Homework №11

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87. To prove that a relation is an equivalence relation, we must show that the relation is reflexive, symmetric, and transitive.

I. Showing that $\sim$ is reflexive.

$x \sim x = x + 2x = 3x$ Thus, $\sim$ is reflexive.

II. Showing that $\sim$ is symmetric.

Suppose that $x \sim y$, then $x + 2y = 3k$ where $k \in \mathbb{Z}$. Then, if we solve the equation for x, we get $x = 3k - 2y$. Now, consider $y + 2x$. Let's substitute x with $3k - 2y$. We get, $y + 6k - 4y = 6k - 3y = 3 \times (2k - y)$. Hence, if $x + 2y$ is divisible by 3, then $y + 2x$ is also divisible by 3 and the relation is symmetric.

III. Showing that $\sim$ is transitive.

Suppose that $x \sim y$ and $y \sim z$. Then $x + 2y = 3k$ where $k \in \mathbb{Z}$ and $y + 2z = 3l$ where $l \in \mathbb{Z}$. Consider the relation on the variables x and z. The relation is $x \sim z = x + 2z$. Now, from the first equation, let's substitute x and from the second one, substitute z. We get that $x = 3k - 2y$ and $z = \dfrac{3l - y}{2}$. Finally, we get: $x + 2z = 3k - 2y + 2 \times \dfrac{3l - y}{2} = 3k - 2y + 3l - y = 3k + 3l - 3y = 3 \times (k + l - 1)$ and $3 \times (k + l - 1)$ is clearly a multiple of 3. Hence, we got that $\sim$ is transitive.

Now, we proved that the relation $\sim$ is reflexive, symmetric, and transitive and thus, the relation $\sim$ is the equivalence relation. $\square$

88. (a) $\Xi(S)$ is a relation on $\mathcal{P}(S)$ such that it takes the powerset of S

(b) It is not symmetric. Let $A =$

(c)

(d)

Bookwork

4.2

2. $\in$, $\notin$, $\subset$, $=$, $\mathcal{P}$

4. (a) It is not. Consider tuples (a, b) and (b, a). For the relation R to be transitive, since $(a, b) \in R$ and $(b, a) \in R$, it must be the case that $(a, a) \in R$. However, $(a, a) \notin R$. Thus, the relation is not transitive. $\square$

   (b) i. It is. Suppose that $x - y = q_1$ and $y - z = q_2$ where $q_1, q_2 \in \mathbb{Q}$. Let's then sum those two equations up, and we get $x - y + y - z = q_1 + q_2$ and finally $x - z = q_1 + q_2$. Hence, we got that $x - z$ is a sum of two rational numbers and thus is rational itself. Therefore, the relation R is transitive. $\square$

   ii. It is not. Consider $x = \sqrt{2}$, $y = 1$, and $z = \sqrt{2} + 1$. Then $x - y = \sqrt{2} - 1$ thus is irrational and $y - z = 1 - (\sqrt{2} + 1) = -\sqrt{2}$ hence, is also irrational. However, $x - z = \sqrt{2} - (\sqrt{2} + 1) = \sqrt{2} - \sqrt{2} - 1 = -1$ which is rational. Therefore, the relation R is not transitive. $\square$

   iii. It is not. Consider $x = 1$, $y = 2$, and $z = 4$. Then $|x - y| = 1$ and $|y - z| = 2$. However, $|x - z| = 3 > 2$. Hence, the relation R is not transitive. $\square$

12. (a) It is reflexive. Suppose that $x \in R \cap S$. Then $x \in R$ and $x \in S$. Since R and S are reflexive, $(x, x) \in R$ and $(x, x) \in S$. Therefore, $(x, x) \in R \cap S$. $\square$

   (b) It is reflexive. Suppose that $x \in R \cup S$. Then, without a loss of generality, let $x \in S$. Now, since S is reflexive, $(x, x) \in S$ and thus, $(x, x) \in R \cup S$. $\square$

   (e) It is transitive. Suppose that $(x, y), (y, z) \in R \cap S$. Then $(x, y), (y, z) \in R$ and $(x, y), (y, z) \in S$. Since R, S are transitive, $(x, z) \in R$ and $(x, z) \in S$. Finally, $(x, z) \in R \cap S$ and $R \cap S$ is transitive. $\square$

   (f) It is transitive. Suppose that $(x, y), (y, z) \in R \cup S$. Then, without a loss of generality, $(x, y), (y, z) \in R$. Since R is transitive, $(x, z) \in R$ and $(x, z) \in R \cup S$. Therefore, $R \cup S$ is transitive. $\square$

4.4

1. (a) Yes, it is an equivalence relation since it satisfies all the criteria: reflexive, transitive, symmetric.

   1. It is reflexive, since $(a, a),\ (b, b),\ (c, c) \in R$.

   2. It is transitive. $(a, a), (a, c) \in R$ and $(a, c) \in R$; $(c, a), (a, a) \in R$ and $(c, a) \in R$; $(c, c), (c, a) \in R$ and $(c, a) \in R$; $(a, c), (c, c) \in R$ and $(a, c) \in R$.

   3. It is symmetric. $(a, a) \in R$ and $(a, a) \in R$; $(b, b) \in R$ and $(b, b) \in R$; $(c, c) \in R$ and $(c, c) \in R$; $(a, c) \in R$ and $(c, a) \in R$; $(c, a) \in R$ and $(a, c) \in R$.

   (b) No, it is not since $(b, a), (a, c) \in R$ but $(b, c) \notin R$ (it is not transitive).

3. It is not. For a relation to be the equivalence relation, it must be reflexive, transitive, symmetric. It is not transitive since if we have three lines x, y, z in the euclidean space and if $x \perp y$ and $y \perp z$, then $y \not\perp z$ (because $y \parallel z$). As a side note, it is not reflexive either since the line cannot be perpendicular to itself.

9 (a)