

# Development of Filter Modeling with Polydispersed Fibers

**Seungkoo Kang and David Y. H. Pui**

*Particle Technology Laboratory  
University of Minnesota*

CFR 52 Review Meeting, Mechanical Engineering  
University of Minnesota  
October 6<sup>th</sup>, 2017



Center for Filtration Research



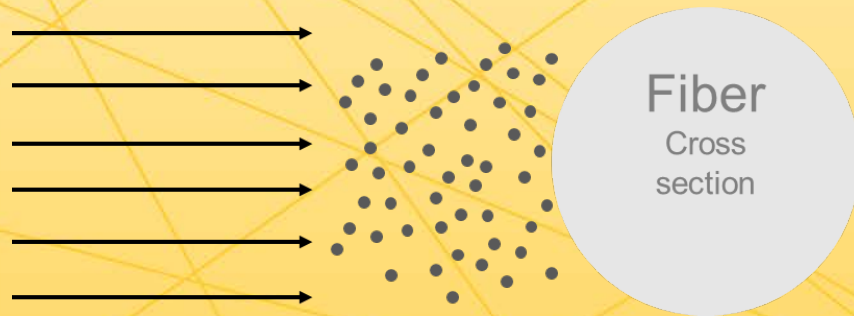
UNIVERSITY OF MINNESOTA

# Outline

- Introduction
- Objectives
- Experimental methods
  - Filter properties
  - Experimental method and setup
- Numerical simulation methods
  - Filter generation
  - Flow field calculation
  - Particle trajectory
- Results
  - Collection efficiency comparison
  - Effect of polydispersity

# Single Fiber Efficiency (SFE) theory

- For the estimation of fibrous filter capture efficiency, SFE theory is generally applied



$$P = 1 - E = \exp\left(\frac{-4\alpha E_{\Sigma} t}{\pi d_f (1 - \alpha)}\right)$$

*t*: filter thickness, *α*: solidity

*d<sub>f</sub>*: fiber diameter, *E<sub>Σ</sub>*: single fiber efficiency

$$E_{\Sigma} = 1 - (1 - E_R)(1 - E_I) (1 - E_D) (1 - E_{DR}) (1 - E_G)$$

*E<sub>R</sub>*: Efficiency for interception

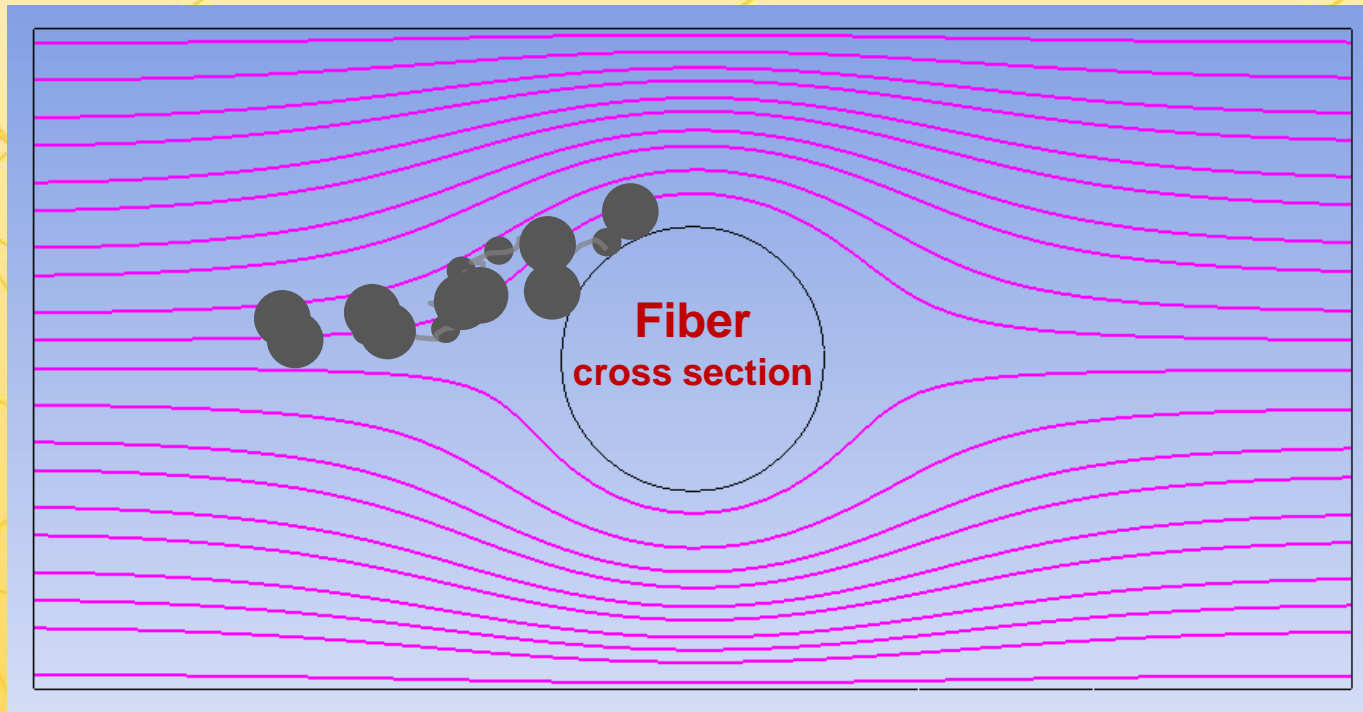
*E<sub>I</sub>*: Efficiency for impaction

*E<sub>D</sub>*: Efficiency for diffusion

*E<sub>DR</sub>*: Efficiency for interception of particles with diffusion

*E<sub>G</sub>*: Efficiency for gravitational setting

# Particle deposition mechanism



## Deposition mechanism:

- Diffusion
- Interception
- Impaction
- Gravitational settling
- Electrostatic attraction

# Analytical expressions for each capture mechanism

SFE Diffusion	Ed
Pich (1965)	$2.27Ku^{-1/3}Pe^{-2/3}(1 + 0.62Kn_fPe^{1/3}Ku^{-1/3})$
Stechkina (1966)	$2.9Ku^{-1/3}Pe^{-2/3} + 0.624Pe^{-1}$
Lee and Liu (1982)	$1.6\left(\frac{1-\alpha}{Ku}\right)^{1/3}Pe^{-2/3}$
Liu and Rubow (1990)	$1.6\left(\frac{1-\alpha}{Ku}\right)^{1/3}Pe^{-2/3}C_d$
Payet et al. (1992)	$1.6\left(\frac{1-\alpha}{Ku}\right)^{1/3}Pe^{-2/3}C_dC_{d'}$
Kirsch and Fuchs (1968)	$2.7Pe^{-2/3}$

$$Kn_f = \frac{2\lambda}{d_f}$$

$$Ku = -0.15\ln\alpha - \frac{3}{4} - \frac{\alpha^2}{4} + \alpha$$

$$C_d = 1 + 0.388Kn_f \left[ \frac{(1-\alpha)Pe}{Ku} \right]^{1/3}$$

$$C_{d'} = \left[ 1 + 1.6 \left[ \frac{1-\alpha}{Ku} \right]^{1/3} Pe^{-2/3} C_d \right]^{-1}$$

SFE Interception	Er
Lee and Gieseke (1980)	$\left(\frac{1-\alpha}{Ku}\right)R^2(1+R)^{\frac{-2}{3(1-\alpha)}}$
Pich (1966)	$\frac{(1+R)^{-1} - (1+R) + 2(1 + 1.996Kn_f)(1+R)\ln(1+R)}{2(-0.75 - 0.5\ln\alpha) + 1.996Kn_f(-0.5 - \ln\alpha)}$
Lee and Liu (1982)	$0.6\left(\frac{1-\alpha}{Ku}\right)\frac{R^2}{1+R}$
Liu and Rubow (1990)	$0.6\left(\frac{1-\alpha}{Ku}\right)\frac{R^2}{1+R}C_r$

$$R = \frac{d_p}{d_f}$$

$$C_r = 1 + \frac{1.996Kn_f}{R}$$

SFE Impaction	Ei
Yeh and Liu (1974)	$\left(\frac{stk}{2Ku^2}\right)J$

$$Stk = \frac{\rho_p d_p^2 C_c U_0}{18\mu d_f} \quad J = (29.6 - 28\alpha^{0.62})R^2 - 27.5R^{2.8}$$



# Equivalent fiber diameter (for SFE theory)

- An **equivalent fiber diameter** is applied to SFE to obtain collection efficiency of filter media

