

Standard Code Library

ONGLU

North Eastern University

August 2021

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初始化

数据结构

ST 表

二维哈希

```
1 ull hs[109][109], pw1[10009], pw2[100009];
2 ull gethash(int lx, int ly, int rx, int ry) {
3     ull hs1 = hs[lx][ly] - pw2[ry - ly + 1] * hs[lx][ry + 1];
4     ull hs2 = hs[rx + 1][ly] - pw2[ry - ly + 1] * hs[rx + 1][ry + 1];
5     return hs1 - pw1[rx - lx + 1] * hs2;
6 }
7 pw1[0] = pw2[0] = 1;
8 for(int i = 1; i <= 1000; i++) pw1[i] = pw1[i - 1] * 19260817;
9 for(int i = 1; i <= 1000; i++) pw2[i] = pw2[i - 1] * 135;
10 for(int i = n; i >= 1; i--) {
11     for(int j = 1; j <= m; j++) {
12         if(i == n) hs[i][j] = sum[i][j] + 2;
13         else hs[i][j] = hs[i + 1][j] * 19260817 + sum[i][j] + 2;
14     }
15 }
16 for(int i = 1; i <= n; i++) {
17     for(int j = m - 1; j; j--) {
18         hs[i][j] = hs[i][j + 1] * 135 + hs[i][j];
19     }
20 }
```

轻重链剖分

```
1 void dfs1(int x, int pre) {
2     siz[x] = 1; mson[x] = 0;
3     dth[x] = dth[pre] + 1;
4     fa[x] = pre;
5     for(auto y : son[x]) if(y != pre) {
6         dfs1(y, x);
7         siz[x] += siz[y];
8         if(!mson[x] || siz[y] > siz[mson[x]])
9             mson[x] = y;
10    }
11 }
12 void dfs2(int x, int pre, int ntp) {
13     id[x] = ++idcnt;
14     ltp[x] = ntp;
15     if(mson[x]) dfs2(mson[x], x, ntp);
16     for(auto y : son[x]) {
17         if(y == mson[x] || y == pre) continue;
18         dfs2(y, x, y);
19     }
20 }
21 void link_modify(int x, int y, int z) {
22     z %= mod;
23     while(ltp[x] != ltp[y]) {
24         dth[ltp[x]] < dth[ltp[y]] && (x ^ y ^ x ^ y);
25         modify(1, n, id[ltp[x]], id[x], 1, z);
26         x = fa[ltp[x]];
27     }
28     dth[x] < dth[y] && (x ^ y ^ x ^ y);
29     modify(1, n, id[y], id[x], 1, z);
30 }
31
32 int link_query(int x, int y) {
33     int ans = 0;
34     while(ltp[x] != ltp[y]) {
35         dth[ltp[x]] < dth[ltp[y]] && (x ^ y ^ x ^ y);
36         ans = (1ll * ans + query(1, n, id[ltp[x]], id[x], 1)) % mod;
37         x = fa[ltp[x]];
38     }
```

```

39     dth[x] < dth[y] && (x ^ y ^ x ^ y);
40     ans = (1ll * ans + query(1, n, id[y], id[x], 1)) % mod;
41     return ans;
42 }

```

线段树合并

搞个动态开点线段树出来

```

1  #define mval(x) tree[x].mval
2  #define mpos(x) tree[x].mpos
3  #define lson(x) tree[x].lson
4  #define rson(x) tree[x].rson
5  struct node {
6      int mpos, mval, lson, rson;
7  } tree[N * 50];
8  void update(int rt) {
9      if(mval(lson(rt)) >= mval(rson(rt))) {
10         mval(rt) = mval(lson(rt));
11         mpos(rt) = mpos(lson(rt));
12     } else {
13         mval(rt) = mval(rson(rt));
14         mpos(rt) = mpos(rson(rt));
15     }
16 }
17 }
18 void modify(int l, int r, int x, int v, int &rt) {
19     if(!rt) rt = ++idtot;
20     if(l == r) {
21         mval(rt) += v;
22         mpos(rt) = l;
23         return;
24     }
25     if(x <= Mid) modify(l, Mid, x, v, lson(rt));
26     else modify(Mid + 1, r, x, v, rson(rt));
27     update(rt);
28 }
29 int merge(int l, int r, int rt1, int rt2) {
30     if(!rt1 || !rt2) return rt1 + rt2;
31     if(l == r) {
32         mval(rt1) += mval(rt2);
33         mpos(rt1) = l;
34         return rt1;
35     }
36     lson(rt1) = merge(l, Mid, lson(rt1), lson(rt2));
37     rson(rt1) = merge(Mid + 1, r, rson(rt1), rson(rt2));
38     update(rt1);
39     return rt1;
40 }

```

二维树状数组

- 矩阵修改, 矩阵查询

查询前缀和公式:

令 $d[i][j]$ 为差分数组, 定义 $d[i][j] = a[i][j] - (a[i-1][j] - a[i][j-1] - a[i-1][j])$

$$\sum_{i=1}^x \sum_{j=1}^y a[i][j] = (x+1) * (y+1) * d[i][j] - (y+1) * i * d[i][j] + d[i][j] * i * j$$

```

1  void modify(int x, int y, int v) {
2      for(int rx = x; rx <= n; rx += rx & -rx) {
3          for(int ry = y; ry <= m; ry += ry & -ry) {
4              tree[rx][ry][0] += v;
5              tree[rx][ry][1] += v * x;
6              tree[rx][ry][2] += v * y;
7              tree[rx][ry][3] += v * x * y;
8          }
9      }
10 }

```

```

11 void range_modify(int x, int y, int xx, int yy, int v) {
12     modify(xx + 1, yy + 1, v);
13     modify(x, yy + 1, -v);
14     modify(xx + 1, y, -v);
15     modify(x, y, v);
16 }
17 int query(int x, int y) {
18     int ans = 0;
19     for(int rx = x; rx; rx -= rx & -rx) {
20         for(int ry = y; ry; ry -= ry & -ry) {
21             ans += (x + 1) * (y + 1) * tree[rx][ry][0]
22                 - tree[rx][ry][1] * (y + 1) - tree[rx][ry][2] * (x + 1)
23                 + tree[rx][ry][3];
24         }
25     }
26     return ans;
27 }
28 int range_query(int x, int y, int xx, int yy) {
29     return query(xx, yy) + query(x - 1, y - 1)
30         - query(x - 1, yy) - query(xx, y - 1);
31 }

```

平衡树

- luogu P3369 【模板】普通平衡树

```

1  #define val(x) tree[x].val
2  #define cnt(x) tree[x].cnt
3  #define siz(x) tree[x].siz
4  #define fa(x) tree[x].fa
5  #define son(x, k) tree[x].ch[k]
6  struct Tree {
7      struct node {
8          int val, cnt, siz, fa, ch[2];
9      } tree[N];
10     int root, tot;
11     int chk(int x) {
12         return son(fa(x), 1) == x;
13     }
14     void update(int x) {
15         siz(x) = siz(son(x, 0)) + siz(son(x, 1)) + cnt(x);
16     }
17     void rotate(int x) {
18         int y = fa(x), z = fa(y), k = chk(x), w = son(x, k ^ 1);
19         son(y, k) = w; fa(w) = y;
20         son(z, chk(y)) = x; fa(x) = z;
21         son(x, k ^ 1) = y; fa(y) = x;
22         update(y); update(x);
23     }
24     void splay(int x, int goal = 0) {
25         while(fa(x) != goal) {
26             int y = fa(x), z = fa(y);
27             if(z != goal) {
28                 //双旋
29                 if(chk(y) == chk(x)) rotate(y);
30                 else rotate(x);
31             }
32             rotate(x);
33         }
34         if(!goal) root = x;
35     }
36     int New(int x, int pre) {
37         tot++;
38         if(pre) son(pre, x > val(pre)) = tot;
39         val(tot) = x; fa(tot) = pre;
40         siz(tot) = cnt(tot) = 1;
41         son(tot, 0) = son(tot, 1) = 0;
42         return tot;
43     }

```

```

44 void Insert(int x) {
45     int cur = root, p = 0;
46     while(cur && val(cur) != x) {
47         p = cur;
48         cur = son(cur, x > val(cur));
49     }
50     if(cur) cnt(cur)++;
51     else cur = New(x, p);
52     splay(cur);
53 }
54 void Find(int x) {
55     if(!root) return ;
56     int cur = root;
57     while(val(cur) != x && son(cur, x > val(cur)))
58         cur = son(cur, x > val(cur));
59     splay(cur);
60 }
61 int Pre(int x) {
62     Find(x);
63     if(val(root) < x) return root;
64     int cur = son(root, 0);
65     while(son(cur, 1))
66         cur = son(cur, 1);
67     return cur;
68 }
69 int Succ(int x) {
70     Find(x);
71     if(val(root) > x) return root;
72     int cur = son(root, 1);
73     while(son(cur, 0))
74         cur = son(cur, 0);
75     return cur;
76 }
77 void Del(int x) {
78     int lst = Pre(x), nxt = Succ(x);
79     splay(lst); splay(nxt, lst);
80     int cur = son(nxt, 0);
81     if(cnt(cur) > 1) cnt(cur)--, splay(cur);
82     else son(nxt, 0) = 0, splay(nxt);
83 }
84 int Kth(int k) {
85     int cur = root;
86     while(1) {
87         if(son(cur, 0) && siz(son(cur, 0)) >= k) cur = son(cur, 0);
88         else if(siz(son(cur, 0)) + cnt(cur) >= k) return cur;
89         else k -= siz(son(cur, 0)) + cnt(cur), cur = son(cur, 1);
90     }
91 }
92 } T;

```

K-D Tree

用方差最大的那一维坐标作为当前的划分点集，然后选取该维度的中位数点划分成左右两个点集。

```

1  #include <bits/stdc++.h>
2  #define pt(x) cout << x << endl;
3  #define Mid ((l + r) / 2)
4  #define low(x, k) tree[x].low[k]
5  #define high(x, k) tree[x].high[k]
6  #define lson(x) tree[x].lson
7  #define rson(x) tree[x].rson
8  using namespace std;
9  int read() {
10     char c; int num, f = 1;
11     while(c = getchar(), !isdigit(c)) if(c == '-') f = -1; num = c - '0';
12     while(c = getchar(), isdigit(c)) num = num * 10 + c - '0';
13     return f * num;
14 }
15 const int N = 5e5 + 1009;
16
17 namespace KD_Tree{

