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Model Based Control of 3 DOF Parallel Delta Robot using Inverse Dynamic Model

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***Abstract* - The objective of this study is to improve the experimental tracking trajectory of the 3 DOF parallel Delta robot by incorporating the nonlinear inverse dynamic model (IDM) of the system in the H∞ control scheme. Experimental results for a desired trajectory of 12 G in the joint space show the performance improvements obtained by the total controller (H∞ +IDM) in comparison to the H∞ used without the incorporation of the IDM. In each case, the PID controller is used for comparative purposes.**

Index Terms – Model based control, H∞ control, Inverse Dynamic Model.

I. INTRODUCTION

In the last two decades, parallel robots have attracted the attention of both academicians and industrials, due to their high-speed handling performances and very high accuracy. These latter come from their parallel structure, composed by several kinematic chains. However, its complexity has some disadvantages, such as a reduced working space, presence of singularities and a high coupling effect between the axes [1][2].

In order to reduce the effect of these disadvantages, a specific control law which allows the exploitation of all the capacities of the Mechatronic system is required [3][4].

Considering the similarity of the structure of the kinematic and dynamic equations between the serial and parallel robots, most of the advanced control laws developed for serial chain robots can also be applied to parallel mechanisms.

The linear H∞ robust control proved in antecedent researches that it is an efficient method to control nonlinear process where robustness and performances are required at the same time [5][6].

In [7][8], two robust H∞ controllers are respectively synthesized by the loop shaping method and the mixed sensitivity problem in order to minimize the undesirable vibrations of the parallel robot Par 2. The robustness of the controllers is tested for different trajectories and load conditions. Simulation and experimental results are presented to compare the effectiveness of the controllers.

In our previous work [9][10][11][12], a linear H∞ controller is designed by mixed sensitivity approach and applied on the three degrees of freedom parallel Delta robot available at the Laboratory of Robotic Systems (LSRO) at EPFL (Ecole Polytechnique Fédérale de Lausanne) (Fig.1). For this purpose, a nonlinear analytical dynamic state model is developed and a tangential linearization procedure is used to obtain a multivariable linear model around a functional point. Real time experiments were performed to compare the centralized H∞ controller with a classical decentralized PID controller. Experimental tracking results show that the performances of the PID compared to those of the H∞ decrease when the movement dynamic is increased. The experiments of the load variation have proven that the H∞ is more robust than the PID.

In order to improve the tracking trajectory of the Delta robot, we propose in this paper the use of the H∞ control with the combination of the inverse dynamic model of the system (a priori torque) on the three degrees parallel Delta robot. Indeed, this method has already been used by J. De Cuyper et al. in [13][14] to improve the tracking performance of an automotive test bench.

In the following, first, the procedure of synthesis of a centralized H∞ controller is presented. Then, an inverse dynamic model of a Delta robot is performed. It depends only on the joint coordinates. This latter is linearized in state space around a functional point. The H∞ controller is synthesized based on this linear model by the algorithm of Glover-Doyle [15]. The synthesized H∞ and classical PID controllers are used in combination with the a priori torque and implemented in the control scheme of the Delta robot. At the end, experimental results are presented and compared to those obtained from control schemes used without the a priori torque.

Fig. 1 The direct drive Delta robot with three degrees of freedom

The paper is organized as follows: In section 2, the basic theory for synthesis of H∞ controller is developed. The nonlinear inverse dynamic model of the Delta robot is given in section 3 and the linear dynamic model is presented in section 4. Section 5 shows the approach used to design the two proposed controllers: the decentralized PID and the centralized H∞. Each controller is used with the combination of the a priori torque. In section 6, the performance experimental results are presented. The conclusion is given in section 7.

II. H∞ ROBUST CONTROL

A. Objective

The objective of the H∞ control is to design a controller which stabilizes the closed loop system and realizes a good trade-off between robustness and performances when the system is in the presence of parametric uncertainties.

The H∞ standard problem is defined by the Fig. 2 [16][17]:

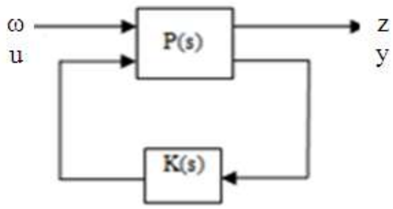

Fig. 2 The H∞ standard problem

Where:

$$\begin{bmatrix} z \\ y \end{bmatrix} = \begin{bmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{22}(s) \end{bmatrix} \begin{bmatrix} \omega \\ u \end{bmatrix} = P(s) \begin{bmatrix} \omega \\ u \end{bmatrix} \quad (1)$$

K is the controller to design.