## **Equivalent fiber diameter**

- Different concepts of mean:**

\*Arithmetic mean:  $\sum \frac{D_f}{n}$

\*Geometric mean:  $\sqrt[n]{D_{f1} D_{f2} \cdots D_{fn}}$

\*Volume-surface average fiber diameter:  $\frac{\sum n D_f^2}{\sum n D_f}$

- Effective fiber diameter:**

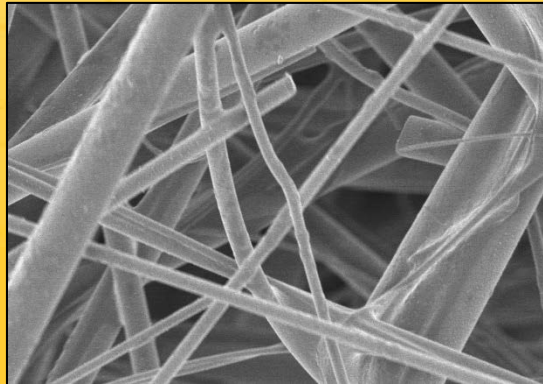
\*Davies:  $D_f = \sqrt{U_\infty \mu t \left( \frac{64 \alpha^{\frac{3}{2}} (1 + 56 \alpha^3)}{\Delta P} \right)}$

**Direct  
measurement  
of fiber size  
(e.g. SEM)**

**Empirical  
expression:  
Approximation  
based on the  
pressure drop  
measurement  
( $\Delta P$ ,  $t$ ,  $\alpha$  are  
required)**

Fiber  
Cross section

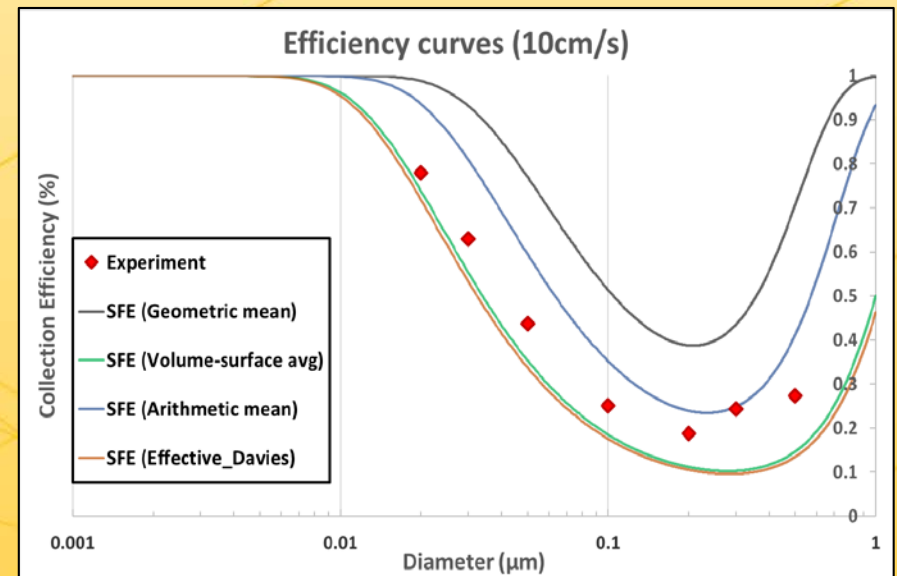
- However, real fibrous filter is composed of a wide range of fibers



# Issues with using an equiv. fiber diameter

- Podgorski (2009) pointed out that “it is impossible to select any one universal mean fiber diameter to describe penetration of nanoparticles with different sizes.”

- Different equivalent fiber diameter definitions affect the efficiency curve

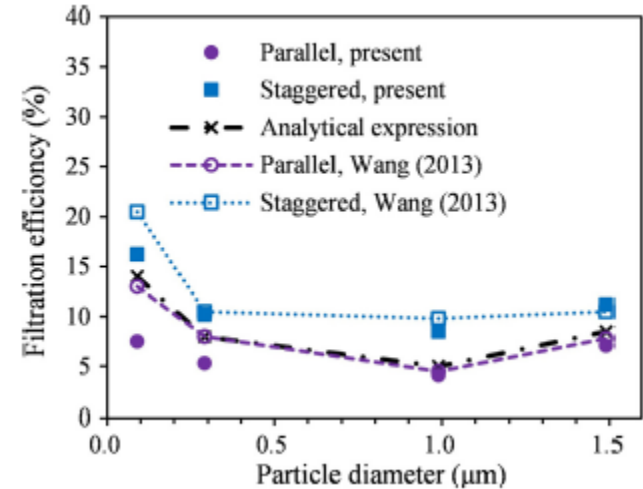
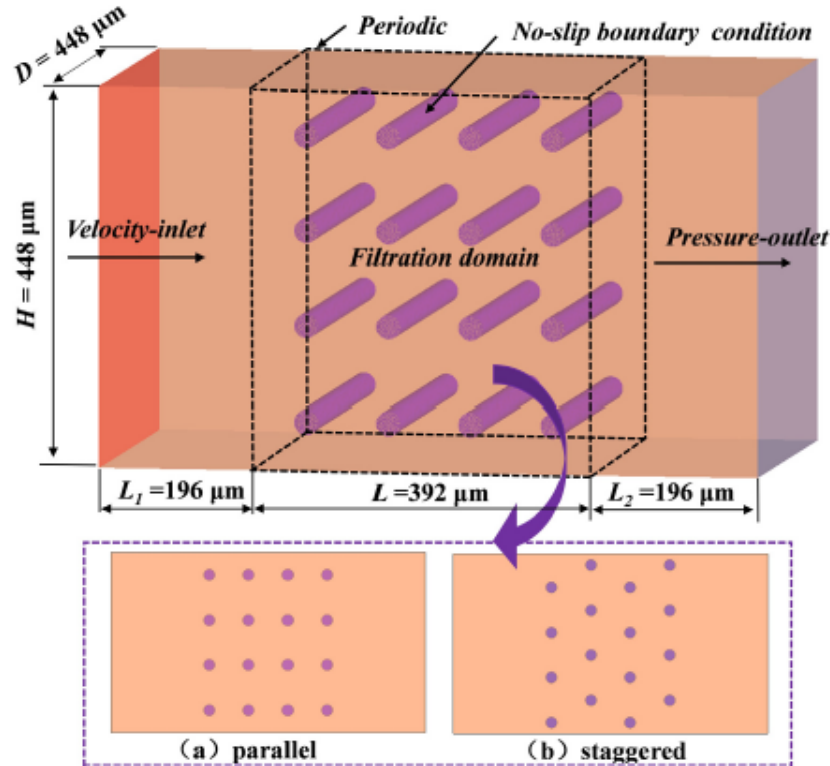


## HF-0012 (Hollingsworth & Vose)

- However, an equivalent fiber diameter is still widely applied to the numerical modeling and SFE theory

# Recent papers using single fiber diameter

Li et al. (2016). Advanced Powder Technology 27: 638-645.

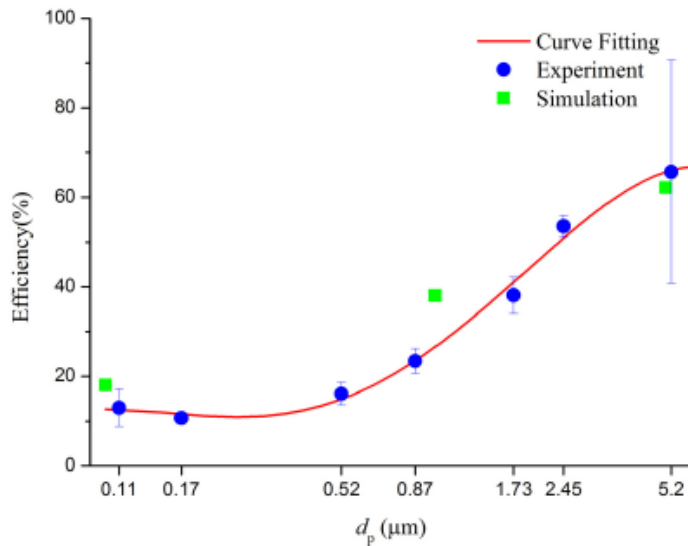
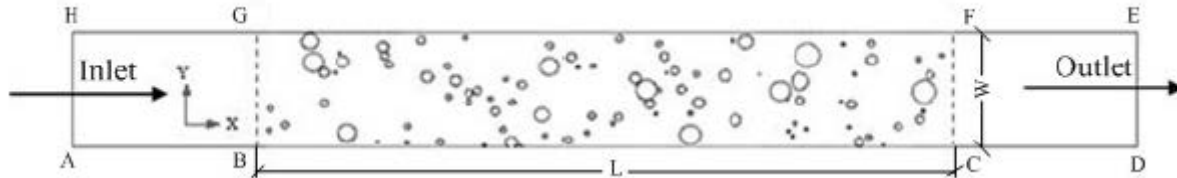