```

```

18
19 const int dimension = 2;
20 struct node {
21     int lson, rson;
22     int low[dimension], high[dimension];
23 } tree[N];
24 struct Point {
25     int id;
26     int v[dimension];
27 } p[N];
28 void update(int rt) {
29     for(int i = 0; i < dimension; i++) {
30         low(rt, i) = high(rt, i) = p[rt].v[i];
31         if(lson(rt)) {
32             low(rt, i) = min(low(rt, i), low(lson(rt), i));
33             high(rt, i) = max(high(rt, i), high(lson(rt), i));
34         }
35         if(rson(rt)) {
36             low(rt, i) = min(low(rt, i), low(rson(rt), i));
37             high(rt, i) = max(high(rt, i), high(rson(rt), i));
38         }
39     }
40 }
41 }
42 int build(int l, int r) {
43     if(l > r) return 0;
44     double av[dimension] = {0};
45     double va[dimension] = {0};
46     for(int i = 0; i < dimension; i++)
47         low(Mid, i) = high(Mid, i) = p[Mid].v[i];
48     for(int i = l; i <= r; i++)
49         for(int j = 0; j < dimension; j++)
50             av[j] += p[i].v[j];
51     for(int i = 0; i < dimension; i++)
52         av[i] /= (double) (r - l + 1);
53     for(int i = l; i <= r; i++)
54         for(int j = 0; j < dimension; j++)
55             va[j] += (p[i].v[j] - av[j]) * (p[i].v[j] - av[j]);
56     int maxdi = 0;
57     for(int i = 1; i < dimension; i++)
58         if(va[i] > va[maxdi])
59             maxdi = i;
60     nth_element(p + l, p + Mid, p + 1 + r, [maxdi](const Point &a, const Point &b) -> int{return a.v[maxdi] <
61     b.v[maxdi]});
62     lson(Mid) = build(l, Mid - 1);
63     rson(Mid) = build(Mid + 1, r);
64     update(Mid);
65     return Mid;
66 }
67 int isIn(const Point &a, const Point &ld, const Point &ru) {
68     for(int i = 0; i < dimension; i++)
69         if(a.v[i] < ld.v[i] || a.v[i] > ru.v[i])
70             return false;
71     return true;
72 }
73 void debug(int rt, int l, int r) {
74     if(l > r) return ;
75     printf("%d\n", p[rt].id);
76     debug(lson(rt), l, Mid - 1);
77     debug(rson(rt), Mid + 1, r);
78 }
79 //只能处理二维
80 void getNodeset(int rt, int l, int r, vector<int> &v, const Point &ld, const Point &ru) {
81     if(l > r) return ;
82     for(int i = 0; i < dimension; i++) {
83         if(low(rt, i) > ru.v[i] || high(rt, i) < ld.v[i]) {
84             return ;
85         }
86     }
87     if(isIn(p[Mid], ld, ru))

```



```

88         v.push_back(p[Mid].id);
89         getNodeset(lson(rt), l, Mid - 1, v, ld, ru);
90         getNodeset(rson(rt), Mid + 1, r, v, ld, ru);
91     }
92 }
93 using namespace KD_Tree;
94 int n, q, root;
95 signed main()
96 {
97     n = read();
98     for(int i = 1; i <= n; i++) {
99         p[i].v[0] = read();
100        p[i].v[1] = read();
101        p[i].id = i - 1;
102    }
103    root = build(1, n);
104    q = read();
105    for(int i = 1; i <= q; i++) {
106        int x = read(), xx = read();
107        int y = read(), yy = read();
108        Point ld, ru;
109        ld.v[0] = x; ld.v[1] = y;
110        ru.v[0] = xx; ru.v[1] = yy;
111        vector<int> v;
112        v.clear();
113        getNodeset(root, 1, n, v, ld, ru);
114        sort(v.begin(), v.end());
115        for(auto x : v)
116            printf("%d\n", x);
117        printf("\n");
118    }
119    return 0;
120 }

```

可持久化数据结构

可持久化 Trie

```

1  namespace Trie {
2      struct node {
3          int ch[2], ed, siz;
4      } tree[N * 40];
5      int tot = 0;
6      int _new() {
7          tot++;
8          tree[tot].ch[0] = 0;
9          tree[tot].ch[1] = 0;
10         tree[tot].ed = tree[tot].siz = 0;
11         return tot;
12     }
13     void init() {
14         tot = 0;
15         rt[0] = _new();
16     }
17     int Insert(int x, int t, int i = 15) {
18         int u = _new(), f = (x >> i) & 1;
19         tree[u] = tree[t];
20         if(i == -1) {
21             ed(u)++;
22             siz(u)++;
23             return u;
24         }
25         son(u, f) = Insert(x, son(t, f), i - 1);
26         siz(u) = siz(son(u, 0)) + siz(son(u, 1));
27         return u;
28     }
29     void print(int u, int now) {
30         if(u == 0) return;
31         for(int i = 1; i <= ed(u); i++) printf("%d ", now);
32         if(son(u, 0)) print(son(u, 0), now * 2);
33         if(son(u, 1)) print(son(u, 1), now * 2 + 1);

```

```

34     }
35     int query(int u1, int u2, int x, int i = 15, int now = 0) {
36         if(i == -1) return now;
37         int f = (x >> i) & 1;
38         if(siz(son(u1, f ^ 1)) - siz(son(u2, f ^ 1)) > 0)
39             return query(son(u1, f ^ 1), son(u2, f ^ 1), x, i - 1, now * 2 + (f ^ 1));
40         else return query(son(u1, f), son(u2, f), x, i - 1, now * 2 + (f));
41     }
42 }

```

主席树（静态第 k 小）

建立权值树，那么 $[l, r]$ 的区间权值树就是第 r 个版本减去第 $l - 1$ 个版本的树。

```

1  #include <iostream>
2  #include <cstdio>
3  #include <algorithm>
4  #include <cmath>
5  #include <assert.h>
6  #define Mid ((l + r) / 2)
7  #define lson (rt << 1)
8  #define rson (rt << 1 | 1)
9  using namespace std;
10 int read() {
11     char c; int num, f = 1;
12     while(c = getchar(), !isdigit(c)) if(c == '-') f = -1; num = c - '0';
13     while(c = getchar(), isdigit(c)) num = num * 10 + c - '0';
14     return f * num;
15 }
16 const int N = 1e7 + 1009;
17 const int M = 2e5 + 1009;
18 struct node {
19     int ls, rs, v;
20 } tree[N];
21 int tb;
22 int n, m, tot, a[M], b[M], rt[M];
23 int _new(int ls, int rs, int v) {
24     tree[++tot].ls = ls;
25     tree[tot].rs = rs;
26     tree[tot].v = v;
27     return tot;
28 }
29 void update(int rt) {
30     tree[rt].v = tree[tree[rt].ls].v + tree[tree[rt].rs].v;
31 }
32 int build(int l, int r) {
33     if(l == r) return _new(0, 0, 0);
34     int x = _new(build(l, Mid), build(Mid + 1, r), 0);
35     update(x);
36     return x;
37 }
38 int add(int l, int r, int p, int rt, int v) {
39     int x = ++tot;
40     tree[x] = tree[rt];
41     if(l == r) {
42         tree[x].v += v;
43         return x;
44     }
45     if(p <= Mid) tree[x].ls = add(l, Mid, p, tree[x].ls, v);
46     else tree[x].rs = add(Mid + 1, r, p, tree[x].rs, v);
47     update(x);
48     return x;
49 }
50 int query(int l, int r, int rt1, int rt2, int k) {
51     if(l == r) return l;
52     if(k <= tree[tree[rt1].ls].v - tree[tree[rt2].ls].v) return query(l, Mid, tree[rt1].ls, tree[rt2].ls, k);
53     else return query(Mid + 1, r, tree[rt1].rs, tree[rt2].rs, k - (tree[tree[rt1].ls].v - tree[tree[rt2].ls].v));
54 }
55 void Debug(int l, int r, int rt) {
56     printf("%d %d %d\n", l, r, tree[rt].v);
57     if(l == r) return;

```

```

58     Debug(l, Mid, tree[rt].ls);
59     Debug(Mid + 1, r, tree[rt].rs);
60 }
61 signed main()
62 {
63     n = read(); m = read();
64     for(int i = 1; i <= n; i++) a[i] = b[i] = read();
65     sort(b + 1, b + 1 + n);
66     tb = unique(b + 1, b + 1 + n) - b - 1;
67     rt[0] = build(1, tb);
68     for(int i = 1; i <= n; i++) {
69         rt[i] = add(1, tb, lower_bound(b + 1, b + 1 + tb, a[i]) - b, rt[i - 1], 1);
70     }
71     for(int i = 1; i <= m; i++) {
72         int l, r, k;
73         l = read(); r = read(); k = read();
74         assert(r - l + 1 >= k);
75         printf("%d\n", b[query(l, tb, rt[r], rt[l - 1], k)]);
76     }
77     return 0;
78 }

```

cdq 分治三维偏序

先按照第一维排序, 然后对第二维归并, 归并时计算左对右的贡献, 先双指针, 满足当前统计出的第二维都有序

```

1  const int N = 1e6 + 1009;
2  struct node{
3      int x, y, z, id, cnt;
4  }a[N], tmp[N];
5  bool operator ==(const node &a, const node &b) {
6      return a.x == b.x && a.y == b.y && a.z == b.z;
7  }
8  int n, m, tot, ans[N], tt[N], tree[N];
9  int ttt[N];
10 bool cmp(node a, node b) {
11     if(a.x == b.x && a.y == b.y) return a.z < b.z;
12     if(a.x == b.x) return a.y < b.y;
13     return a.x < b.x;
14 }
15 void add(int x, int y) {
16     for( ; x <= m; x += x & -x)
17         tree[x] += y;
18 }
19 int query(int x) {
20     int ans = 0;
21     for( ; x; x -= x & -x)
22         ans += tree[x];
23     return ans;
24 }
25 void cdq(int l, int r) {
26     if(l == r) return ;
27     cdq(l, Mid); cdq(Mid + 1, r);
28     int i = l, j = Mid + 1, now = l - 1;
29     while(i <= Mid && j <= r) {
30         if(a[i].y <= a[j].y) {
31             tmp[++now] = a[i];
32             add(a[i].z, a[i].cnt);
33             i++;
34         } else {
35             tmp[++now] = a[j];
36             ans[a[j].id] += query(a[j].z);
37             j++;
38         }
39     }
40     while(i <= Mid) {
41         tmp[++now] = a[i];
42         add(a[i].z, a[i].cnt);
43         i++;
44     }
45     while(j <= r) {

```

```

46         tmp[++now] = a[j];
47         ans[a[j].id] += query(a[j].z);
48         j++;
49     }
50     for(int i = l; i <= Mid; i++) add(a[i].z, -a[i].cnt);
51     for(int i = l; i <= r; i++) a[i] = tmp[i];
52 }
53 main()
54 {
55     n = read(); m = read();
56     for(int i = 1; i <= n; i++) {
57         a[i].x = read();
58         a[i].y = read();
59         a[i].z = read();
60         a[i].cnt = 1;
61     }
62     sort(a + 1, a + 1 + n, cmp);
63     for(int i = 1; i <= n; i++) {
64         if(i == 1 || !(a[i] == a[i - 1])){
65             a[++tot] = a[i];
66         }else a[tot].cnt += a[i].cnt;
67     }
68     for(int i = 1; i <= tot; i++) a[i].id = i, ttt[i] = a[i].cnt;
69     cdq(1, tot);
70     for(int i = 1; i <= tot; i++) tt[ans[i] + ttt[i] - 1] += ttt[i];
71     for(int i = 0; i < n; i++) printf("%d\n", tt[i]);
72     return 0;
73 }

```

数学

数论

欧拉函数

性质

1 和任何数互质。

$$+ \phi(1) = 1 + \phi(p) = p - 1 (p \text{ 为质数}) + \phi(x \times p) = \phi(x) \times p (p \mid x), \phi(x \times p) = \phi(x) \times p (p \nmid x)$$

线性欧拉函数筛

```

1  int phi[N], f[N], pri[N], tot;
2  void getphi() {
3      int k;
4      phi[1] = 1;
5      for(int i = 2; i < N; i++) {
6          if(!f[i]) phi[pri[++tot] = i] = i - 1;
7          for(int j = 1; j <= tot && (k = i * pri[j]) < N; j++) {
8              f[k] = 1;
9              if(i % pri[j]) phi[k] = phi[i] * (pri[j] - 1);
10             else {
11                 phi[k] = phi[i] * pri[j];
12                 break;
13             }
14         }
15     }
16 }

```

$O(\sqrt{n})$ 求欧拉函数

```

1  int getphi(int x) {
2      int phi = 1;
3      for(int i = 2; i * i <= x; i++) if(x % i == 0) {
4          while(x % i == 0) {
5              if(x / i % i == 0) phi = phi * i;
6              else phi = phi * (i - 1);
7              x /= i;
8          }
9      }
10     if(x > 1) phi *= x - 1;

