The : and :: Relations

$$\frac{\texttt{triv x}}{\texttt{x} : \mathcal{T}riv} \tag{1}$$

$$\frac{\texttt{triv x}}{\texttt{x} :: \mathcal{B}ox(\mathcal{T}riv)} \tag{2}$$

$$\frac{\texttt{int x}}{\texttt{x} : \mathit{Int}} \tag{3}$$

$$\frac{\texttt{int x}}{\texttt{x} :: \mathcal{B}ox(\mathit{Int})} \tag{4}$$

$$\frac{\texttt{x} :: \mathcal{B}ox(\mathcal{T})}{\texttt{immut x} :: \mathit{IBox}(\mathcal{T})} \tag{5}$$

$$\frac{\texttt{x} :: \mathcal{T}}{\texttt{ref x} :: \mathcal{B}ox(\mathcal{R}ef(\mathcal{T}))} \tag{6}$$

$$\frac{\texttt{x} :: \mathcal{T}}{\texttt{\& x} : \mathcal{R}ef(\mathcal{T})} \tag{7}$$

$$\frac{\texttt{x} : \mathcal{R}ef(\mathcal{T})}{\texttt{x @} : \mathcal{T}} \tag{8}$$

$$\frac{\texttt{x} : \mathcal{R}ef(\mathcal{T})}{\texttt{x @} :: \mathcal{T}} \tag{9}$$

$$\frac{\texttt{x} :: \mathcal{T} \qquad \texttt{y} :: \mathcal{U} \qquad \mathcal{T} <:: \mathcal{U} \qquad \texttt{x} : \mathcal{V}}{\texttt{x := y} : \mathcal{V}} \tag{10}$$

Fonts

`X,x` - Concrete syntax
X, x - Syntax variables
$\mathcal{X}, \mathcal{x}$ - Type variables

$\texttt{x} : \mathcal{T}$ is read as "expression `x` is of type $\mathcal{T}$ and is in an *r-context*."
$\texttt{x} :: \mathcal{T}$ is read as "variable `x` is of type $\mathcal{T}$ and is in an *l-context*."
The operators could also be referred to as the "r-type of" and "l-type of" operators.

l-context denotes everything that is *assignable* (indicated as a storable memory). r-context, on the other hand, denotes everything that is *expressible* (can be produced by an expression).

There is no r-value (e.g. expression) of the type $\mathcal{B}ox(\mathcal{T})$.

We omit rules for $\mathcal{C}on$ types as they only operate on r-values.

We omit rules for $\mathcal{F}un$ types as they only accept r-values. Any variable and/or primitive type has both r-value and l-value (when it comes to primitive types, only r-value). In all cases, the r-value part of the actual parameter is passed when the function is being called.

The <:: Relation

$$\frac{}{\mathcal{B}ox(\mathcal{T}riv) <:: \mathcal{B}ox(\mathcal{T}riv)} \tag{11}$$

$$\frac{}{\mathcal{B}ox(\mathit{Int}) <:: \mathcal{B}ox(\mathit{Int})} \tag{12}$$

$$\frac{\mathcal{T} <:: \mathcal{B}ox(\mathcal{U})}{\mathcal{T} <:: \mathit{IB}ox(\mathcal{U})} \tag{13}$$

$$\frac{\mathcal{B}ox(\mathcal{T}) <:: \mathit{IB}ox(\mathcal{U})}{\mathit{IB}ox(\mathcal{T}) <:: \mathit{IB}ox(\mathcal{U})} \tag{14}$$

$$\frac{\mathcal{T} <:: \mathcal{U}}{\mathcal{R}ef\ \mathcal{T} <:: \mathcal{R}ef\ \mathcal{U}} \tag{15}$$

<:: specifies the relationship between two container types.

$\mathcal{B}ox?$ statement checks for both mutable and immutable containers. In other words, $\mathcal{B}ox(\mathit{Int})$ and $\mathit{IB}ox(\mathit{Int})$ would both satisfy the $\mathcal{B}ox?$ condition.