- CFD-DPM (Discrete Phase Modeling)
- Parallel vs staggered
- Collection efficiency of multi-fiber filters is obtained



# Papers using polydisperse fibers

Bin et al. (2016). Aerosol and Air Quality Research (Article in press)

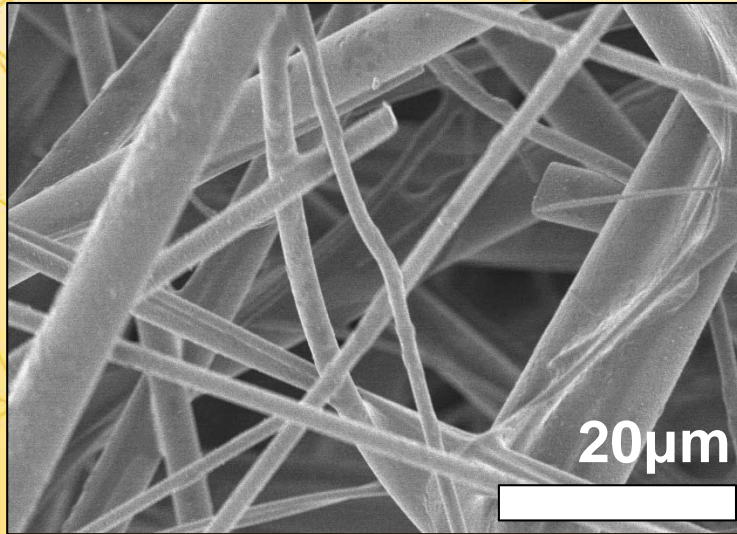


- Lattice Boltzmann method
- Randomly located polydisperse fibers
- **Limitations:**
  - Smallest particle challenging filter is **100 nm** (Capture by **diffusion** cannot be well understood)
  - Only three particle sizes are investigated, and only one of them is submicron

# Objectives

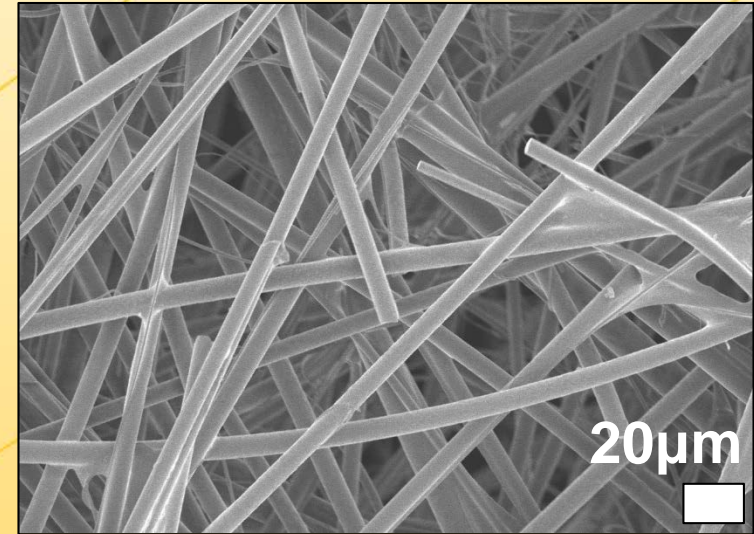
- To develop a method to generate fibrous filter domain composed of polydisperse fibers using commercial filter media (H & V)
- To develop a numerical model of nanoparticle filtration by fibrous filters
- To compare the collection efficiency between numerical model and experimental data with challenging particle diameter from 3nm up to 500 nm
- To investigate the effect of equivalent fiber diameters on predicted collection efficiency

# Filter properties (Hollingsworth & Vose)



HF-0012	
Fiber distribution	size ( $\mu\text{m}$ )
Arithmetic mean	3.3
Geometric mean	2.5
Volume-surface average	5.1
Effective Fiber diameter (Davies)	5.3

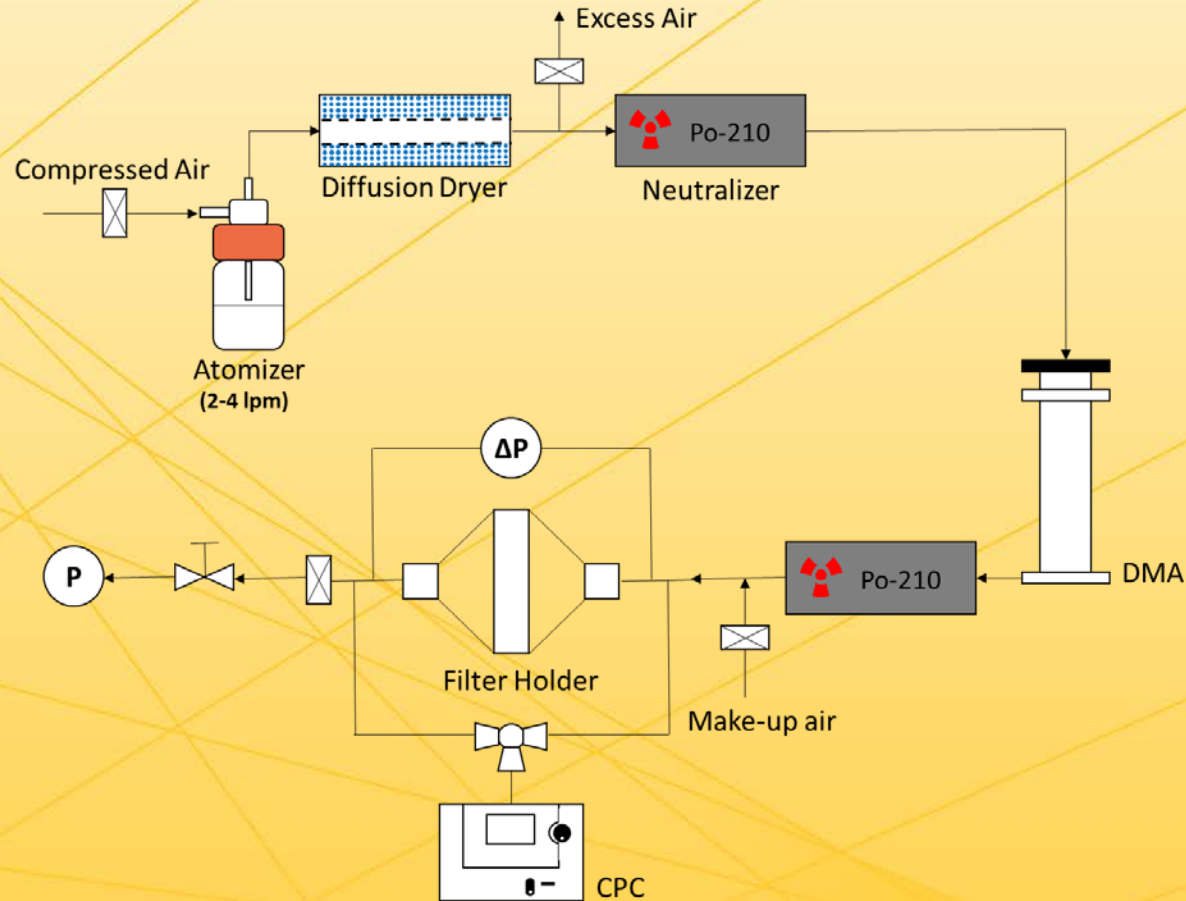
Filter Parameters	
Thickness (mm)	0.74
Solidity	0.039
DOP % Penetration (0.3 $\mu\text{m}$ at 5.3 cm/s)	79.9
Pressure drop (Pa) @ 5.3 cm/s	13



HF-0493	
Fiber distribution	size ( $\mu\text{m}$ )
Arithmetic mean	5.7
Geometric mean	4.5
Volume-surface average	5.6
Effective Fiber diameter (Davies)	8.3

Filter Parameters	
Thickness (mm)	0.36
Solidity	0.076
DOP % Penetration (0.3 $\mu\text{m}$ at 5.3 cm/s)	88
Pressure drop (Pa) @ 5.3 cm/s	8