```

```

11     return phi;
12 }

```

排列组合

斯特林近似求组合 (≥ 15 时收敛)

精度容易不够, 推荐使用 python Demical 类

$$\ln n! \simeq n \ln n - n + \frac{1}{6} \ln(8n^3 + 4n^2 + n + \frac{1}{30}) + \frac{1}{2} \ln \pi$$

```

1 double lnfac(int n) {
2     return n * log(n) - n + 1.0 / 6 * log(8 * n * n * n + 4 * n * n + n + 1.0 / 30) + 0.5 * log(acos(-1.0));
3 }
4 double C(int n, int m) {
5     return exp(lnfac(n) - lnfac(n - m) - lnfac(m));
6 }

```

Lucas 定理

$$\binom{n}{m} = \binom{n \bmod p}{m \bmod p} \times \binom{n/p}{m/p}$$

```

1 int C(int n, int m) {
2     if(m > n) return 0;
3     if(n < mod) return 1ll * fac[n] * inv[n - m] % mod * inv[m] % mod;
4     else return 1ll * C(n / mod, m / mod) * C(n % mod, m % mod) % mod;
5 }

```

Min-Max 容斥

$$\max(S) = \sum_{T \subseteq S} (-1)^{|T|-1} \min(T)$$

逆元

线性推

```

1 inv[1] = inv[0] = 1;
2 for(int i = 2; i < N; i++) inv[i] = (1ll * mod - mod / i) * inv[mod % i] % mod;

```

费马小定理 (模数为质数)

```

1 int inv(int x) {
2     return Pow(x % mod, mod - 2);
3 }

```

exgcd(ap 互质)

```

1 int inv(int x) {
2     int x, y;
3     exgcd(x, y, a, p);
4     return (x % p + p) % p;
5 }

```

拓展欧几里得

求解的是类似 $ax + by = \gcd(a, b)$ 的一组解。

```

1 void exgcd(int &x, int &y, int a, int b) {
2     if(b == 0) return (void)(x = 1, y = 0);
3     exgcd(y, x, b, a % b);
4     y = y - a / b * x;
5 }

```

拓展中国剩余定理

拓展中国剩余定理用于解决同余方程组。

$$x \equiv a_i \pmod{b_i}$$

构造 $M_k = lcm_{i=1}^{k-1} b_i$

假设前面的解为 p 显然新解 $p + M_k \times y$ 仍然是前面方程的解。

exgcd 求出 $M_k \times x + b_i \times y = gcd(M_k, b_i)$ 的解。

于是 $p' = p + x \times M_k \times (a_i - p) / gcd(M_k, b_i)$ 。

实际处理的时候可以直接让 $b_i = b_i / gcd(b_i, M_k)$ 防止溢出。

```
1  #define long long ll
2  ll gcd(ll a, ll b) {
3      return b == 0 ? a : gcd(b, a % b);
4  }
5  ll lcm(ll a, ll b) {
6      return a / gcd(a, b) * b;
7  }
8  ll exgcd(ll &x, ll &y, ll a, ll b) {
9      if(b == 0) return x = 1, y = 0, a;
10     ll t = exgcd(y, x, b, a % b);
11     y -= a / b * x;
12     return t;
13 }
14 inline ll mul(ll x, ll y, ll mod){
15     return (x * y - (ll)((long double)x / mod * y) * mod + mod) % mod;
16 }
17
18 ll excrt(ll n, ll *a, ll *b) {
19     ll ans = a[1], M = b[1];
20     for(ll i = 2; i <= n; i++) {
21         ll c = ((a[i] - ans) % b[i] + b[i]) % b[i], x, y;
22         ll t = exgcd(x, y, M, b[i]), pb = b[i] / t;
23         if(c % t != 0) return -1;
24         x = mul(x, c / t, pb);
25         ans = ans + x * M;
26         M = M * pb;
27         ans = (ans % M + M) % M;
28     }
29     return ans;
30 }
```

Miller_rabbin 素数测试

```
1  namespace Isprime{
2      ll mul(ll x, ll y, ll mod){
3          return (x * y - (ll)((long double)x / mod * y) * mod + mod) % mod;
4      }
5      ll Pow(ll a, ll p, ll mod) {
6          ll ans = 1;
7          for( ; p; p >>= 1, a = mul(a, a, mod))
8              if(p & 1)
9                  ans = mul(ans, a, mod);
10         return ans % mod;
11     }
12     int check(ll P){
13         const ll test[11] = {0, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29};
14         if(P == 1) return false;
15         if(P > 6 && P % 6 != 1 && P % 6 != 5) return false;
16         ll k = 0, t = P - 1;
17         while(!(t & 1)) k++, t >>= 1;
18         for(int i = 1; i <= 10 && test[i] <= P; i++) {
19             if(P == test[i]) return true;
20             ll nxt, a = Pow(test[i], t, P);
21             for(int j = 1; j <= k; j++) {
22                 nxt = mul(a, a, P);
23                 if(nxt == 1 && a != 1 && a != P - 1) return false;
24                 a = nxt;
25             }
26         }
27     }
```

```

25         }
26         if(a != 1) return false;
27     }
28     return true;
29 }
30 }

```

多项式

结论

1. 自然数幂之和 $s(n) = \sum_{i=0}^n i^k$ 是关于 n 的 $k+1$ 次多项式

拉格朗日插值法

令拉格朗日函数

$$l_i(x) = \prod_{j \neq i} \frac{x - x_j}{x_i - x_j}$$

注意到这个函数有一些性质：

1. 次数为 n
2. 在 $x = x_i$ 位置值为 1, $x = x_j (j \neq i)$ 位置值为 0

于是可以凑出唯一的多项式表达式为：

$$f(x) = \sum_{i=0}^n y_i \prod_{j \neq i} \frac{x - x_j}{x_i - x_j}$$

如果要取模的话得求逆元，逆元先求好分母再一起求即可。

```

1  int interpolation(int *x, int *y, int n) {
2      int f = 0;
3      for(int i = 1; i <= n; i++) {
4          int s1 = 1, s2 = 1;
5          for(int j = 1; j <= n; j++) {
6              if(i != j) {
7                  s1 = 1ll * s1 * (x[i] - x[j] + mod) % mod;
8                  s2 = 1ll * s2 * (x[j] - x[i] + mod) % mod;
9              }
10         }
11         f = (f + 1ll * y[i] * s1 % mod * inv(s2) % mod) % mod;
12     }
13     return f;
14 }

```

FFT 快速傅里叶变换

FFT 的想法是把第 k 号位置变成 $f(\omega_n^k)$ ，注意到 $\omega_n^k = -\omega_n^{k+n/2}$ ，于是可以进行变换。

几条公式：

$$\omega_n^n = 1$$

$$\omega_n^k = \omega_{2n}^{2k}$$

$$\omega_{2n}^{k+n} = -\omega_{2n}^k$$

蝴蝶变换：相邻的位置为二进制的 reverse

DFT 变换公式 ($DFT(f)$ 为矩阵)：

令

$$G(x) = a_0 + a_2x + a_4x^2 + \dots$$

$$H(x) = a_1 + a_3x + a_5x^3 + \dots$$

则有

$$\begin{aligned}f(x) &= G(x^2) + x \times H(x^2) \\DFT(f(\omega_n^k)) &= DFT(G(\omega_{n/2}^k) + \omega_n^k \times DFT(H(\omega_{n/2}^k))) \\DFT(f(\omega_n^{k+n/2})) &= DFT(G(\omega_{n/2}^k) - \omega_n^k \times DFT(H(\omega_{n/2}^k)))\end{aligned}$$

$DFT(G(\omega_{n/2}^k), DFT(H(\omega_{n/2}^k)))$ 可递归计算

NTT 快速数论变换

NTT 使用原根代替复数进行运算。

原根 g 的重要性质: $g^t \equiv k \pmod n, t \in [0, n-2], k$ 遍取 $1 \sim n-1$

原根存在的充要条件是: 模数 $n = 2, 4, p^\alpha, 2p^\alpha$ (p 为奇质数)。

对于一个质数 $p = qn + 1 (n = 2^m)$, 原根满足性质 $g^{qn} \equiv 1 \pmod p$ 。

它满足和复数近似的性质, 我们把 q 看成复数中的 2π , 就可以套用 FFT 实现 NTT 了。

$$g_n^n \equiv 1, g_n^n \equiv -1$$

通常取

$$p = 1004535809 = 7 \times 479 \times 2^{21} + 1, g = 3$$

$$p = 998244353 = 7 \times 17 \times 2^{23} + 1, g = 3$$

```
1  const int P = 998244353, G = 3, Gi = 332748118;
2  struct Complex {double x, y;};
3  Complex operator+(const Complex &a, const Complex &b) {return (Complex) {a.x + b.x, a.y + b.y};}
4  Complex operator-(const Complex &a, const Complex &b) {return (Complex) {a.x - b.x, a.y - b.y};}
5  Complex operator*(const Complex &a, const Complex &b) {return (Complex) {a.x * b.x - a.y * b.y, a.x * b.y + a.y *
    ↪ b.x};}
6  namespace Polynomial {
7      const double Pi = acos(-1.0);
8      int rev[N];
9      template <typename T>
10     void change(T *y, int n) {
11         for(int i = 0; i < n; i++)
12             rev[i] = (rev[i >> 1] >> 1) | ((i & 1) ? (n >> 1) : 0);
13         for(int i = 0; i < n; i++)
14             if(i < rev[i])
15                 swap(y[i], y[rev[i]]);
16     }
17     void FFT(Complex *A, int n, int type) {
18         //type = 1 DFT
19         //type = -1 IDFT
20         //确保 n 是 2 的幂次
21         change(A, n);
22         for(int m = 1; m < n; m <= 1) {
23             Complex Wn = (Complex) {cos(Pi / m), type * sin(Pi / m)};
24             for(int i = 0; i < n; i += 2 * m) {
25                 Complex w = (Complex) {1.0, 0};
26                 for(int j = 0; j < m; j++, w = w * Wn) {
27                     Complex x = A[i + j], y = w * A[i + j + m];
28                     A[i + j] = x + y;
29                     A[i + j + m] = x - y;
30                 }
31             }
32         }
33         if(type == -1) {
34             for(int i = 0; i < n; i++)
35                 A[i].x = A[i].x / n;
36         }
37     }
38     void NTT(int *A, int n, int type) {
39         //type = 1 DFT
40         //type = -1 IDFT
41         change(A, n);
```



```

42     for(int m = 1; m < n; m <= 1) {
43         int Wn = Pow(type == 1 ? G : Gi, (P - 1) / (m < 1));
44         for(int i = 0; i < n; i += 2 * m) {
45             int w = 1;
46             for(int j = 0; j < m; j++, w = 1ll * w * Wn % P) {
47                 int x = A[i + j], y = 1ll * w * A[i + j + m] % P;
48                 A[i + j] = (x + y) % P;
49                 A[i + j + m] = (x - y + P) % P;
50             }
51         }
52     }
53     if(type == -1) {
54         int inv = Pow(n, P - 2);
55         for(int i = 0; i < n; i++)
56             A[i] = 1ll * A[i] * inv % P;
57     }
58 }
59
60 }
61 //以下代码加在主函数内
62 limit = 1;
63 while(limit <= n + m) limit <= 1;
64 Polynomial :: FFT(A, limit, 1);
65 Polynomial :: FFT(B, limit, 1);
66 for(int i = 0; i < limit; i++) A[i] = A[i] * B[i];
67 Polynomial :: FFT(A, limit, -1);

```

FWT 快速沃尔什变换

FWT 用于计算下列多项式

$$C[k] = \sum_{i \oplus j = k} A[i] \times B[j]$$

先通过 FWT 将 A, B 变为 $FWT(A), FWT(B)$, 这样有 $FWT(C) = FWT(A) \times FWT(B)$ 。
当然位运算符不同的时候对应的变换形式也需要改变。