ω, is the exogenous inputs and z is the outputs to control.

u, is the output of the controller K and y is the measurements available for the controller.

The original system G(s) augmented with the weighting function matrices W_S and W_T containing all the specifications of performance and robustness constitute the augmented system P.

This formalism is used to analyze the transfer matrix function between ω and z:

$$z = T_{z\omega}(P, K)\omega \quad (2)$$

with:

$$T_{z\omega} = \begin{bmatrix} W_S S \\ W_T T \end{bmatrix}$$

Where, $(S = (I+G(s)K(s))^{-1})$ represents the sensitivity matrix and $(T = I-S)$ is the complementary sensitivity matrix.

So, the problem of H∞ standard problem is to determine a stabilizing controller K for the augmented plant P, which makes the transfer $T_{z\omega}$ internally stable and verifies the condition norm of (3):

$$\left\| T_{z\omega} \right\|_\infty \le 1 \quad (3)$$

with:

$$\left\| T_{z\omega} \right\|_\infty = \sup_{\omega} \overline{\sigma}(T_{z\omega}(j\omega))$$

$\overline{\sigma}$: represents the maximum singular value of $T_{z\omega}$.

The constraint (3) is called the mixed sensitivity problem [18], it implies to satisfy two conditions:

$$\overline{\sigma}(S(j\omega)) \le \left| W_S^{-1}(j\omega) \right| \quad (4)$$

$$\overline{\sigma}(T(j\omega)) \le \left| W_T^{-1}(j\omega) \right| \quad (5)$$

All the performance and robustness specifications are defined in the constraints $\left| W_S^{-1}(j\omega) \right|$ and $\left| W_T^{-1}(j\omega) \right|$.

The constraint (4) implies that to have a better performance (lower tracking error), S must be made small and the condition (5) implies that to obtain robustness stability, the matrix function T should be made small in frequency region where the model uncertainty is large.

Because of the complementarities of S and T, (S + T = I), these matrix functions cannot be made small at the same frequency. So, the solution is to synthesize a controller with a large gain at low frequency (from 0 to a bandwidth B) which makes S small in this region, and a low gain outside this band which makes T small in the high frequencies. The problem becomes of shaping S and T by a suitable choice of the weighting function matrices.

B. The Choice of the Weighting Matrices

The weighting matrices ($\left| W_S^{-1}(j\omega) \right|$) and ($\left| W_T^{-1}(j\omega) \right|$) are the desired upper bounds on the magnitude of the closed loop transfer matrices S and T.

To obtain S small at low frequencies, each component of the weighting matrix $W_S(s)$ must be a low pass filter [19].

We put:

$$Ws(s) = W_{s11}(s) I_d$$

I_d is the identity matrix, and:

$$W_{s11}(s) = \frac{s + m}{K(s + n)} \quad (6)$$

The parameter m is chosen to be greater than n and ($K > \sqrt{2}$). At the pulse ω_c, given by (7), W_{s11} has a gain of -3dB.

$$\omega_c = \sqrt{\frac{2m^2 - K^2 n^2}{K^2 - 2}} \quad (7)$$

It is obvious that to have a large bandwidth, K must be small and m greater than n.

The model uncertainties are large at high frequencies. So, W_T imposes to T to decrease in this region with a certain roll-off (-40dB/decade at least).

We put:

$$W_T(s) = W_{T11}(s) I_d$$

W_{T11} can take this form:

$$W_{T11}(s) = \frac{s^2}{k} \tag{8}$$

C. Resolution of the standard problem

The H∞ standard problem solving has been well investigated in 1988 with the Glover-Doyle algorithm [15] which proposes a solution in the state space very close to the LQG (Resolution by Riccati equations). It requires the satisfaction of four conditions and allows the designing of controller which stabilizes internally the closed loop system and ensures that:

$$\left\| T_{z\omega} \right\|_\infty \leq 1$$

III. DYNAMIC MODEL

The Inverse Dynamic Model (IDM) of the Delta robot performed by A. Codourey in [20][21] has the particularity to depend not only of the joint coordinates but also to the kinematics of the tool. In order to implement an H∞ controller in the joint space, we perform from this model, an IDM which depends only on the joint variables.

