① (a) $x = \pi = 3.14159265358 98$

$x^* = \frac{22}{7} = 3.14285714285 7143$

Absolute error $= |x - x^*| = 0.00126448927$

Relative error $= \frac{|x-x^*|}{|x|} = 4.024994356143$

(b) $x = \sqrt{2} = 1.41421356237 31$, $x^* = 1.414$

$A.E = |x - x^*| = 0.0002135623731$

$R.E = \frac{|x-x^*|}{|x|} = 0.0001510114022253$

(c) $x = 8! = 40320$, $x^* = 39900$

$A.E = |x - x^*| = 420$

$R.E = \frac{|x-x^*|}{|x|} = 0.0104166667$

② (a) $x = \pi$, $tol = 10^{-4}$

$R.E = \frac{|x-x^*|}{|x|} \le 0.0001 \Rightarrow \frac{|\pi - x^*|}{|\pi|} \le 0.0001 \rightarrow |\pi - x^*| \le (0.0001)\pi$

$\Rightarrow -(0.0001)\pi \le \pi - x^* \le (0.0001)\pi \Rightarrow -\pi-(0.0001)\pi \le -x^* \le (0.0001)\pi - \pi$

$\Rightarrow (-1.0001)\pi \le -x^* \le -0.9999\pi$

$\Rightarrow 0.9999\pi \le x^* \le 1.0001\pi$ ~~$\Rightarrow 3.14127849432$~~

$\Rightarrow 3.14127849432 \le x^* \le 3.14190681285 5152$

(b) $x = e$

$R.E = \frac{|x-x^*|}{|x|} \le 0.0001 \Rightarrow \frac{|e - x^*|}{|e|} \le 0.0001$

$\Rightarrow 0.9999e \le x^* \le 1.0001e$

$\Rightarrow 2.7180100003 \le x^* \le 2.71855365 66$

(c) $x = \sqrt{3}$

$R.E = \frac{|x-x^*|}{|x|} \le 0.0001 \Rightarrow \frac{|\sqrt{3} - x^*|}{\sqrt{3}} \le 0.0001$

$\Rightarrow (0.9999)\sqrt{3} \le x^* \le (1.0001)\sqrt{3}$

$\Rightarrow 1.731877602488 \le x^* \le 1.7322240126 4963$

(d) $x = \sqrt[3]{7}$, $R.E = \frac{|\sqrt[3]{7} - x^*|}{\sqrt[3]{7}} \le 0.0001 \Rightarrow (0.9999)\sqrt[3]{7} \le x^* \le (1.0001)\sqrt[3]{7}$

$\Rightarrow 1.912739889 \le x^* \le 1.9131224759$

③ 3-digit Arithmetic

(a)

$x^* = \sqrt{3} + (\sqrt{5} + \sqrt{7}) = 1.73 + (2.24 + 2.65) = 6.62$

$x = \sqrt{3} + (\sqrt{5} + \sqrt{7}) = 6.61387009$

(Exact value)

$A.E = |x - x^*| = 0.00612991$

$R.E = \dfrac{|x - x^*|}{|x|} = 0.000926826489 87$

(b) $x = (121 - 0.327) - 119 = 1.673$

$x^* = (121 - 0.327) - 119 = 121 - 119 = \cancel{1.673}\ 2$

$\therefore A.E = |x - x^*| = 0.327$

$R.E = \dfrac{|x - x^*|}{|x|} = 0.1954 5726$

(c) $x = -10\pi + 6e - \dfrac{3}{62} = -15.1546 2266$

$x^* = (-10\pi) + (6e) - (3/62) = (-31.4) + (16.3) - (0.05) = -15.2$

$A.E = |x - x^*| = 0.0454$

$R.E = |x - x^*|/|x| = 0.002995$

(d) $x = \dfrac{\pi - 22/7}{1/17} = -0.021496318$

$x^* = \dfrac{3.14 - 3.14}{0.0588} = 0.00$

$|x - x^*| = 0.021496318 = 0.0215$

$\dfrac{|x - x^*|}{|x|} = \dfrac{\cancel{0.021496318}\ 0.0215}{0.0214963} = 1.0002$

