
Real Analysis

Assignment №9

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5.2.3 (a)

$$\boxed{h'(x) = \lim_{x \to c} \frac{h(x) - h(c)}{x - c} = \lim_{x \to c} \frac{\frac{1}{x} - \frac{1}{c}}{x - c} = \lim_{x \to c} -\frac{1}{cx} = -\frac{1}{x^2}} \quad \square$$

(b) Assuming $g(c) \neq 0$, we have:

$$\boxed{\left(\frac{f}{g}\right)'(c) = f'(c)\frac{1}{g(c)} + \left(-\frac{1}{(g(c))^2} g'(c)f(c)\right) = \frac{f'(c)g(c) - g'(c)f(c)}{(g(c))^2}} \quad \square$$

(c) Assuming $g(c) \neq 0$, we have:

$$
\begin{aligned}
\left(\frac{f}{g}\right)'(c) &= \lim_{x \to c} \frac{\left(\frac{f}{g}\right)(x) - \left(\frac{f}{g}\right)(c)}{x - c} \\
&= \lim_{x \to c} \frac{\frac{f(x)}{g(x)} - \frac{f(c)}{g(x)} + \frac{f(c)}{g(x)} - \frac{f(c)}{g(c)}}{x - c} \\
&= \lim_{x \to c} \frac{\frac{f(x)}{g(x)} - \frac{f(c)}{g(c)}}{x - c} \\
&= \lim_{x \to c} \frac{f(x)g(c) - f(c)g(x)}{g(x)g(c)(x - c)} \\
&= \lim_{x \to c} \frac{g(c)\Big(f(x) - f(c)\Big) - f(c)\Big(g(x) - g(c)\Big)}{g(x)g(c)(x - c)} \\
&= \lim_{x \to c} \frac{g(c)}{g(x)g(c)} \times \lim_{x \to c} \frac{f(x) - f(c)}{x - c} - \lim_{x \to c} \frac{f(c)}{g(x)g(c)} \times \lim_{x \to c} \frac{g(x) - g(c)}{x - c} \\
&= \frac{g(c)}{\Big(g(c)\Big)^2} \times f'(c) - \frac{f(c)}{\Big(g(c)\Big)^2} \times g'(c) \\
&= \boxed{\frac{g(c)f'(c) - f(c)g'(c)}{\Big(g(c)\Big)^2}} \quad \square
\end{aligned}
$$

5.2.7 Placeholder

5.3.1 (a) Placeholder

5.3.3

5.3.7

5.3.11 (a) Placeholder