$a \in S, b \in S$ 可以表示为 $a|b \in S$

FWT 为线性变换 $\sum FWT(F) = FWT(\sum F)$

与卷积

当 $\oplus = and$ 的时候

$$FWT(A) = (FWT(A_0) + FWT(A_1), FWT(A_1))$$

$$FWT(A) = A(\text{长度为 } 1)$$

$$IFWT(A) = (IFWT(A_0) - IFWT(A_1), IFWT(A_1))$$

或卷积

当 $\oplus = or$ 的时候

$$FWT(A) = (FWT(A_0), FWT(A_0) + FWT(A_1))$$

$$FWT(A) = A(\text{长度为 } 1)$$

$$IFWT(A) = (IFWT(A_0), IFWT(A_1) - IFWT(A_0))$$

异或卷积

当 $\oplus = xor$ 的时候

$$FWT(A) = (FWT(A_0) + FWT(A_1), FWT(A_0) - FWT(A_1))$$

$$FWT(A) = A(\text{长度为 } 1)$$

$$IFWT(A) = (\frac{IFWT(A_0) + IFWT(A_1)}{2}, \frac{IFWT(A_0) - IFWT(A_1)}{2})$$

```

1 namespace Polynomial {
2     void FWT_or(int *A, int n, int type) {
3         for(int m = 1; m < n; m <= 1) {
4             for(int i = 0; i < n; i += 2 * m) {
5                 for(int j = 0; j < m; j++) {
6                     A[i + j + m] = (1ll * A[i + j + m] + A[i + j] * type + mod) % mod;
7                 }
8             }
9         }
10    }

```

```

9     }
10    }
11    void FWT_and(int *A, int n, int type) {
12        for(int m = 1; m < n; m <= 1) {
13            for(int i = 0; i < n; i += 2 * m) {
14                for(int j = 0; j < m; j++) {
15                    A[i + j] = (1ll * A[i + j + m] * type + A[i + j] + mod) % mod;
16                }
17            }
18        }
19    }
20    void FWT_xor(int *A, int n, int type) {
21        int inv_2 = Pow(2, mod - 2);
22        for(int m = 1; m < n; m <= 1) {
23            for(int i = 0; i < n; i += 2 * m) {
24                for(int j = 0; j < m; j++) {
25                    int x = A[i + j], y = A[i + j + m];
26                    A[i + j] = (1ll * x + y) * (type == 1 ? 1 : inv_2) % mod;
27                    A[i + j + m] = (1ll * x - y + mod) * (type == 1 ? 1 : inv_2) % mod;
28                }
29            }
30        }
31    }
32 }

```

子集卷积

子集卷积求的是下面一个式子:

$$c_k = \sum_{i|j=k, i \& j=0} a_i \times b_j$$

就是把集合 k 划分成两个集合。

后面那个与的条件通过 $|k| = |i| + |j|$ 干掉, 加一维集合元素个数, 就变成了

$$c[i + j][mask_k] = \sum_{i|j=k} a[i][mask_i] \times b[j][mask_j]$$

这个可以用 FWT 算。

```

1 namespace ssc{
2     int f[21][1 << 21], g[21][1 << 21], ans[21][1 << 21];
3     void subset_convolution(int *A, int *B, int *C, int n, int lim) {
4         // memset(f, 0, sizeof(f));
5         // memset(g, 0, sizeof(g));
6         for(int i = 0; i < lim; i++) f[__builtin_popcount(i)][i] = A[i];
7         for(int i = 0; i < lim; i++) g[__builtin_popcount(i)][i] = B[i];
8         for(int i = 0; i <= n; i++) FWT_or(f[i], lim, 1), FWT_or(g[i], lim, 1);
9         for(int i = 0; i <= n; i++)
10             for(int j = 0; j <= i; j++)
11                 for(int k = 0; k < lim; k++)
12                     ans[i][k] = (ans[i][k] + 1ll * f[j][k] * g[i - j][k] % mod) % mod;
13         for(int i = 0; i <= n; i++) FWT_or(ans[i], lim, -1);
14         for(int i = 0; i < lim; i++) C[i] = ans[__builtin_popcount(i)][i];
15     }
16 }

```

群论

结论

1. 子群检验法: 群 G 是群 H 的子群的充分必要条件: 对于所有元素 h, g , 只需检查 $g^{-1} \cdot h \in H$ 。

线性代数

矩阵运算全家桶

```
1 struct mat {
2     int g[5][5], n, m;
3 };
4 void operator+=(mat &a, const mat &b) {
5     if(a.n != b.n || a.m != b.m) cerr << "+= size error" << endl, exit(0);
6     for(int i = 1; i <= a.n; i++)
7         for(int j = 1; j <= a.m; j++) {
8             a.g[i][j] = (a.g[i][j] + b.g[i][j]);
9             if(a.g[i][j] >= mod) a.g[i][j] -= mod;
10        }
11 }
12 void operator-=(mat &a, const mat &b) {
13     if(a.n != b.n || a.m != b.m) cerr << "-= size error" << endl, exit(0);
14     for(int i = 1; i <= a.n; i++)
15         for(int j = 1; j <= a.m; j++) {
16             a.g[i][j] -= b.g[i][j];
17             if(a.g[i][j] < 0) a.g[i][j] += mod;
18        }
19 }
20 mat operator+(const mat &a, const mat &b) {
21     if(a.n != b.n || a.m != b.m) cerr << "+ size error" << endl, exit(0);
22     mat c;
23     c.n = a.n; c.m = a.m;
24     for(int i = 1; i <= a.n; i++)
25         for(int j = 1; j <= a.m; j++) {
26             c.g[i][j] = (a.g[i][j] + b.g[i][j]);
27             if(c.g[i][j] >= mod) c.g[i][j] -= mod;
28        }
29     return c;
30 }
31 mat operator-(const mat &a, const mat &b) {
32     if(a.n != b.n || a.m != b.m) cerr << "- size error" << endl, exit(0);
33     mat c;
34     c.n = a.n; c.m = a.m;
35     for(int i = 1; i <= a.n; i++)
36         for(int j = 1; j <= a.m; j++) {
37             c.g[i][j] = (a.g[i][j] - b.g[i][j]);
38             if(c.g[i][j] < 0) c.g[i][j] += mod;
39        }
40     return c;
41 }
42 mat operator*(const mat &a, const mat &b) {
43     if(a.m != b.n) cerr << "* size error" << endl, exit(0);
44     mat c;
45     c.n = a.n; c.m = b.m;
46     for(int i = 1; i <= a.n; i++) {
47         for(int j = 1; j <= b.m; j++) {
48             c.g[i][j] = 0;
49             for(int k = 1; k <= a.m; k++) {
50                 c.g[i][j] = c.g[i][j] + 1ll * a.g[i][k] * b.g[k][j] % mod;
51                 if(c.g[i][j] >= mod) c.g[i][j] -= mod;
52             }
53         }
54     }
55     return c;
56 }
57 mat Pow(mat a, int p) {
58     if(a.n != a.m) cerr << "* size error" << endl, exit(0);
59     mat ans;
60     ans.n = ans.m = a.n;
61     memset(ans.g, 0, sizeof(ans.g));
62     for(int i = 1; i <= ans.n; i++) ans.g[i][i] = 1;
63     for(; p; p >>= 1, a = a * a)
64         if(p & 1)
65             ans = ans * a;
66     return ans;
67 }
```

高斯消元

```
1 namespace Gauss {
2     int n, m;
3     double g[N][N];
4     int iszero(double x) {return fabs(x) < eps;}
5     void exchange(int i, int j) {
6         for(int k = 1; k <= m; k++)
7             swap(g[i][k], g[j][k]);
8     }
9     void minus(int i, int j, double t) {
10        for(int k = 1; k <= m; k++)
11            g[j][k] -= g[i][k] * t;
12    }
13    void div(int i, double d) {
14        for(int k = 1; k <= m; k++)
15            g[i][k] /= d;
16    }
17    void solve() {
18        for(int i = 1; i <= n; i++) {
19            if(iszero(g[i][i])) {
20                for(int j = i + 1; j <= n; j++) {
21                    if(!iszero(g[j][i])) {
22                        exchange(i, j);
23                        break;
24                    }
25                }
26                if(iszero(g[i][i])) continue;
27            }
28            div(i, g[i][i]);
29            for(int j = 1; j <= n; j++) if(i != j && !iszero(g[j][i])){
30                minus(i, j, g[j][i]);
31            }
32        }
33    }
34 }
```

图论

树论

树的直径

模板: POJ - 1985

- 两遍 DFS

```
1 void dfs(int x, int fa) {
2     for(int i = 0; i < E[x].size(); i++) {
3         int y = E[x][i].ver;
4         int w = E[x][i].val;
5         if(y == fa) continue;
6         d[y] = d[x] + w;
7         if(d[y] > d[c]) c = y;
8         dfs(y, x);
9     }
10 }
11 signed main()
12 {
13     n = read();
14     for(int i = 1; i < n; i++) {
15         int x = read(), y = read(), w = read();
16         E[x].push_back((Edge) {y, w});
17         E[y].push_back((Edge) {x, w});
18     }
19     dfs(1, 0);
20     d[c] = 0;
21     dfs(c, 0);
22     printf("%d\n", d[c]);
23 }
```

```

23     return 0;
24 }

```

- 树形 DP

```

1  void dfs(int x, int fa) {
2      d1[x] = d2[x] = 0;
3      for(int i = 0; i < E[x].size(); i++) {
4          int y = E[x][i].ver;
5          int w = E[x][i].val;
6          if(y == fa) continue;
7          dfs(y, x);
8          int t = d1[y] + w;
9          if(t > d1[x]) {
10             d2[x] = d1[x];
11             d1[x] = t;
12         } else if(t > d2[x]) {
13             d2[x] = t;
14         }
15     }
16     d = max(d, d1[x] + d2[x]);
17 }
18 signed main()
19 {
20     n = read();
21     for(int i = 1; i < n; i++) {
22         int x = read(), y = read(), w = read();
23         E[x].push_back((Edge) {y, w});
24         E[y].push_back((Edge) {x, w});
25     }
26     dfs(1, 0);
27     printf("%d\n", d);
28     return 0;
29 }

```

求 LCA

- 树链剖分

```

1  namespace Tree {
2      int siz[N], mson[N], ltp[N], fa[N], dth[N];
3      vector<int> son[N];
4      void dfs1(int x, int pre) {
5          siz[x] = 1;
6          mson[x] = 0;
7          fa[x] = pre;
8          dth[x] = dth[pre] + 1;
9          for(auto y : son[x]) if(y != pre) {
10             dfs1(y, x);
11             if(mson[x] == 0 || siz[y] > siz[mson[x]]) mson[x] = y;
12         }
13     }
14     void dfs2(int x, int pre, int tp) {
15         ltp[x] = tp;
16         if(mson[x]) dfs2(mson[x], x, tp);
17         for(auto y : son[x]) if(y != pre && y != mson[x]) {
18             dfs2(y, x, y);
19         }
20     }
21     void init() {
22         dfs1(1, 0);
23         dfs2(1, 0, 1);
24     }
25     int LCA(int x, int y) {
26         while(ltp[x] != ltp[y]) {
27             if(dth[ltp[x]] > dth[ltp[y]]) x = fa[ltp[x]];
28             else y = fa[ltp[y]];
29         }
30         return dth[y] > dth[x] ? x : y;

