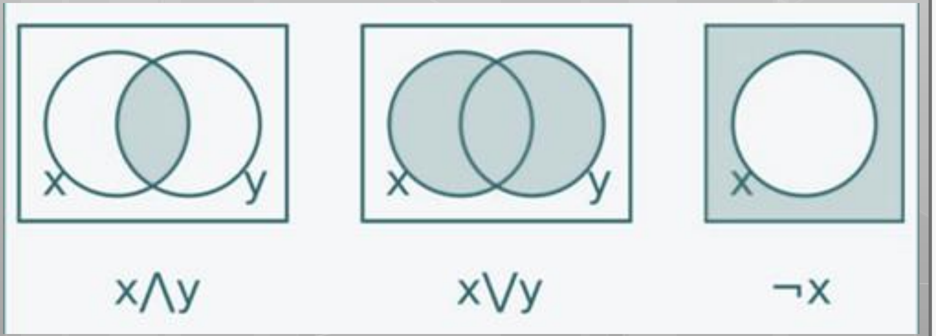

Boolean Algebra

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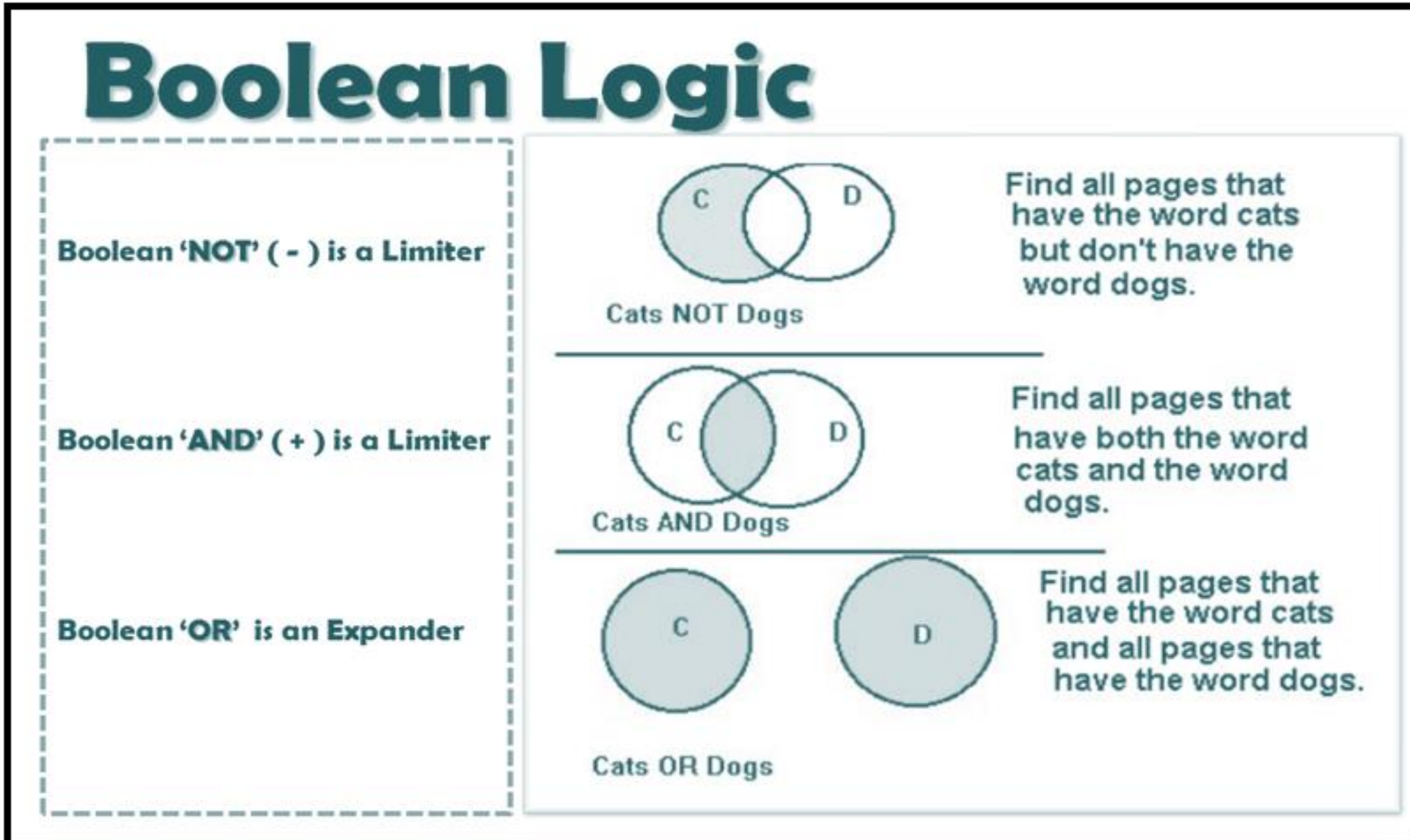
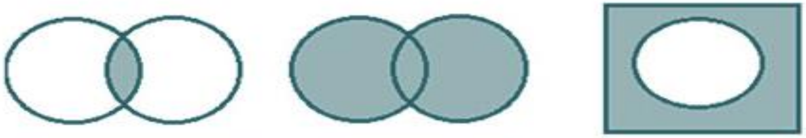
Introduction

