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Tut-8

Q5 $\int_0^2 \frac{1}{x+4}\,dx$ $\quad 10^{-4}$

$n, h = ?$

Trapezoidal

$$\left|\frac{h^3}{12} f''(\xi)\right| \le 10^{-4}$$

$$f(x) = \frac{1}{x+4}, \quad f'(x) = \frac{-1}{(x+4)^2}$$

$$f''(x) = \frac{2}{(x+4)^3}$$

$$\left|\frac{h^3}{12} \cdot \frac{2}{(\xi+4)^3}\right| \le 10^{-4}$$

$$\left|\frac{h^3}{\cancel{12}\,6} \cdot \frac{\cancel{2}}{(4)^3}\right| \le 10^{-4}$$

$$|h^3| \le 384 \times 10^{-4}$$

$$h^3 \le 0.0384$$

$$h \le 0.3373731$$

$$n = \frac{b-a}{h} = \frac{2}{h}$$

$$n \ge 5.928$$

$$\boxed{n = 6} \quad \boxed{h = 0.33}$$

Q11 $t = \int_{10}^{5} \frac{10}{-v\sqrt{v}}\,du$ $\quad n = 2$

Comp. Simpson $\quad h = \frac{10-5}{2} = 2.5$

$$t = \frac{2.5}{3}\left[\frac{10}{(-10)\sqrt{10}} + 4 \times \frac{10}{-(7.5)(\sqrt{7.5})} + \frac{10}{(-5)\sqrt{5}}\right]$$

$= \frac{2.5}{3}\left[\frac{1}{\sqrt{10}} + \right.$

$= \frac{2.5}{3}\left[0.31623 + 1.947458 + 0.894427\right]$

$= \underline{2.6317625}$

Trap. $h\left(\frac{f(x_0)+f(x_1)}{2}\right)$

Error $= -\frac{h^3}{12} f''(\xi)$

Simpson. $\frac{h}{3}\left(f(x_0) + 4 f(x_1) + f(x_2)\right)$

Error $= -\frac{(b-a)^5}{120 \times 4!} f^{(4)}(\xi)$

Comp Trap $\frac{h}{2}\left(f(x_0) + f(x_n) + 2\sum_{i=1}^{n-1} f(x_i)\right)$

Err. $= -\frac{h^2 (b-a)}{12} f''(\xi)$

Comp. Simp $\frac{h}{3}\left(f(x_0) + f(x_n) + 4 \times \text{odd terms} + 2 \times \text{even}\right)$

Err. $= -\frac{(b-a)}{180} h^4 f^{iv}(\xi)$

Q1 (a) $I = \int_{-.25}^{.25} (\cos x)^2 \, dx$

Trap. $= \frac{0.5}{2}\left[\cos(-.25) + \cos(.25)\right] = \underline{.484456}$

.2448563824

Simp $= \frac{0.5}{2\times3}\left[\cos(.25)+\cos(-.25)+4\cos(0)\right]$

$= \frac{.98964}{2} = 0.49482$

(b) $\int_e^{e+1} \frac{1}{x\ln x}\,dx$

trap. $= \frac{1}{2}\left[\frac{1}{e\ln e}+\frac{1}{(e+1)\ln(e+1)}\right]$

$= \frac{1}{2}\left[\frac{1}{e}+\frac{1}{(e+1)(1.313)}\right]$

$= (0.5)(0.36788 + .204789)$

$= .2863345$

Simp. $= \frac{1}{2\times3}\left[\frac{1}{e\ln e}+\frac{1}{(e+1)\ln(e+1)}+4\cdot\frac{1}{(e+\frac{1}{2})\ln(e+\frac{1}{2})}\right]$

$= \frac{1}{2\times3}(0.36788 + 0.204789 + 1.06335)$

$= \frac{0.54534}{2} = .27267$

(c) $\int_{-.25}^{.25}\cos^2 x\,dx = \int_{-.25}^{.25}\frac{1+\cos 2x}{2}\,dx = \left[\frac{x}{2}+\frac{\sin 2x}{4}\right]_{-.25}^{.25}$

