School of Mathematics, Thapar Institute of Engineering & Technology, Patiala

UMA007 : Numerical Analysis
Assignment 8
Numerical Integration

1. Approximate the following integrals using the trapezoidal and Simpson's formulas and compare with exact values.

(a) $I = \int_{-0.25}^{0.25} (\cos x)^2 \, dx.$

(b) $\int_{e}^{e+1} \frac{1}{x \ln x} \, dx.$

2. Approximate the integral $\int_{1}^{1.5} x^2 \ln x dx$ using the (non-composite) trapezoidal rule. Give a rigorous error bound on this approximation.

3. The Trapezoidal rule applied to $\int_{0}^{2} f(x)dx$ gives the value 4, and Simpson's rule gives the value 2. What is $f(1)$?

4. Evaluate

$$I = \int_{-1}^{1} \frac{dx}{1+x^2}$$

using trapezoidal and Simpson's rule with 8 subintervals. Compare with the exact value of the integral.

5. The quadrature formula $\int_{0}^{2} f(x)dx = c_0 f(0) + c_1 f(1) + c_2 f(2)$ is exact for all polynomials of degree less than or equal to 2. Determine c_0, c_1, and c_2.

6. Find the constants c_0, c_1, and x_1 so that the quadrature formula

$$\int_{0}^{1} f(x)dx = c_0 f(0) + c_1 f(x_1)$$

has the highest possible degree of precision.

7. Determine the values of n and h required to approximate

$$\int_{0}^{2} \frac{1}{x+4} \, dx$$

to within 10^{-4}. Use composite Trapezoidal and composite Simpson's rule.

8. A car laps a race track in 84 seconds. The speed of the car at each 6-second interval is determined by using a radar gun and is given from the beginning of the lap, in feet/second, by the entries in the following table.

Time	0	6	12	18	24	30	36	42	48	54	60	66	72	78	84
Speed	124	134	148	156	147	133	121	109	99	85	78	89	104	116	123

How long is the track?

9. Evaluate the integral

$$\int_{-1}^{1} e^{-x^2} \cos x \, dx$$

by using the Gauss-Legendre two and three point formulas.

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10. Determine constants a, b, c, and d that will produce a quadrature formula

$$\int_{-1}^{1} f(x)dx = af(-1) + bf(1) + cf'(-1) + df'(1)$$

that has degree of precision 3.

11. A particle of mass m moving through a fluid is subjected to a viscous resistance R, which is a function of the velocity v. The relationship between the resistance R, velocity v, and time t is given by the equation

$$t = \int\limits_{v(t_0)}^{v(t)} \frac{m}{R(u)}\, du$$

Suppose that $R(v) = -v\sqrt{v}$ for a particular fluid, where R is in newtons and v is in meters/second. If $m = 10$ kg and $v(0) = 10$ m/s, approximate the time required for the particle to slow to $v = 5$ m/s.