Boolean Algebra provides the operations and rules for working with the set {0, 1}. The three operations in Boolean Algebra that we will use most are Complementation, the Boolean sum and Boolean product.


Boolean Logic
Boolean 'NOT' (-) is a Limiter
Boolean 'AND' (+) is a Limiter
Boolean 'OR' is an Expander
C
D
Cats NOT Dogs
Find all pages that have the word cats but don't have the word dogs.
C
D
Cats AND Dogs
Find all pages that have both the word cats and the word dogs.
C
D
Cats OR Dogs
Find all pages that have the word cats and all pages that have the word dogs.

Conjunction	$x \wedge y$	AND	Multiplication
Disjunction	x V y	OR	Addition
Negation	! x	NOT	1 - x



Conjunction Disjunction Negation

AND		
X	Y	$Z = X \cdot Y$
0	0	0
0	1	0
1	0	0
1	1	1

OR		
X	Y	$Z = X + Y$
0	0	0
0	1	1
1	0	1
1	1	1

NOT	
X	$Z = \overline{X}$
0	1
1	0

List of axioms and theorems:

Identity	$A + 0 = A$	$A.\ 1 = A$
Complement	$A + A' = 1$	$A.\ A' = 0$
Commutative	$A + B = B + A$	$A.\ B = B.\ A$
Assosiative	$A + (B + C) = (A + B) + C$	$A.\ (B.\ C) = (A.\ B).\ C$
Distributive	$A.\ (B + C) = A.\ B + A.\ C$	$A + (B.\ C) = (A + B).\ (A + C)$
Null Element	$A + 1 = 1$	$A.\ 0 = 0$
Involution	$(A')' = A$	
Indempotency	$A + A = A$	$A.\ A = A$
Absorption	$A + (A.\ B) = A$	$A.\ (A + B) = A$
Distributive	$A + A'.\ B = A + B$	$A' + A.B = A' + B$
De Morgan's	$(A + B)' = A'.\ B'$	$(A.\ B)' = A'.\ B'$

Example 1: Minimize the following expression by use of Boolean rules.

$F = (A+B+C)(A+B'+C)(A+B+C')$

Let $X = A+B$

$F = (X+C)(X+C')(A+B'+C)$ — Commutative Law

$F = (X+CC')(A+B'+C)$ — Distributive Law

$F = (X+0)(A+B'+C)$

$F = X.(A+B'+C)$

$F = (A+B)(A+B'+C)$

$F = A+B.(B'+C)$ — Distributive Law

$F = A+B.B'+BC$

$F = A+0+BC$

$F = A+BC$

Example 2: Minimize the following expression by use of Boolean rules.