The IDM of Delta robot is given by (9):

$$\Gamma = I_b \ddot{\alpha} - KK_m \ddot{X} + KG_n - G_b \tag{9}$$

Γ, represents the joint torques vector;

$\alpha = [\alpha_1, \alpha_2, \alpha_3]^T$, represents the coordinates of the robot in joint space;

$X = [x, y, z]^T$, represents the coordinates of the robot in operational space.

$\dot{\alpha}$: is the joint velocity vector;

$\dot{X}$ **:** is the operational velocity vector;

$\ddot{\alpha} = [\ddot{\alpha}_1 \quad \ddot{\alpha}_2 \quad \ddot{\alpha}_3]^T$, represents the acceleration joint vector;

$\ddot{X} = [\ddot{x} \quad \ddot{y} \quad \ddot{z}]^T$, represents the vector of operational acceleration;

I_b : inertia diagonal matrix of the arms (reported to the motors);

KG_n is the travelling plate gravity effects;

G_b: is the arm gravity effects;

I , K, K_m, G_n and G_b are defined in [20] .

In order to develop from (9), a differential equation depending only on the joint positions, we must find a relationship between $\ddot{x}$ and $\ddot{\alpha}$. This latter can be given from the Direct Geometric Model (DGM):

$$X = f(\alpha)$$

The robot kinematic model is obtained by deriving the DGM equation:

$$\dot{X} = \begin{pmatrix} \frac{\partial f_x}{\partial \alpha_1} & \frac{\partial f_x}{\partial \alpha_2} & \frac{\partial f_x}{\partial \alpha_3} \\ \frac{\partial f_y}{\partial \alpha_1} & \frac{\partial f_y}{\partial \alpha_2} & \frac{\partial f_y}{\partial \alpha_3} \\ \frac{\partial f_z}{\partial \alpha_1} & \frac{\partial f_z}{\partial \alpha_2} & \frac{\partial f_z}{\partial \alpha_3} \end{pmatrix} \dot{\alpha} = J(\alpha)\dot{\alpha} \tag{10}$$

J: represents the robot Jacobian matrix.

A second derivation leads to:

$$\ddot{X} = J\ddot{\alpha} + D\dot{\alpha}_i^2 + E\dot{\alpha}_{ij} \tag{11}$$

with:

$$\dot{\alpha}_i^2 = [\dot{\alpha}_1^2 \quad \dot{\alpha}_2^2 \quad \dot{\alpha}_3^2]^T, \ \dot{\alpha}_{ij} = [\dot{\alpha}_1\dot{\alpha}_2 \quad \dot{\alpha}_1\dot{\alpha}_3 \quad \dot{\alpha}_2\dot{\alpha}_3]^T,$$

$$D = \begin{bmatrix} \frac{\partial}{\partial \alpha_1}\left(\frac{\partial f_x}{\partial \alpha_1}\right) & \frac{\partial}{\partial \alpha_2}\left(\frac{\partial f_x}{\partial \alpha_2}\right) & \frac{\partial}{\partial \alpha_3}\left(\frac{\partial f_x}{\partial \alpha_3}\right) \\ \frac{\partial}{\partial \alpha_1}\left(\frac{\partial f_y}{\partial \alpha_1}\right) & \frac{\partial}{\partial \alpha_2}\left(\frac{\partial f_y}{\partial \alpha_2}\right) & \frac{\partial}{\partial \alpha_3}\left(\frac{\partial f_y}{\partial \alpha_3}\right) \\ \frac{\partial}{\partial \alpha_1}\left(\frac{\partial f_z}{\partial \alpha_1}\right) & \frac{\partial}{\partial \alpha_2}\left(\frac{\partial f_z}{\partial \alpha_2}\right) & \frac{\partial}{\partial \alpha_3}\left(\frac{\partial f_z}{\partial \alpha_3}\right) \end{bmatrix} \tag{12}$$

