

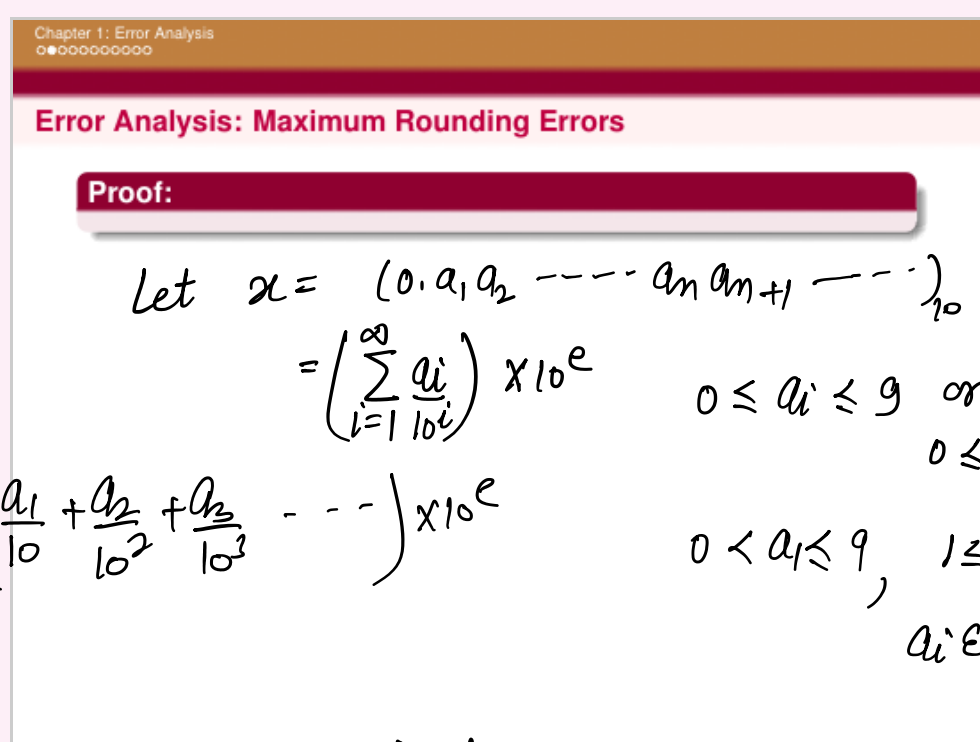
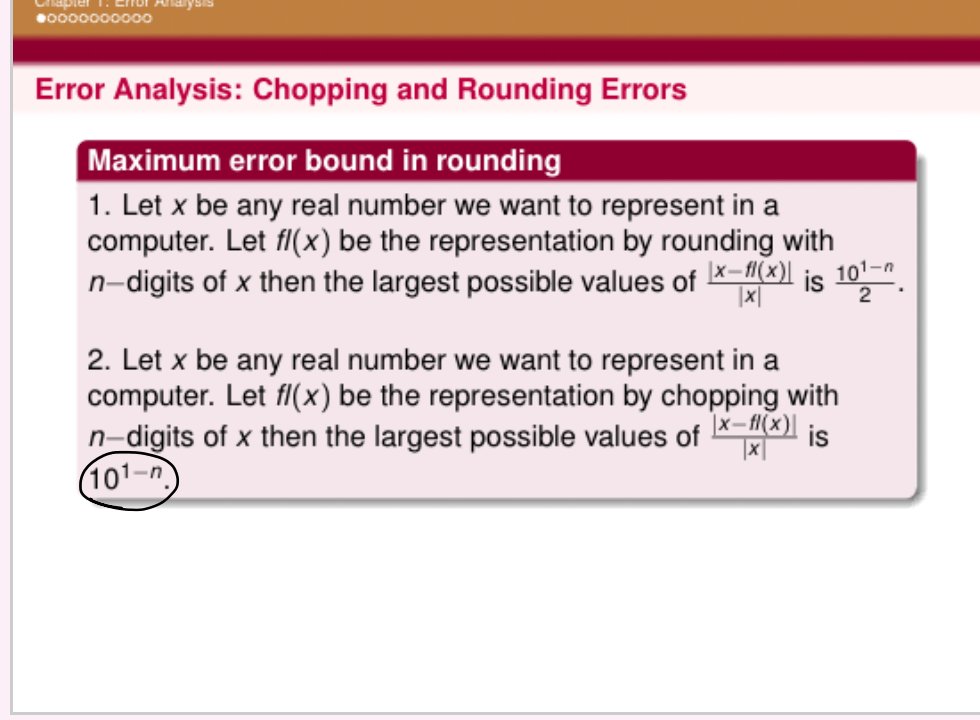
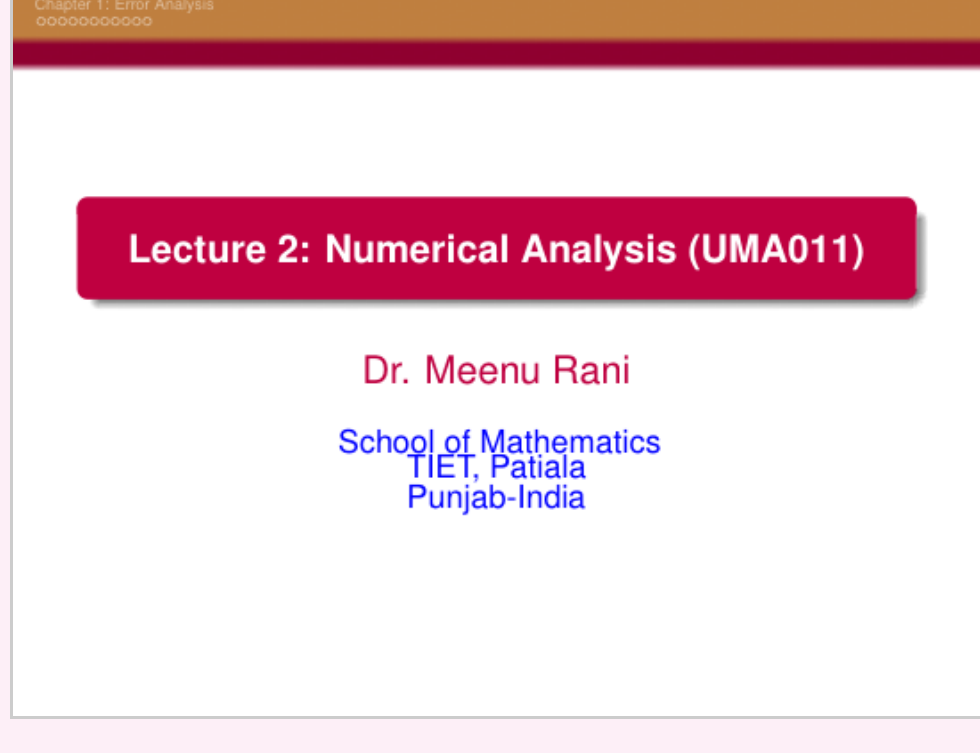
# lecture-2