$G = (A+B)(A+B')(A'+B)(A'+B')$

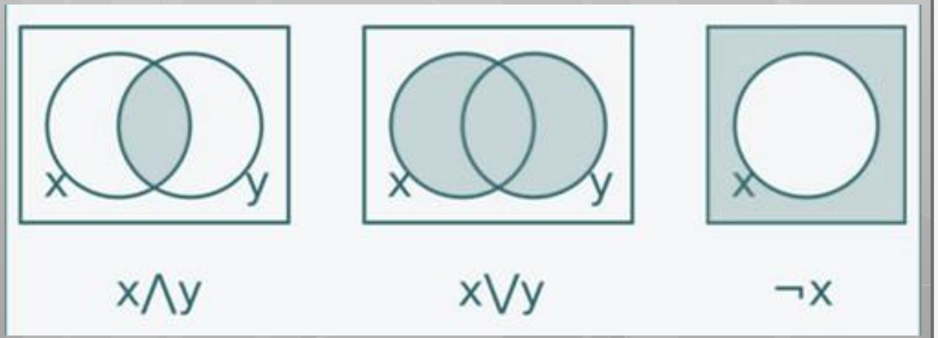
$G = (A+BB')(A'+BB')$ Distributive Law

$G = (A+0)(A'+0)$

$G = A.A'$

$G = 0$


Thank you!



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Standard Forms

Canonical forms are basic forms obtained from the truth table of the function. These forms are usually not used to represent the function as they are cumbersome to write and it is preferable to represent the function in the least number of literals possible. There are two types of standard forms:

1. **Sum of Products (SOP)** A boolean expression involving AND terms with one or more literals each, OR'ed together.
2. **Product of Sums (POS)** A boolean expression involving OR terms with one or more literals each, AND'ed together.

SOP (sum of products)	POS (product of sums)
1. A method of describing a Boolean expression using a set of min terms or product terms.	1. A method of describing a Boolean expression using a set of max terms or sum terms.
2. We write the product terms for each input combination that gives high (1) output.	2. We write the sum terms for each input combination that gives low (0) output.
3. We take the input variables if the value is 1 and write the complement of the variable if the value is 0 when writing the min terms.	3. We take the input variables if the value is 0 and write the complement of the variable if the value is 1 when writing the max terms.
4. Final expression is obtained by adding the relevant product terms.	4. Final expression is obtained by multiplying the relevant sum terms.

Canonical Forms: Minterms and Maxterms

x	y	z	minterm	designation	maxterm	designation
0	0	0	$\bar{x}\,\bar{y}\,\bar{z}$	m_0	$x+y+z$	M_0
0	0	1	$\bar{x}\,\bar{y}\,z$	m_1	$x+y+\bar{z}$	M_1
0	1	0	$\bar{x}\,y\,\bar{z}$	m_2	$x+\bar{y}+z$	M_2
0	1	1	$\bar{x}\,y\,z$	m_3	$x+\bar{y}+\bar{z}$	M_3
1	0	0	$x\,\bar{y}\,\bar{z}$	m_4	$\bar{x}+y+z$	M_4
1	0	1	$x\,\bar{y}\,z$	m_5	$\bar{x}+y+\bar{z}$	M_5
1	1	0	$x\,y\,\bar{z}$	m_6	$\bar{x}+\bar{y}+z$	M_6
1	1	1	$x\,y\,z$	m_7	$\bar{x}+\bar{y}+\bar{z}$	M_7
			(AND terms)		(OR terms)	

For the given truth table, Minimize the SOP expression:

A	B	Y
0	0	0
0	1	1
1	0	0
1	1	1

$Y = A'B + AB$
$Y = B(A + A')$
$Y = B$

Simplify the expression:

$Y(A, B) = \sum m(0, 2, 3)$
$Y = m_0 + m_2 + m_3$
$Y = A'B' + AB' + AB$
$Y = B'(A + A') + AB$
$Y = B' + AB$
$Y = A + B'$ **Distributive Law**

