School of Mathematics, Thapar Institute of Engineering & Technology, Patiala

UMA007 : Numerical Analysis
Assignment 5
Iterative Techniques in Matrix Algebra

1. Find l_∞ and l_2 norms of the vectors.

(a) $x = (3, -4, 0, \frac{3}{2})^t$.

(b) $x = (\sin k, \cos k, 2^k)^t$ for a fixed positive integer k.

2. The following linear system $Ax = b$ has x as the actual solution and $\bar{x}$ as an approximate solution. Compute $\|x - \bar{x}\|_\infty$ and $\|A\bar{x} - b\|_\infty$. Also compute $\|A\|_\infty$.

$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= 1 \\ 2x_1 + 3x_2 + 4x_3 &= -1 \\ 3x_1 + 4x_2 + 6x_3 &= 2, \\ x &= (0, -7, 5)^t \\ \bar{x} &= (-0.2, -7.5, 5.4)^t. \end{aligned}$$

3. Find the first two iterations of Jacobi and Gauss-Seidel using $x^{(0)} = 0$.

$$\begin{aligned} 4.63x_1 - 1.21x_2 + 3.22x_3 &= 2.22 \\ -3.07x_1 + 5.48x_2 + 2.11x_3 &= -3.17 \\ 1.26x_1 + 3.11x_2 + 4.57x_3 &= 5.11. \end{aligned}$$

4. The linear system

$$\begin{aligned} x_1 - x_3 &= 0.2 \\ -\frac{1}{2}x_1 + x_2 - \frac{1}{4}x_3 &= -1.425 \\ x_1 - \frac{1}{2}x_2 + x_3 &= 2 \end{aligned}$$

has the solution $(0.9, -0.8, 0.7)^T$.

(a) Is the coefficient matrix strictly diagonally dominant?

(b) Compute the spectral radius of the Gauss-Seidel iteration matrix.

(c) Perform four iterations of the Gauss-Seidel iterative method to approximate the solution.

(d) What happens in part (c) when the first equation in the system is changed to $x_1 - 2x_3 = 0.2$?

5. Check whether you can apply the Jacobi and Gauss-Seidel iterative techniques to solve the following linear system.

$$\begin{aligned} 2x_1 + 3x_2 + x_3 &= -1 \\ 3x_1 + 2x_2 + 2x_3 &= 1 \\ x_1 + 2x_2 + 2x_3 &= 1. \end{aligned}$$

6. Find the first two iterations of the SOR method with $\omega = 1.1$ for the following linear system, using $x^{(0)} = 0$.

$$\begin{aligned} 4x_1 + x_2 - x_3 &= 5 \\ -x_1 + 3x_2 + x_3 &= -4 \\ 2x_1 + 2x_2 + 5x_3 &= 1. \end{aligned}$$

7. Compute the condition numbers of the following matrices relative to $\|.\|_\infty$.

(a) $\begin{bmatrix} 3.9 & 1.6 \\ 6.8 & 2.9 \end{bmatrix}$

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(b) $\begin{bmatrix} 0.04 & 0.01 & -0.01 \\ 0.2 & 0.5 & -0.2 \\ 1 & 2 & 4 \end{bmatrix}$.

8. The linear system $Ax = b$ given by

$$\begin{bmatrix} 1 & 2 \\ 1.0001 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 3.0001 \end{bmatrix}$$

has solution $(1, 1)^t$. Use seven-digit rounding arithmetic to find the solution of the perturbed system

$$\begin{bmatrix} 1 & 2 \\ 1.000011 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3.00001 \\ 3.00003 \end{bmatrix}$$

Is matrix A ill-conditioned? What does this say about the linear system?

9. Determine the largest eigenvalue and the corresponding eigenvector of the following matrix correct to three decimals using the power method with $x^{(0)} = (-1, 2, 1)^t$ using the power method.

$$\begin{bmatrix} 1 & -1 & 0 \\ -2 & 4 & -2 \\ 0 & -1 & 2 \end{bmatrix}.$$

10. Use the inverse power method to approximate the most dominant eigenvalue of the matrix until a tolerance of 10^{-2} is achieved with $x^{(0)} = (1, -1, 2)^t$.

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}.$$

11. Find the eigenvalue of matrix nearest to 3

$$\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

using inverse power method.