Joint probability distribution 2

Joint probability distribution_2

A function $f(x,y)$ is a joint probability mass function of the random variables X and Y if

1. $\underbrace{f(x,y) \geq 0}$ for all (x,y)
2. $\sum_y \sum_x f(x,y) = 1$
3. $P(X=x, Y=y) = f(x,y)$

(discrete case)

Example The joint distribution of X and Y is given by

$$f(x,y) = \frac{x+y}{21} \; ; \; \underline{x = 1,2,3} \; ; \; \underline{y = 1,2}$$

1) Find Marginal distribution of X and Y,
2) Find conditional distribution of Y given X=3.

Ans

1)

X\Y	1	2	Total
1	2/21	3/21	5/21
2	3/21	4/21	7/21
3	4/21	5/21	9/21
total	9/21	12/21	

x	1	2	3
$p(x)$	5/21	1/3	9/21

marginal distribution of X

$$p(x) = \sum_y f(x,y)$$

y	1	2	

marginal distribution of Y $\rightarrow$

$P(y)$	$\frac{9}{21}$	$\frac{12}{21}$	

$$f(y) = \sum_x f(x,y)$$

2) Conditional distribution of Y given $X=3$, $P(Y/X=3) = \frac{P(3,y)}{P(x=3)}$

$Y/X=3$	1	2
conditional distribution $P(Y/X=3)$	$\frac{4/21}{9/21}$	$\frac{12}{21}$

$= \frac{4}{9}$

Example

$$f(x,y) = \begin{cases} \frac{2}{3} y^2 e^{-xy} & \text{for } (x,y) \in [0,\infty) \times [1,2] \\ 0 & \text{otherwise} \end{cases}$$

find conditional density function of X given $Y=2$

Soln The conditional density function

$$f(x/y=2) = \frac{f(x,2)}{f_Y(2)}$$

$\quad\quad = \frac{2}{3} \cdot 4 e^{-2x} =$

$$f(x,2) = \frac{}{3} \cdots$$

$$f_Y(y) = \int_{-\infty}^{\infty} f(x,y)\,dx = \int_0^{\infty} \frac{2}{3} y^2 e^{-xy}\,dx$$

$$= \left[-\frac{2}{3} y e^{-xy}\right]_0^{\infty}$$

$$= \frac{2}{3} y$$

$$f_Y(2) = \frac{4}{3}$$

Therefore the conditional probability density function $f(x/y=2) = \begin{cases} \frac{2/3\, 2^2 e^{-2x}}{4/3} = 2e^{-2x} & \text{if } x = [0,\infty) \\ & x [1,2] \\ 0 & \text{otherwise} \end{cases}$

<u>Ex</u> X and Y with j.d.f.

ⓐ $f(x,y) = \begin{cases} 6\, e^{-2x} e^{-3y} & 0 < x < \infty,\ 0 < y < \infty \\ 0 & \text{otherwise} \end{cases}$

are X and Y independent?

ⓑ $f(x,y) = \begin{cases} 24xy & 0 < x < 1,\ 0 < y < 1 \\ & 0 < x+y < 1 \\ 0 & \text{otherwise} \end{cases}$

Sol^n — ⓐ ...

Sol. (a) $f_X(x) = \int_0^{\infty} 6 e^{-2x} e^{-3y} \, dy$ for $0 < x$

$$= 2 e^{-2x}$$

and $f_Y(y) = 3 e^{-3y}$

We see that $f_{XY}(x,y) = f_X(x) f_Y(y)$ for all x, y

Therefore X and Y are independent

(b) Since the domain

$0 < x < 1$, $0 < y < 1$, $0 < x + y < 1$

cannot be written in the form $A \times B$ for any intervals A and B, therefore X and Y are dependent.