```

```

31     }
32 }

```

● 倍增

```

1  namespace Tree {
2      vector<int> son[N];
3      int root, fa[N][31], dth[N];
4      void dfs(int x, int pre) {
5          fa[x][0] = pre;
6          dth[x] = dth[pre] + 1;
7          for(int i = 1; i <= 30; i++)
8              fa[x][i] = fa[fa[x][i-1]][i-1];
9          for(auto y : son[x]) if(y != pre)
10             dfs(y, x);
11     }
12     void init() {
13         dfs(root, 0);
14     }
15     int LCA(int x, int y) {
16         if(dth[x] > dth[y]) swap(x, y);
17         for(int i = 30; ~i; i--)
18             if(dth[fa[y][i]] >= dth[x])
19                 y = fa[y][i];
20         if(x == y) return x;
21         for(int i = 30; ~i; i--)
22             if(fa[y][i] != fa[x][i]) {
23                 x = fa[x][i];
24                 y = fa[y][i];
25             }
26         return fa[x][0];
27     }
28 }

```

树上启发式合并

长春站的痛.jpg

- 先递归计算轻儿子的答案
- 计算重儿子的答案，并且保留重儿子的状态数组
- 把其他所有轻儿子的答案加到状态数组中，更新当前点的答案

```

1  void dfs1(int x, int pre) {
2      siz[x] = 1;
3      mson[x] = 0;
4      for(auto y : son[x]) if(y != pre) {
5          dfs1(y, x);
6          siz[x] += siz[y];
7          if(!mson[x] || siz[y] > siz[mson[x]]) mson[x] = y;
8      }
9  }
10 void add(int x, int pre, int v) {
11     cnt[col[x]] += v;
12     if(cnt[col[x]] > Mx) Mx = cnt[col[x]], sum = col[x];
13     else if(cnt[col[x]] == Mx) sum += col[x];
14     for(auto y : son[x]) {
15         if(y == pre || y == Son) continue;
16         add(y, x, v);
17     }
18 }
19 void dfs2(int x, int pre, int keep) {
20     for(auto y : son[x]) {
21         if(y == pre || y == mson[x]) continue;
22         dfs2(y, x, 0);
23     }
24     if(mson[x]) dfs2(mson[x], x, 1), Son = mson[x];
25     add(x, pre, 1); Son = 0;
26     ans[x] = sum;
27     if(!keep) add(x, pre, -1), sum = 0, Mx = 0;
28 }
29 }

```

图论

第 k 短路

模板: HDU-6351

估值函数: $h(x) = f(x) + g(x)$, 其中 $f(x)$ 为从起点到现在的距离, $g(x)$ 为起点到当前点的最短路。

```
1  bool operator<(const node &a, const node &b) {
2      return a.f + a.g > b.f + b.g;
3  }
4  priority_queue<node> q;
5  signed main()
6  {
7      n = read(); m = read();
8      for(int i = 1; i <= m; i++) {
9          int x, y, w;
10         x = read(); y = read(); w = read();
11         E[x].push_back((Edge) {y, w});
12         re[y].push_back((Edge) {x, w});
13     }
14     s = read(); t = read(); k = read();
15     memset(dis, 0x3f, sizeof(dis)); dis[t] = 0;
16     q.push((node) {t, 0, 0});
17     while(q.size()) {
18         int x = q.top().x, d = q.top().f;
19         q.pop();
20         if(dis[x] < d) continue;
21         for(int i = 0; i < re[x].size(); i++) {
22             int y = re[x][i].y, w = re[x][i].w;
23             if(dis[y] > dis[x] + w) {
24                 dis[y] = dis[x] + w;
25                 q.push((node) {y, dis[y], 0});
26             }
27         }
28     }
29     for(int i = 1; i <= n; i++) cnt[i] = k;
30     cnt[s]++;
31     q.push((node) {s, 0, dis[s]});
32     while(q.size()) {
33         int x = q.top().x, f = q.top().f, g = q.top().g;
34         q.pop();
35         if(cnt[x] == 0) continue;
36         cnt[x]--;
37         if(x == t && cnt[x] == 0) {
38             printf("%lld\n", f);
39             return 0;
40         }
41         for(int i = 0; i < E[x].size(); i++) {
42             int y = E[x][i].y, w = E[x][i].w;
43             q.push((node) {y, f + w, dis[y]});
44         }
45     }
46     printf("-1\n");
47     return 0;
48 }
```

二分图匹配

结论

最大匹配数: 最大匹配的匹配边的数目

最小点/边覆盖数: 选取最少的点/边, 使任意一条边至少有一个点被选择 / 点至少连有一条边。

最大独立数: 选取最多的点, 使任意所选两点均不相连

最小路径覆盖数: 对于一个 DAG (有向无环图), 选取最少条路径, 使得每个顶点属于且仅属于一条路径。路径长可以为 0 (即单个点)。

1. 最大匹配数 = 最小点覆盖数 (这是 Konig 定理)
2. 最大匹配数 = 最大独立数

3. 最小路径覆盖数 = 顶点数 - 最大匹配数
4. 原图的最大团 = 补图的最大独立集原图的最大独立集 = 补图的最大团
5. 最小边覆盖 = 顶点数 - 最大匹配数

在一般图中：

最小不相交路径覆盖：每个点拆点为 $2x - 1, 2x$ ，那么一条边 (x, y) ，则连边 $(2x - 1, 2y)$ ，答案是 $n - \text{maxmatch}$

最小可相交路径覆盖：跑一遍传递闭包，按传递闭包上的边建边之后转化为最小不相交路径覆盖。

二分图最大匹配的必须边：

在完备匹配中：

匹配边从左到右方向，非匹配边从右到左方向，则一条边为必须边当且仅当边在最大匹配中，并且边所连的两个点不在同一个强连通分量中。

在非完备匹配中：

匈牙利算法

```

1  int dfs(int x) {
2      for(int i = head[x]; i; i = nxt[i]) {
3          int y = ver[i];
4          if(vis[y]) continue;
5          vis[y] = 1;
6          if(!match[y] || dfs(match[y])) {
7              match[y] = x;
8              return true;
9          }
10     }
11     return false;
12 }
13 for(int i = 1; i <= n; i++) {
14     memset(vis, 0, sizeof(vis));
15     if(dfs(i)) ans++;
16 }

```

KM 算法二分图最大权匹配

KM 算法只支持二分图最大权完美匹配，若图不一定存在完美匹配，注意补 0 边和补点。

KM 算法引入了顶标的概念，用 $la[x]$ 和 $lb[x]$ 分别保存两侧点的顶标，顶标必须满足大于所有边。
 每次对每个点进行循环匹配，匹配中统计一个 delta 表示最小的权值使得一条边可以加入。
 然后修改顶标再继续匹配。

```

1  int la[N], lb[N], va[N], vb[N], delta, match[N], g[N][N], n;
2  int dfs(int x) {
3      va[x] = 1;
4      for(int y = 1; y <= n; y++) {
5          if(!vb[y]) {
6              if(la[x] + lb[y] - g[x][y] == 0) {
7                  vb[y] = 1;
8                  if(!match[y] || dfs(match[y])) {
9                      match[y] = x;
10                     return true;
11                 }
12             } else delta = min(delta, la[x] + lb[y] - g[x][y]);
13         }
14     }
15     return false;
16 }
17 void work() {
18     for(int i = 1; i <= n; i++)
19         for(int j = 1; j <= n; j++)
20             g[i][j] = read();
21     memset(match, 0, sizeof(match));
22     for(int i = 1; i <= n; i++) {
23         la[i] = g[i][1];
24         lb[i] = 0;

```



```

25     for(int j = 2; j <= n; j++)
26         la[i] = max(la[i], g[i][j]);
27 }
28 for(int i = 1; i <= n; i++) {
29     while(true) {
30         memset(va, 0, sizeof(va));
31         memset(vb, 0, sizeof(vb));
32         delta = 0x3f3f3f3f;
33         if(dfs(i)) break;
34         for(int j = 1; j <= n; j++) {
35             if(va[j]) la[j] -= delta;
36             if(vb[j]) lb[j] += delta;
37         }
38     }
39 }
40 long long ans = 0;
41 for(int i = 1; i <= n; i++)
42     ans += g[match[i]][i];
43 printf("%lld\n", ans);
44 }

```

网络流

Dinic 算法

```

1  const int inf = 0x3f3f3f3f;
2  queue<int> q;
3  int d[N];
4  int bfs() {
5      memset(d, 0, sizeof(int) * (t + 10)); d[s] = 1;
6      while(q.size()) q.pop(); q.push(s);
7      while(q.size()) {
8          int x = q.front(); q.pop();
9          for(int i = head[x]; i; i = nxt[i]) {
10             if(d[ver[i]]) continue;
11             if(edge[i] <= 0) continue;
12             d[ver[i]] = d[x] + 1;
13             q.push(ver[i]);
14         }
15     }
16     return d[t];
17 }
18 int dinic(int x, int flow) {
19     if(x == t) return flow;
20     int k, res = flow;
21     for(int i = head[x]; i && res; i = nxt[i]) {
22         if(d[ver[i]] != d[x] + 1 || edge[i] <= 0) continue;
23         k = dinic(ver[i], min(res, edge[i]));
24         if(k == 0) d[ver[i]] = 0;
25         edge[i] -= k;
26         edge[i ^ 1] += k;
27         res -= k;
28     }
29     return flow - res;
30 }

```

EK 算法费用流

```

1  //反向边 cost 为负数, 容量为 0
2  int SPFA() {
3      queue<int> q; q.push(s);
4      memset(dis, 0x3f, sizeof(dis)); dis[s] = 0;
5      memset(vis, 0, sizeof(vis)); vis[s] = 1;
6      q.push(s); flow[s] = 0x3f3f3f3f;
7      while(q.size()) {
8          int x = q.front();
9          vis[x] = 0; q.pop();
10         for(int i = head[x]; i; i = nxt[i]) {
11             if(edge[i] <= 0) continue;
12             if(dis[ver[i]] > dis[x] + cost[i]) {

```

```

13         dis[ver[i]] = dis[x] + cost[i];
14         pre[ver[i]] = i;
15         flow[ver[i]] = min(flow[x], edge[i]);
16         if(!vis[ver[i]]) {
17             q.push(ver[i]);
18             vis[ver[i]] = 1;
19         }
20     }
21 }
22 }
23 return dis[t] != 0x3f3f3f3f;
24 }
25 void update() {
26     int x = t;
27     while(x != s) {
28         int i = pre[x];
29         edge[i] -= flow[t];
30         edge[i ^ 1] += flow[t];
31         x = ver[i ^ 1];
32     }
33     maxflow += flow[t];
34     minncost += dis[t] * flow[t];
35 }

```

Dinic 算法费用流

```

1 int SPFA() {
2     while(q.size()) q.pop(); q.push(s);
3     memset(d, 0x3f, sizeof(int) * (n + 10)); d[s] = 0;
4     memset(vis, 0, sizeof(int) * (n + 10)); vis[s] = 1;
5     while(q.size()) {
6         int x = q.front(); q.pop();
7         vis[x] = 0;
8         for(int i = head[x]; i; i = nxt[i]) {
9             if(edge[i] <= 0) continue;
10            if(d[ver[i]] > d[x] + cost[i]) {
11                d[ver[i]] = d[x] + cost[i];
12                if(!vis[ver[i]]) {
13                    vis[ver[i]] = 1;
14                    q.push(ver[i]);
15                }
16            }
17        }
18    }
19    return d[t] != 0x3f3f3f3f;
20 }
21 int dinic(int x, int flow) {
22     if(x == t) return flow;
23     vis[x] = 1;
24     int k, res = flow;
25     for(int i = head[x]; i && res; i = nxt[i]) {
26         if(vis[ver[i]]) continue;
27         if(d[ver[i]] != d[x] + cost[i] || edge[i] <= 0) continue;
28         k = dinic(ver[i], min(edge[i], res));
29         if(!k) d[ver[i]] = -1;
30         edge[i] -= k;
31         edge[i ^ 1] += k;
32         res -= k;
33         mincost += cost[i] * k;
34     }
35     vis[x] = 0;
36     return flow - res;
37 }