Tuesday, July 26, 2022

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lecture-2



let us approximate  $x$  up to  $n$ -digit with rounding then

$$fl(x) = \begin{cases} (0.a_1a_2 \dots a_n)_{10} \times 10^e & ; a_{n+1} < 5 \\ ((0.a_1a_2 \dots a_n) + (0.\underbrace{00\dots0}_{n+1}))_{10} \times 10^e & ; 5 \leq a_{n+1} \leq 9 \end{cases}$$

$0.6359\overline{87}$   
 $0.63599$

$$= \begin{cases} \left(\sum_{i=1}^n \frac{a_i}{10^i}\right) \times 10^e & ; a_{n+1} < 5 \\ \left(\sum_{i=1}^n \frac{a_i}{10^i} + \frac{1}{10^n}\right) \times 10^e & ; 5 \leq a_{n+1} \leq 9 \end{cases}$$

A.E. Case I  $a_{n+1} < 5$

$$|x - fl(x)| = \left| \left( \sum_{i=n+1}^{\infty} \frac{a_i}{10^i} \right) \times 10^e \right|$$

$a < b$   
 $a+c < b+c$

$$= \left| \frac{a_{n+1}}{10^{n+1}} + \sum_{i=n+2}^{\infty} \frac{a_i}{10^i} \right| \times 10^e$$

$a < 5$   
 $a+c < 5+c$

$$< \left| \frac{5}{10^{n+1}} + \sum_{i=n+2}^{\infty} \frac{a_i}{10^i} \right| \times 10^e$$

$a_{n+1} < 5$

$$\leq \left| \frac{4}{10^{n+1}} + \sum_{i=n+2}^{\infty} \frac{9}{10^i} \right| \times 10^e$$

or  
 $a_{n+1} \leq 4$

$$= \left| \frac{4}{10^{n+1}} + 9 \left( \sum_{i=n+2}^{\infty} \frac{1}{10^i} \right) \right| \times 10^e$$

$$\frac{a}{1-x} = \left| \frac{4}{10^{n+1}} + 9 \frac{\frac{1}{10^{n+2}}}{1 - \frac{1}{10}} \right| \times 10^e$$

$$= \left| \frac{4}{10^{n+1}} + 9 \times \frac{1}{10^{n+2}} \times \frac{10}{9} \right| \times 10^e$$

$\frac{10}{2} \times \frac{1}{10^{n+2}} \times 10^e$

$$= \left| \frac{4}{10^{n+1}} + \frac{10}{10^{n+2}} \right| \times 10^e = \left| \frac{5}{10^{n+1}} \right| \times 10^e = \frac{1}{2} \times 10^{e-n}$$

Case II  $5 \leq a_{n+1} \leq 9$

$$A.E. = |x - fl(x)| = \left| \sum_{i=n+1}^{\infty} \frac{a_i}{10^i} - \frac{1}{10^n} \right| \times 10^e$$

