Principal Disjunctive Normal form ①

↓

Sum of Product
(V) (∧)

Elementary operation $(- \wedge -) \vee \underbrace{(- \wedge -)}$

Raw Eg. of | Notation elementary operations.

Obtain the Principal Disjunctive Normal form first follow the following prerequiste.

(1) Construct a touth table for given proposition.

(2) Example $(A \wedge B) \overset{\text{sum of}}{\vee} (A \overset{\text{products}}{\wedge} C)$ Sum of products (V) (∧)

proposition 1 proposition 2

(V)
combined with this symbol

Called as Compound proposition.

- This type of propositions are formed by taking the disjunction of conjuctions of some combination of variables & their negations

* The individual conjunctions that make up the disjunctive normal forms are called minterms.

Methods to Construct DNF

(1) Construct truth table for proposition

(2) Use the rows of truth table where the proposition is True to construct minterms

  (a) If the variable is true, Use propositional variable in minterm

  (b) If variable is false, use the negation of the variable in minterm.

(3) Connect the minterms with V's (OR's)

Simple Example: $(p \rightarrow q)$

p	q	$p \rightarrow q$
T	T	T ✓ — 1
T	F	F
F	T	T ✓ — 2
F	F	T ✓ — 3

$p \rightarrow q$ is True, when

(i) p is True, q is True

(ii) p is false, q " "

(iii) p " " , q is false

∴ Disjunctive Normal form. is then

$(p \wedge q) \vee (\sim p \wedge q) \vee (\sim p \wedge \sim q)$

~~②~~ Example $q \vee (p \vee \neg q) \longrightarrow$ three variables

Main two variables ← (p, q), ¬q

p	q	¬q	p∨¬q	q∨(p∨¬q)	
T	T	F	T	T ←	p ∧ q
T	F	T	T	T ←	p ∧ ¬q
F	T	F	F	T ←	¬p ∧ q
F	F	T	T	T ←	¬p ∧ ¬q

Answer.

DNF: $(p \wedge q) \vee (p \wedge \neg q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q)$

$\underset{\text{AND (product)}}{\uparrow}$ $\underset{\text{sum}}{\uparrow}$ ⟶ This is logic in DNF

③ ~~¬p~~

$$(p \wedge \neg q \wedge \neg r) \vee (q \wedge r)$$

p	q	r	¬q	¬r	(p∧¬q∧¬r) P_1	q∧r P_2	$P_1 \vee P_2$
T	T	T	F	F	F	T	~~F~~ T
T	T	F	F	T	F	F	~~F~~ F
T	F	T	T	F	F	F	~~F~~ F
F	T	T	F	F	F	T	~~F~~ T
F	F	T	T	F	F	F	~~F~~ F
F	T	F	F	T	F	F	~~F~~ F
T	F	F	T	T	T	F	~~F~~ T
T	F	F	T	T	F	F	~~F~~ F

Answer

DNF:

$(p \wedge q \wedge r) \vee (\neg p \wedge q \wedge r) \vee (p \wedge \neg q \wedge \neg r)$ Ans

Elementary Product ($\wedge$): DNF Case

*. Product of the variables & their negations in a given proposition.

Some simple Example's :- $p \wedge q$ (Standard Example)

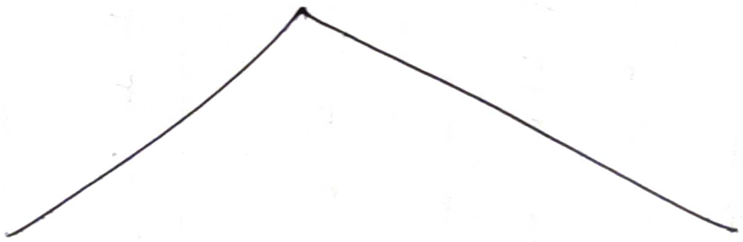
$p \wedge \sim q$, $\sim p \wedge q$, $\sim p \wedge \sim q$.

Elementary Sum ($\vee$): CNF Case

* Sum of the Variables & their negations.

Examples: $p \vee q$, $p \vee \sim q$, $\sim p \vee q$, $p \vee q \vee r$.

NORMAL FORM



DNF

↓

Sum of Elementary products.

Ex: $(p \wedge \sim q) \vee (\sim p \wedge q)$

CNF

Products of Elementary Sums

Ex: $(p \vee \sim q) \wedge (\sim p \vee q)$

Method to solve Normal form

1. Remove the symbols like &, → by using logical Equivalence.

2. ELiminate "~" symbol before sum & products by using De Morgan's Law.

3 Apply | or use Distributive Law.

Note for Logical equivalence use in given proposition (Validate for step 1).

Like (i) If we encountered $p \to q$, then write $\sim p \vee q$.

