School of Mathematics, Thapar Institute of Engineering & Technology, Patiala

UMA007 : Numerical Analysis
Assignment 3
Roots of Non-linear Equations-B

1. Use secant method to find solutions accurate to within 10^{-3} for the following problems.

(a) $-x^3 - \cos x =$ with $x_0 = -1$ and $x_1 = 0$.

(b) $x - \cos x = 0, \ \ x \in [0, \pi/2]$.

2. Use Newton's method to find solutions accurate to within 10^{-3} to the following problems.

(a) $x - e^{-x} = 0$ for $0 \le x \le 1$.

(b) $2x \cos 2x - (x-2)^2 = 0$ for $2 \le x \le 3$ and $3 \le x \le 4$.

3. A calculator is defective: it can only add, subtract, and multiply. Use the equation $1/x = 1.732$, the Newton's Method, and the defective calculator to find $1/1.732$ correct to 4 decimal places.

4. The function $f(x) = \sin x$ has a zero on the interval $(3, 4)$, namely, $x = \pi$. Perform three iterations of Newton's method to approximate this zero, using $x_0 = 4$. Determine the absolute error in each of the computed approximations. What is the apparent order of convergence?

5. Use Newton's method to approximate, to within 10^{-4}, the value of x that produces the point on the graph of $y = x^2$ that is closest to $(1, 0)$.

6. (a) Apply Newton's method to the function

$$f(x) = \begin{cases} \sqrt{x}, & x \ge 0 \\ -\sqrt{-x}, & x < 0 \end{cases}$$

with the root $\alpha = 0$. What is the behavior of the iterates? Do they converge, and if so, at what rate?

(b) Do the same but with

$$f(x) = \begin{cases} \sqrt[3]{x^2}, & x \ge 0 \\ -\sqrt[3]{x^2}, & x < 0 \end{cases}$$

7. Use Newton's method and the modified Newton's method to find a solution of

$$\cos(x + \sqrt{2}) + x(x/2 + \sqrt{2}) = 0, \quad \text{for } -2 \le x \le -1$$

accurate to within 10^{-3}.

8. Apply the Newton's method with $x_0 = 0.8$ to the equation $f(x) = x^3 - x^2 - x + 1 = 0$, and verify that the convergence is only of first-order. Further show that root $\alpha = 1$ has multiplicity 2 and then apply the modified Newton's method with $m = 2$ and verify that the convergence is of second-order.

9. The function $f(x) = \tan \pi x - 6$ has a zero at $\dfrac{\arctan 6}{\pi} \approx 0.447431543$. Use ten iterations of each of the following methods to approximate this root. Which method is most successful and why?

(a) Bisection method in interval [0,1].

(b) Secant method with $x_0 = 0$ and $x_1 = 0.48$.

(c) Newton's method with $x_0 = 0.4$.

10. Suppose α is a zero of multiplicity m of f, where $f^{(m)}$ is continuous on an open interval containing α. Show that the fixed-point method $x = g(x)$ with the following g has second-order convergence:

$$g(x) = x - m\frac{f(x)}{f'(x)}.$$

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11. It costs a firm $C(q)$ dollars to produce q grams per day of a certain chemical, where

$$C(q) = 1000 + 2q + 3q^{2/3}.$$

The firm can sell any amount of the chemical at \$4 a gram. Find the break-even point of the firm, that is, how much it should produce per day in order to have neither a profit nor a loss. Use the Newton's method and give the answer to the nearest gram.

12. The circle below has radius 1, and the longer circular arc joining A and B is twice as long as the chord AB. Find the length of the chord AB, correct to four decimal places. Use Newton's method.