The: and:: Relations

$$\frac{\text{triv } x}{x : \textit{Triv}} \tag{1}$$

 $\frac{\text{Fonts}}{\mathbf{x} :: \mathcal{B}o\chi(\mathit{Triv})} \tag{2}$

 ${\tt X}, {\tt x}$ - Concrete syntax

X, x - Syntax variables

 X,χ - Type variables

$$\frac{\text{int } x}{x : Int} \tag{3}$$

$$\frac{\text{int } \times}{\times :: \mathcal{B}o\chi(Int)} \tag{4}$$

$$\frac{\mathsf{x} :: \mathcal{B} o \chi(\mathcal{T})}{\mathsf{immut} \; \mathsf{x} :: \mathit{I} \mathcal{B} o \chi(\mathcal{T})} \tag{5}$$

$$\frac{\mathsf{x} :: \mathcal{T}}{\mathsf{ref} \mathsf{x} :: \mathcal{B}o\chi(\mathcal{R}ef(\mathcal{T}))} \tag{6}$$

$$\frac{\mathsf{x} :: \mathcal{T}}{\mathsf{\&} \; \mathsf{x} : \mathcal{R}ef(\mathcal{T})} \tag{7}$$

$$\frac{\mathsf{x} : \operatorname{Ref}(\mathcal{B}\mathit{o}\chi(T))}{\mathsf{x} \ \mathsf{0} : T} \tag{8}$$

$$\frac{\mathsf{x}: \mathit{Ref}\left(\mathit{IBox}(\mathcal{T})\right)}{\mathsf{x} \ \mathsf{0}: \mathcal{T}} \tag{9}$$

$$\frac{\mathsf{x}: \mathit{Ref}(\mathcal{T})}{\mathsf{x} \ \mathfrak{0} :: \mathcal{T}} \tag{10}$$

$$\frac{\mathsf{x} :: \mathcal{T} \qquad \mathsf{y} :: \mathcal{U} \qquad \mathcal{T} <:: \mathcal{U} \qquad \mathsf{x} : \mathcal{V}}{\mathsf{x} := \mathsf{y} : \mathcal{V}} \tag{11}$$

 $\mathbf{x}: \mathcal{T}$ is read as "expression \mathbf{x} is of type \mathcal{T} and is in an r-context."

x :: T is read as "variable x is of type T and is in an *l-context*."

The operators could also be referred to as the "r-type of" and "l-type of" operators.

l-context denotes everything that is *assignable* (indicated as a storable memory). r-context, on the other hand, denotes everything that is *expressible* (can be produced by an expression).

There is no r-value (e.g. expression) of the type $\mathcal{B}o\chi(\mathcal{T})$.

For now, we omit rules for Con types as they only operate on r-values.

For now, we omit rules for $\mathcal{F}un$ types as they only accept r-values. Any variable and/or primitive type has both r-value and l-value (when it comes to primitive types, only r-value). In all cases, the r-value part of the actual parameter is passed when the function is being called.

The <:: Relation

$$\overline{\mathcal{B}o\chi(\mathit{Triv}) < :: \mathcal{B}o\chi(\mathit{Triv})} \tag{12}$$

$$\frac{}{\mathcal{B}o\chi(Int) < :: \mathcal{B}o\chi(Int)} \tag{13}$$

$$\frac{T < :: \mathcal{B}o\chi(\mathcal{U})}{T < :: I\mathcal{B}o\chi(\mathcal{U})} \tag{14}$$

$$\frac{\mathcal{B}o\chi(\mathcal{T}) < :: I\mathcal{B}o\chi(\mathcal{U})}{I\mathcal{B}o\chi(\mathcal{T}) < :: I\mathcal{B}o\chi(\mathcal{U})}$$
(15)

$$\frac{\mathcal{T} < :: \mathcal{U}}{\text{Ref } \mathcal{T} < : \text{Ref } \mathcal{U}} \tag{16}$$

 $\mathcal{B}o\chi$? statement checks for both mutable and immutable containers. In other words, $\mathcal{B}o\chi(Int)$ and $Immut(\mathcal{B}o\chi(Int))$ would both satisfy the $\mathcal{B}o\chi$? condition.