# Experimental method and setup

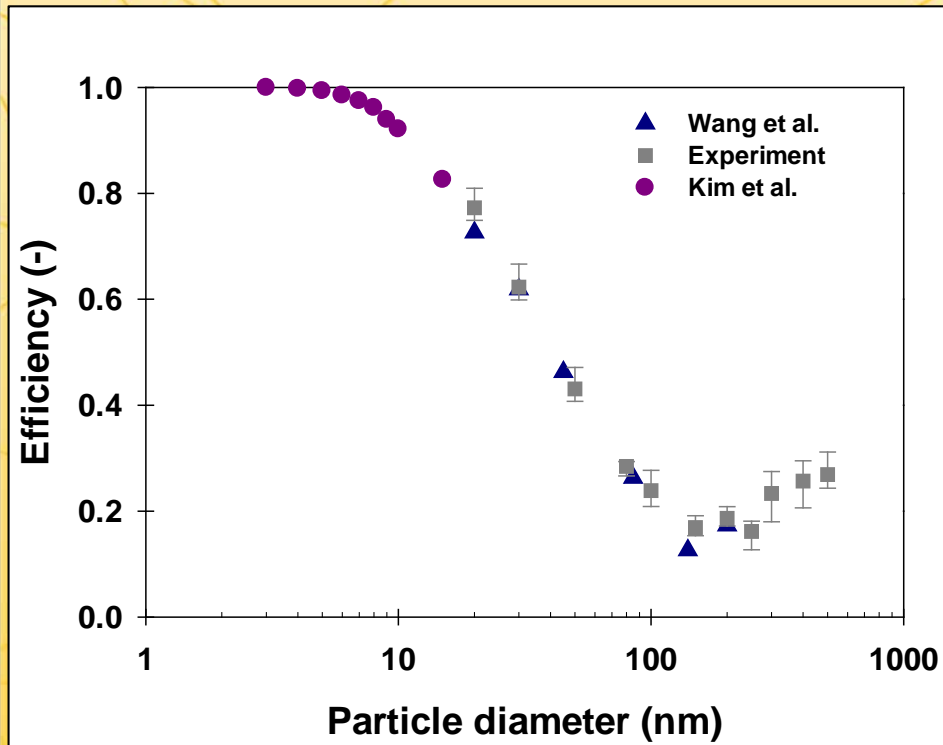


- KCl particles (20, 30, 50, 80, 100, 150, 200, 300, and 500 nm) are used to challenge the filter media (HF-0012 and HF-0493)
- Particle size from 3 to 20nm was obtained from Kim et al. (2008) to be compared with the simulation results

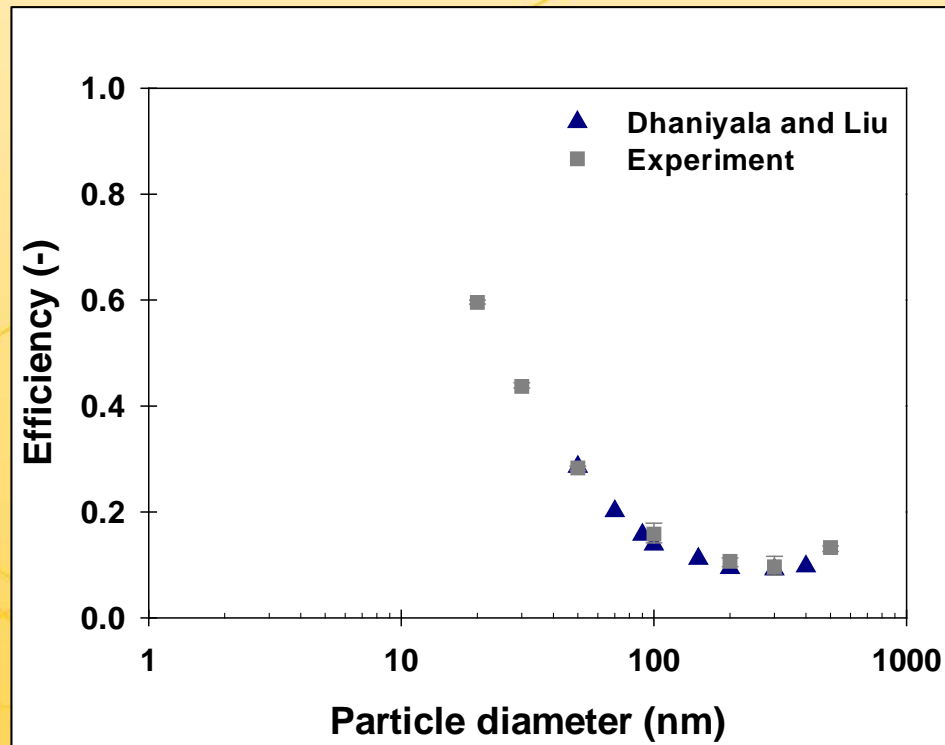


# Experimental results comparison

## HF-0012 10cm/s



## HF-0493 10cm/s



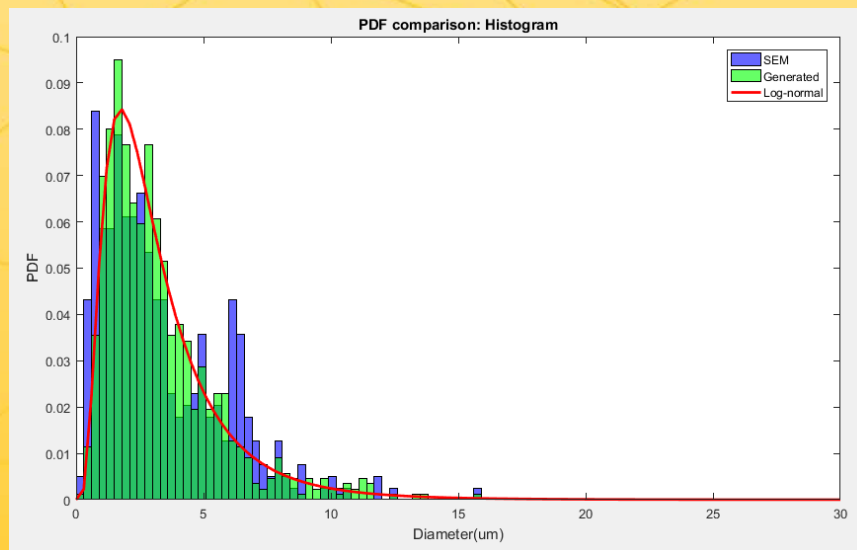
- Kim et al.: **Silver** particles (3-20nm)
- Wang et al.: **NaCl** particles (20-200nm)
- Experiment: **KCl** particles (20 – 500nm)

- Dhaniyala and Liu: **DOS** (50-400nm)
- Experiment: **KCl** particles (20 – 500nm)

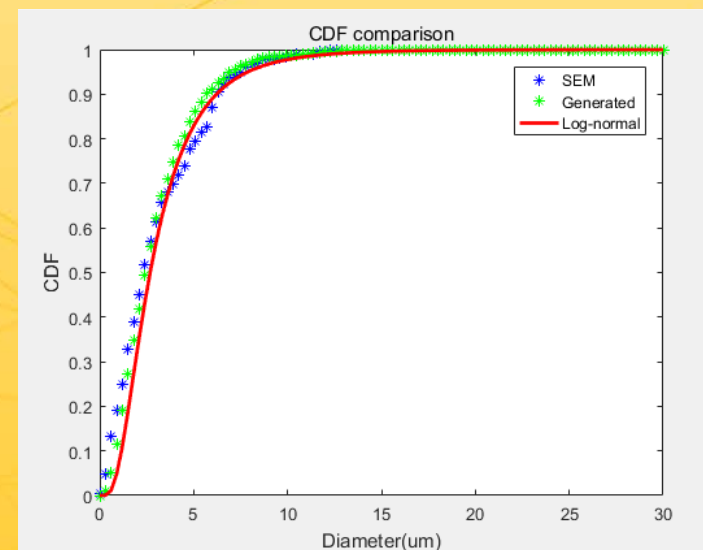
- Dhaniyala and Liu (1999). Journal of the IEST 42(1): 32-40.
- Wang et al. (2007). Journal of Nanoparticle Research 9: 109-115.
- Kim et al. (2007). Journal of Nanoparticle Research 9: 117-125.

