Getting Started with LaTeX

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1. Render the following text in LaTeX as exactly as you can:

   The quadratic formula asserts that if $a, b, c \in \mathbb{N}$ and $a \neq 0$ then the roots of the polynomial $ax^2 + bx + c$ are given by

   $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

   The case of the cubic $ax^3 + bx^2 + cx + d$ is more complicated, and one must first compute $\Delta_0 = b^2 - 3ac, \Delta_1 = 2b^3 - 9abc + 27ad^2$ and

   $$C = \sqrt[3]{\frac{\Delta_1 \pm \sqrt{\Delta_1^2 - 4\Delta_0^3}}{2}}.$$

   In this case, one finds that one root of the cubic is given by

   $$x = \frac{1}{3a}(b + C + \frac{\Delta_0}{C}).$$

   A similar formula exists for the roots of the quartic $ax^4 + bx^3 + cx^2 + dx + e$, however it is even more complicated. Remarkably, Abel and Ruffini proved in 1824 that no similar formula exists for the roots of the quintic $ax^5 + bx^4 + cx^3 + dx^2 + ex + f$, or indeed for any polynomial of degree ≥ 5.

2. How convinced are you by the evidence and argument presented that there are more odd numbered houses than even numbered houses? Which do you find more convincing, the argument or the numerical evidence? [Your answer here should probably be about 4 sentences.]

   I think both proofs were reasonable. I would say that the argument was stronger here. This is not because the bell curve analysis was irrelevant, but rather because tbe argument was a lot simpler (subjective opinion) to explain in human words without using math, statistics, bell curves or deviations. Thus, simpler is better and I go with the argument here. I was pretty convinced as I thought of the same as soon as James said that odd-numbered houses were a bit more than the even-numbered ones.

3. Suppose you looked at all numbers from 1 to 1000000. What percentage of them do you think start with an odd number?

It will be 55.5556%. To see why it's true, let's jot down the list of odd-digit-starting numbers for different ranges.

$$\begin{gathered} 5 \text{ for numbers in the range } 1-9 \\ 50 \text{ for numbers in the range } 10-99 \\ 500 \text{ for numbers in the range } 100-999 \\ 5000 \text{ for numbers in the range } 1000-9999 \\ 50000 \text{ for numbers in the range } 10000-99999 \\ 500000 \text{ for numbers in the range } 100000-999999 \\ 1 \text{ for } 1000000 \end{gathered}$$

We get, $5 + 50 + 500 + 5000 + 50000 + 500000 + 1 = 555556$ and finally, $555556/1000000 \times 100\% = 55.5556\%$.

Here is a program to confirm our proof. It is written in **Haskell**, a purely functional programming language which is based upon **Lambda calculus** and is used by a lot of mathematicians worldwide.

```haskell
-- Importing digitToInt function from Data.Char module
import Data.Char(digitToInt)


-- | Function to check the oddity of the first digit
isOddFirst :: Int -> Bool
isOddFirst x
    | firstDigit `mod` 2 == 1 = True
    | otherwise = False
    where
        firstDigit = digitToInt $ head $ show x

-- Function to count the numbers that start with an odd digit
oddStartingNums :: [Int] -> Int
oddStartingNums xs = length $ filter isOddFirst xs


main = do
    let bigInt = oddStartingNums [1..1000000] :: Int
    let pctg = show (fromIntegral bigInt * 100 / 1000000) ++ "%"

    print bigInt -- 555556
    putStr pctg -- 55.5556% (indeed, it's 55.5556% !!)
```

4. Various theorems and laws. Apart from that, I learned that something that seems true might not be true, vice versa.

5. Conjecture 17 (otherwise known as the examples of Moessner's process). I found it interesting how these square numbers were generated. It should also be noted that the same idea (with a bit of modification) could generate cubic and quartic power numbers and even factorials!

   Here is the explanation for the conjecture.

Conjecture 17 states: Write down the positive integers, delete every second, and form the partial sums of those remaining.

Let's list the natural numbers up to 20 (we could continue indefinitely).

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

Now, let's cross out every second number. We get:

1 ~~2~~ 3 ~~4~~ 5 ~~6~~ 7 ~~8~~ 9 ~~10~~ 11 ~~12~~ 13 ~~14~~ 15 ~~16~~ 17 ~~18~~ 19 ~~20~~

If we list the partial sums, we get:

1 4 9 16 25 36 49 64 81

Here is why it happens:

It's clear that numbers 1, 3, 5, 7 ... etc. form the arithmetic progression where the common difference is 2. Then we can write the following:

$$\sum_{i=1}^{n} 2n - 1 = \frac{1 + 2n - 1}{2} \times n = n \times n = n^2$$

Thus, this proves that all the partial sums are the squares.

A Page for Feedback