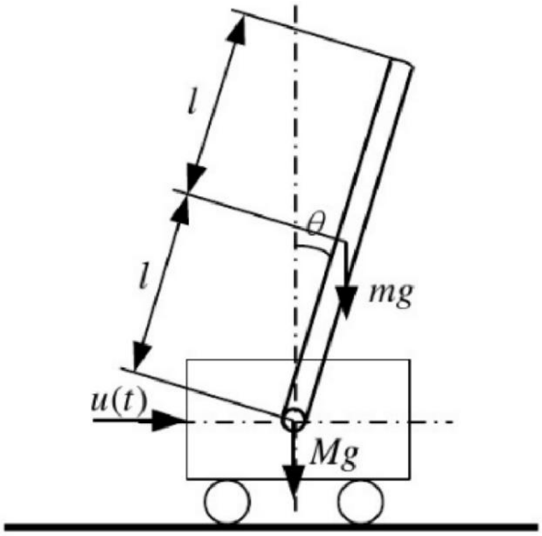
Mathematical modeling of linear inverted pendulum

Inverted pendulum system is an ideal controlled object in the study of control theory. The following mathematical modeling of inverted pendulum is carried out to facilitate a deeper understanding of inverted pendulum.

Based on the classical control theory and Newton's second law of motion, this paper makes a mechanical analysis of the inverted pendulum system, establishes the dynamic equations of the carriage (the slider above the inverted pendulum) in the horizontal motion and the pendulum bar in the vertical position, and obtains the transfer function of the system through reasonable linearization and Laplace transformation. Finally, the block diagram of control system is built by using Simulink.



In the picture :

u(t): force added to the car

M: Quality of cars

J: Moment of inertia of pendulum rod

x: Car position

N: Interaction force between swing rod and trolley

l: The length of the rocker shaft to the center of mass

According to Newton's law of motion and the law of rigid body motion, we can see:

The rotation equation of the swing rod around its center of gravity is:

$$J\ddot{\theta} = N_y l \sin\theta - N_x l \cos\theta \qquad (1)$$

The X direction of the interaction force between the swing rod and the trolley is:

$$N_x = m\frac{d^2}{d^2 t}(x + l\sin\theta) \qquad (2)$$

According to the differential algorithm, we get:

$$N_x = m\ddot{x} + ml\ddot{\theta}\cos\theta - \mathrm{ml}\dot{\theta}^2\sin\theta \qquad (3)$$

The Y direction of the interaction force between the swing rod and the trolley is:

$$N_y = mg + m\frac{d^2}{d^2t}(l\cos\theta) \qquad (4)$$

According to the differential algorithm,

$$N_{\mathrm{y}} = mg - ml\ddot{\theta}\sin\theta - \mathrm{ml}\dot{\theta}^2\cos\theta \qquad (5)$$

According to the differential algorithm, the movement of the trolley in horizontal direction is as follows:

$$\mathrm{u}(t) - N_{\mathrm{x}} = M\frac{d^2x}{d^2t} \qquad (6)$$

In addition, considering that the inclination angle of pendulum bars is only adjusted between them, the approximate ones are:

$$\dot{\theta}^2 \approx 0$$
$$\sin\theta \approx \theta$$
$$\cos\theta \approx 1$$

Therefore, we simplify (1), (3) and (5) the following equations:

$$J\ddot{\theta} = N_{\mathrm{y}}l\theta - N_x l \qquad (7)$$

$$N_{\mathrm{x}} = m\ddot{x} + ml\ddot{\theta} \qquad (8)$$

$$N_{\mathrm{y}} = mg - \mathrm{ml}\ddot{\theta}\theta \qquad (9)$$

Combined with (6) (7) (8) (9), the following equations of motion can be obtained:

$$(M + \mathrm{m})\ddot{x} + ml\ddot{\theta} = u(t) \qquad (10)$$

$$(J + \mathrm{ml}^2)\ \ddot{\theta} + ml\ddot{x} = mgl\theta \qquad (11)$$

After finishing,

$$\ddot{x} = -\frac{m^2gl^2}{(M+m)J + Mml^2}\theta + \frac{J + \mathrm{ml}^2}{(M+\mathrm{m})J + Mml^2}u \qquad (12)$$

$$\ddot{\theta} = \frac{(M+\mathrm{m})mgl}{(M+m)J + Mml^2}\theta - \frac{\mathrm{ml}}{(M+\mathrm{m})J + Mml^2}u \qquad (13)$$

After measurement,

M=0.2275Kg, m=0.0923Kg, l=0.185m, J=0.004212 are obtained.

