1. Lexiographic ordering

2. Hasse diagram

3. Lattices

4. Boolean Algebra

Procedure for lexicographic ordering

We have to find largest 'i' such that following inequality should hold's true

$$\boxed{K_{i-1} < K_i}$$

After Achieving first step, we will find another largest 'j' other than 'i' such that

$$\boxed{K_{i-1} < K_j}$$

Now Interchange K_{i-1} & K_j

Just do the Reverse ordering of a given no. or digit i.e $K_i, K_{i+1}, \ldots, K_n$

Ques. find the permutation next to 32574

Sol^n^: First we have to find the largest 'i' Such that $K_{i-1} < K_i$ — (1)

K_1	K_2	K_3	K_4	K_5
3	2	5	7	4

here largest 'i' mean's $i = 5$

i.e $K_5 = 4$.

$\therefore$ 'i-1' = $5-1 = 4$ i.e $K_4 = 7$

Now inequality (1) $K_{i-1} < K_i$

$7 \not< 4$ doesn't hold true

Now for $i=5$ inequality does n't hold true

Take $i=4$. ; it mean $K_4 = 7$.

$i-1 = 4-1 = 3$ i.e $K_3 = 5$

Now inequality (1) hold true as

i.e $K_{i-1} < K_i$ $5 < 7$ ✓

Second step :- choose largest 'j' Again select $j=5$.

it means $K_j = 4$ (for $j=5$)

Now $i-1 = 3$ i.e $K_3 = 5$.

condition $K_{i-1} < K_j$

$5 \not< 4$ does not hold true

Now select $j=4$, $K_j = K_4 = 7$

$i-1 = 3$; $K_3 = 5$

i.e $\boxed{K_{i-1} < K_j \Rightarrow 5 < 7}$ ✓

Now $\boxed{i=4,\ J=4}$

Next step: $K_{i-1} = K_3 = 5$ ⌉ interchange
$K_j = K_4 = 7$ ⌋

i.e $3\,2\,5\,7\,4 \xrightarrow{\text{interchange}} 3\,2\,7\,5\,4.$

Last step: Reverse ordering the digit i.e $\overset{K_1}{3}\overset{K_2}{2}\overset{K_3}{7}\overset{K_4}{5}\overset{K_5}{4}$ Reverse ordering.

for $K_i, K_{i+1}, K_{i+2}, \ldots, K_n$

Here $i=4$, $i+1=5$

i.e $\boxed{3\,2\,7\,4\,5}$ Ans

Hasse Diagram: Representation of Partial ordered set (POSET).

Steps for Creating Hasse Diagram.

(1) Create vertex for every Element.

(2) If $\boxed{a\,R\,b}$, then draw edge from a to b

(3) Remove self loop also transititive edges from vertexes.

Note: Hasse Diagram is in always upward direction.

(4)

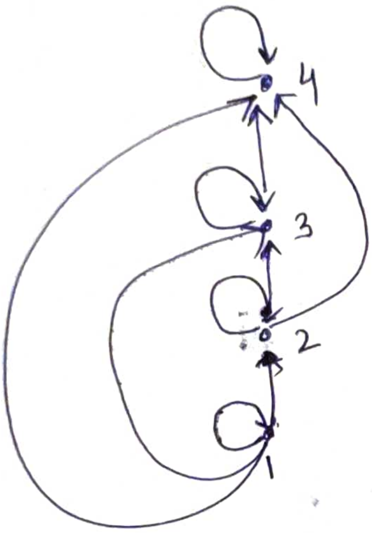
General Question) Example : Construct the Hasse Diagram for

$(\{1,2,3,4\} ; \leq)$

elements. Relations

check first $1 \leq 1,\ 2 \leq 2,\ 3 \leq 3,\ 4 \leq 4$ (holds true)

$\therefore$



Hasse Diagram is always in upward direction

First Hasse Diagram But not final.

Also
$$\left\{\begin{matrix} 1 \leq 2, & 1 \leq 3, & 1 \leq 4 \\ 2 \leq 3, & 2 \leq 4 & \\ 3 \leq 4 & & \end{matrix}\right\}$$

Now we have to Remove $\boxed{\text{self loop}}$ & $\boxed{\text{transitive edges}}$

