

Lecture-3

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Lecture 3: Numerical Analysis (UMA011)

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Error Analysis: Significant figures

Examples:

1) Add the following nos
$$\begin{array}{r} \checkmark 6.4546 \times 10^3 \\ 0.5483 \times 10^7 \\ \hline 0.00004546 \times 10^7 \\ 0.5483 \times 10^7 \\ \hline 0.54834546 \times 10^7 \rightarrow 0.5483 \times 10^7 \checkmark \end{array}$$

Error Analysis: Significant figures

Examples:

2) Subtract
$$\begin{array}{r} -0.5424 \times 10^{-99} \checkmark \rightarrow 4 \text{ s.d.} \\ +0.5452 \times 10^{-99} \checkmark \rightarrow 4 \text{ s.d.} \\ \hline -0.0028 \times 10^{-99} \\ = -0.2800 \times 10^{-101} \end{array}$$
loss of significance.
3) Multiply
$$\begin{array}{r} 0.1111 \times 10^74 \\ 0.2000 \times 10^80 \\ \hline 0.2222 \times 10^154 \\ = 0.2222 \times 10^153 \end{array}$$

Error Analysis: Loss of Significance

Loss of Significance:

one of most common error producing calculations involve the cancellation of s.d. due to subtraction of two nearly equal nos.
$$\begin{array}{r} 100007.23 \\ 100007.21 \\ \hline 0.02 \end{array}$$

→ 1 s.d.

Error Analysis: Loss of Significance

Examples:

If $x = 0.3721478693$ and $y = 0.3720230572$, then what is the relative error in the computation of $x - y$ using 5-decimal digits of accuracy?

$$\begin{aligned} X &= x - y = 0.3721478693 - 0.3720230572 \\ X &= 0.0001248121 \checkmark \rightarrow \text{Exact diff} \\ fl(x) &= 0.37215 \quad fl(y) = 0.37202 \\ X^* &= fl(x) - fl(y) = 0.00013 \rightarrow \text{App. diff} \\ A.E. &= |X - X^*| = |0.0001248121 - 0.00013| \\ &= 0.0000051879 \\ R.E. &= \frac{|X - X^*|}{|X|} = \frac{0.0000051879}{0.0001248121} = 4.15\% \checkmark \end{aligned}$$

1) Nested Arithmetic → evaluating the polynomial at a pt

2) Rationalizing $\frac{1}{\sqrt{x-y}} \times \frac{\sqrt{x+y}}{\sqrt{x+y}} = \frac{1}{\sqrt{x-y}} \times \frac{\sqrt{x+y}}{\sqrt{x+y}}$

3) Series Expansion.

Error Analysis: Finite-digit-Arithmetic

Finite-digit-Arithmetic:

$$\begin{aligned} n\text{-digits } x + y &\rightarrow m\text{ s.d.} \quad m < n \quad (32.79) \rightarrow 4 \text{ s.d.} \\ fl(x) + fl(y) &= m \text{ s.d.} \quad x = 3.7129 \\ n\text{ s.d. } x \oplus y \oplus z & \quad fl(x) = 3.71 \\ fl(x) + fl(y) + fl(z) & \quad fl(x) = 3.71 \\ \downarrow \quad \downarrow \quad \downarrow & \quad fl(x) + fl(y) + fl(z) = () \\ n\text{ s.d. } \quad u & \quad (3.71) + fl(z) = () \\ \sqrt{x} + \sqrt{y} + \sqrt{z} & \quad \sqrt{x} + \sqrt{y} + \sqrt{z} \\ (1.41 + 1.73) + 2.24 & \quad (1.41 + 1.73) + 2.24 \\ 3.14 + 2.24 &= 5.38 \checkmark \end{aligned}$$

Error Analysis: Nested Arithmetic

Example:

Evaluate $f(x) = 1.5 + 3.2x - 6.1x^2 + x^3$ at $x = 4.71$ using 3-digit arithmetic directly and with nesting.

$$\begin{aligned} 3\text{-digit directly} \quad f(4.71) &= 1.5 + 3.2(4.71) - 6.1(4.71)^2 + (4.71)^3 \\ &= 1.5 + 15.1 - 6.1(22.2) + 4.71(22.2) \\ &= 1.5 + 15.1 - 135 + 105 = -13.4 \checkmark \\ 3\text{-digit with nesting} \quad f(x) &= 1.5 + 3.2x - 6.1x^2 + x^3 \\ &= 1.5 + x(3.2 - 6.1x + x^2) \\ &= 1.5 + x(3.2 + x(-6.1 + x)) \\ f(4.71) &= 1.5 + 4.71(3.2 + 4.71(-6.1 + 4.71)) \\ &= 1.5 + 4.71(3.2 + 4.71(-1.39)) \\ &= 1.5 + 4.71(3.2 - 6.55) = 1.5 + 4.71(-3.35) \\ f(4.71) &= -14.263899 \checkmark \end{aligned}$$