Substituting them in (12) (13) can be obtained:

$$\ddot{x} = -1.383\theta + 3.568u$$

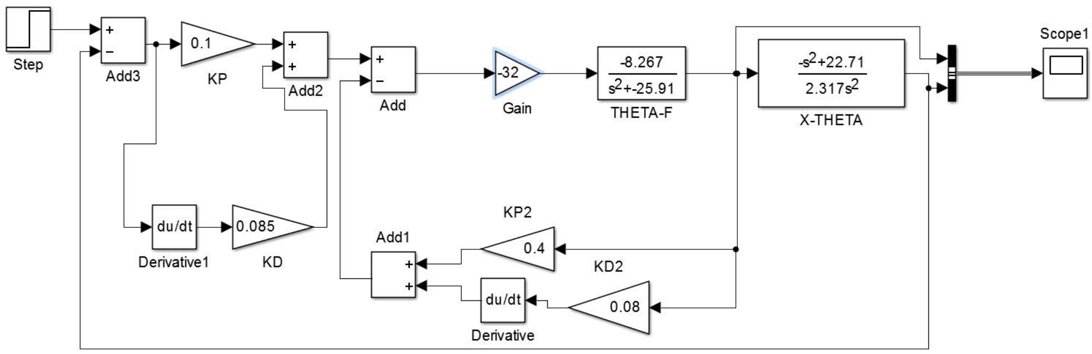
$$\ddot{\theta} = 25.91\theta - 8.267\mathrm{u}$$

The Laplasse transform is used to obtain the following transfer functions:

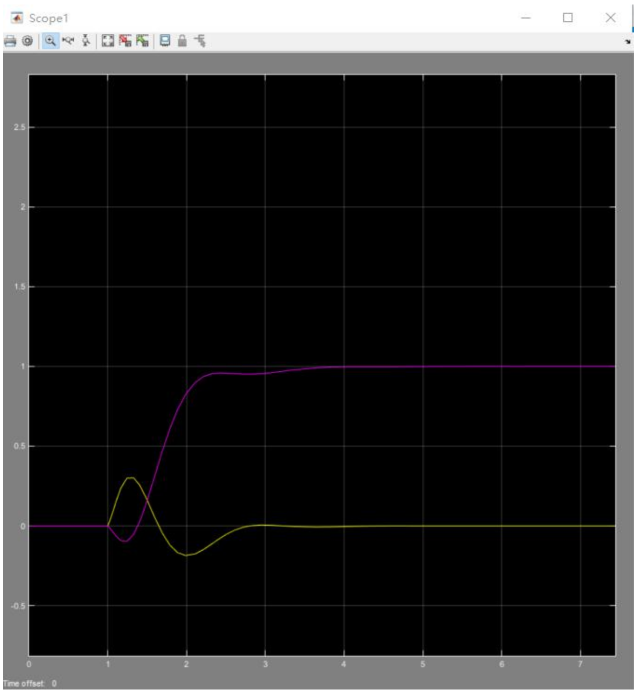
$$\frac{X(\mathrm{s})}{\theta(s)} = \frac{-s^2 + 22.71}{2.317s^2}$$

$$\frac{\theta(s)}{u(s)} = \frac{-8.267}{s^2 - 25.91}$$

Next, use Simulink to build the system block diagram.



The response waveform is as follows:


Scope1
2.5
2
1.5
1
0.5
0
-0.5
0
1
2
3
4
5
6
7
Time offset: 0