A	B	Y
0	0	1
0	1	0
1	0	1
1	1	1

Simplify the expression:

A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

F = A'BC' + AB'C' + AB'C + ABC' + ABC
F = A'BC' + AB' (C + C') + AB (C + C')
F = A'BC' + AB' + AB
F = A'BC' + A(B' + B)
F = A'BC' + A
F = BC' + A

Distributive Law

or

$F = m_2 + m_4 + m_5 + m_6 + m_7$

or

$F = \sum m(2, 4, 5, 6, 7)$

For the given truth table, Minimize the POS expression:

A	B	Y
0	0	1
0	1	0
1	0	1
1	1	0

$Y = (A+B') \,.\, (A'+B')$
$Y = B'+A\,A'$
$Y = B'$

or

$Y = \prod (M_1, M_3)$

or

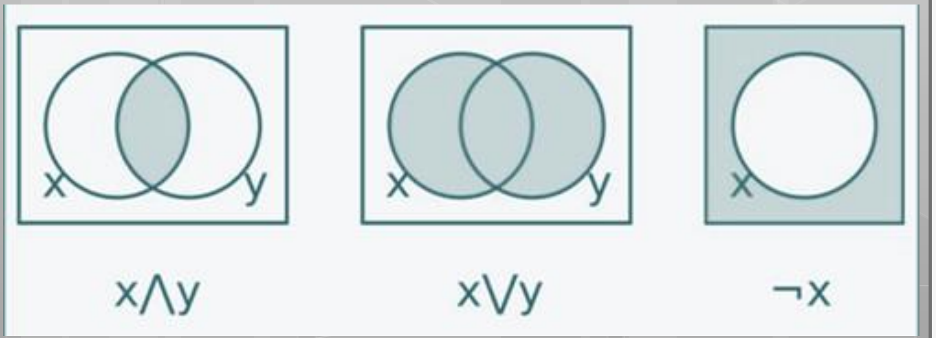
$Y = \prod M(1, 3)$

Simplify the expression:

A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

$F = (A + B + C).(A + B + C').(A + B' + C')$
$F = (A + B + CC').(A + B' + C')$
$F = (A + B).(A + B' + C')$
$F = A + B.(B' + C')$
$F = A + B\,B' + BC'$
$F = A + BC'$
$F = (A + B)\,(A + C')$


Thank you!



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K-Maps

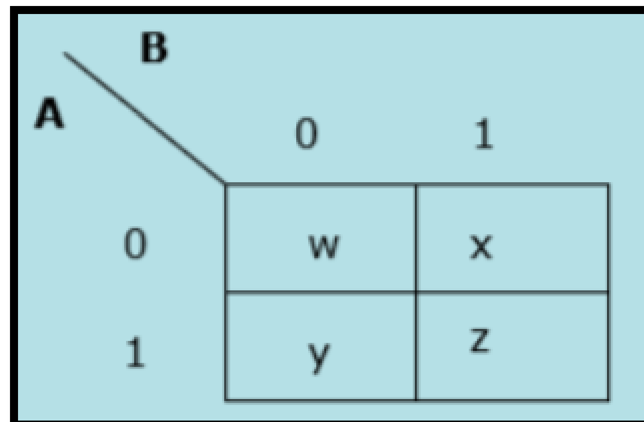
The Karnaugh map (K-map) is a method of simplifying Boolean algebra expressions.

Example 1

An arbitrary truth table is taken below

A	B	A operation B
0	0	w
0	1	x
1	0	y
1	1	z

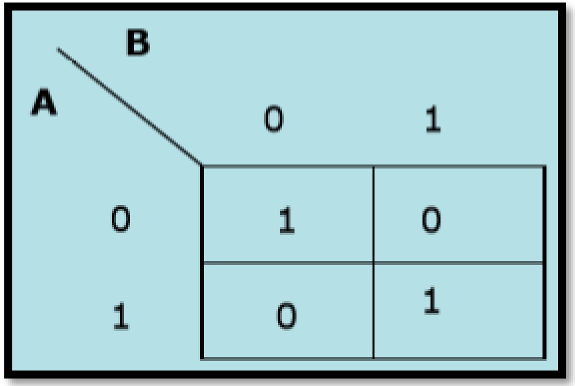
Now we will make a k-map for the above truth table



K-Maps

Example 2

Now we will make a K-map for the expression: **AB+ A'B'**



Simplification Using K-map

K-map uses some rules for the simplification of Boolean expressions by combining together adjacent cells into single term. The rules are described below –

Rule 1 – Any cell containing a zero cannot be grouped.

