CS 576 Assignment 1 - Analysis Report

Introduction

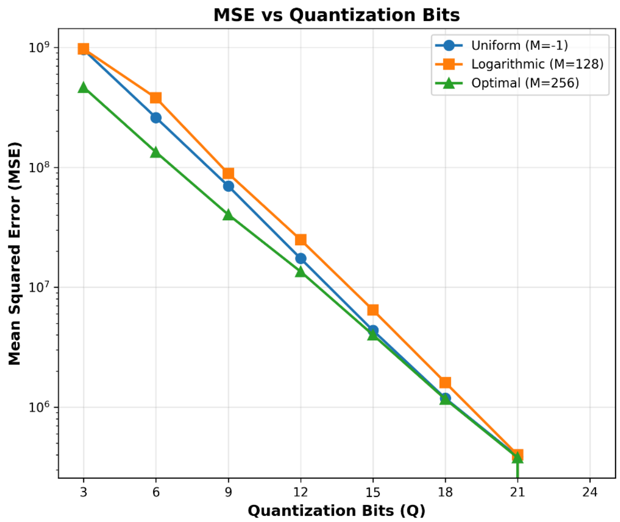
For this assignment, I implemented three different quantization methods and tested them on the Lena image. I compared uniform quantization (M=-1), logarithmic quantization with pivot at 128 (M=128), and optimal quantization (M=256) across different bit depths from Q=3 to Q=24.

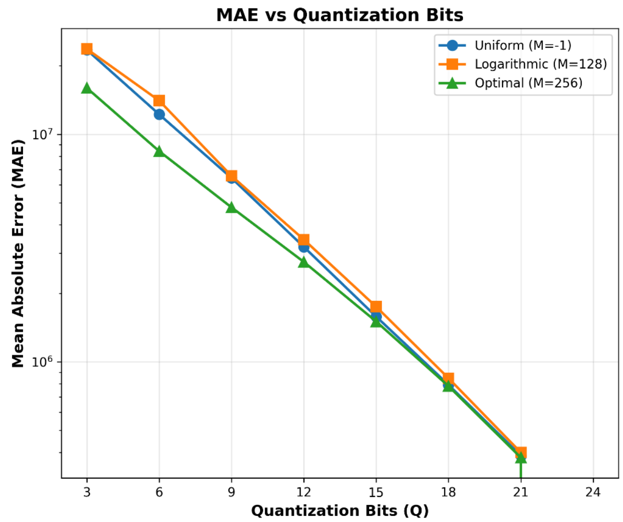
Results

Here are the MSE values I got from running my program:

Q	Uniform MSE	Log (M=128) MSE	Optimal MSE
3	958,316,100	970,917,224	465,311,505
6	259,893,444	379,493,628	133,913,495
9	69,839,940	88,837,553	40,308,724
12	17,369,348	24,883,364	13,504,741
15	4,358,756	6,455,953	4,007,114
18	1,184,940	1,606,106	1,166,116
21	386,988	399,444	379,420
24	0	0	0

Graphs





Observations

Optimal Quantization Works Best

The first thing I noticed is that optimal quantization (M=256) consistently gives the lowest MSE across all Q values. At Q=3 (which is only 1 bit per channel), optimal quantization cuts the error almost in half compared to uniform - that's a 51% improvement! Even at Q=9 where we have 3 bits per channel, optimal is still about 42% better than uniform.

I think this makes sense because the optimal method uses the Lloyd-Max algorithm which looks at the actual pixel values in the image and places the quantization levels where pixels actually exist. Uniform quantization just divides the 0-255 range evenly without caring about the image content.

Logarithmic Quantization Didn't Work Well

Something interesting is that logarithmic quantization actually performed worse than uniform for the Lena image. At Q=9, logarithmic had an MSE of 88 million while uniform only had 69 million. This was surprising at first, but after thinking about it, I realized it makes sense.

Logarithmic quantization works by putting more bins near the pivot point (M=128 in my tests). But Lena is a pretty balanced image - the pixels are spread fairly evenly across the brightness range. So by using logarithmic spacing, we're basically wasting precision in some areas while not having enough in others. If I had tested on a really dark or really bright image, logarithmic might have done better.

Error Decreases Exponentially

Looking at the graph, you can see that the error drops really fast as Q increases. Each time we add more bits, the MSE roughly gets divided by 3-4. For example:

- Q=3 → Q=6: MSE goes from 958M to 259M (about 1/4)
- Q=6 → Q=9: MSE goes from 259M to 69M (about 1/4)
- Q=9 → Q=12: MSE goes from 69M to 17M (about 1/4)

This makes sense because every bit we add doubles the number of quantization levels, which should roughly halve the maximum error.

Diminishing Returns at Higher Bits

One trend I noticed is that optimal quantization helps a lot more when we have fewer bits. At Q=3, optimal is 51% better than uniform. But at Q=15, it's only 8% better, and by Q=21, it's basically the same.

I think this happens because when we already have a lot of quantization levels (like 128 levels per channel at Q=21), both methods are precise enough that it doesn't matter much how you place the bins. The bins are so small that they capture the pixel values well either way.

All Methods Converge

At Q=24, all three methods have MSE=0, which makes sense because that's 8 bits per channel - the same as the original image. So there's no quantization happening at all. Even at Q=21 (7 bits per channel), all three methods give very low error (under 400K), so at that point the differences between methods become pretty negligible.

Discussion

Why Different Methods Perform Differently

Based on my results, I think the key difference between these methods is how they handle the actual distribution of pixel values:

- **Uniform** doesn't look at the image at all, just splits 0-255 evenly
- **Logarithmic** assumes a certain distribution (more pixels near the pivot)

- **Optimal** actually analyzes the image and adapts

For Lena, which has a fairly balanced distribution, uniform does okay but optimal does much better because it finds where the actual pixels are. Logarithmic does worse because its assumption about the distribution doesn't match Lena.

When to Use Each Method

Based on what I saw:

- Use **optimal** when you need the best quality at low bit rates ($Q \leq 12$) and can afford the extra computation
- Use **uniform** when you need something simple or when you're already at high bit rates ($Q \geq 18$)
- Use **logarithmic** only if you know your image has a skewed distribution (very dark or very bright)

Limitations

There are some limitations to my analysis:

1. I only tested on one image (Lena). Different images might give different results.
2. I only tried M=128 for logarithmic. Testing M=0 or M=255 on darker/brighter images might show better results.
3. MSE doesn't always match what looks good visually to humans.

If I had more time, I would test on a variety of images with different characteristics (dark images, bright images, high contrast, etc.) to see if the patterns hold.

Conclusions

From this assignment, I learned that:

1. Optimal quantization is significantly better than uniform at low bit rates, with improvements of 40-50% at Q=3-9
2. Logarithmic quantization isn't always better - it depends on matching the image distribution
3. Error decreases exponentially with more bits, so each additional bit helps less and less
4. At high bit rates ($Q \geq 18$), the quantization method doesn't matter much because all methods are already very accurate
5. Data-driven approaches like optimal quantization work better than fixed assumptions

Overall, this assignment showed me that simple assumptions (like uniform bins) don't always give the best results, and that adapting to the actual image content can make a big difference, especially when working with limited bits.