④ Percentage error in $A = \dfrac{\text{Absolute error in } A}{A} \times 100$

$= \dfrac{\delta A}{A} \times 100 = 0.5$

$\Rightarrow \delta A = \dfrac{1}{200} A = \dfrac{1}{200} (\pi r^2)$

Now percentage error in $r = \dfrac{\delta r}{r} \times 100$

$= \dfrac{100}{r} \times \delta r = \dfrac{100}{r} \times \dfrac{\delta A}{\partial A / \partial r}$

$\left[\delta A = \dfrac{\partial A}{\partial r} \times \delta r \Rightarrow \delta r = \dfrac{\delta A}{\partial A / \partial r} \right]$

$= \dfrac{100}{r} \times \dfrac{\left(\frac{1}{200} \times \pi r^2\right)}{2\pi r} = \dfrac{1}{4} = 0.25$ [25 %]

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(5) $\frac{x^2}{3} + \frac{123}{4}x - \frac{1}{6} = 0$

$\Rightarrow$ ~~as~~ $a = \frac{1}{3}$, $b = \frac{123}{4}$, $c = -\frac{1}{6}$

Using 4-digit ~~A~~rithmetic, $a = 0.3333$, $b = 30.75$, $c = -0.1667$

$$x_1^* = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

Now $\sqrt{b^2 - 4ac} = \sqrt{(30.75)(30.75) - (4.000)(0.3333)(-0.1667)}$

$= \sqrt{945.6 + 0.2222} = \sqrt{945.8} = 30.75$

$$x_1^* = \frac{-30.75 + 30.75}{(2.000)(0.3333)} = 0.000$$

$$x_2^* = \frac{-b - \sqrt{b^2 - 4ac}}{2a} = \frac{-30.75 - 30.75}{(2.000)(0.3333)} = \frac{-61.50}{0.6666} = -92.26$$

Exact Solutions

$\frac{x^2}{3} + \frac{123}{4}x - \frac{1}{6} = 0 \Rightarrow 4x^2 + 369x - 2 = 0$

$$x = \frac{-369 \pm \sqrt{(369)^2 - 4(4)(-2)}}{2(4)(-2)} = \frac{-369 \pm \sqrt{136193}}{8}$$

$$= \frac{-369 \pm 369.0433579}{8} = 0.005419735788, \quad -92.25541974$$

$\therefore x_1 = 0.005419735788$

$x_2 = -92.25541974$

A.E in x_1 $\quad |x_1 - x_1^*| = 0.000001264212$

R.E in x_1 $\quad \frac{|x_1 - x_1^*|}{|x_1|} = \frac{0.000001264212}{0.005419735788} = 0.00023326081$

A.E in x_2 $\quad |x_2 - x_2^*| = 0.00458026$

R.E in x_2 $\quad \frac{|x_2 - x_2^*|}{|x_2|} = \frac{0.00458026}{92.25541974} = 0.0000496475980$

Rewrite x_1, $\quad x_1^* = \frac{-b + \sqrt{b^2 - 4ac}}{2a} = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \times \frac{-b - \sqrt{b^2 - 4ac}}{-b - \sqrt{b^2 - 4ac}}$

$= \frac{-2c}{b + \sqrt{b^2 - 4ac}} = \frac{(-2.000)(-0.1667)}{30.75 + 30.75} = \frac{0.3334}{61.50} = 0.0054$

(a) $y = x^3 - 7x^2 + 8x - 0.35$

At $x = 1.37$ and 3-digit chopping arithmetic

$$y^* = (1.37)^3 - 7(1.37)^2 + 8(1.37) - 0.35$$

$$= [(1.37)(1.37)(1.37)] - (7.00)[(1.37)(1.37)] + (8.00)(1.37) - 0.35$$

$$= [(1.87)(1.37)] - (7.00)(1.87) + (10.9) - 0.35$$

$$= 2.56 - 13.0 + 10.9 - 0.35$$

$$= -10.4 + 10.9 - 0.35 = 0.50 - 0.35 = 0.15$$

Exact value,

$$y = (1.37)^3 - 7(1.37)^2 + 8(1.37) - 0.35 = 0.043053$$

$$R.E = \frac{|y-y^*|}{|y|} = \frac{|0.043053 - 0.15|}{|0.043053|} = \frac{0.106947}{0.043053} = \cancel{0.161359} = 2.484078$$

Percent Error $= R.E \times 100 =$ 248.4078 %.