$$E = \begin{bmatrix} \frac{\partial}{\partial \alpha_1}\left(\frac{\partial f_x}{\partial \alpha_2}\right) + \frac{\partial}{\partial \alpha_2}\left(\frac{\partial f_x}{\partial \alpha_1}\right) & \frac{\partial}{\partial \alpha_3}\left(\frac{\partial f_x}{\partial \alpha_1}\right) + \frac{\partial}{\partial \alpha_1}\left(\frac{\partial f_x}{\partial \alpha_3}\right) & \frac{\partial}{\partial \alpha_2}\left(\frac{\partial f_x}{\partial \alpha_3}\right) + \frac{\partial}{\partial \alpha_3}\left(\frac{\partial f_x}{\partial \alpha_2}\right) \\ \frac{\partial}{\partial \alpha_1}\left(\frac{\partial f_y}{\partial \alpha_2}\right) + \frac{\partial}{\partial \alpha_2}\left(\frac{\partial f_y}{\partial \alpha_1}\right) & \frac{\partial}{\partial \alpha_3}\left(\frac{\partial f_y}{\partial \alpha_1}\right) + \frac{\partial}{\partial \alpha_1}\left(\frac{\partial f_y}{\partial \alpha_3}\right) & \frac{\partial}{\partial \alpha_2}\left(\frac{\partial f_y}{\partial \alpha_3}\right) + \frac{\partial}{\partial \alpha_3}\left(\frac{\partial f_y}{\partial \alpha_2}\right) \\ \frac{\partial}{\partial \alpha_1}\left(\frac{\partial f_z}{\partial \alpha_2}\right) + \frac{\partial}{\partial \alpha_2}\left(\frac{\partial f_z}{\partial \alpha_1}\right) & \frac{\partial}{\partial \alpha_3}\left(\frac{\partial f_z}{\partial \alpha_1}\right) + \frac{\partial}{\partial \alpha_1}\left(\frac{\partial f_z}{\partial \alpha_3}\right) & \frac{\partial}{\partial \alpha_2}\left(\frac{\partial f_z}{\partial \alpha_3}\right) + \frac{\partial}{\partial \alpha_3}\left(\frac{\partial f_z}{\partial \alpha_2}\right) \end{bmatrix} \tag{13}$$

Replacing (11) in (9), this leads to:

$$\Gamma = M(\alpha)\ddot{\alpha} + N(\alpha)\dot{\alpha}_i^2 + L(\alpha)\dot{\alpha}_{ij} + H(\alpha) \tag{14}$$

with;

$$M(\alpha) = I_b - KK_m J, \ N(\alpha) = -KK_m D$$
$$L(\alpha) = -KK_m E, \ H(\alpha) = KG_n - G_b$$

IV. LINEAR MODEL

The design of a linear H∞ controller for the Delta robot requires the determination of a linear dynamic model. The linearization of (14) around an operating point was performed in Matlab. The linear model is represented in state space by its matrices (A, B, C):

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 20.019 & -4 & -4 & 0 & 0 & 0 \\ -4 & 20.019 & -4 & 0 & 0 & 0 \\ -4 & -4 & 20.019 & 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 53.84 & 15.13 & 15.13 \\ 15.13 & 53.84 & 15.13 \\ 15.13 & 15.13 & 53.84 \end{bmatrix}$$

$$C=\begin{bmatrix}1&0&0&0&0&0\\0&1&0&0&0&0\\0&0&1&0&0&0\end{bmatrix}$$

V. Design of PID and H∞ Controllers

The H∞ and PID controllers are implemented on the direct drive Delta Robot, available at the Laboratory of Robotic Systems of the Swiss Federal Institute of Technology in Lausanne (EPFL). The control system is implemented on a PC (Core Duo 2,1 Ghz) which is performed under windows XP and a real time extension (RTX from IntervalZero Inc.). The two controllers are implemented with the association of the feed forward pre computed torque Γ_a (a priori torque, i.e., the incorporation of the IDM in the control scheme).

In order to show the performance improvements brought by the association of the a priori torque to the control schemes of the two controllers (H∞ and PID), the experimental results obtained are compared to those realized by the control schemes used without the a priori torque (Γ_a=0).

The desired trajectory in the joint space is shown in Fig. 3. It is used with an acceleration of 12G (G is the gravitational acceleration equal to 9.81 N/kg).

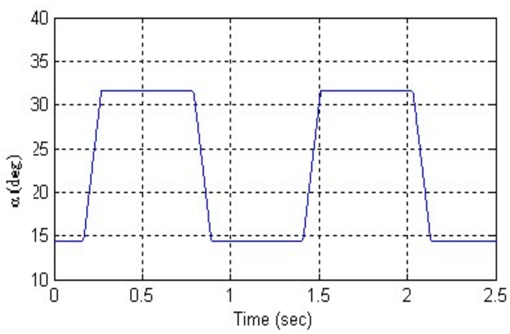

Fig.3 Experimental desired trajectory in the joint space.

