School of Mathematics, Thapar Institute of Engineering & Technology, Patiala

UMA007 : Numerical Analysis

Assignment 7

Newton's Forward and backward Interpolation

1. Verify that the polynomials $P(x) = 5x^3 - 27x^2 + 45x - 21$, $Q(x) = x^4 - 5x^3 + 8x^2 - 5x + 3$ interpolate the data

x	1	2	3	4
y	2	1	6	47

   and explain why this does not violate the uniqueness part of the theorem on existence of polynomial interpolation.

2. The following data are given for a polynomial $P(x)$ of unknown degree.

x	0	1	2	3
$f(x)$	4	9	15	18

   Determine the coefficient of x^3 in $P(x)$ if all fourth-order forward differences are 1.

3. Construct the interpolating polynomial that fits the following data using Newton's forward and backward difference interpolation. Hence find the values of $f(x)$ at $x = 0.15$ and 0.45.

   (a)

x	0	0.1	0.2	0.3	0.4	0.5
$f(x)$	-1.5	-1.27	-0.98	-0.63	-0.22	0.25

   (b)

x	0	0.2	0.4	0.6
$f(x)$	15.0	21.0	30.0	51.0

4. Suppose that $f(x) = \cos x$ to be approximated on $[0, 1]$ by an interpolating polynomial on $n + 1$ equally spaced points. What step size h ensure that linear interpolation gives an absolute error of at most 10^{-6} for all $x \in [0, 1]$.

5. A fourth-degree polynomial $P(x)$ satisfies $\Delta^4 P(0) = 24$, $\Delta^3 P(0) = 6$, and $\Delta^2 P(0) = 0$, where $\Delta P(x) = P(x+1) - P(x)$. Compute $\Delta^2 P(10)$.