# Fiber generation

- Fiber size distribution measured by SEM images follows log normal distribution
- Figures shows the fiber size distribution from HF-0012 (Blue color – Measured by SEM) and from randomly generated one (Orange color – Generated by the code developed)
- Their distributions are well agreed as shown in the picture



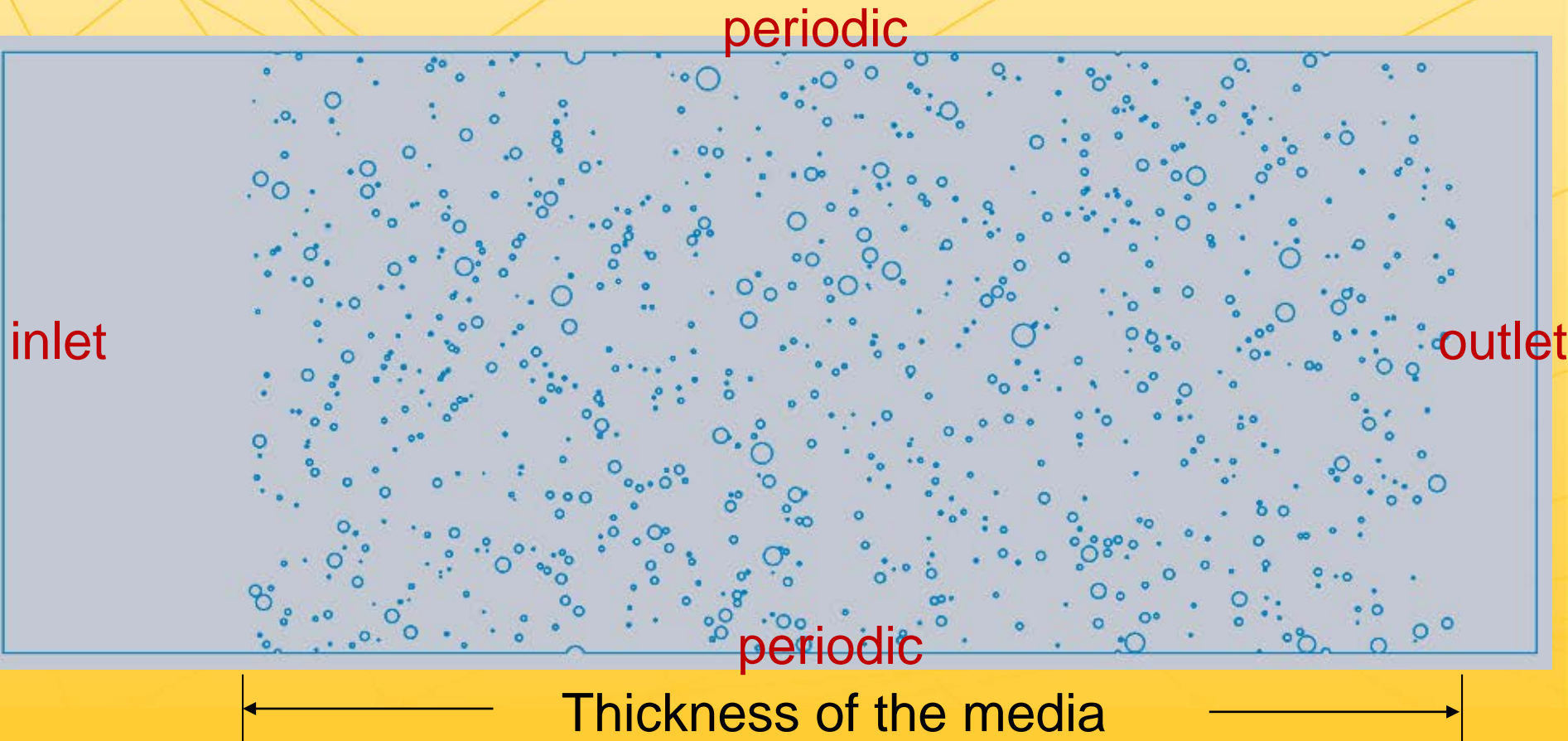
Fiber size distribution  
(Probability density function)



Fiber size distribution  
(Cumulative size distribution)

# Fiber arrangement and boundary condition

- Fibers are randomly generated and arranged in the real size of the filter domain until the **target solidity** is achieved



# Example of fiber distribution for HF-0012

- Three fiber domains below have the same domain size and solidity
- Fibers are randomly generated until target solidity ( $\alpha = 0.04$ ) is achieved