A \ BC	00	01	11	10
0	1	0	1	0
1	0	1	1	1

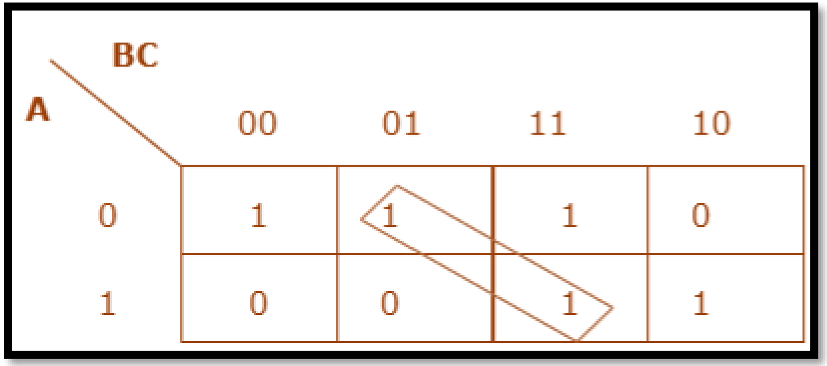
Wrong grouping

Rule 2 – Groups must contain 2n cells (n starting from 1).

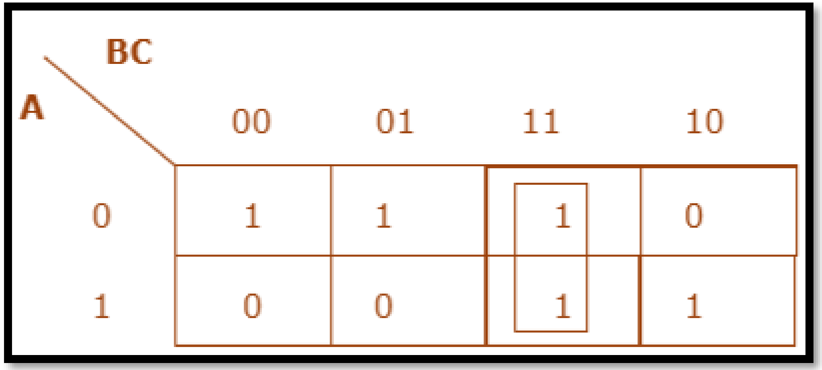
A \ BC	00	01	11	10
0	1	0	1	0
1	0	1	1	1

Wrong grouping

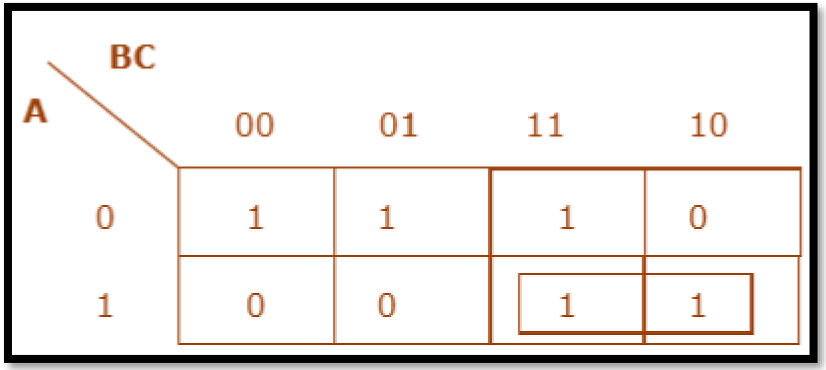
Rule 3 – Grouping must be horizontal or vertical, but must not be diagonal.