POSET ⟶ R, A, T ✓✓

↓ then we no need to show in Hasse Diagram

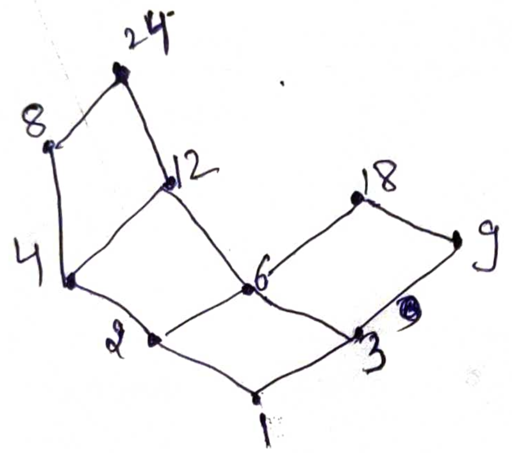
Final Hasse Diagram

Another examples for Hasse

Q $\{1, 2, 3, 4, 6, 8, 9, 12, 18, 24\}$ such that (a, b) $a | b$ or a divides b

Sol^n $A = \{1, 2, 3, 4, 6, 8, 9, 12, 18, 24\}$ such $a | b$. Relation

$\boxed{\text{1 divides by all}}$



Lattices

Lattices: A partial ordered set (POSET) in which every ~~ele~~ pair of elements has both least upper bound & as well greatest lower bound is known as Lattice.

NOTE !- # If in POSET, each & every pair of elements have LUB (supremum) exists then that POSET is Join ($\vee$ Symbol) - Semi Lattice

$\forall\, x, y \in S,\ LUB(x,y) \neq \phi$

If in POSET, each & every pair of elts have GLB (infimum) exists than that POSET is known as Meet ($\wedge$ Symbol) - Semi Lattice

Logic

$\forall\, x, y \in S,\ GLB(x,y) \neq \phi$

upper bound $\rightarrow$ Least value

Lower bound $\rightarrow$ Maximum value

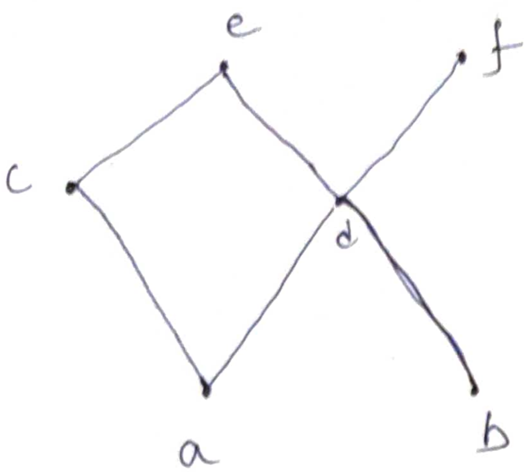
case (i) <u>Upper Bound</u> :- These are the elements that are greater than or equal to all the elements in a Subset 'A' of POSET 'S'.

<u>LOWER Bound</u> :- In this case, it contains all the elements that are lower than or equal to all the elements in a Subset 'A' of POSET 'S'.

<u>Least upper bound</u> :- It is one of the upper bound elements which is less than all the other upper bound elements.

<u>Greatest lower bound</u> :- It is one of the lower bound elements which is greater than all the lower bound elts.

Practice Hasse Diagram's for LUB, GLB.



Cases

(a) A = {c, d}

UB = e,

Min. ← LUB = e

LB = a

Max. ← GLB = a

(b) A = {a, b}

$\boxed{\text{UB}}$ = e, f, d

L$\boxed{\text{UB}}$ = d

LB = ϕ

GLB = ϕ

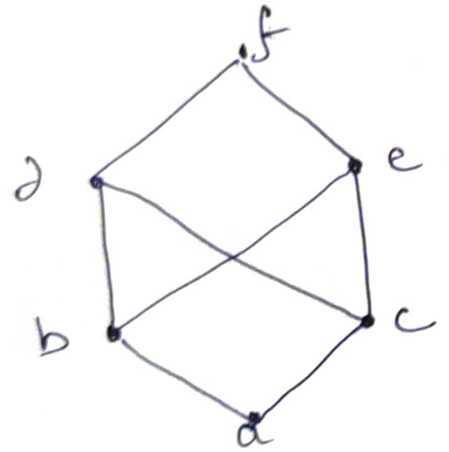
(c) A = {e, f},

UB = ϕ

LUB = ϕ

LB = a, d, b

GLB = d



(a). A = {d, e}

UB = f

LUB = f

LB = b, a, c

GLB = ϕ

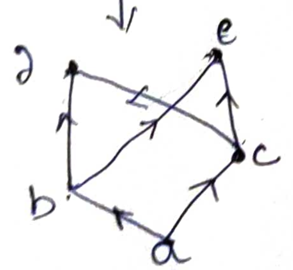
A = {b, c}

UB = d, f, e

LUB = ϕ

LB = a

GLB = a



b c
same level
but no connection
b/w them

Maximal & Minimal Elements: Let (A, ≤) be a POSET. An element M∈A is called a maximal element of A if there is no element x in A such that M<x.

An element m∈A is called a minimal element of A if there is no element x in A such that x<m.