(b) $y = ((x-7)x + 8)x - 0.35$

At $x = 1.37$ and 3-digit chopping arithmetic

$$y^* = ((1.37 - 7.00)(1.37) + 8.00)(1.37) - 0.35$$

$$= ((-5.63)(1.37) + 8.00)(1.37) - 0.35$$

$$= ((-7.71) + 8.00)(1.37) - 0.35$$

$$= (0.29)(1.37) - 0.35 = 0.397 - 0.35 = 0.047$$

Exact $y = 0.043053$

$$R.E = \frac{|y-y^*|}{|y|} = \frac{|0.043053 - 0.047|}{0.043053} = 0.091677$$

Percent Error $= R.E \times 100 = 9.16\%$

Error is less in part (b) as compared to part (a)

$\therefore$ Nesting loop is better.

(8) $f(x) = \dfrac{1}{1-3x^2}$, $f'(x) = \dfrac{6x}{(1-3x^2)^2}$

Now At $x = 0.577$ and 3-digit chopping arithmetic,

$$f'(x) = \frac{(6.00)(0.577)}{[1.00 - (3.00)(0.577)^2]^2} = \frac{3.46}{[1.00 - (3.00)(0.332)]^2}$$

$$= \frac{3.46}{[1.00 - 0.996]^2} = \frac{3.46}{(0.004)^2} = \frac{3.46}{0.00} \to \infty$$

At $x = 0.577$ and 4-digit chopping arithmetic

$$f'(x) = \frac{6x}{(1-3x^2)^2} = \frac{(6.000)(0.5770)}{[1.000-(3.000)(0.5770)^2]^2}$$

$$= \frac{3.462}{[1.000-(3.000)(0.3329)]^2} = \frac{3.462}{[1.000-0.9987]^2}$$

$$= \frac{3.462}{[0.0013]^2} = \frac{3.462}{0.000} \to \infty$$

Yes, we have difficulties in evaluating the $f'(x)$ at $x = 0.577$ using 3 and 4-digit arithmetic with chopping.

(9) $l = 3\,cm$, $w = 4\,cm$, $h = 5\,cm$

$2.5 < l < 3.5$

$3.5 < w < 4.5$

$4.5 < h < 5.5$

$l > 2.5 \Rightarrow l = 2.51 \Rightarrow l = 3$
$l < 3.5 \Rightarrow l = 3.4999 \Rightarrow l = 3$

$w > 3.5 \Rightarrow w = 3.51\ldots \Rightarrow w = 4$
$w < 4.5 \Rightarrow w = 4.4999 \Rightarrow w = 4$

$h > 4.5 \Rightarrow h = 4.51 \Rightarrow h = 5$
$h < 5.5 \Rightarrow h = 5.4999 \Rightarrow h = 5$

Volume $= lwh$

$\therefore (2.5)(3.5)(4.5) < lwh < (3.5)(4.5)(5.5)$

$$\boxed{39.375 < \text{Volume} < 86.625}$$

Surface Area $= 2[lw + lh + wh]$

Now $(2.5)(3.5) < lw < (3.5)(4.5) \Rightarrow 8.75 < lw < 15.75$

$(2.5)(4.5) < lh < (3.5)(5.5) \Rightarrow 11.25 < lh < 19.25$

$(3.5)(4.5) < wh < (4.5)(5.5) \Rightarrow 15.75 < wh < 24.75$

Now $(8.75 + 11.25 + 15.75) < lw + lh + wh < 15.75 + 19.25 + 24.75$

$35.75 < lw + lh + wh < 59.75$

$\Rightarrow 2(35.75) < 2(lw + lh + wh) < 2(59.75)$

$$\boxed{71.5 < \text{Surface Area} < 119.5}$$

$\therefore$ Best Lower bound for Volume $= 39.375$

" Upper " " " $= 86.625$

" Lower " " Surface Area $= 71.5$

" Upper " " " $= 119.5$

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$$x = \frac{x_0 y_1 - x_1 y_0}{y_1 - y_0}, \quad x = x_0 - \frac{(x_1 - x_0) y_0}{y_1 - y_0}$$

(a) Now $x = x_0 - \frac{(x_1 - x_0) y_0}{y_1 - y_0} = \frac{x_0 (y_1 - y_0) - (x_1 - x_0) y_0}{y_1 - y_0}$

$$= \frac{x_0 y_1 - \cancel{x_0 y_0} - x_1 y_0 + \cancel{x_0 y_0}}{y_1 - y_0} = \frac{x_0 y_1 - x_1 y_0}{y_1 - y_0}$$

$\therefore$ Both the formulas are algebraically correct.

