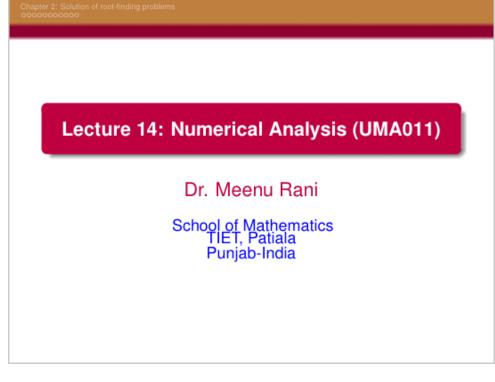
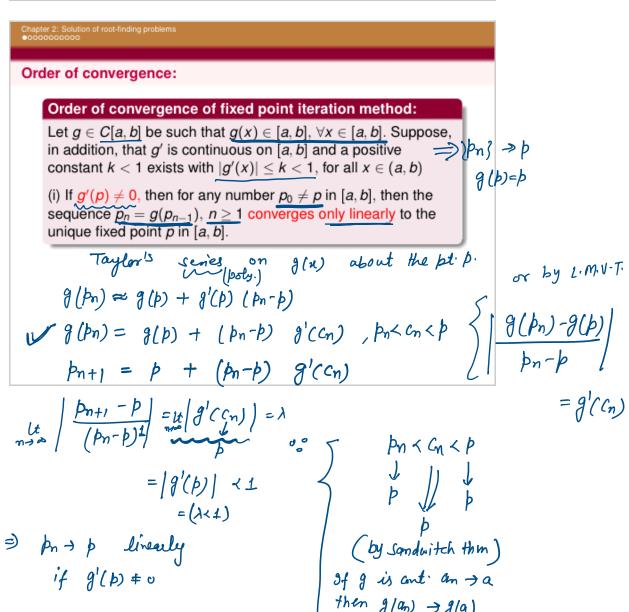
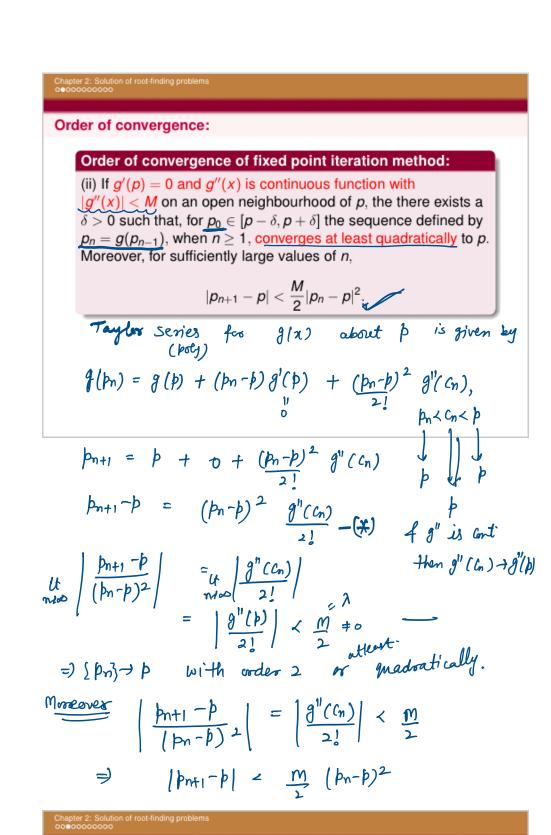
Monday, August 29, 2022









Order of convergence of fixed point iteration method:

In general, if
$$g'(p) = 0$$
, $g''(p) = 0$, ..., $g^{m-1}(p) = 0$, then the sequence defined by $p_n = g(p_{n-1})$, when $n \ge 1$, converges at least of order m to p .

In general, if $g'(p) = 0$, $g''(p) = 0$, ..., $g^{m-1}(p) = 0$, then the sequence defined by $p_n = g(p_{n-1})$, when $n \ge 1$, converges at least of order m to p .

In $g''(p) = 0$, $g''(p) = 0$

Order of convergence of Newton's method:

The Sequence generated by N-M is given by

$$p_n = p_{n-1} - \frac{f(p_{n-1})}{f'(p_{n-1})}, n \ge 1$$

$$p_n = g(p_{n-1}) \rightarrow p \text{ (which converges to p)}$$
Here
$$g(x) = x - \frac{f(x)}{f'(x)}, g(y) = p - \frac{f(y)}{f'(y)} \neq 0$$

$$= p - \frac{g'(x)f'(x) - f(x)f''(x)}{(f'(x))^2}$$

$$= 1 - \frac{(f'(x))^2 - f(x)f''(x)}{(f'(x))^2} = \frac{f(x)f''(x)}{(f'(x))^2}$$

$$= x - x + \frac{f(x)f''(x)}{(f'(x))^2} = \frac{f(x)f''(x)}{(f'(x))^2}$$

$$= y - x + \frac{f(x)f''(x)}{(f'(x))^2} = \frac{f(x)f''(x)}{(f'(x))^2}$$

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 $g'(p) = f(p)f''(p) = 0 \Rightarrow p_n \rightarrow p$ with vodes atteast 2 or atteast quadraticity

$$g'(x) = \frac{2}{3} - \frac{2}{3} \frac{a}{x^{3}}$$

$$\text{fut } p = a^{3}$$

$$g'(p) = \frac{2}{3} - \frac{2}{3} \frac{a}{a^{2/3}} = 0$$

$$g''(x) = 0 - \frac{2}{3} a \frac{(-3)}{x^{4}} = \frac{2a}{x^{4}}$$

$$\text{fut } p = a^{3/3}$$

$$\text{fut } p = a^{3/3}$$

$$g''(a^{3/3}) = \frac{2a}{(a^{3/3})^{4}} \neq 0$$

$$= x_{n} \rightarrow p \text{ quadratically or order is } 2$$

Exercise: What is the order of convergence of the iteration

 $x_{n+1} = \frac{x_n(x_n^2 + 3a)}{3x_n^2 + a}, \ a \in \mathbb{R}$

as it converges to the fixed point
$$p = \sqrt{a}$$
?
The iterates $x_{n+1} = 2 - (1+c)x_n + cx_n^3$ converges to $p =$

The iterates $x_{p+1} = 2 - (1+c)x_p + cx_p^3$ converges to p=1for some values of constant c (provided that x_0 is sufficiently close to p). For what values of c, if any, convergence is quadratic.