$= (.25 + .23971)$

$= .48971$

$\int_e^{e+1}\frac{1}{x\ln x}\,dx = \left[\ln(\ln x)\right]_e^{e+1} = .272514$

AE. (a) Error bound (trap.) $= -\frac{h^3}{12}f''(\xi) = \frac{(.5)^3(1)}{12} = .01041\overline{7}$

$$\text{Ad. err. (trap)} = |0.48971 - .48446| = \underline{0.00525}$$

$$\text{Error bound (Simp)} = \frac{-(b-a)^5}{120 \times 4!} f^4(\xi)$$

$$= \frac{(0.5)^5}{120 \times 4!} \times (1) = \underline{1.08 \times 10^{-5}}$$

$$\text{Actual error} = |1.48482 - .48971| = \underline{5.11 \times 10^{-3}}$$

(b)
$$\underset{(trap)}{E.B} = \frac{(1)^3}{12} \max\left[\frac{d^2}{dx^2}\left(\frac{1}{x\ln x}\right)\right]$$

$$= \frac{1}{12} \max\left[\frac{d}{dx}\left(\frac{-(1+\ln x)}{(x(\ln x))^2}\right)\right]$$

$$= \frac{1}{12} \max\left[\frac{(x\ln x)^2(-1/x) + (1+\ln x)^2\, 2(x\ln x)}{(x\ln x)^4}\right]$$

$$= \frac{1}{12} \max\left[\frac{2x\ln x(1+\ln x)^2 - x(\ln x)^2}{(x\ln x)^4}\right]$$

$$= \frac{1}{12}(0.34851) = \underline{0.0290425}$$

$$\underset{(trap)}{\text{Ad error}} = |.272514 - .2863345| = \underline{0.01382}$$

$$\underset{(Simp)}{E.B} = \frac{(1)^5}{120 \times 4!} \max\left[\frac{24\ln^4 x + 90\ln^3 x + 70\ln^2 x + 60\ln x + 24}{x^5 \ln^5 x}\right]$$

$$= \frac{1}{120 \times 4!}(1.536252) = \underline{5.334 \times 10^{-4}}$$

$$\underset{(Simp)}{\text{Ad. err.}} = |.272514 - .272671| = \underline{1.56 \times 10^{-4}}$$

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Q 2

$$\int_0^2 f(x)\,dx = c_0 f(0) + c_1 f(1) + c_2 f(2)$$

$$\int_0^2 1 \cdot dx = c_0 + c_1 + c_2 = 2 \quad -(1)$$

$$\int_0^2 x\,dx = c_1 + 2c_2 = 2 \quad -(2)$$

$$\int_0^2 x^2\,dx = c_1 + 4c_2 = \frac{8}{3} \quad -(3)$$

$$(3) - (2) \quad 2c_2 = \frac{2}{3} \Rightarrow \boxed{c_2 = \frac{1}{3}}$$

$$\boxed{c_1 = \frac{1}{3}} \qquad \boxed{c_0 = \frac{4}{3}}$$

Q 3

$$\int_0^1 f(x)\,dx = c_0 f(0) + c_1 f(x_1)$$

$$\int_0^1 1 \cdot dx = c_0 + c_1 = 1 \quad -(1)$$

$$\int_0^1 x\,dx = c_1 x_1 = 1/2 \quad -(2)$$

$$\int_0^1 x^2\,dx = c_1 x_1^2 = 1/3 \quad -(3)$$

$$\frac{1}{2} \cdot x_1 = \frac{1}{3} \Rightarrow \boxed{x_1 = \frac{2}{3}}$$

$$c_1 \cdot \frac{2}{3} = \frac{1}{2} \Rightarrow \cancel{c_1 = \frac{4}{3}} \quad \boxed{c_1 = \frac{3}{4}}$$

$$\boxed{c_0 = \frac{1}{4}}$$

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Q4 $4x^2 + 9y^2 = 36 \Rightarrow \frac{x^2}{9} + \frac{y^2}{4} = 1$

$$y = \sqrt{\frac{36 - 4x^2}{9}} = f(x)$$

$$f'(x) = \frac{1}{3} \cdot \frac{1}{2} \frac{(-8x)}{\sqrt{36-4x^2}} = \frac{-4x}{3\sqrt{36-4x^2}}$$

$$I = \int_a^b \sqrt{1 + \frac{16x^2}{9(36-4x^2)}} \, dx$$

$$I = \int_{-2}^{2} \sqrt{1 + \frac{4x^2}{9(9-x^2)}} \, dx$$

Simpson's

subi = 4 $\therefore h = \frac{(4)}{4} = 1$

$$I = \frac{1}{3}\left[f(-2) + 4f(-1) + 2f(0) + 4f(1) + f(2)\right]$$

$$= \frac{1}{3}\left[1.16428 + 1.6274 \times 8 + 2 + 1.16428\right]$$

$$= \underline{4.182587}$$

subi = 6 $h = 4/6 = 2/3$

$$\cancel{I = \frac{2}{9}\left[f(-2) + 4f(-1) + 2f(0) + 4f(1) + f(2)\right]}$$

$$= \frac{2}{9}\left[f(-2) + 4f(-1.33) + 2f(-.67) + 4f(0) + 2f(.67) + 4f(1.33) + f(2)\right]$$