```

无源汇上下界可行流

$x \rightarrow y$, 则 s 向 y , s 向 x 连 l , x 向 y 连 $r - l$, 有可行流的条件是 s 出边全满流, 解通过残量网络构造出。

```

1 for(int i = 1; i <= m; i++) {
2     int x = read(), y = read();
3     int l = read(), r = read();

```

```

4     low[i] = l;
5     add(x, y, r - l); add(y, x, 0);
6     id[i] = tot;
7     add(s, y, l); add(y, s, 0);
8     add(x, t, l); add(t, x, 0);
9 }
10 while(bfs())
11     dinic(s, inf);
12 int f = 1;
13 for(int i = head[s]; i; i = nxt[i]) {
14     f &= (edge[i] == 0);
15 }
16 printf("%s\n", f ? "YES" : "NO");
17 if(!f) return 0;
18 for(int i = 1; i <= m; i++) {
19     printf("%d\n", edge[id[i]] + low[i]);
20 }

```

连通性算法

Tarjan 强连通分量

$dfn[x]$: dfs 序。

$low[x]$: 追溯值, 指 x 的子树内部, 通过一条非树边能到达的最小的 dfn 值。

如果 $dfn[x] == low[x]$, 当前栈中, x 以后的元素为一个强连通。

```

1 void tarjan(int x) {
2     low[x] = dfn[x] = ++dfncnt;
3     s[++t] = x; vis[x] = 1;
4     for(int i = head[x]; i; i = nxt[i]) {
5         if(!dfn[ver[i]]) {
6             tarjan(ver[i]);
7             low[x] = min(low[x], low[ver[i]]);
8         } else if(vis[ver[i]]) {
9             low[x] = min(low[x], dfn[ver[i]]);
10        }
11    }
12    if(dfn[x] == low[x]) {
13        int z = -1;
14        ++sc;
15        while(z != x) {
16            scc[s[t]] = sc;
17            siz[sc]++;
18            vis[s[t]] = 0;
19            z = s[t];
20            t--;
21        }
22    }
23 }
24 //从任意点开始跑, 但是注意如果图不连通, 需要每个点跑一次
25 for(int i = 1; i <= n; i++)
26     if(!dfn[i])
27         tarjan(i);

```

点双连通

Tarjan 割点判定

```

1 int cut[N];
2 namespace v_dcc {
3     int root, low[N], dfn[N], dfntot;
4     void tarjan(int x) {
5         low[x] = dfn[x] = ++dfntot;
6         int flag = 0;
7         for(int i = head[x]; i; i = nxt[i]) {
8             int y = ver[i];
9             if(!dfn[y]) {
10                 tarjan(y);
11                 low[x] = min(low[x], low[y]);

```

```

12         if(low[y] >= dfn[x]) {
13             flag++;
14             if(x != root || flag > 1) cut[x] = 1;
15         }
16     } else low[x] = min(low[x], dfn[y]);
17 }
18 }
19 }
20 void getcut() {
21     for(int i = 1; i <= n; i++)
22         if(!dfn[i])
23             tarjan(root = i);
24 }
25 }

```

求点双连通分量

点双连通分量比较复杂，一个点可能存在于多个点双连通分量当中，一个点删除与搜索树中的儿子节点断开时，不能在栈中弹掉父亲点，但是父亲点属于儿子的 v-dcc。

```

1  int cut[N];
2  vector<int> dcc[N];
3  namespace v_dcc {
4      int s[N], t, root;
5      int es[N], et;
6      void tarjan(int x) {
7          dfn[x] = low[x] = ++dfntot;
8          s[++t] = x;
9          if(x == root && head[x] == 0) {
10              dcc[++dc].clear();
11              dcc[dc].push_back(x);
12              return;
13          }
14          int flag = 0;
15          for(int i = head[x]; i; i = nxt[i]) {
16              int y = ver[i];
17              if(!dfn[y]) {
18                  tarjan(y);
19                  low[x] = min(low[x], low[y]);
20                  if(low[y] >= dfn[x]) {
21                      flag++;
22                      if(x != root || flag > 1) cut[x] = true;
23                      dcc[++dc].clear();
24                      int z = -1;
25                      while(z != y) {
26                          z = s[t--];
27                          dcc[dc].push_back(z);
28                      }
29                      dcc[dc].push_back(x);
30                  }
31              } else low[x] = min(low[x], dfn[y]);
32          }
33      }
34      void get_cut() {
35          for(int i = 1; i <= n; i++)
36              if(!dfn[i])
37                  tarjan(root = i);
38      }
39  }

```

边双连通

搜索树上的点 x ，若它的一个儿子 y ，满足严格大于号 $low[y] > dfn[x]$ ，那么这条边就是桥。

注意由于会有重边，不能仅仅考虑他的父亲编号，而应该记录入边编号。

```

1  namespace e_dcc {
2      int low[N], dfn[N], dfntot;
3      vector<int> E[N];
4      void tarjan(int x, int in_edge) {
5          low[x] = dfn[x] = ++dfntot;

```

```

6         for(int i = head[x]; i; i = nxt[i]) {
7             int y = ver[i];
8             if(!dfn[y]) {
9                 tarjan(y, i);
10                low[x] = min(low[x], low[y]);
11                if(low[y] > dfn[x])
12                    bridge[i] = bridge[i ^ 1] = true;
13            } else if(i != (in_edge ^ 1))
14                //注意运算优先级
15                low[x] = min(low[x], dfn[y]);
16        }
17    }
18    void getbridge() {
19        for(int i = 1; i <= n; i++)
20            if(!dfn[i])
21                tarjan(i, 0);
22    }
23    void dfs(int x) {
24        dcc[x] = dc;
25        for(int i = head[x]; i; i = nxt[i]) {
26            if(!dcc[ver[i]] && !bridge[i]) {
27                dfs(ver[i]);
28            }
29        }
30    }
31    void getdcc() {
32        for(int i = 1; i <= n; i++) {
33            if(!dcc[i]) {
34                ++dc;
35                dfs(i);
36            }
37        }
38    }
39    void getgraphic() {
40        for(int x = 1; x <= n; x++) {
41            for(int i = head[x]; i; i = nxt[i]) {
42                if(dcc[ver[i]] != dcc[x]) {
43                    E[dcc[x]].push_back(dcc[ver[i]]);
44                    E[dcc[ver[i]]].push_back(dcc[x]);
45                }
46            }
47        }
48    }
49 }

```

2-SAT

2-SAT 用于解决每个变量的 01 取值问题，用于判断是否存在一种不冲突取值方法。

建边方法：假如选了 A 之后， B 的取值**确定**，那么就 A 的这个取值向 B 的这个取值建边，否则不要建边。

判定方法：如果， $\exists A$ ，使得 A 和 $\neg A$ 在同一个强连通分量里面，说明不存在一种合法取值，否则存在。

输出方案：自底向上确定每个变量的取值，由于 tarjan 求解强连通分量是自底向上，所以编号比较小的强连通是位于 DAG 底部的。

基于 tarjan 的方案输出就变得十分简单了，只要判断一个点和对立节点哪个 scc 的编号小就行了。

例如： $A \rightarrow B \rightarrow C$ ，那么 C 的编号最小。

```

1  for(int i = 1; i <= m; i++) {
2      int x = read() + 1, y = read() + 1;
3      int w = read();
4      char c[10];
5      scanf("%s", c + 1);
6      if(c[1] == 'A') {
7          if(w) {
8              add(2 * x - 0, 2 * x - 1);
9              add(2 * y - 0, 2 * y - 1);
10         } else {
11             add(2 * x - 1, 2 * y - 0);
12             add(2 * y - 1, 2 * x - 0);

```

```

13     }
14 }
15 if(c[1] == 'O') {
16     if(w) {
17         add(2 * x - 0, 2 * y - 1);
18         add(2 * y - 0, 2 * x - 1);
19     } else {
20         add(2 * x - 1, 2 * x - 0);
21         add(2 * y - 1, 2 * y - 0);
22     }
23 }
24 if(c[1] == 'X') {
25     if(w) {
26         add(2 * x - 0, 2 * y - 1);
27         add(2 * x - 1, 2 * y - 0);
28         add(2 * y - 0, 2 * x - 1);
29         add(2 * y - 1, 2 * x - 0);
30     } else {
31         add(2 * x - 0, 2 * y - 0);
32         add(2 * x - 1, 2 * y - 1);
33         add(2 * y - 0, 2 * x - 0);
34         add(2 * y - 1, 2 * x - 1);
35     }
36 }
37 }
38 for(int i = 1; i <= 2 * n; i++)
39     if(!dfn[i])
40         tarjan(i);
41 for(int i = 1; i <= n; i++) {
42     if(scc[2 * i - 0] == scc[2 * i - 1]) {
43         printf("NO\n");
44         return 0;
45     }
46 }
47 printf("YES\n");
48 //2 * x - a -> 2 * y - b 的边表示, 假如 x 取值为 a, 那么 y 的取值必须为 b
49
50 //输出方案
51 for(int i = 2; i <= 2 * n; i += 2) {
52     if(scc[i - 0] == scc[i - 1]) {
53         printf("NO\n");
54         return 0;
55     } else ans[(i + 1) / 2] = scc[i - 1] < scc[i - 0];
56 }

```

计算几何

结构体定义

```

1  const double Pi = acos(-1.0);
2  const double eps = 1e-11;
3  struct line;
4  struct Point;
5  struct Point {
6      double x, y;
7      Point() : x(0), y(0) {}
8      Point(double x, double y) : x(x), y(y) {}
9      Point(const line &l);
10 };
11 struct line{
12     Point s, t;
13     line() {}
14     line(const Point &s, const Point &t) : s(s), t(t) {}
15 };
16
17 struct circle{
18     Point c;
19     double r;
20     circle() : c(Point(0,0)), r(0) {}
21     circle(const Point &c, double r) : c(c), r(r) {}

```

```

22     Point point(double a) {
23         return Point(c.x + cos(a)*r, c.y + sin(a)*r);
24     }
25 };
26 typedef Point Vector;
27 Point operator+(const Point &a, const Point &b) { return Point(a.x + b.x, a.y + b.y); }
28 Point operator-(const Point &a, const Point &b) { return Point(a.x - b.x, a.y - b.y); }
29 Point operator*(const Point &a, const double &c) { return Point(c * a.x, c * a.y); }
30 Point operator/(const Point &a, const double &c) { return Point(a.x / c, a.y / c); }
31 Point :: Point(const line &l) { *this = l.t - l.s; }
32 // 三态函数
33 int sgn(double x) {
34     if(fabs(x) < eps) return 0;
35     else return x < 0 ? -1 : 1;
36 }
37 bool operator == (const Point& a, const Point& b) { return !sgn(a.x - b.x) && !sgn(a.y - b.y); }
38 // 点积
39 double dot(const Vector& a, const Vector& b) { return a.x * b.x + a.y * b.y; }
40 // 叉积
41 double det(const Vector& a, const Vector& b) { return a.x * b.y - a.y * b.x; }
42 double cross(const Point& s, const Point& t, const Point& o = Point()) { return det(s - o, t - o); }

```

基本操作

```

1 // 点到原点距离
2 double abs(const Point &a){ return sqrt(a.x * a.x + a.y * a.y); }
3 // 点旋转 theta 角度
4 Point rot(const Point &a, double theta){ return Point(a.x * cos(theta) - a.y * sin(theta), a.x * sin(theta) + a.y *
    ↪ cos(theta)); }
5 // 逆时针旋转 90 度
6 Point rotCCW90(const Point &a) { return Point(-a.y, a.x); }
7 // 顺时针旋转 90 度
8 Point rotCW90(const Point &a) { return Point(a.y, -a.x); }
9 // 点的幅角
10 double arg(const Point &a){
11     double t = atan2(a.y, a.x);
12     return t < 0 ? t + 2 * Pi:t;
13 }