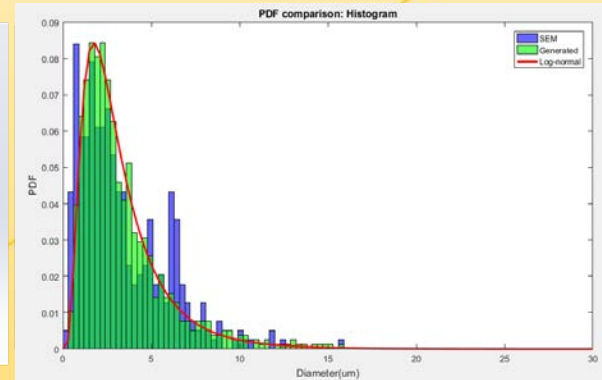
**Case1:** Number of fibers: 782

**Filter A**

$\alpha$ : 4%

Thickness: 740  $\mu\text{m}$

1



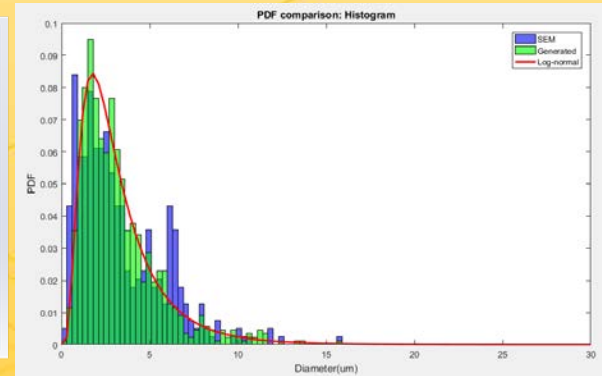
**Case2:** Number of fibers: 874

**Filter A**

$\alpha$ : 4%

Thickness: 740  $\mu\text{m}$

2



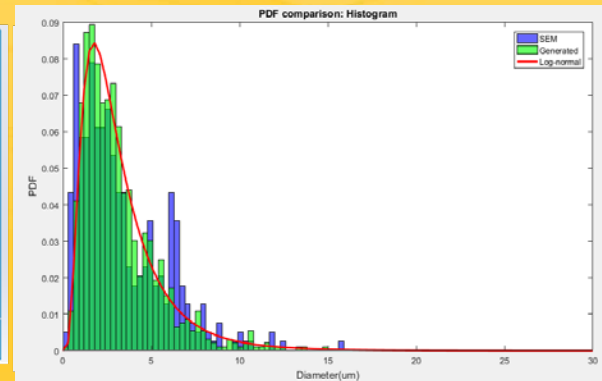
**Case3:** Number of fibers: 929

**Filter A**

$\alpha$ : 4%

Thickness: 740  $\mu\text{m}$

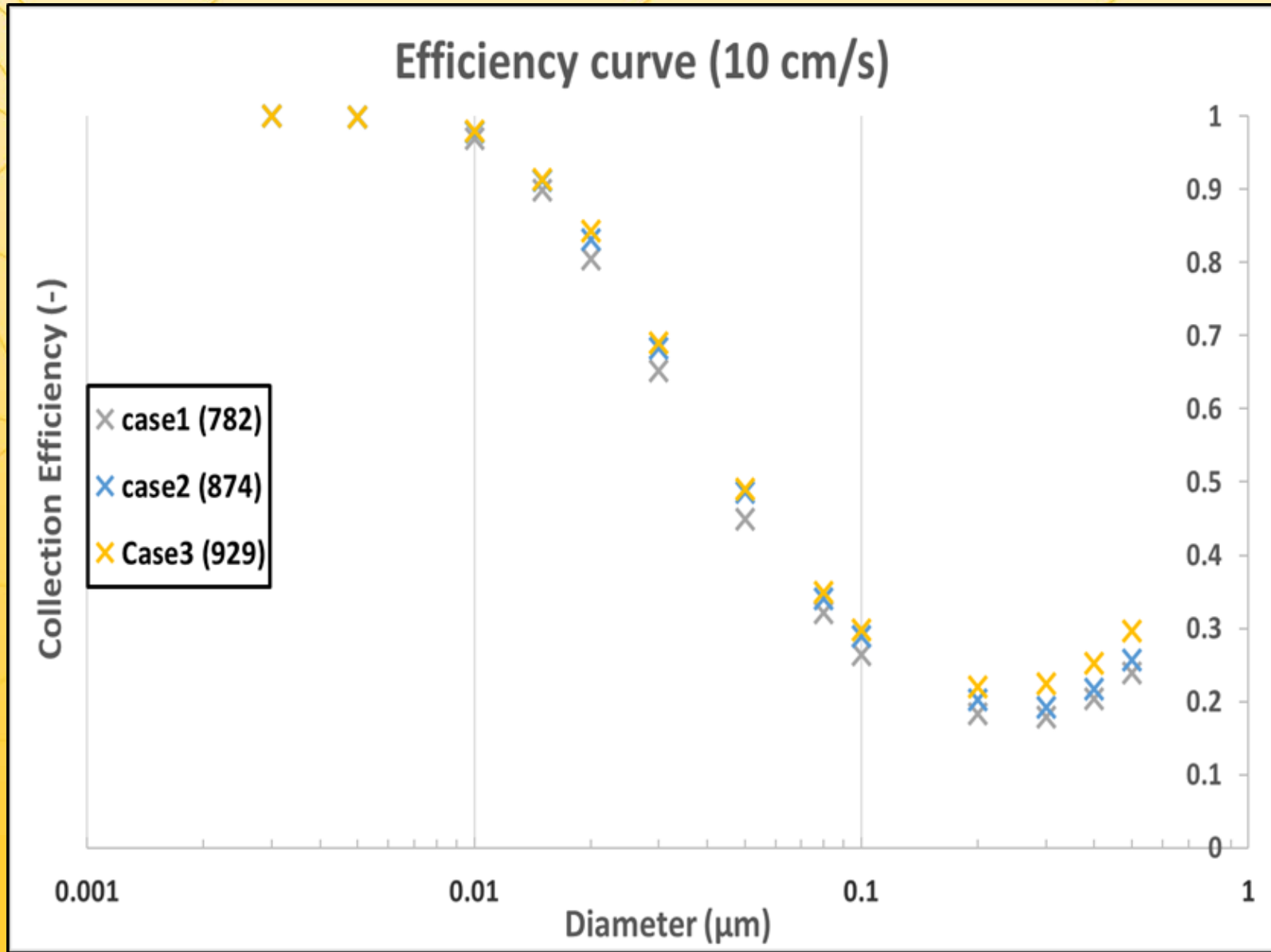
3





# Example of fiber distribution for HF-0012

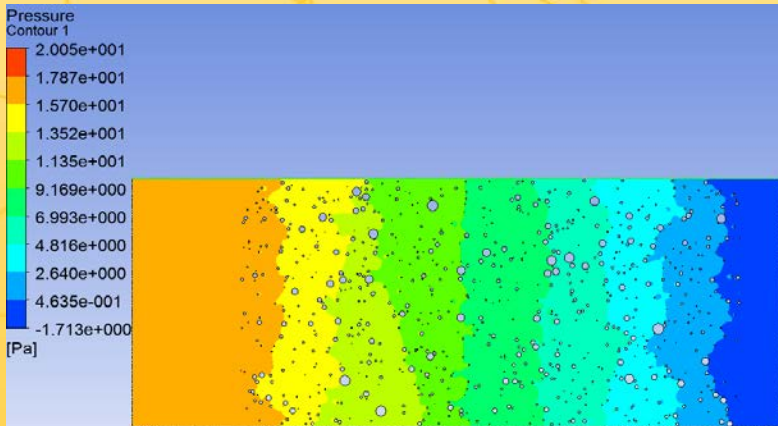
- Three fiber domains below have the same domain size and solidity
- Fibers are randomly generated until target solidity ( $\alpha = 0.04$ ) is achieved



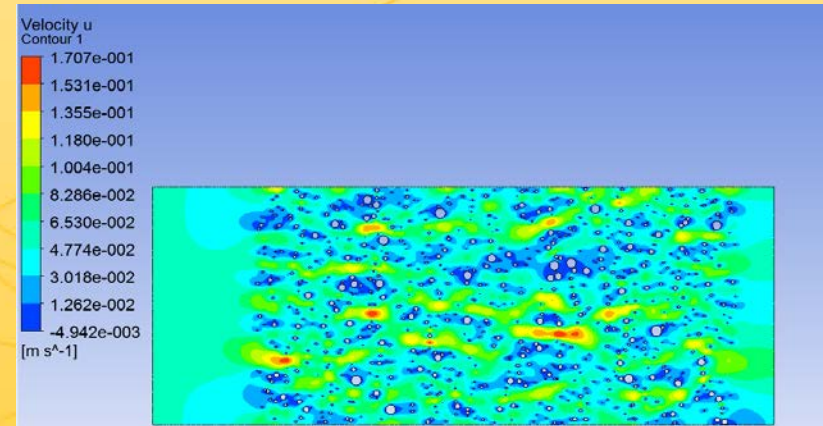
# Flow field calculation: CFD

- Continuity and momentum equation in the Stokes regime were numerically solved (Fluent)