$$= \frac{2}{9}\left[1.16428 \times 2 + 8 \times 1.05328 + 4 \times 1.011478 + 4\right]$$

$$= \frac{8.355872}{2} = \underline{4.1779}$$

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Trapezoidal

subi = 4 $\quad h = 1$

$$I = \frac{1}{2}\left[f(-2) + 2f(-1) + 2f(0) + 2f(1) + f(2)\right]$$

$$= \frac{1}{2}\left[1.16428 \times 2 + 4 \times 1.0274 + 2\right]$$

$$= 4.21908$$

subi = 6 $\quad h = 2/3$

$$I = \frac{2}{6}\left[f(-2) + 2f(-1.33) + 2f(-.67) + 2f(0) + 2f(.67) + 2f(1.33) + f(2)\right]$$

$$= \frac{1}{3}\left[2 \times 1.16428 + 4 \times 1.05328 + 4 \times 1.011478 + 2\right]$$

$$= 4.195864$$

Q 6

(a) $f(x) = \cos x - x^2$

$f'(x) = -\sin x - 2x$

$$x' = x + \frac{\cos x - x^2}{2x + \sin x} = \frac{x^2 + x\sin x + \cos x}{2x + \sin x}$$

$x_0 = 0.5$

$x_1 = 0.9242$

$x_2 = 0.82910$

$x_3 = 0.82415$

$x_4 = 0.82413$

$$A = 4\int_0^{.82413} (\cos x - x^2)^{1/2}\, dx$$

(b) $h = \frac{(.82413)}{6} = 0.137355$

$$A = 4 \times \frac{0.137355}{2}\left[f(0) + 2\left(f(.137355) + f(.27471) + f(.412065) + f(.54942) + f(.686775)\right) + f(.82413)\right]$$

$$A = (.27471)\left[1 + 2(.98576 + .94183 + .864 + .74227 + .549213) + .002347\right]$$
$$= \underline{2.51867}$$

Q7 $b - a = 84$, $h = 6$ $\Rightarrow n = 14$

distance = speed × time = $\cancel{dt}$ dt

$v = \frac{ds}{dt} \Rightarrow s = \int_a^b v\,dt$

Comp. Simp.

$$I = \frac{6}{3}\left[f(0) + f(84) + 4[f(6) + f(18) + f(30) + f(42) + f(54) + f(66) + f(78)] + 2[f(12) + f(24) + f(36) + f(48) + f(60) + f(72)]\right]$$

$$= 2[124 + 123 + 4(134 + 156 + 133 + 109 + 85 + 89 + 116) + 2(148 + 147 + 121 + 99 + 78 + 104)]$$

$$= 2(247 + 3288 + 1394) = \boxed{9858\ ft}$$

Q8 $I = \int_0^1 \frac{\sin x}{2+x}\,dx = I_1 + I_2$

$$= \int_0^{1/2} \frac{\sin x}{2+x}\,dx + \int_{1/2}^1 \frac{\sin x}{2+x}\,dx$$

Substitute

$x = \frac{t+1}{4}$ & $x = \frac{z+3}{4}$ in I_1 & I_2

$$I_1 = \int_{-1}^{1} \frac{\sin\left(\frac{t+1}{4}\right)}{1+\frac{t+1}{4}} \cdot \frac{dt}{4} = \int_{-1}^{1} \frac{\sin\left(\frac{t+1}{4}\right)}{t+5}\, dt$$

$$= \cancel{f\left(\frac{-1}{\sqrt{3}}\right) + f\left(\frac{1}{\sqrt{3}}\right)} = \cancel{0.105466 + 0.384197}$$

$$\cancel{= \underline{0.489663}}$$

$$I_2 = \int_{-1}^{1} \frac{\sin\left(\frac{z+3}{4}\right)}{1+\frac{z+3}{4}} \cdot \frac{dz}{4} = \int_{-1}^{1} \frac{\sin\left(\frac{z+3}{4}\right)}{z+7}\, dz$$

$$\cancel{I = I} \quad = f\left(\frac{-1}{\sqrt{3}}\right) + f\left(\frac{1}{\sqrt{3}}\right)$$

$$I = I_1 + I_2$$

$$= (0.023847 + 0.068885)$$

$$+ (0.08864 + 0.102911)$$

$$= \underline{0.284283}$$