A. PID controller

The Delta robot is considered as a linear system and each of its joint is controlled by a decentralised PID controller with constant gains combined with the a priori torque Γ_a. The gain parameters of the controller have been adjusted experimentally to Kp = 500, Ki =3000, and Kd =5, which guarantees a good bandwidth and good tracking performance.

B. Design of the H∞ controller

To design the H∞ controller by the mixed sensitivity approach, we use the linear dynamic model of the Delta robot represented in state space by its (A,B,C) matrices. This model is augmented by the weighting function matrices W_s and W_T. The controller H∞ is synthesized in Matlab using the algorithm of Glover-Doyle [15].

The synthesized controller $K_{H\infty}$ is implemented in the control scheme of the Delta robot combined with the a priori torque Γ_a, as shown in Fig.4.

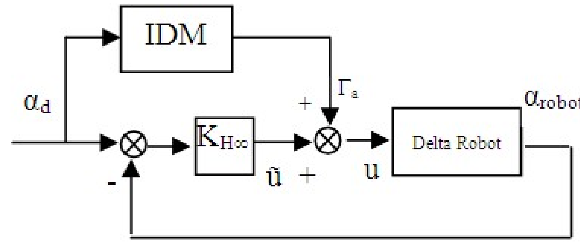

Fig. 4 H∞ Control scheme combined with the a priori torque Γ_a.

α_d: is the desired trajectory in the joint space.

The total control:

$$u = \tilde{u} + \Gamma_a \tag{15}$$

$$\Gamma_a = M(\alpha_d)\ddot{\alpha}_d + N(\alpha_d)\dot{\alpha}_{id}^{\,2} + L(\alpha_d)\dot{\alpha}_{ij_d} + H(\alpha_d) \tag{16}$$

α_d , $\dot{\alpha}_d$ and $\ddot{\alpha}_d$ are respectively the desired positions, velocities and accelerations in the joint space.

$$\dot{\alpha}_{id}^{\,2} = [\dot{\alpha}_{1d}^{\,2} \quad \dot{\alpha}_{2d}^{\,2} \quad \dot{\alpha}_{3d}^{\,2}]^T \tag{17}$$

$$\dot{\alpha}_{i\,jd} = \left[\dot{\alpha}_{1d}\dot{\alpha}_{2d} \quad \dot{\alpha}_{1d}\dot{\alpha}_{3d} \quad \dot{\alpha}_{2d}\dot{\alpha}_{3d}\right]^T \tag{18}$$

$\tilde{u}$: represents the output of the H∞ controller.

The parameters of the weighting matrices are selected experimentally in order to have the best performances in terms of minimum tracking errors:

$$W_{s11}(s) = \frac{s + 10^{-4}}{143\,(s + 10^{-4})}, \quad W_{T11}(s) = \frac{s^2}{800000}$$

VI. Experimental Results

The experimental tracking errors for 12G of the H∞ and the PID controllers combined with the a priori torque Γ_a are represented in Fig.5 (for the three articulations). For comparison purposes, Fig. 6 shows the experimental tracking errors of the two controllers used without the a priori torque (Γ_a=0).

The Table 1 summarizes the performances obtained by the H∞ and the PID controllers for the two cases. The used performance index is the maximum tracking error.

As we can see, for each case, the results obtained for the H∞ controller are more satisfactory than those of PID. And for the two controllers, we can notice the performance improvements provided by the association of the a priori torque.

TABLE I
Performances Comparison Between the H∞ and PID Controllers

Error peaks (deg)	**Hinf.**		**PID**	
	Γ_a = 0	***with Γ_a***	***Γ_a = 0***	***with Γ_a***
Art.1	0.51	0.19	0.63	0.31
Art2	0.48	0.19	0.61	0.33
Art3	0.42	0.28	0.57	0.35