$$\nabla \cdot u = 0$$
$$\nabla p - \mu \nabla^2 u = 0$$



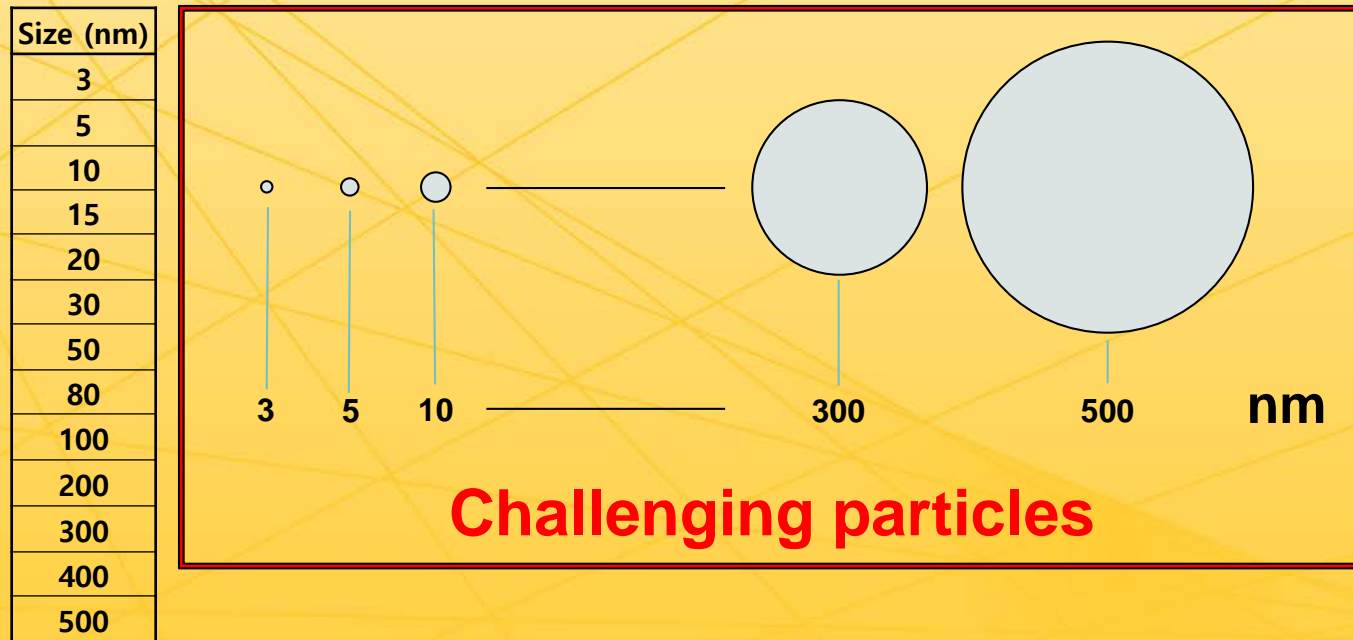
Pressure drop



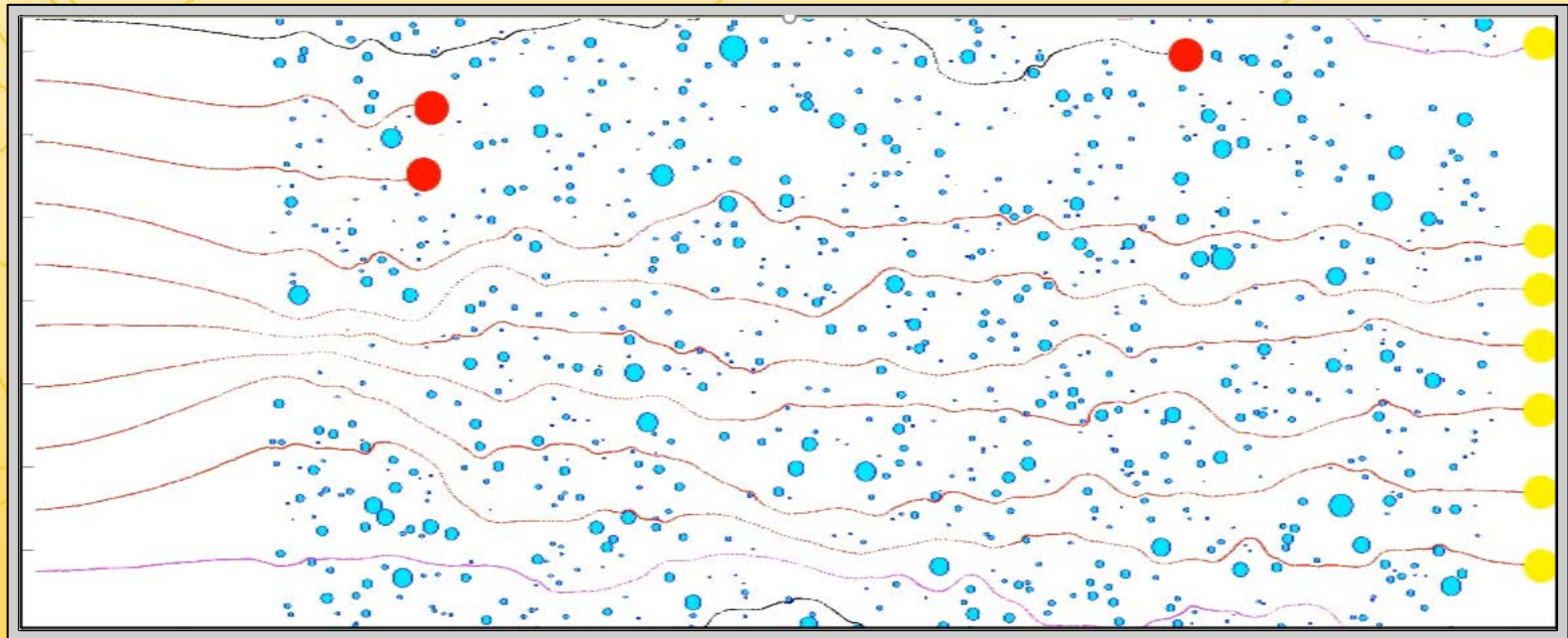
Streamline

# Particle trajectory

- In-house JAVA code is applied to solve the Langevin equation
- Collection efficiency is obtained using the particles from 3 nm up to 500 nm



# Example particle trajectory



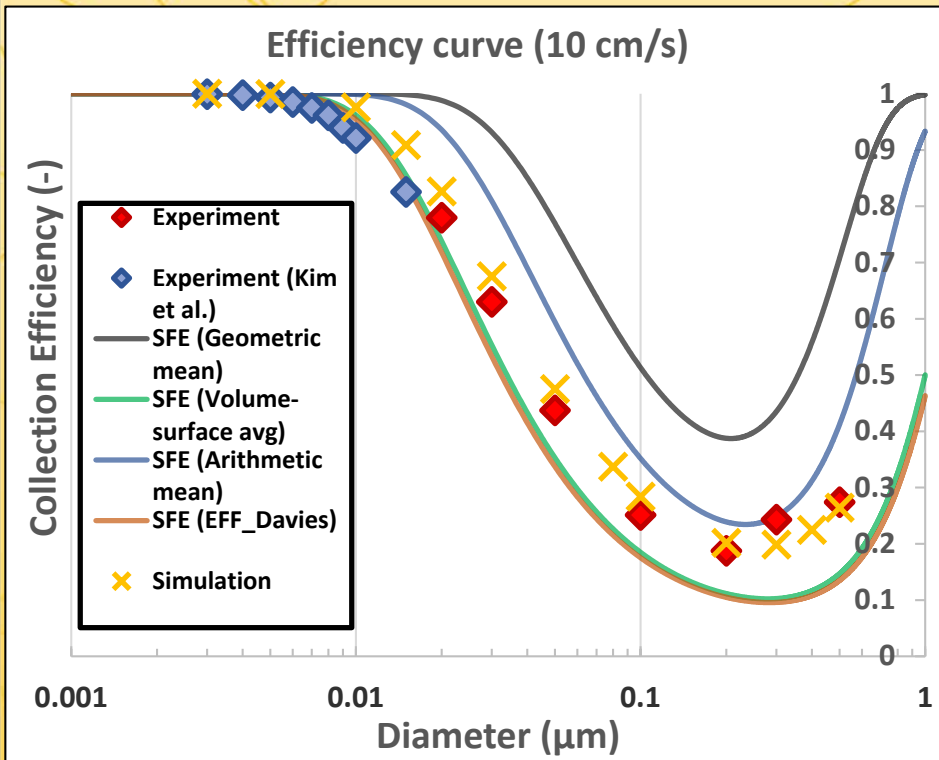
**HF-0012 Domain**  
**Face velocity: 10 cm/s**  
**200nm particles**

