Homework №12

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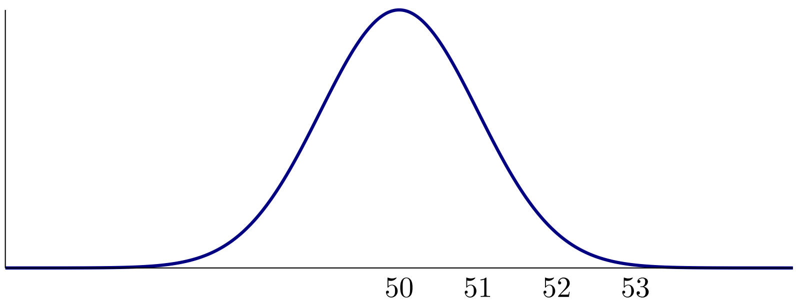
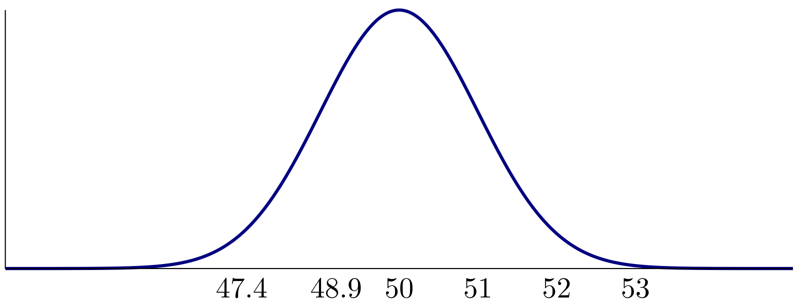
17.1 (a) Since the sample size is 30, it is sufficient to say that the sampling distribution of the sample mean is normal with the sample mean $\overline{x} = \mu = 50$ and the standard deviation $\frac{\sigma}{\sqrt{n}} = \frac{15}{\sqrt{225}} = 1$. Below is the sketch that shows that 50 is the standard deviation, and 51, 52, and 53 are points for 1, 2, and 3 standard deviations respectively.



(b) The result is towards the lower tail of the curve and point 48.9 is close to the center. Since $\mu = 50$, value like 48.9 would not be a good evidence as it is really close to 50. On the other hand, value 47.4 is too low as it is almost within 2 standard deviation from the mean. It therefore provides a better evidence that $\mu < 50$. Below is the sketch.



17.3 $H_0: \ \mu = 50$. $H_a: \ \mu < 50$.
$H_a: \ \mu < 50$ because the teacher think that poor attitudes are partly caused by the decline in scores.

17.11 (a) $z = \dfrac{\overline{x} - \mu}{\sigma/\sqrt{n}} = \dfrac{48.9 - 50}{15/\sqrt{225}} \approx -1.10.$
Now, using Table A, we get that $P(z < -1.10) = 0.1357$. Since $P(z < -1.10) = 0.1357 > 0.05 > 0.01$, it is not statistically significant at either $\alpha = 0.05$ or $\alpha = 0.01$.

(b) $z = \dfrac{\overline{x} - \mu}{\sigma/\sqrt{n}} = \dfrac{47.4 - 50}{15/\sqrt{225}} \approx -2.60.$
Now, using Table A, we get that $P(z < -2.60) = 0.0047$. Since $P(z < -1.10) = 0.0047 < 0.01 < 0.05$, it is statistically significant at both $\alpha = 0.05$ or $\alpha = 0.01$.

(c) Outcomes tell us that $\overline{x} = 47.3$ is the strong evidence against the null hypothesis since the P-value is 0.0047 which is really close to 0.

Outcomes tell us that $\overline{x} = 48.9$ is not the strong evidence against the null hypothesis since the P-value is 0.1357 which is not too close to 0.

17.14 We start with the STATE part.

STATE

$H_0: \ \mu = 10.1$ and $H_a: \ \mu \neq 10.1$

SOLVE

$$\overline{x} = \frac{10.08 + 9.89 + 10.05 + 10.16 + 10.21 + 10.11}{6} \approx 10.0833.$$
$$z = \frac{\overline{x} - \mu}{\sigma/\sqrt{n}} = \frac{10.0833 - 10.1}{0.1/\sqrt{6}} \approx -0.41.$$
$P(z < -0.41 \text{ or } z > 0.41) = 2 \times P(z > 0.41) = 2 \times 0.3409 = 0.6818.$

CONCLUDE

Since $P(z < -0.41 \text{ or } z > 0.41) = 0.6818 > 0.5$, there is not a sufficient evidence that the true conductivity is not 10.1 (aka fail to reject H_0).

17.16 Notice that $z \approx 1.88$. Then $P(z > 1.88) = P(z < -1.88) = 0.0301$.
For $\alpha = 0.05$, $P < \alpha$ and therefore, is statistically significant.
For $\alpha = 0.01$, $P > \alpha$ and therefore, is not statistically significant.

17.17 Notice that $z \approx 1.88$. Then $P(z > 1.88 \text{ or } z < -1.88) = P(z < -1.88) = 2 \times 0.0301 - 0.0602$.
For $\alpha = 0.05$, $P > \alpha$ and therefore, is not statistically significant.
For $\alpha = 0.01$, $P > \alpha$ and therefore, is not statistically significant.

17.30 (a) The null hypothesis says that the population mean is equal to 15. $H_0: \ \mu = 15$.

(b) $z = \dfrac{\overline{x} - \mu}{\sigma/\sqrt{n}} = \dfrac{15.3 - 15}{8.5/\sqrt{463}} \approx 0.76$.

(c) The P-value is $P(z > 0.76) = P(z < -0.76) = 0.2236$. Now, since $P(z > 0.76) = 0.2236 > 0.05$, there is not a strong evidence that the students study more than 15 hours per week on average.

17.33 The P-value is the probability that the test statistic would take a value as or more extreme than observerd, assuming H_0 is true, by chance error alone.

The student is wrong as the P-value is not the probability of null hypothesis being true.

17.44 (a) $z_{\alpha/2} = z_{0.05} = 1.645$.
Then, the confidence interval is $\overline{x} \pm z_{\alpha/2} \times \dfrac{\sigma}{\sqrt{n}}$. Therefore, the confidence interval is from 123.162 to 128.978.

(b) $z = \dfrac{\overline{x} - \mu}{\sigma/\sqrt{n}} = \dfrac{126.07 - 128}{15/\sqrt{72}} \approx -1.09$. $P(z < -1.09 or Z > 1.09) = 2 \times P(z > 1.9) = 2.1379 = 0.2758$. Since $P(z < -1.09 or Z > 1.09) = 0.2758 > 0.10$, it is not statistically significant.

(c) $z = \dfrac{\overline{x} - \mu}{\sigma/\sqrt{n}} = \dfrac{126.07 - 129}{15/\sqrt{72}} \approx -1.66$. $P(z < -1.66 \text{ or } Z > 1.66) = 2 \times P(z > 1.66) = 2.0485 = 0.0970$. Since $P(z < -1.66 \text{ or } Z > 1.66) = 0.0970 < 0.10$, it is statistically significant.