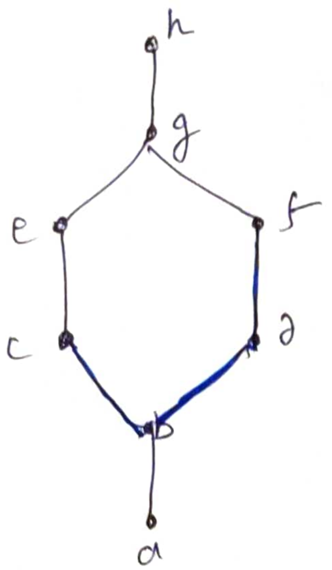
Consider a Poset (S, R). (8)

Def :- The Poset (S, R) is called a lattice, iff it is a meet semilattice & a join semilattice.

meet → GLB $(\wedge)$

join → LUB $(\vee)$

Example from Hasse or Poset, we need to check whether it is Lattice or not ??



check this Hasse diagram, whether it is Lattice or not ??

Sol^n. $\forall x, y \in S \Rightarrow GLB \neq \phi$ (meet)

Illy $\forall x, y \in S \Rightarrow LUB \neq \phi$ (Join).

consider all the pairs {∵ check for all elements}.

incomparable set elts.

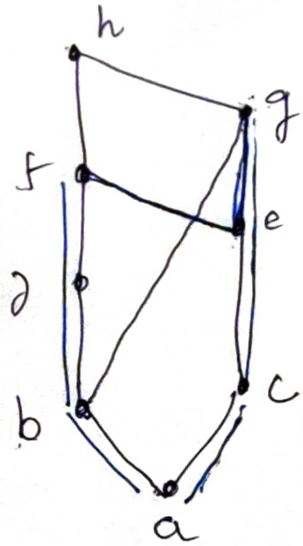
$(f, e) \Rightarrow GLB = b$
$LUB = g$

$(c, d) \Rightarrow GLB = b$
$LUB = g$

$(e, d) \Rightarrow GLB = b$
$LUB = g$

$(c, f) \Rightarrow GLB = b$
$LUB = g$

∴ Lattice



consider the pair (f, g)

$GLB(f, g) = \phi$ No single pt exist. {Two Lowers}

$LUB(f, g) = h$

No Meet semi Lattice ∴ Not Lattice

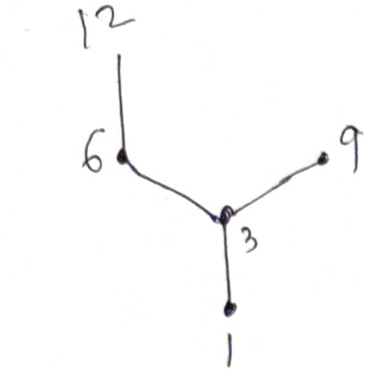
Determine whether these Posets are lattices.

(a) $(\{1,3,6,9,12\}, |)$

(b) $(P(S), \supseteq)$
↓
superset

↓ Make Hasse

Sol^n. (a)



Now check → it is Lattice or not.

$(9,12)$

$GLB(9,12) = 3$

$LUB(9,12) = \phi$

Not meet any pt.

$\therefore$ Not Lattice.

Sol^n. b:

$R = \{(a,b) \mid a \supseteq b\}$

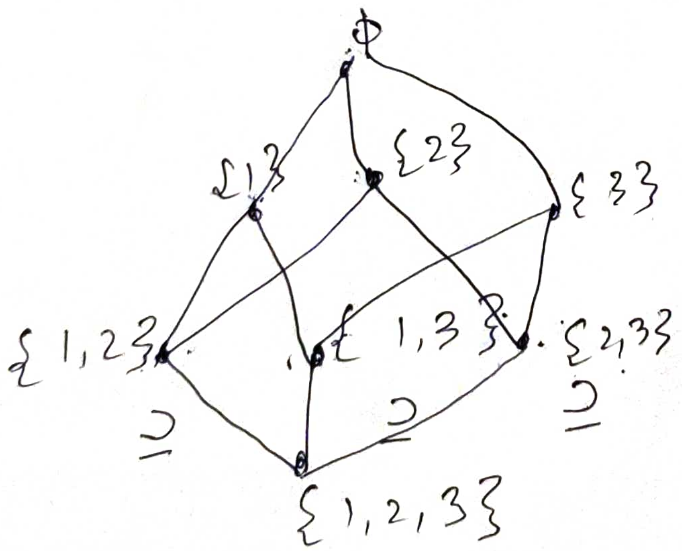
a is superset of b.

Simple Example: $S = \{1,2,3\}$

$P(S) = \{\phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$.

↓

Superset Acc. to Def.



Lattice

GLB, LUB } exist.