Cosine Similarity and Its Applications in AI

David Oniani
Luther College
oniada01@luther.edu

April 19, 2020

Abstract

Choosing the right metric [1] can be crucial to designing performant artificial intelligence models. Thousands of packages and libraries have been built and written just for providing these metrics. Cosine similarity is one of many metrics used extensively in natural language processing and artificial intelligence tasks. The paper will introduce the technique and discuss its advantages and disadvantages as well as compare it to other approaches. Additionally, sample implementations of the above-mentioned approaches will also be provided.

Table of Contents

1 Introduction

There are a number of approaches for comparing whether two texts are semantically similar to each other. Cosine similarity is one of those methods. In order to understand how cosine similarity works, let us first discuss the modeling of a text document and cosine similarity in general and then proceed by its applications in text document similarity tasks.

1.1 Document Modeling

There are several ways in which a text document can be modeled. This includes a bag of words modeling, where the frequency of a term in a text document represents its weight and therefore, more frequent words are deemed more "important." The whole idea behind a text document modeling is to quantify the textual data into the numeric data (usually, vectors). Once the numeric data is obtained, we can then apply various text semantic similarity techniques in order to compare whether two documents are similar to each other.

1.2 Cosine Similarity

Cosine similarity is a measure of similarity between two non-zero vectors of an inner product space that measures the cosine of the angle between them [2]. If we have two vectors $\vec{a}$ and $\vec{b}$, then the cosine of these two vectors is $\vec{a} \cdot \vec{b}$ which is equal to $\|\vec{a}\| \left\|\vec{b}\right\| \cos\theta$ where θ is the angle between these vectors (Euclidean dot product formula). It can also be represented as the product of two vectors. Therefore if $\vec{a} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$, then $\vec{a} \cdot \vec{b}$ is also equal to $a_1b_1 + a_2b_2 + a_3b_3$.

Some of you might be wondering where the cosine comes from. The answer lies in the law of cosines. A great walkthrough is available on the proof Wikipedia page [3].

1.3 Example

As an example, consider two vectors $\vec{a} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} 5 \\ 12 \end{bmatrix}$. Then the size of a is $\sqrt{1^2 + 2^4} = 5$ and the size of b is $\sqrt{3^2 + 4^2} = 13$. Then $\vec{a} \cdot \vec{b} = 15 + 48 = 63$ and $|\vec{a}| \times |\vec{b}| = 5 * 13 = 65$. Therefore, the cosine of an angle between these vectors is $\dfrac{63}{65}$ which is the similarity measure between these two vectors.

2 Applications

Cosine similarity is one of the most commonly used approaches in text document similarity.

Suppose that we have two documents D_1 and D_2 modeled as term vectors $\vec{t_1}$ and $\vec{t_2}$ respectively. Then the similarity of two documents corresponds to the correlation between the vectors and can be quantified as a cosine of the angle between the vectors. The formula would be

$$SIM(D_1, D_2) = \frac{\vec{t_1} \cdot \vec{t_2}}{|\vec{t_a}| \times |\vec{t_b}|}.$$

As a result, the similarity value is non-negative, bounded by the closed interval $[0, 1]$.

Below find a Python implementation of cosine similarity.

```python
import numpy as np


def cosine_similarity(a: np.array, b: np.array) -> float:
    """Returns the cosine similarity value of two vectors."""

    dot_product: float = np.dot(a, b)
    size_a: float = np.linalg.norm(a)
    size_b: float = np.linalg.norm(b)
    similarity: float = dot_product / (size_a * size_b)

    return similarity


if __name__ == "__main__":
    a: np.array = np.array([1, 0, 1, 0, 1, 1, 1, 1, 1, 1, 1])
    b: np.array = np.array([0, 2, 1, 0, 1, 3, 0, 0, 0, 1, 0])
    print(cosine_similarity(a, b))  # Prints out 0.5
```

Figure 1: Sample Python Implementation of Cosine Similarity.

The program above is fairly simplistic (yet performant) and as shown, it takes two vectors and prints out the cosine similarity.

3 Performance and Comparison to Other Approaches

Performance of cosine similarity can be altered by how the text document is vectorized [4] (e.g., tf-idf vectorization might yield better results than Word2Vec vectorization). This means that the right vectorization strategy could drastically increase the performance, while the poor vectorization will, unsurprisingly, result in a poor performance.

There are many other approaches for finding semantic similarity between the texts. The recent advancements in the domains of artificial intelligence and natural language processing allowed for development of models such as BERT [5], BioBERT [6], and Universal Sentence Encoder (USE) [7] can be used for high-accuracy text semantic similarity score computations. These approaches are thought to, on average, outperform the cosine similarity. That said, there are

4 Current Work

Cosine similarity is very much used in the modern, state-of-the-art papers, such as ones cited in the paper. Its flexibility allows one to apply it in virtually any setting, as long as the documents can be represented as vectors (also known as term vectors). Therefore, cosine similarity is still a trendy tool.

5 Summary

We have introduced cosine similarity and briefly covered its history. We have discussed the importance of cosine similarity in the domains of artificial intelligence and natural language processing and its prevalence in the state-of-the-art work. Sample Python implementation of the algorithm was also provided alongside with small illustrious examples depicting the cosine similarity procedure.

References

[1] Rachel Thomas and David Uminsky. "The Problem with Metrics is a Fundamental Problem for AI". In: (2020). URL: https://arxiv.org/abs/2002.08512.

[2] Wikipedia The Free Encyclopedia. *Cosine similarity*. URL: https://en.wikipedia.org/wiki/Cosine_similarity.

[3] Wikipedia The Free Encyclopedia. *Cosine Formula for Dot Product*. URL: https://proofwiki.org/wiki/Cosine_Formula_for_Dot_Product.

[4] Pinky Sitikhu et al. "A Comparison of Semantic Similarity Methods for Maximum Human Interpretability". In: (2019).

[5] Iulia Turc et al. "Well-Read Students Learn Better: On the Importance of Pre-training Compact Models". In: *arXiv preprint arXiv:1908.08962v2* (2019).

[6] Jinhyuk Lee et al. "BioBERT: a pre-trained biomedical language representation model for biomedical text mining". In: *Bioinformatics* (Sept. 2019). ISSN: 1367-4803. DOI: 10.1093/bioinformatics/btz682. URL: https://doi.org/10.1093/bioinformatics/btz682.

[7] Daniel Cer et al. "Universal Sentence Encoder". In: (2018). eprint: arXiv:1803.11175.