Wrong diagonal grouping



Proper vertical grouping



Proper horizontal grouping

Rule 4 – Groups must be covered as largely as possible.

A \ BC	00	01	11	10
0	1	0	1	0
1	1	1	1	1

Insufficient grouping

A \ BC	00	01	11	10
0	1	0	1	0
1	1	1	1	1

Proper grouping

Rule 5 – If 1 of any cell cannot be grouped with any other cell, it will act as a group itself.

A \ BC	00	01	11	10
0	$\boxed{1}$	0	$\boxed{1}$	0
1	0	$\boxed{1}$	0	$\boxed{1}$

Proper grouping

Rule 6 – Groups may overlap but there should be as few groups as possible.

A \ BC	00	01	11	10
0	0	0	1	1
1	1	1	1	1

Proper grouping

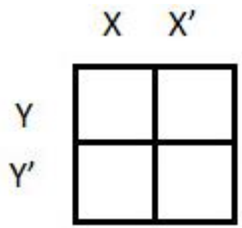
Rule 7 – The leftmost cell/cells can be grouped with the rightmost cell/cells and the topmost cell/cells can be grouped with the bottommost cell/cells.

A \ BC	00	01	11	10
0	1	0	0	1
1	1	0	0	1

Proper grouping

2 variable K-maps

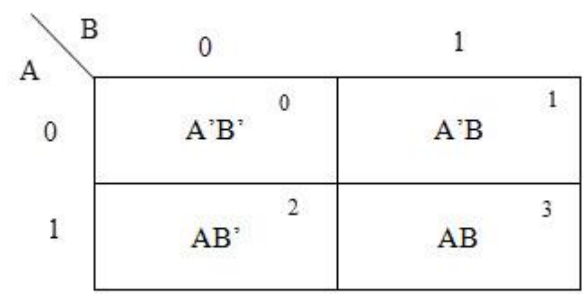
There are 4 cells in the 2-variable k-map. It will look like:



The following table shows the positions of all the possible outputs of 2-variable Boolean function on a K-map.

A	B	Possible Outputs	Location on K-map
0	0	A'B'	0
0	1	A'B	1
1	0	AB'	2
1	1	AB	3

A general representation of a 2 variable K-map plot is shown below.



2 variable K-maps

A. SOP: -

A \ B	$\overline{B}$ 0	B 1
$\overline{A}$ 0	$\overline{A}.\overline{B}$	$\overline{A}.B$
A 1	$A.\overline{B}$	$A.B$

B. POS: -

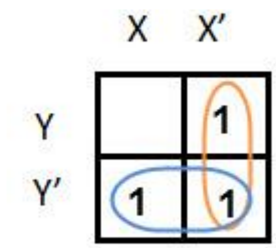
A \ B	B 0	$\overline{B}$ 1
A 0	$A+B$	$A+\overline{B}$
$\overline{A}$ 1	$\overline{A}+B$	$\overline{A}+\overline{B}$

Simplify the given 2-variable Boolean equation by using K-map.
F = X Y’ + X’ Y + X’Y’

First, let’s construct the truth table for the given equation,

X	Y	F
0	0	1
0	1	1
1	0	1
1	1	0

We put 1 at the output terms given in equation.



After grouping the variables, the next step is determining the minimized expression.

So the reduced equation will be **X’ + Y’**.

3 variable K-maps

For a 3-variable Boolean function, there is a possibility of 8 output min terms. The general representation of all the min terms using 3-variables is shown below:

A	B	C	Output Function	Location on K-map
0	0	0	A'B'C'	0
0	0	1	A'B'C	1
0	1	0	A'BC'	2
0	1	1	A'BC	3
1	0	0	AB'C'	4
1	0	1	AB'C	5
1	1	0	ABC'	6
1	1	1	ABC	7

A typical plot of a 3-variable K-map is shown below. It can be observed that the positions of columns 10 and 11 are interchanged so that there is only change in one variable across adjacent cells. This modification will allow in minimizing the logic.

