Lecture 1: Numerical Analysis (UMA011)

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General Information

Books:

1. Richard L. Burden, J. Douglas Faires, and Annette M. Burden, Numerical Analysis, 10th edition, 2015.
2. K. Atkinson and W. Han, Elementary Numerical Analysis, 3rd edition, John Willey and sons, 2004.
3. Brian Bradie, A Friendly Introduction to Numerical Analysis, Pearson Publishers, 2006.
4. Steven C. Chapra and Raymond P. Canale, Numerical Methods for Engineers, McGraw-Hill Higher Education; 6th edition, 2010.

$x^2 - 9 = 0$

$x = \pm 3$ ✓

$x = 2.9$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

✓ $a_{100}x^{100} + a_{99}x^{99} - - - a_0 = 0$

98

✓ $\sin x - e^x = 0$

x

error

Advantage

↓

find roots

Numerical Techniques.

disadvantage

approximate

Error Analysis

Floating point representation of numbers

Let x be any real no., then it can be represented as an infinite sequence of the digits.

$$x = 0.a_1 a_2 a_3 a_4 \ldots\ldots a_n a_{n+1} \ldots\ldots$$

Exact No.

n- bit computer

$$\left(-2^{n-1},\ 2^{n-1}-1\right)$$

32-bit computer

$$\left(-2^{31},\ 2^{31}-1\right)$$

$$\left\{ \begin{array}{l} \frac{2}{3} = 0.66666\ldots \\ \frac{1}{2} = 0.50000\ldots \end{array} \right.$$

$$\frac{1}{3} = 0.3333\ldots$$

$$fl(x) = 0.a_1a_2 \ldots a_n$$

$$x = \underbrace{(0.a_1a_2 \ldots a_n a_{n+1} \ldots)}_{\text{mantissa}}{}_{10} \times 10^{e}$$

10 → base

e → exponent

$$fl(x) = (0.a_1a_2 \ldots a_n)_{10} \times 10^{e}$$

for e.g. $\quad 42.965 = 4\times10^{1} + 2\times10^{0} + 9\times10^{-1} + 6\times10^{-2} + 5\times10^{-3}$

$$\frac{4}{10}$$

$$= 10^{2}\left(\frac{4}{10} + \frac{2}{10^2} + \frac{9}{10^3} + \frac{6}{10^4} + \frac{5}{10^5}\right) = \underbrace{(0.42965)_{10}}_{\text{Normal form}} \times 10^{2}$$

$$-0.00234 = -\left(2\times10^{-3} + 3\times10^{-4} + 4\times10^{-5}\right)$$

$$= -10^{-2}\,(0.234) \checkmark \longleftrightarrow \text{Normal form}$$

$$= -0.0234\times10^{-1} \checkmark$$

$$= -0.00234\times10^{0} \checkmark$$

Error Analysis

Normal form

A non-zero floating pt. no. is in the normal form if the value of the mantissa lies in $(-1, -0.1]$ or $[0.1, 1)$ ie $(0.a_1 a_2 a_3 \cdots a_n)_{10} \times 10^e$

$$0 \le a_i \le 9, \quad 2 \le i \le n$$

$$a_1 \geq 1$$

There are m & M s.t. $-m \le e \le M$

Error Analysis

Overflow and Underflow

An overflow is obtained when a no. is too large to fit into the floating pt. system in use ie $e > M$

$\frac{8}{3} = 2.6666 - - -$

An underflow is obtain " " " small to " " " ie $e < -m$

-0.0000000000002