(ii) $p \leftrightarrow q = p \to q \wedge q \to p$ (CNF)

& [illegible]

$(p \wedge q) \vee (\sim p \wedge \sim q)$ (DNF)

(iii) D.Laws emorgan's : $\sim(p \wedge q) = \sim p \vee \sim q$

$\sim(p \vee q) = \sim p \wedge \sim q$

(iv) Distributive Law's : $p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$

$p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$

Example: Convert the following proposition to both DNF & CNF

$$\begin{tikzcd} & \text{DNF ?} \\ \sim(p\wedge q)\longleftrightarrow p\vee q \arrow[ur] \arrow[dr] & \\ & \text{CNF ?} \end{tikzcd}$$

DNF : $\sim(p\wedge q)\longleftrightarrow p\vee q$

Case

$[\sim(p\wedge q)\wedge(p\vee q)]\vee$

(i) $[\sim(\sim(p\wedge q))\wedge\sim(p\vee q)]$

$$\begin{tikzcd} (p \arrow[d] & \longleftrightarrow & q) \arrow[d] \\ \sim(p\wedge q) & & p\vee q \end{tikzcd}$$

Ans: $(p\wedge q)\vee(\sim p\wedge\sim q)$

$\{\sim\sim p = p\}$

(ii) $[(\sim p\vee\sim q)\wedge(p\vee q)]\vee[(p\wedge q)\wedge(\sim p\wedge\sim q)]$

$\vee[(p\wedge q\wedge\sim p\wedge\sim q)]$

Sum

Elementary product.

Elementary products

$(\sim p\wedge p)\vee(\sim p\wedge q)$

$(\sim q\wedge p)\vee(\sim q\wedge q)$

Sums

$\therefore$ Required Answer is DNF.($\vee$)

$p\wedge(q\vee r)\equiv(p\wedge q)\vee(p\wedge r)$

CNF Case : $\sim(p\wedge q) \longleftrightarrow p\vee q$

$$\Big[\underbrace{\sim(p\wedge q)}_{p} \rightarrow \underbrace{(p\vee q)}_{q}\Big] \wedge$$

(i) $$\Big[\underbrace{(p\vee q)}_{p} \rightarrow \underbrace{\sim(p\wedge q)}_{q}\Big]$$

$$\boxed{\begin{array}{l} p:\ \sim(p\wedge q) \\ q:\ (p\vee q) \\ \text{CNF}:\ p\rightarrow q \wedge q\rightarrow p \end{array}}$$

Now Remove → sign.

$\because p\rightarrow q \rightarrow \sim p\vee q$

ii) $$\Big[\sim(\sim(p\wedge q))\vee(p\vee q)\Big] \wedge \Big[\sim(p\vee q)\vee(\sim(p\wedge q))\Big]$$

→ To Remove(~)

iii) $$\Big[(p\wedge q)\vee(\overbrace{p\vee q}^{r})\Big] \wedge \Big[\sim(p\vee q)\vee\sim(p\wedge q)\Big]$$

D. Law

Demorgan Law

$\because (p\wedge q)\vee r = (p\vee r)\wedge(q\vee r)$

iii) $$\Big[(p\vee p\vee q)\wedge(q\vee p\vee q)\Big] \wedge \Big[(\sim p\wedge\sim q)\vee(\sim p\vee\sim q)\Big]$$

products

iv) $$\Big[\underbrace{(p\vee p\vee q)\wedge(q\vee p\vee q)}\Big] \wedge \Big[\sim p\vee(\sim p\vee\sim q)\wedge\sim q\vee(\sim p\vee\sim q$$

~~Sums~~ Elementary sums

product

v) $$\Big[(p\vee p\vee q)\wedge(q\vee p\vee q)\Big] \wedge \Big[(\sim p\vee\sim p\vee\sim q)\wedge(\sim q\vee\sim p\vee\sim q)\Big]$$

Elementary Sums

$\therefore$ Required form in CNF ($\wedge$)

Well formed formula

Def:- Any expression (Compound proposition) that obeys the Syntactic rules of propositional logic is Called as well formed formula. or WFF

OR

A statement or expression or proposition which consists of variables (capital letters), parentheses and connective symbols, then a recursive definition of a statement is known as well-formed formula

Following rules need to be taken care of while developing well formed formula's.

(1) A statement ~~alone~~ variable is well-formed formula.

(2) If A is well-formed formula, then $\neg A$ is also a well-formed formula.

(3) If A & B are well-formed formulas, then $(A \land B)$, $(A \lor B)$, $(A \rightarrow B)$ + $(A \rightleftarrows B)$ are also well-formed formulas.
Biconditional connective

(4) If any expression/statement/string of symbols containing the statement variables, connectives + parenthesis is a wff.

for Eg: $P \longrightarrow$ 2 is a prime number

$Q \longrightarrow$ 8 is a Even number

then $[P \wedge Q)$, $(P \vee Q)$, $(P \longrightarrow Q)$, $(P \rightleftarrows Q)$,

$\neg P$, $\neg Q$ are w.f.f.

If, I write, $\neg P \vee Q \longrightarrow$ Not a wff. ∵ parenthesis are missing

$[\neg P \vee Q) \longrightarrow (\vee P) \longrightarrow$ this is not. (∵ Not represent any thing

$\uparrow$ $\neg P$ is wff