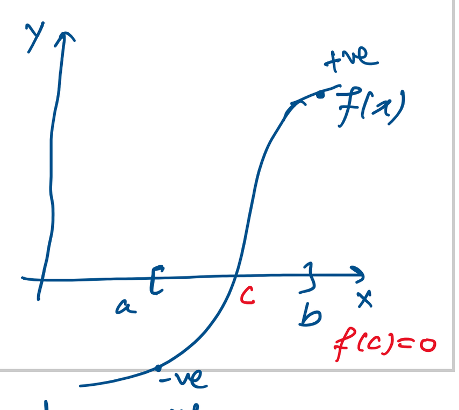
R.E. $\frac{|x_n-p|}{|p|}$

Root-finding problem

$f(x)=0$

Intermediate Value Theorem (IVT)

Let $f(x)$ be a continuous function on $[a,b]$ and $f(a)*f(b)<0$, then $\exists$ (there exists) a no. $c\in(a,b)$ s.t. $f(c)=0$.
↓ root of $f(x)=0$



$f(0)=$ +ve

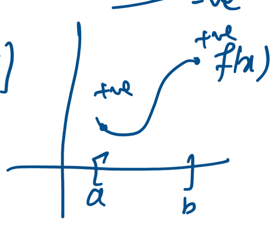
$f(1)=$ +ve

$f(2)=$ −ve $[1,2]$

4.2

4, 4.5 $[4.5,5]$

$[4,5]$



Root-finding problem

find the root of $f(x)=0$ by B.M.

Bisection method: Procedure

$f(x)=0$ $f(x)=0$ 1, 3, 5 $[0,2]$

Use IVT to find the interval.

$f(0)=$ +ve

$f(1)=$ +ve

$f(2)=$ +ve

$f(3)=$ −ve

+ve −ve $[2,3]$

By Bisection method. $x_1=\frac{2+3}{2}=2.5$

$[2,2.5]$, $[2.5,3]$
+ve +ve; +ve −ve

check the sign of $f(2.5)$ = +ve

from IVT root lie in $[2.5,3]$

By Bisection method

$x_2=\frac{2.5+3}{2}=2.75$

$[2.5,2.75]$ and $[2.75,3]$
+ve +ve

check sign of $f(2.75)$

from IVT

x_1, x_2, x_3, x_4, x_5, x_6, x_7

$|x_1-x_2|<10^{-2}$

$|x_2-x_3|<10^{-2}$

x_4 0.01

10^{-2} $x=2.7529152$

$x^*=2.751239$

2.753

$x_1=\frac{a+b}{2}$

$[a,x_1]$ or $[x_1,b]$

$[a,x_1]$

$x_2=\frac{a+x_1}{2}$

$[a,x_2]$ or $[x_2,x_1]$
−ve +ve

$x_3=\frac{x_2+x_1}{2}$

$[x_2,x_3]$ & $[x_3,x_1]$