The : and :: Relations

Fonts
$\texttt{X},\texttt{x}$ - Concrete syntax
X,x - Syntax variables
$\mathcal{X},\mathcal{x}$ - Type variables

$$\frac{\texttt{triv x}}{\texttt{x} : \mathcal{Triv}} \tag{1}$$

$$\frac{\texttt{triv x}}{\texttt{x} :: \mathcal{Box}(\mathcal{Triv})} \tag{2}$$

$$\frac{\texttt{int x}}{\texttt{x} : \mathit{Int}} \tag{3}$$

$$\frac{\texttt{int x}}{\texttt{x} :: \mathcal{Box}(\mathit{Int})} \tag{4}$$

$$\frac{\mathsf{x} :: \mathcal{Box}(\mathcal{T})}{\texttt{immut}\ \mathsf{x} :: \mathit{Immut}(\mathcal{Box}(\mathcal{T}))} \tag{5}$$

$$\frac{\mathsf{x} :: \mathcal{T}}{\texttt{ref}\ \mathsf{x} :: \mathcal{Box}(\mathcal{Ref}(\mathcal{T}))} \tag{6}$$

$$\frac{\mathsf{x} :: \mathcal{T}}{\texttt{\&}\ \mathsf{x} : \mathcal{Ref}(\mathcal{T})} \tag{7}$$

$$\frac{\mathsf{x} : \mathcal{Ref}(\mathcal{T})}{\mathsf{x}\ \texttt{@} : \mathcal{T}} \tag{8}$$

$$\frac{\mathsf{x} : \mathcal{Ref}(\mathcal{T})}{\mathsf{x}\ \texttt{@} :: \mathcal{T}} \tag{9}$$

$$\frac{\mathsf{x} :: \mathcal{T} \qquad \mathsf{y} :: \mathcal{U} \qquad \mathcal{T} <: \mathcal{U} \qquad \mathsf{x} : \mathcal{V}}{\mathsf{x}\ \texttt{:=}\ \mathsf{y} : \mathcal{V}} \tag{10}$$

$\texttt{x} : \mathcal{T}$ is read as "expression $\texttt{x}$ is of type $\mathcal{T}$ and is in an *r-context.*"
$\texttt{x} :: \mathcal{T}$ is read as "variable $\texttt{x}$ is of type $\mathcal{T}$ and is in an *l-context.*"
The operators could also be referred to as the "r-type of" and "l-type of" operators.

l-context denotes everything that is *assignable* (indicated as a storable memory). r-context, on the other hand, denotes everything that is *expressible* (can be produced by an expression).

There is no r-value (e.g. expression) of the type $\mathcal{Box}(\mathcal{T})$.

We omit rules for $\mathcal{Con}$ types as they only operate on r-values.

We omit rules for $\mathcal{Fun}$ types as they only accept r-values. Any variable and/or primitive type has both r-value and l-value (when it comes to primitive types, only r-value). In all cases, the r-value part of the actual parameter is passed when the function is being called.

The <: Relation

$$\frac{}{\mathcal{Box}(\mathcal{Triv}) <: \mathcal{Box}(\mathcal{Triv})} \tag{11}$$

$$\frac{}{\mathcal{Box}(Int) <: \mathcal{Box}(Int)} \tag{12}$$

$$\frac{\mathcal{T} <: \mathcal{Box}(\mathcal{U})}{\mathcal{T} <: Immut(\mathcal{Box}(\mathcal{U}))} \tag{13}$$

$$\frac{\mathcal{Box}(\mathcal{T}) <: Immut(\mathcal{U})}{Immut(\mathcal{Box}(\mathcal{T})) <: Immut(\mathcal{U})} \tag{14}$$

$$\frac{\mathcal{T} <: \mathcal{U} \qquad \mathcal{Box}?(\mathcal{T})}{\mathcal{Ref}\ \ \mathcal{T} <: \mathcal{Ref}\ \ \mathcal{U}} \tag{15}$$

$\mathcal{Box}?$ statement checks for both mutable and immutable containers. In other words, $\mathcal{Box}(Int)$ and $Immut(\mathcal{Box}(Int))$ would both satisfy the $\mathcal{Box}?$ condition.