Real Analysis

Assignment №3

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2.4.1 (a) Let us first show that (x_n) is monotonically decreasing. We can use induction for this proof.

Base case: $x_1 = 3$ and $x_2 = \dfrac{1}{4 - x_1} = \dfrac{1}{1} = 1$. It follows that $x_2 - x_1 = 1 - 3 = -2 < 0$. Hence, the base case is satisfied.

Inductive step: suppose $x_n - x_{n+1} > 0$. We now have to show that $x_{n+1} - x_{n+2} > 0$.

$$\begin{aligned} x_{n+1} - x_{n+2} &= \frac{1}{4 - x_n} - \frac{1}{4 - x_{n+1}} \\ &= \frac{x_n - x_{n+1}}{(4 - x_n)(4 - x_{n+1})} > 0 \end{aligned}$$

Thus, by assuming that $x_n - x_{n+1} > 0$, we got that $x_{n+1} - x_{n+2} > 0$ as well. Hence, $\forall n \in \mathbb{N}, x_n > x_{n+1}$. Additionally, (x_n) is a bounded sequence since $\forall n \in \mathbb{N}, 0 < x_n < 5$. Finally, by **Monotone Convergence Theorem**, we get that (x_n) converges.

□

(b) As $\lim x_n$ exists, let $\lim x_n = X$. Then $\forall \epsilon > 0, \exists N \in \mathbb{N}$ s.t. if $n \geq N, |x_n - X| < \epsilon$. Now, since $n + 1 > n \geq N$, we get that $|x_{n+1} - X| < \epsilon$ and hence, $\lim x_{n+1} = X$.

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2.5.1 klasjdlkasks

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