```

线

```

1 // 是否平行
2 bool parallel(const line &a, const line &b) {
3     return !sgn(det(a.t - a.s, b.t - b.s));
4 }
5 // 直线是否相等
6 bool l_eq(const line& a, const line& b) {
7     return parallel(a, b) && parallel(line(a.s, b.t), line(b.s, a.t));
8 }

```

点与线

```

1 // 点是否在线段上, <= 包含端点
2 bool p_on_seg(const Point &p, const line &seg) {
3     return !sgn(det(p - seg.s, p - seg.t)) && sgn(dot(p - seg.s, p - seg.t)) < 0;
4 }
5 // 点到直线距离
6 double dist_to_line(const Point &p, const line &l) {
7     return fabs(cross(l.s, l.t, p)) / abs(Point(l));
8 }
9 // 点到线段距离
10 double dist_to_seg(const Point &p, const line &l) {
11     if (l.s == l.t) return abs(p - l.s);
12     Vector vs = p - l.s, vt = p - l.t;
13     if (sgn(dot(Point(l), vs)) < 0) return abs(vs);
14     else if (sgn(dot(Point(l), vt)) > 0) return abs(vt);
15     else return dist_to_line(p, l);
16 }

```

线与线

```
1 // 直线交点, 需保证存在
2 Point l_intersection(const line& a, const line& b) {
3     double s1 = det(Point(a), b.s - a.s), s2 = det(Point(a), b.t - a.s);
4     return (b.s * s2 - b.t * s1) / (s2 - s1);
5 }
6 // 线段和直线是否有交 1 = 规范, 2 = 不规范
7 int s_l_cross(const line &seg, const line &line) {
8     int d1 = sgn(cross(line.s, line.t, seg.s));
9     int d2 = sgn(cross(line.s, line.t, seg.t));
10    if ((d1 ^ d2) == -2) return 1; // proper
11    if (d1 == 0 || d2 == 0) return 2;
12    return 0;
13 }
14 // 线段的交 1 = 规范, 2 = 不规范
15 // 如果是不规范相交, p_on_seg 函数要改成 <=
16 int s_cross(const line &a, const line &b, Point &p) {
17     int d1 = sgn(cross(a.t, b.s, a.s)), d2 = sgn(cross(a.t, b.t, a.s));
18     int d3 = sgn(cross(b.t, a.s, b.s)), d4 = sgn(cross(b.t, a.t, b.s));
19     if ((d1 ^ d2) == -2 && (d3 ^ d4) == -2) { p = l_intersection(a, b); return 1; }
20     if (!d1 && p_on_seg(b.s, a)) { p = b.s; return 2; }
21     if (!d2 && p_on_seg(b.t, a)) { p = b.t; return 2; }
22     if (!d3 && p_on_seg(a.s, b)) { p = a.s; return 2; }
23     if (!d4 && p_on_seg(a.t, b)) { p = a.t; return 2; }
24     return 0;
25 }
```

多边形

```
1 #define nxt(i) ((i + 1) % s.size())
2 typedef vector<Point> Polygon
3 // 多边形面积
4 double poly_area(const Polygon &s){
5     double area = 0;
6     for(int i = 1; i < s.size() - 1; i++)
7         area += cross(s[i], s[i + 1], s[0]);
8     return area / 2;
9 }
10 // 多边形是否为凸多边形
11 int is_convex(const Polygon &s) {
12     int x = 1;
13     int sg = 0;
14     for(int i = 1; i < s.size(); i++) {
15         if(!sg && sgn(det(s[i] - s[i - 1], s[nxt(i)] - s[i])) != 0) {
16             sg = sgn(det(s[i] - s[i - 1], s[nxt(i)] - s[i]));
17         }
18         x &= (sgn(det(s[i] - s[i - 1], s[nxt(i)] - s[i])) == sg) ||
19             (sgn(det(s[i] - s[i - 1], s[nxt(i)] - s[i])) == 0);
20     }
21     return x;
22 }
23 // 点是否在多边形中 0 = 在外部 1 = 在内部 -1 = 在边界上
24 int p_in_poly(Point p, const Polygon &s){
25     int cnt = 0;
26     for(int i = 0; i < s.size(); i++) {
27         Point a = s[i], b = s[nxt(i)];
28         if (p_on_seg(p, line(a, b))) return -1;
29         //p 在多边形边上
30         if (sgn(a.y - b.y) <= 0) swap(a, b);
31         if (sgn(p.y - a.y) > 0) continue;
32         if (sgn(p.y - b.y) <= 0) continue;
33         //一条边包含它较低的点, 不包含较高的点
34         cnt += sgn(cross(b, a, p)) > 0;
35         //如果 p 在这条线段左边
36     }
37     return bool(cnt & 1);
38 }
```


两圆的公切线

在 res 中存放的线上的两点分别是在 c1,c2 上的切点。

```
1 int tangent(const circle &C1, const circle &C2, vector<line> &res){
2     double d = abs(C1.c - C2.c);
3     if(d < eps) return 0;
4
5     int c=0;
6     // 内公切线
7     if(C1.r + C2.r < d - eps){
8         double t = acos((C1.r + C2.r) / d);
9         res.push_back(line(C1.c + rot(C1.r / d * (C2.c - C1.c), t), C2.c + rot(C2.r / d * (C1.c - C2.c), t)));
10        res.push_back(line(C1.c + rot(C1.r / d * (C2.c - C1.c), -t), C2.c + rot(C2.r / d * (C1.c - C2.c), -t)));
11        c += 2;
12    } else if(C1.r + C2.r < d + eps){
13        Point p = C1.c + C1.r / d * (C2.c - C1.c);
14        res.push_back(line(p, p + rot(C2.c - C1.c, Pi / 2)));
15        c++;
16    }
17
18    // 外公切线
19    if(abs(C1.r - C2.r) < d - eps){
20        double t1 = acos((C1.r - C2.r) / d), t2 = Pi - t1;
21        res.push_back(line(C1.c + rot(C1.r / d * (C2.c - C1.c), t1), C2.c + rot(C2.r / d * (C1.c - C2.c), -t2)));
22        res.push_back(line(C1.c + rot(C1.r / d * (C2.c - C1.c), -t1), C2.c + rot(C2.r / d * (C1.c - C2.c), t2)));
23        c+=2;
24    } else if(abs(C1.r - C2.r) < d + eps){
25        Point p = C1.c + C1.r / d * (C2.c - C1.c);
26        res.push_back(line(p, p + rot(C2.c - C1.c, Pi / 2)));
27        c++;
28    }
29
30    return c;
31 }
```

tips

- atan2(y, x) 函数: 点 (x, y) 到原点的方位角, 值域在 $(-\pi, \pi)$ 在一二象限为正, 三四象限为负。

字符串

字符串哈希

```
1 namespace String {
2     const int x = 135;
3     const int p1 = 1e9 + 7, p2 = 1e9 + 9;
4     ull xp1[N], xp2[N], xp[N];
5     void init_xp() {
6         xp1[0] = xp2[0] = xp[0] = 1;
7         for(int i = 1; i < N; i++) {
8             xp1[i] = xp1[i - 1] * x % p1;
9             xp2[i] = xp2[i - 1] * x % p2;
10            xp[i] = xp[i - 1] * x;
11        }
12    }
13    struct HashString {
14        char s[N];
15        int length, subsize;
16        bool sorted;
17        ull h[N], hl[N];
18        ull init(const char *t) {
19            if(xp[0] != 1) init_xp();
20            length = strlen(t);
21            strcpy(s, t);
22            ull res1 = 0, res2 = 0;
23            h[length] = 0;
24            for(int j = length - 1; j >= 0; j--) {
25                #ifdef ENABLE_DOUBLE_HASH
```

```

26         res1 = (res1 * x + s[j]) % p1;
27         res2 = (res2 * x + s[j]) % p2;
28         h[j] = (res1 << 32) | res2;
29     #else
30         res1 = res1 * x + s[j];
31         h[j] = res1;
32     #endif
33 }
34 return h[0];
35 }
36 //获取子串哈希, 左闭右开
37 ull get_substring_hash(int left, int right) {
38     int len = right - left;
39     #ifdef ENABLE_DOUBLE_HASH
40         unsigned int mask32 = ~(0u);
41         ull left1 = h[left] >> 32, right1 = h[right] >> 32;
42         ull left2 = h[left] & mask32, right2 = h[right] & mask32;
43         return (((left1 - right1 * xp1[len] % p1 + p1) % p1) << 32) |
44             (((left2 - right2 * xp2[len] % p2 + p2) % p2));
45     #else
46         return h[left] - h[right] * xp[len];
47     #endif
48 }
49 void get_all_subs_hash(int sublen) {
50     subsize = length - sublen + 1;
51     for (int i = 0; i < subsize; ++i)
52         hl[i] = get_substring_hash(i, i + sublen);
53     sorted = 0;
54 }
55
56 void sort_substring_hash() {
57     sort(hl, hl + subsize);
58     sorted = 1;
59 }
60
61 bool match(ull key) const {
62     if (!sorted) assert (0);
63     if (!subsize) return false;
64     return binary_search(hl, hl + subsize, key);
65 }
66 };
67 }

```

Trie

```

1 namespace trie {
2     int t[N][26], sz, ed[N];
3     int _new() {
4         sz++;
5         memset(t[sz], 0, sizeof(t[sz]));
6         return sz;
7     }
8     void init() {
9         sz = 0;
10        _new();
11        memset(ed, 0, sizeof(ed));
12    }
13    void Insert(char *s, int n) {
14        int u = 1;
15        for(int i = 0; i < n; i++) {
16            int c = s[i] - 'a';
17            if(!t[u][c]) t[u][c] = _new();
18            u = t[u][c];
19        }
20        ed[u]++;
21    }
22    int find(char *s, int n) {
23        int u = 1;
24        for(int i = 0; i < n; i++) {
25            int c = s[i] - 'a';
26            if(!t[u][c]) return -1;

```

```

27         u = t[u][c];
28     }
29     return u;
30 }
31 }

```

KMP 算法

```

1  namespace KMP {
2      void get_next(char *t, int m, int *nxt) {
3          int j = nxt[0] = 0;
4          for(int i = 1; i < m; i++) {
5              while(j && t[i] != t[j]) j = nxt[j - 1];
6              nxt[i] = j += (t[i] == t[j]);
7          }
8      }
9      vector<int> find(char *t, int m, int *nxt, char *s, int n) {
10         vector<int> ans;
11         int j = 0;
12         for(int i = 0; i < n; i++) {
13             while(j && s[i] != t[j]) j = nxt[j - 1];
14             j += s[i] == t[j];
15             if(j == m) {
16                 ans.push_back(i - m + 1);
17                 j = nxt[j - 1];
18             }
19         }
20         return ans;
21     }
22 }