A \ BC	00	01	11	10
0	A'B'C' 0	A'B'C 1	A'BC 3	A'BC' 2
1	AB'C' 4	AB'C 5	ABC 7	ABC' 6

3 variable K-maps

A. SOP: -

A \ BC	$\bar{B}\,\bar{C}$ 0 0	$\bar{B}\,C$ 0 1	$B\,C$ 1 1	$B\,\bar{C}$ 1 0
$\bar{A}$ 0	$\bar{A}\,\bar{B}\,\bar{C}$ 0	$\bar{A}\,\bar{B}\,C$ 1	$\bar{A}\,B\,C$ 3	$\bar{A}\,B\,\bar{C}$ 2
A 1	$A\,\bar{B}\,\bar{C}$ 4	$A\,\bar{B}\,C$ 5	$A\,B\,C$ 7	$A\,B\,\bar{C}$ 6

AB \ C	$\bar{C}$ 0	C 1
$\bar{A}\,\bar{B}$ 00	$\bar{A}\,\bar{B}\,\bar{C}$ 0	$\bar{A}\,\bar{B}\,C$ 1
$\bar{A}\,B$ 01	$\bar{A}\,B\,\bar{C}$ 2	$\bar{A}\,B\,C$ 3
$A\,B$ 11	$A\,B\,\bar{C}$ 6	$A\,B\,C$ 7
$A\,\bar{B}$ 10	$A\,\bar{B}\,\bar{C}$ 4	$A\,\bar{B}\,C$ 5

B. POS: -

A \ B+C	$B+C$ 0 0	$B+\bar{C}$ 0 1	$\bar{B}+\bar{C}$ 1 1	$\bar{B}+C$ 1 0
A 0	$A+B+C$ 0	$A+B+\bar{C}$ 1	$A+\bar{B}+\bar{C}$ 3	$A+\bar{B}+C$ 2
$\bar{A}$ 1	$\bar{A}+B+C$ 4	$\bar{A}+B+\bar{C}$ 5	$\bar{A}+\bar{B}+\bar{C}$ 7	$\bar{A}+\bar{B}+C$ 6

A+B \ C	C 0	$\bar{C}$ 1
$A+B$ 0 0	$A+B+C$ 0	$A+B+\bar{C}$ 1
$A+\bar{B}$ 0 1	$A+\bar{B}+C$ 2	$A+\bar{B}+\bar{C}$ 3
$\bar{A}+\bar{B}$ 1 1	$\bar{A}+\bar{B}+C$ 6	$\bar{A}+\bar{B}+\bar{C}$ 7
$\bar{A}+B$ 1 0	$\bar{A}+B+C$ 4	$\bar{A}+B+\bar{C}$ 5

Simplify the given 3-variable Boolean equation by using K-map.

F = X’ Y’ Z’ + X’ Y’ Z + X’ Y Z’ + X Y’ Z’ + X Y Z

First, let’s construct the truth table for the given equation,

We put 1 at the output terms given in equation.

X \ YZ	00	01	11	10
0	1	1		1
1	1		1	

X	Y	Z	F
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

After grouping the variables, the next step is determining the minimized expression.

So the reduced equation will be **X Y Z + X’ Z’ + X’ Y’ + Y’ Z’**.

4 variable K-maps

There are 16 possible min terms in case of a 4-variable Boolean function. The general representation of minterms using 4 variables is shown below:

A	B	C	D	Output function	K-map location
0	0	0	0	A' B' C' D'	0
0	0	0	1	A' B' C' D	1
0	0	1	0	A' B' C D'	2
0	0	1	1	A' B' C D	3
0	1	0	0	A' B C' D'	4
0	1	0	1	A' B C' D	5
0	1	1	0	A' B C D'	6
0	1	1	1	A' B C D	7
1	0	0	0	A B' C' D'	8
1	0	0	1	A B' C' D	9
1	0	1	0	A B' C D'	10
1	0	1	1	A B' C D	11
1	1	0	0	A B C' D'	12
1	1	0	1	A B C' D	13
1	1	1	0	A B C D'	14
1	1	1	1	A B C D	15

A typical 4-variable K-map plot is shown below. It can be observed that both the columns and rows of 10 and 11 are interchanged.

