

Lecture-6

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Chapter 2: Solution of root-finding problem

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Lecture 6: Numerical Analysis (UMA011)

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- chapter :-
- ① Max error bound in Rounding chopping.
  - ② Nested Arithmetic.  $\rightarrow$  poly evaluation open the series exp & use nested.
  - ③ Roots of quadratic equations.
  - ④ stability  $\rightarrow$  write Algorithm find c.n. at each Step modification.

Chapter 2: Solution of root-finding problem

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$f(x)=0$   
 $x=?$   
 $\neq$

$x-3=0 \checkmark$   
 $x=3 \checkmark$   
 $x^2-2x+3=0$   
 $x=\frac{-b \pm \sqrt{b^2-4ac}}{2a} \checkmark \checkmark$   
 $x^2-2x+1=0$   
 $x=1,1$

$x^{10}+9x^9+2x+1=0$   
 $\sin x + \cos x - e^x = 0$

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Root-finding problem

Methods for root-finding problem:  
To find a solution of an equation  $f(x) = 0$ , we discuss the following four methods:

1 Bisection method  $\checkmark$

2 Fixed point iteration  $\checkmark$

3 Newton method  $\checkmark$

4 Secant method  $\checkmark$

Iterative Methods

$x_0, x_1, x_2, \dots, x_n$   
 $A \in \dots$   
 $R \in \dots$   
 $|x_n - b|$   
 $|f(x)|$

$f(x)=0$   
 $f(x_0) \approx 0$   
 $x_0 \rightarrow$  guess  
 $x_1 \rightarrow$   
 $x_2 \rightarrow$   
 $x_3 \rightarrow$

$\rightarrow$  app  
 $\rightarrow$  (b)  $\rightarrow$  exact soln.

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Root-finding problem

Intermediate Value Theorem (IVT)

Let  $f(x)$  be a continuous function on  $[a, b]$  and  $f(a) \neq f(b) < 0$ , then  $\exists$  a no.  $c \in (a, b)$  s.t.  $f(c) = 0$ .  
 $\downarrow$   
root of  $f(x)=0$

$f(a) = +ve$   
 $f(1) = +ve$   
 $f(2) = -ve$   $[1, 2]$

$f(x)=0$   
 $f(a)=0$   
 $f(b)=0$   
 $f(c)=0$

$[4, 2]$   
 $[4, 5]$   
 $[4.5, 5]$   
 $[4, 5)$

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Root-finding problem

find the root of  $f(x)=0$  by B.M.  $\checkmark$

Bisection method: Procedure

$f(x)=0$   
Use IVT to find the interval.  
 $f(0) = +ve$   
 $f(1) = +ve$   
 $f(2) = +ve$   
 $f(3) = -ve$

$f(x)=0$   
 $[3, 5]$   
 $[2, 3]$   
 $[2.5, 3]$   
 $[2.5, 2.5]$   
 $[2.5, 2.5]$   
 $[2.5, 2.5]$

By Bisection method.  $x_1 = \frac{2+3}{2} = 2.5$

check the sign of  $f(2.5) = +ve$   
from IVT root lie in  $[2.5, 3]$

By Bisection method  
 $x_2 = \frac{2.5+3}{2} = 2.75$   
 $[2.5, 2.75]$  and  $[2.75, 3]$   
check sign of  $f(2.75)$

from IVT

$x_1$   
 $x_2$   
 $x_3$   
 $x_4$   
 $x_5$   
 $x_6$   
 $x_7$   
 $x_8$   
 $x_9$   
 $x_{10}$   
 $x_{11}$   
 $x_{12}$   
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 $x_{97}$   
 $x_{98}$   
 $x_{99}$   
 $x_{100}$

$x_1(x_2) \neq 10^{-2}$   
 $|x_2 - x_3| \neq 10^{-2}$   
 $0.01$   
 $2.753$

$x = \frac{2.75 + 2.9152}{2}$   
 $x^* = 2.751239$

$x_1 = \frac{a+b}{2}$   
 $[a, x_1]$  or  $[x_1, b]$   
 $[a, x_1]$   
 $x_2 = \frac{a+x_1}{2}$   
 $[a, x_2]$  or  $[x_2, x_1]$   
 $x_3 = \frac{x_2+x_1}{2}$   
 $[x_2, x_3]$  or  $[x_3, x_1]$

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Root-finding problem

find the root of  $f(x)=0$  by B.M.  $\checkmark$

Bisection method: Procedure

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Use IVT to find the interval.  
 $f(0) = +ve$   
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from IVT root lie in  $[2.5, 3]$

By Bisection method  
 $x_2 = \frac{2.5+3}{2} = 2.75$   
 $[2.5, 2.75]$  and  $[2.75, 3]$   
check sign of  $f(2.75)$

from IVT

$x_1$   
 $x_2$   
 $x_3$   
 $x_4$   
 $x_5$   
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 $[a, x_2]$  or  $[x_2, x_1]$   
 $x_3 = \frac{x_2+x_1}{2}$   
 $[x_2, x_3]$  or  $[x_3, x_1]$