```

manacher 算法

```

1  namespace manacher {
2      char s[N];
3      int p[N], len;
4      void getp(string tmp) {
5          len = 0;
6          for(auto x : tmp) {
7              s[len++] = '#';
8              s[len++] = x;
9          }
10         s[len++] = '#';
11         memset(p, 0, sizeof(int) * (len + 10));
12         int c = 0, r = 0;
13         for(int i = 0; i < len; i++) {
14             if(i <= r) p[i] = min(p[2 * c - i], r - i);
15             else p[i] = 1;
16             while(i - p[i] >= 0 && i + p[i] < len && s[i - p[i]] == s[i + p[i]])
17                 p[i]++;
18             if(i + p[i] - 1 > r) {
19                 r = i + p[i] - 1;
20                 c = i;
21             }
22         }
23         for(int i = 0; i < len; i++) p[i]--;
24     }
25     void getp(char *tmp, int n) {
26         len = 0;
27         for(int i = 0; i < n; i++) {
28             s[len++] = '#';
29             s[len++] = tmp[i];
30         }
31         s[len++] = '#';
32         memset(p, 0, sizeof(int) * (len + 10));
33         int c = 0, r = 0;
34         for(int i = 0; i < len; i++) {
35             if(i <= r) p[i] = min(p[2 * c - i], r - i);
36             else p[i] = 1;
37             while(i - p[i] >= 0 && i + p[i] < len && s[i - p[i]] == s[i + p[i]])

```

```

38         p[i]++;
39         if(i + p[i] - 1 > r) {
40             r = i + p[i] - 1;
41             c = i;
42         }
43     }
44     for(int i = 0; i < len; i++) p[i]--;
45 }
46 int getlen() {
47     return *max_element(p, p + len);
48 }
49 int getlen(string s) {
50     getp(s);
51     return getlen();
52 }
53 }

```

AC 自动机

```

1  struct ac_automaton {
2      int t[N][26], danger[N], tot, fail[N];
3      int dp[N][N];
4      void init() {
5          tot = -1;
6          _new();
7      }
8      int _new() {
9          tot++;
10         memset(t[tot], 0, sizeof(t[tot]));
11         danger[tot] = 0;
12         fail[tot] = 0;
13         return tot;
14     }
15     void Insert(const char *s) {
16         int u = 0;
17         for(int i = 0; s[i]; i++) {
18             if(!t[u][mp[s[i]]]) t[u][s[i] - 'a'] = _new();
19             u = t[u][mp[s[i]]];
20         }
21         danger[u] = 1;
22     }
23     void build() {
24         queue<int> q;
25         for(int i = 0; i < 26; i++) {
26             if(t[0][i]) {
27                 fail[i] = 0;
28                 q.push(t[0][i]);
29             }
30         }
31         while(q.size()) {
32             int u = q.front(); q.pop();
33             danger[u] |= danger[fail[u]];
34             for(int i = 0; i < 26; i++) {
35                 if(t[u][i]) {
36                     fail[t[u][i]] = t[fail[u]][i];
37                     q.push(t[u][i]);
38                 } else t[u][i] = t[fail[u]][i];
39             }
40         }
41     }
42     int query(const char *s) {
43         memset(dp, 0x3f, sizeof(dp));
44         int n = strlen(s);
45         dp[0][0] = 0;
46         for(int i = 0; i < n; i++) {
47             for(int j = 0; j <= tot; j++) if(!danger[j]) {
48                 for(int k = 0; k < 26; k++) if(!danger[t[j][k]]) {
49                     dp[i + 1][t[j][k]] = min(dp[i + 1][t[j][k]], dp[i][j] + (s[i] - 'a' != k));
50                 }
51             }
52         }
53     }

```

```

53     int ans = 0x3f3f3f3f;
54     for(int i = 0; i <= tot; i++) if(!danger[i]) {
55         ans = min(ans, dp[n][i]);
56     }
57     return ans == 0x3f3f3f3f ? -1 : ans;
58 }
59 };

```

杂项

int128

```

1  typedef __uint128_t u128;
2  inline u128 read() {
3      static char buf[100];
4      scanf("%s", buf);
5      // std::cin >> buf;
6      u128 res = 0;
7      for(int i = 0; buf[i]; ++i) {
8          res = res << 4 | (buf[i] <= '9' ? buf[i] - '0' : buf[i] - 'a' + 10);
9      }
10     return res;
11 }
12 inline void output(u128 res) {
13     if(res >= 16)
14         output(res / 16);
15     putchar(res % 16 >= 10 ? 'a' + res % 16 - 10 : '0' + res % 16);
16     //std::cout.put(res % 16 >= 10 ? 'a' + res % 16 - 10 : '0' + res % 16);
17 }

```

Java, BigInteger

```

1  public BigInteger add(BigInteger val)    返回当前大整数对象与参数指定的大整数对象的和
2  public BigInteger subtract(BigInteger val)  返回当前大整数对象与参数指定的大整数对象的差
3  public BigInteger multiply(BigInteger val)  返回当前大整数对象与参数指定的大整数对象的积
4  public BigInteger divide(BigInteger val)    返回当前大整数对象与参数指定的大整数对象的商
5  public BigInteger remainder(BigInteger val)  返回当前大整数对象与参数指定的大整数对象的余
6  public int compareTo(BigInteger val)      返回当前大整数对象与参数指定的大整数对象的比较结果, 返回值是 1, -1, 0, 分别表示当前大整数对象大
   于, 小于或等于参数指定的大整数。
7  public BigInteger abs()                  返回当前大整数对象的绝对值
8  public BigInteger pow(int exponent)      返回当前大整数对象的 exponent 次幂。
9  public String toString()                 返回当前大整数对象十进制的字符串表示。
10 public String toString(int p)            返回当前大整数对象 p 进制的字符串表示。
11 public BigInteger negate()               返回当前大整数的相反数。

```

奇技淫巧

****_builtin_ 内建函数 ****

- `__builtin_popcount(unsigned int n)` 该函数是判断 `n` 的二进制中有多少个 1
- `__builtin_parity(unsigned int n)` 该函数是判断 `n` 的二进制中 1 的个数的奇偶性
- `__builtin_ffs(unsigned int n)` 该函数判断 `n` 的二进制末尾最后一个 1 的位置, 从一开始
- `__builtin_ctz(unsigned int n)` 该函数判断 `n` 的二进制末尾后面 0 的个数, 当 `n` 为 0 时, 和 `n` 的类型有关
- `__builtin_clz(unsigned int x)` 返回前导的 0 的个数

真·popcount

```

1  int _popcount(int x) {
2      return __builtin_popcount(x & (0ull - 1)) + __builtin_popcount(x >> 32);
3  }

```

随机数种子

```
1 srand(std :: chrono :: system_clock :: now().time_since_epoch().count());
```

T(5) 求任意 int log2

```
1 inline int LOG2_1(unsigned x){
2     static const int tb[32]={0,9,1,10,13,21,2,29,11,14,16,18,22,25,3,30,8,12,20,28,15,17,24,7,19,27,23,6,26,5,4,31};
3     x|=x>>1; x|=x>>2; x|=x>>4; x|=x>>8; x|=x>>16;
4     return tb[x*0x07C4ACDDu>>27];
5 }
```

O(1) 求 2 的整幂次 log2

```
1 inline int LOG2(unsigned x){ //x=2^k
2     static const int tb[32]={31,0,27,1,28,18,23,2,29,21,19,12,24,9,14,3,30,26,17,22,20,11,8,13,25,16,10,7,15,6,5,4};
3     return tb[x*263572066u>>27];
4 }
```

开启编译优化

```
1 作者: qwqwqer
2 链接: https://www.zhihu.com/question/264251178/answer/2155420801
3 来源: 知乎
4 著作权归作者所有。商业转载请联系作者获得授权, 非商业转载请注明出处。
```

```
6 #pragma GCC optimize(2)
7 #pragma GCC optimize(3)
8 #pragma GCC optimize("Ofast")
9 #pragma GCC optimize("inline")
10 #pragma GCC optimize("-fgcse")
11 #pragma GCC optimize("-fgcse-lm")
12 #pragma GCC optimize("-fipa-sra")
13 #pragma GCC optimize("-ftree-pre")
14 #pragma GCC optimize("-ftree-vrp")
15 #pragma GCC optimize("-fpeephole2")
16 #pragma GCC optimize("-ffast-math")
17 #pragma GCC optimize("-fsched-spec")
18 #pragma GCC optimize("unroll-loops")
19 #pragma GCC optimize("-falign-jumps")
20 #pragma GCC optimize("-falign-loops")
21 #pragma GCC optimize("-falign-labels")
22 #pragma GCC optimize("-fdevirtualize")
23 #pragma GCC optimize("-fcaller-saves")
24 #pragma GCC optimize("-fcrossjumping")
25 #pragma GCC optimize("-fthread-jumps")
26 #pragma GCC optimize("-funroll-loops")
27 #pragma GCC optimize("-fwhole-program")
28 #pragma GCC optimize("-freorder-blocks")
29 #pragma GCC optimize("-fschedule-insns")
30 #pragma GCC optimize("inline-functions")
31 #pragma GCC optimize("-ftree-tail-merge")
32 #pragma GCC optimize("-fschedule-insns2")
33 #pragma GCC optimize("-fstrict-aliasing")
34 #pragma GCC optimize("-fstrict-overflow")
35 #pragma GCC optimize("-falign-functions")
36 #pragma GCC optimize("-fcse-skip-blocks")
37 #pragma GCC optimize("-fcse-follow-jumps")
38 #pragma GCC optimize("-fsched-interblock")
39 #pragma GCC optimize("-fpartial-inlining")
40 #pragma GCC optimize("no-stack-protector")
41 #pragma GCC optimize("-freorder-functions")
42 #pragma GCC optimize("-findirect-inlining")
43 #pragma GCC optimize("-fhoist-adjacent-loads")
44 #pragma GCC optimize("-frerun-cse-after-loop")
45 #pragma GCC optimize("inline-small-functions")
46 #pragma GCC optimize("-finline-small-functions")
47 #pragma GCC optimize("-ftree-switch-conversion")
48 #pragma GCC optimize("-foptimize-sibling-calls")
49 #pragma GCC optimize("-fexpensive-optimizations")
50 #pragma GCC optimize("-funsafe-loop-optimizations")
51 #pragma GCC optimize("inline-functions-called-once")
52 #pragma GCC optimize("-fdelete-null-pointer-checks")
```

快速乘

```
1 ll mul(ll x, ll y, ll mod){
2     return (x * y - (ll)((long double)x / mod * y) * mod + mod) % mod;
3 }
4 ll mul(ll a, ll b, ll MOD) {
5     __int128 x = a, y = b, m = MOD;
6     return (ll)(x * y % m);
7 }
```

子集枚举

枚举 s 的子集

```
1 for(int i = s; i; i = (i - 1) & s))
```

枚举所有大小为 r 的集合

```
1 for(int s = (1 << r) - 1; s < (1 << n); ) {
2     int x = s & -s;
3     int y = s + x;
4     s = ((y ^ s) >> __builtin_ctz(x) + 2) | y;
5 }
```

mt19937_64 随机数生成器

```
1 std::mt19937_64 rng(std::chrono::steady_clock::now().time_since_epoch().count());
2 template <typename T>
3 T rd(T l, T r) {
4     std::uniform_int_distribution<T> u(l, r);
5     return u(rng);
6 }
7 template <>
8 double rd<double>(double l, double r) {
9     std::uniform_real_distribution<double> u(l, r);
10    return u(rng);
11 }
```

tips:

- 如果使用 sort 比较两个函数，不能出现 $a < b$ 和 $a > b$ 同时为真的情况，否则会运行错误。
- 多组数据清空线段树的时候，不要忘记清空全部数组（比如说 lazytag 数组）。
- 注意树的深度和节点到根的距离是两个不同的东西，深度是点数，距离是边长，如果求 LCA 时用距离算会出错。
- 连通性专题：注意判断 $dfn[x]$ 和 $low[y]$ 的关系时是否不小心两个都达成 low 了
- 推不等式确定范围的时候，仅需要考虑所有不等式限定的范围，然后判断左端点是否大于右端点，不要加额外的臆想条件。
- 矩阵快速幂如果常数十分大的时候，可以考虑 unordered_map 保存结果，可以明显加速。
- ****__builtin_popcount**** 只支持 unsigned int 型，不支持 long long!!!!!!