**● Captured**  
**● Penetrate**

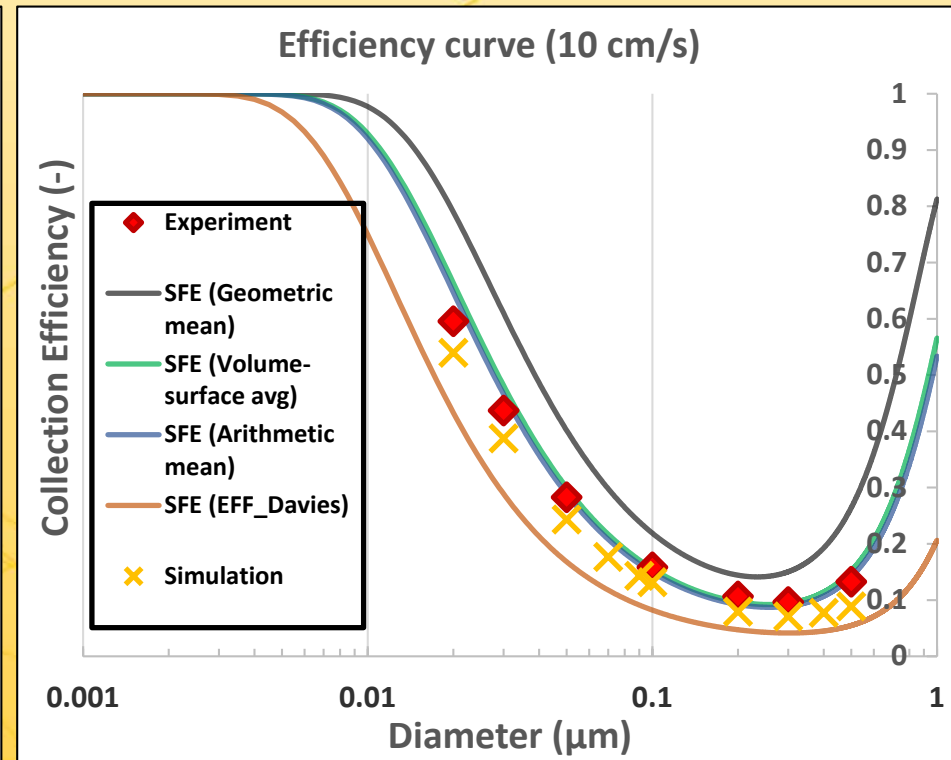


# Filtration efficiency comparison

HF-0012 10cm/s



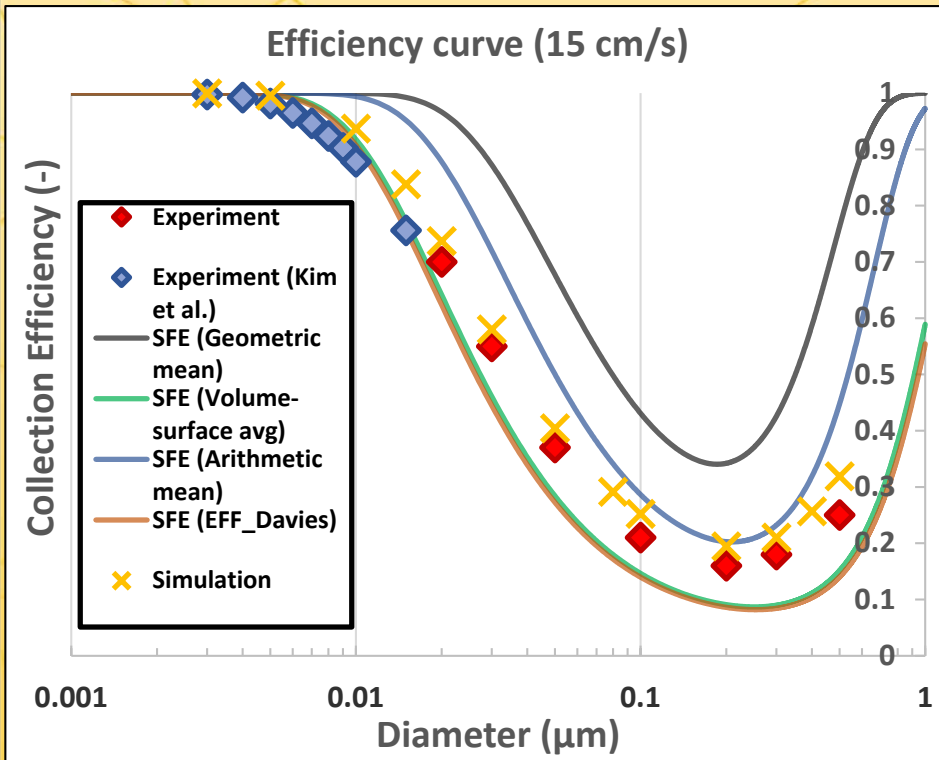
HF-0493 10cm/s



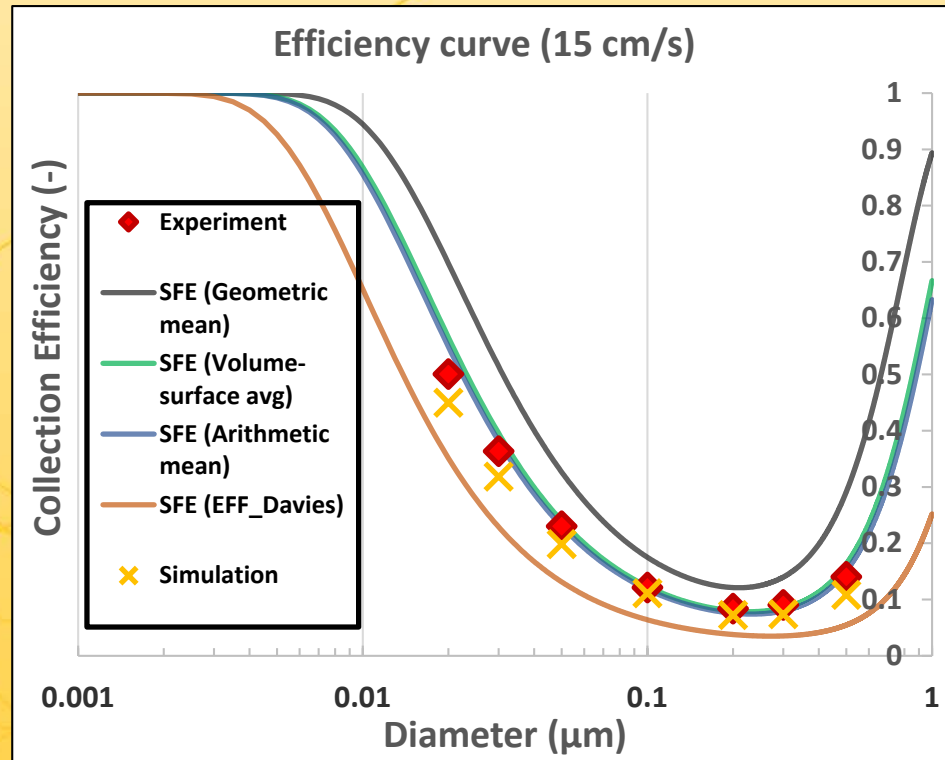
- The relative accuracy of estimated efficiency curve using equivalent fiber diameters varies with the filter media
- Simulation results agree well with experimental data for both filters

# Filtration efficiency comparison

HF-0012 15cm/s



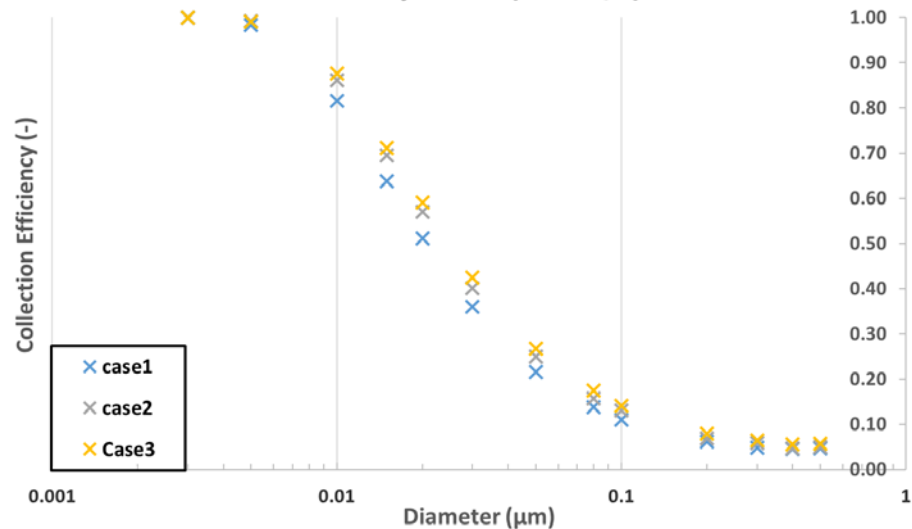
HF-0493 15cm/s



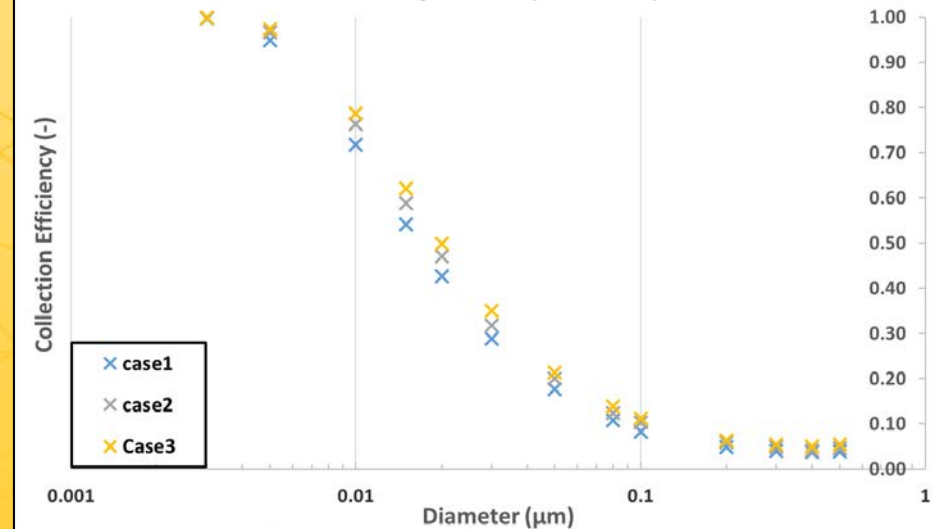
# Effect of polydispersity: same average diameter

case	Thickness (mm)	Solidity (-)	Mean ( $\mu\text{m}$ )	Min ( $\mu\text{m}$ )	Max ( $\mu\text{m}$ )	std ( $\mu\text{m}$ )	# fibers	COV (SD/mean)
1	1	0.1	15	3.5	65.0	7	227	0.5
2			15	7.9	30.7	4	269	0.3
3			15	12.5	18.4	1	283	0.1

Efficiency curve (10 cm/s)



Efficiency curve (15 cm/s)



# **Applications using the developed method will help us better understand/design filters**

- Air/Liquid filtration
- Electret filter
- Particle loading on filter media
- Pleated filter



# Summary

- We have developed a numerical simulation of nanoparticle filtration by fibrous filter media
- Collection efficiencies for two different filters were measured experimentally with challenging particles from 3nm up to 500nm, and the efficiencies agree well with those from simulation
- SFE theory is a good method for estimating the trends of collection efficiency for fibrous filters, but the estimated efficiency can deviate depending on the fiber diameter definitions and SFE equations for each capture mechanism
- The relative accuracy of predicted collection efficiency, using an equivalent fiber diameter can be altered for the filter media
- The degrees of polydispersity affect the collection efficiency