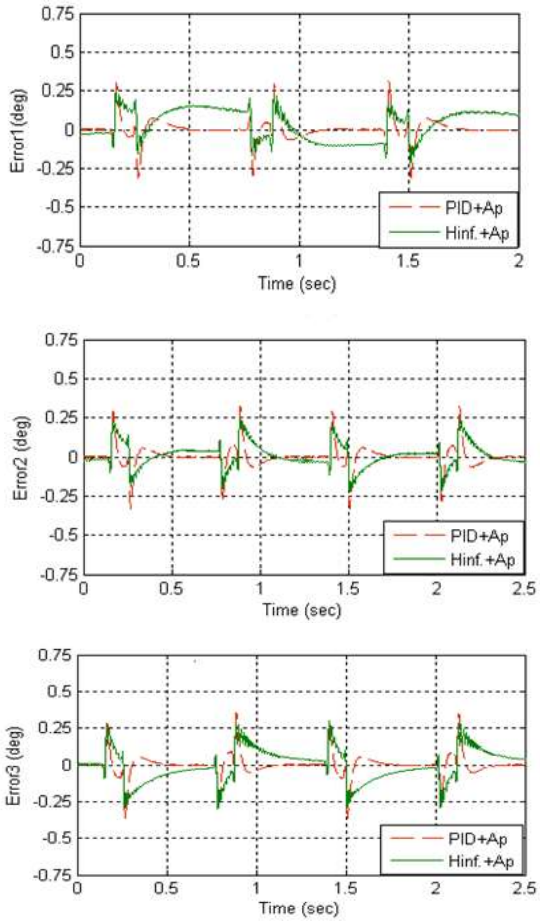

Fig.5 Experimental tracking errors for 12G with H∞ controller (solid line) and the PID controller (dashed line) combined with the a priori torque Γ_a.

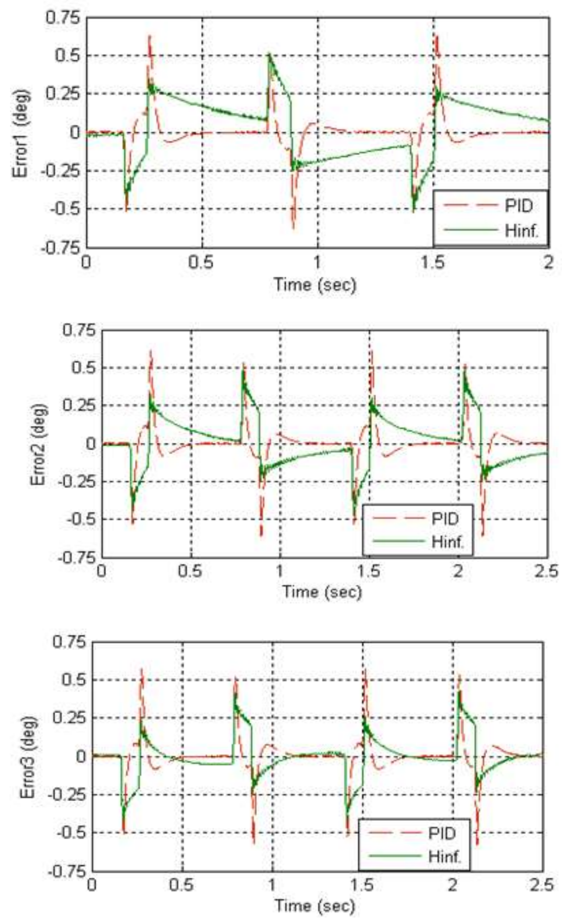

Fig.6 Experimental tracking errors for 12G with H∞ controller (solid line) and the PID controller (dashed line) used without the association of the a priori torque ($\Gamma_a = 0$).

VII. Conclusion

In this work, we propose the application of the H∞ control to the parallel Delta. The controller is used in combination with the IDM of the system (a priori torque).

The design of the controller required the determination of a linear dynamic model of the Delta robot from its nonlinear inverse dynamic model.

The H∞ controller is synthesized based on this linear model by mixed sensitivity approach in which the sensitivity function matrix S and the complementary sensitivity function matrix T are taken into account.

The H∞ controller is implemented in the control scheme of the Delta robot with the combination of the a priori torque.

Experimental results show on the one hand that the centralized H∞ controller has the best performances compared to the decentralized PID controller and on the other hand that the association of the a priori torque in the control scheme improves the performance of the H∞ considerably. The maximum tracking errors of the H∞ with a priori torque is 58% of the H∞ without the a priori torque.

Further research will focus on considering more complex trajectories for pick and place in tool space and improving the performance of the Delta robot by using multi-models and multi-objectives control approaches.

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