$|a-b|$   
 $= |b-a|$

$$= \left| \frac{1}{10^n} - \sum_{i=n+1}^{\infty} \frac{a_i}{10^i} \right| \times 10^e$$

$5 \leq a_{n+1}$   
 $-5 \geq -a_{n+1}$

$$= \left| \frac{1}{10^n} - \frac{a_{n+1}}{10^{n+1}} - \sum_{i=n+2}^{\infty} \frac{a_i}{10^i} \right| \times 10^e$$

$\geq -a_{n+1} \leq -5$

$$\leq \left| \frac{1}{10^n} - \frac{5}{10^{n+1}} - \sum_{i=n+2}^{\infty} \frac{a_i}{10^i} \right| \times 10^e$$

$|a-b-c|$   
 $\leq |a-b|$

$$\leq \left| \frac{1}{10^n} - \frac{5}{10^{n+1}} \right| \times 10^e$$

$$= \left| \frac{10-5}{10^{n+1}} \right| \times 10^e = \frac{5}{10^{n+1}} \times 10^e$$

$$= \frac{10^{e-n}}{2}$$

$$R.E. = \frac{|x - fl(x)|}{|x|} \leq \frac{10^{e-n}}{2|x|}$$

$$|x| = |(0.a_1a_2 \dots a_n a_{n+1} \dots)_{10} \times 10^e|$$

$$\geq |(0.100 \dots 0)_{10} \times 10^e|$$

$$= (0.1) \times 10^e = \frac{1}{10} \times 10^e$$

$|x| \geq 10^{e-1} \Rightarrow \frac{1}{|x|} \leq \frac{1}{10^{e-1}}$

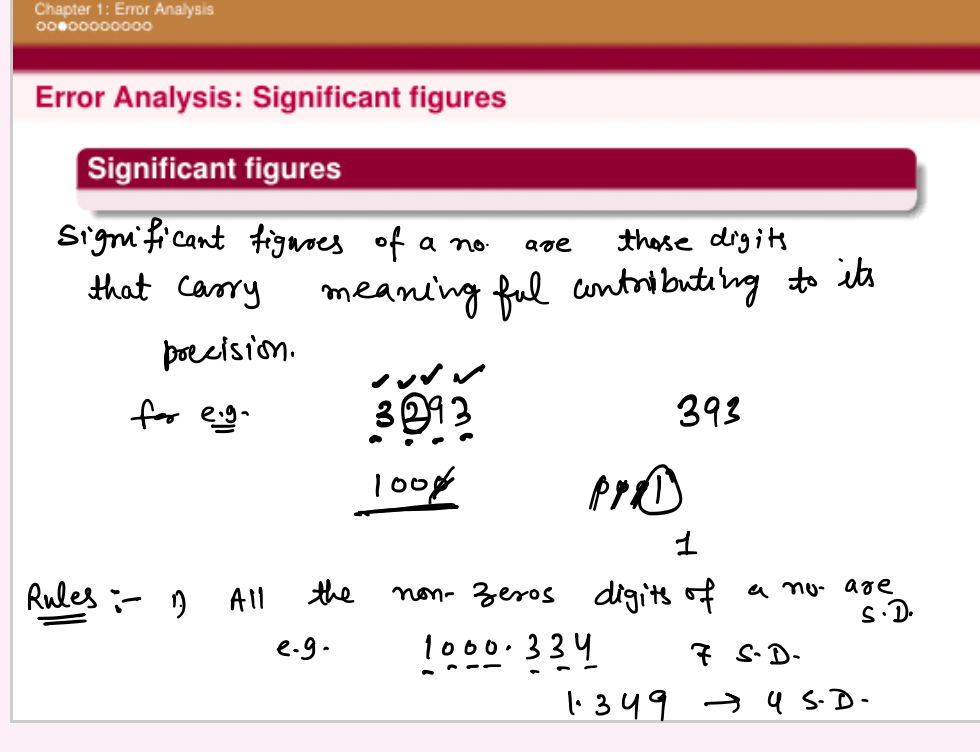
R.E.

$$\frac{|x - fl(x)|}{|x|} \leq \frac{10^{e-n}}{2} \left( \frac{1}{|x|} \right)$$

$$= \frac{10^{e-n}}{2} \times \frac{1}{10^{e-1}}$$

$$= \frac{10^{1-n}}{2}$$

$$R.E. \leq \frac{10^{1-n}}{2}$$



2) Zeros between non-zero digits are S.D.  
100009, 7 S.D.

3) Leading zeros to the left of the first non-zero digits are not S.D.  
000001234 -> 4 S.D.

4) Trailing zeros that are also to the right of a decimal pt. in a no are S.D.

for e.g. 100000 -> 6 S.D.

100 = 1 x 10^2 (1.00 x 10^2) 3 S.D.

100 = 1 x 10^2 -> 1 S.D. 1.0 x 10^2 2 S.D.

100 = 1 x 10^2 -> 1 S.D. 1 x 10^2 -> 1 S.D.