AB \ CD	00	01	11	10
00	0 A' B' C' D'	1 A' B' C' D	3 A' B' C D	2 A' B' C D'
01	4 A' B C' D'	5 A' B C' D	7 A' B C D	6 A' B C D'
11	12 A B C' D'	13 A B C' D	15 A B C D	14 A B C D'
10	8 A B' C' D'	9 A B' C' D	11 A B' C D	10 A B' C D'

4 variable K-maps

A. SOP: -

AB \ CD	$\overline{C}\,\overline{D}$ 00	$\overline{C}D$ 01	CD 11	$C\overline{D}$ 10
$\overline{A}\,\overline{B}$ 00	$\overline{A}\,\overline{B}\,\overline{C}\,\overline{D}$ 0	$\overline{A}\,\overline{B}\,\overline{C}D$ 1	$\overline{A}\,\overline{B}CD$ 3	$\overline{A}\,\overline{B}C\overline{D}$ 2
$\overline{A}B$ 01	$\overline{A}B\overline{C}\,\overline{D}$ 4	$\overline{A}B\overline{C}D$ 5	$\overline{A}BCD$ 7	$\overline{A}BC\overline{D}$ 6
AB 11	$AB\overline{C}\,\overline{D}$ 12	$AB\overline{C}D$ 13	$ABCD$ 15	$ABC\overline{D}$ 14
$A\overline{B}$ 10	$A\overline{B}\,\overline{C}\,\overline{D}$ 8	$A\overline{B}\,\overline{C}D$ 9	$A\overline{B}CD$ 11	$A\overline{B}C\overline{D}$ 10

B. POS: -

A+B \ C+D	$C+D$ 00	$C+\overline{D}$ 01	$\overline{C}+\overline{D}$ 11	$\overline{C}+D$ 10
$A+B$ 00	$A+B+C+D$ 0	$A+B+C+\overline{D}$ 1	$A+B+\overline{C}+\overline{D}$ 3	$A+B+\overline{C}+D$ 2
$A+\overline{B}$ 01	$A+\overline{B}+C+D$ 4	$A+\overline{B}+C+\overline{D}$ 5	$A+\overline{B}+\overline{C}+\overline{D}$ 7	$A+\overline{B}+\overline{C}+D$ 6
$\overline{A}+\overline{B}$ 11	$\overline{A}+\overline{B}+C+D$ 12	$\overline{A}+\overline{B}+C+\overline{D}$ 13	$\overline{A}+\overline{B}+\overline{C}+\overline{D}$ 15	$\overline{A}+\overline{B}+\overline{C}+D$ 14
$\overline{A}+B$ 10	$\overline{A}+B+C+D$ 8	$\overline{A}+B+C+\overline{D}$ 9	$\overline{A}+B+\overline{C}+\overline{D}$ 11	$\overline{A}+B+\overline{C}+D$ 10

Simplify the given 4-variable Boolean equation by using K-map.

F (W, X, Y, Z) = (1, 5, 12, 13)

WX \ YZ	00	01	11	10
		1		
		1		
	1	1		

By preparing k-map, we can minimize the given Boolean equation as

So the reduced equation will be **F = W X Y' + W 'Y' Z.**

$F(A,B,C,D) = \Sigma(0,1,2,5,8,9,10)$

POS

AB \ CD			
1 (0)	1 (1)	0 (3)	1 (2)
0 (4)	1 (5)	0 (7)	0 (6)
0 (12)	0 (13)	0 (15)	0 (14)
1 (8)	1 (9)	0 (11)	1 (10)

$F = (C'+D')(A'+B')(B'+D)$


Thank you!