(b) $x_0 = 1.31$, $y_0 = 3.24$, $x_1 = 1.93$, $y_1 = 4.76$, Use 3-digit Rounding Arithmetic

$$x = \frac{x_0 y_1 - x_1 y_0}{y_1 - y_0} = \frac{(1.31)(4.76) - (1.93)(3.24)}{4.76 - 3.24} \quad \text{—(1)}$$

$$= \frac{6.24 - 6.25}{1.52} = \frac{-0.01}{1.52} = -0.007$$

$$x = x_0 - \frac{(x_1 - x_0) y_0}{y_1 - y_0} = 1.31 - \frac{(1.93 - 1.31)(3.24)}{4.76 - 3.24}$$

$$= 1.31 - \frac{(0.62)(3.24)}{1.52} = 1.31 - \frac{2.01}{1.52} = 1.31 - 1.32 = -0.01 \quad \text{—(2)}$$

Exact Values $x = \frac{x_0 y_1 - x_1 y_0}{y_1 - y_0} = \frac{(1.31)(4.76) - (1.93)(3.24)}{4.76 - 3.24}$

$$= \frac{6.2356 - 6.2532}{1.52} = \frac{-0.0176}{1.52} = -0.0115789$$

$$x = x_0 - \frac{(x_1 - x_0) y_0}{y_1 - y_0} = (1.31) - \frac{(1.93 - 1.31)(3.24)}{4.76 - 3.24}$$

$$= 1.31 - \frac{(0.62)(3.24)}{1.52} = 1.31 - \frac{2.0088}{1.52} = 1.31 - 1.321578947$$

$$= -0.0115789$$

(2) is better than (1). It is so because in (1), we have subtractive cancellation due to subtraction of two nearly equal numbers. So there is a loss of significance in (1).

(11) (a) ~~Form~~ $\sum_{i=1}^{n} \sum_{j=1}^{i} a_i b_j$ — (1)

For each i, there are i multiplications in (1)

$\therefore \sum_{i=1}^{n}$ have n iterations $\Rightarrow$ Total Multiplications $= \sum_{i=1}^{n} i$

$= 1+2+3+\dots+n = \frac{n(n+1)}{2}$

There are $i-1$ additions for each i in (1)

$\therefore \sum_{i=1}^{n} (i-1) = \frac{n(n-1)}{2}$

Also $(n-1)$ more additions ($\because$ of $\sum_{i=1}^{n}$)

$\therefore$ Total additions $= \frac{n(n-1)}{2} + n - 1 = \frac{(n+2)(n-1)}{2}$

(b) Equivalent form

$\sum_{i=1}^{n} \left(a_i \sum_{j=1}^{i} b_j\right)$

For each i, $i-1$ additions

1 Multiplication

$\therefore$ Total additions $= \sum_{i=1}^{n} (i-1) + (n-1) = \frac{(n+2)(n-1)}{2}$

$\downarrow$ more additions

Total Multiplications $= \sum_{i=1}^{n} (1) = n$

$\therefore$ New form $\sum_{i=1}^{n}\left(a_i \sum_{j=1}^{i} b_j\right)$ reduces no. of multiplications.

(12) Algorithm

$P(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$

$P(x_0) = a_0 + x_0\left(a_1 + x_0\left(a_2 + \dots + x_0\left(a_{n-1} + x_0 a_n\right)\right)\right)$

Input: $n, a_0, a_1, \dots, a_n, x_0$

Output $y = P(x_0)$

Step 1: Set $y = a_n$

Step 2: for $i = n-1, n-2, \dots, 0$

Set $y = x_0 y + a_i$

Step 3: Output (y)

STOP

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3) Input: $n, x_0, x_1, \ldots, x_n, x$

output: P

Step 1: Set $P = x - x_0$; $i = 1$

Step 2: while $P \neq 0$ and $i \leq n$,

Set $P = P \cdot (x - x_i)$

$i = i + 1$

Step 3: Output (P)

STOP