Homework №3

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3.4 (a) C is the mean and B is the median (since the distribution is right-skewed).

(b) B is both the mean and the median (since the distribution is symmetric).

(c) A is the mean and B is the median (since the distribution is left-skewed).

3.6 (a) The area for 99.7% corresponds to the three standard deviations and therefore, the range for lengths that cover almost all (99.7%) of this distribution is from $35.8 - 3 \times 2.1$ to $35.8 + 3 \times 2.1$. That is the range from 29.5 to 42.1.

(b) Notice that $33.7 = 35.8 - 2.1$. Therefore, the datapoint is located one deviation to the left from the center. Hence, we got that $\frac{32}{2}\% = 16\%$ of women over 20 have the arm length less than 33.7cm.

3.7 (a) According to the $68 - 95 - 99.7$ rule, it will be between $852 - 2 \times 82$ and $852 + 2 \times 82$. That is, between 688 and 1016.

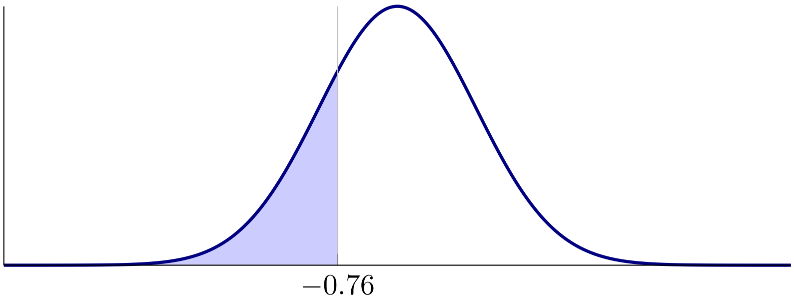
(b) According to the $68 - 95 - 99.7$ rule, it will be $852 - 2 \times 82 = 688$ (this is since 95% leaves us with 2.5% on both sides and we need the left one).

3.8 $z_{\text{Idonna}} = \dfrac{x - \mu}{\sigma} = \dfrac{670 - 514}{118} = 1.32$

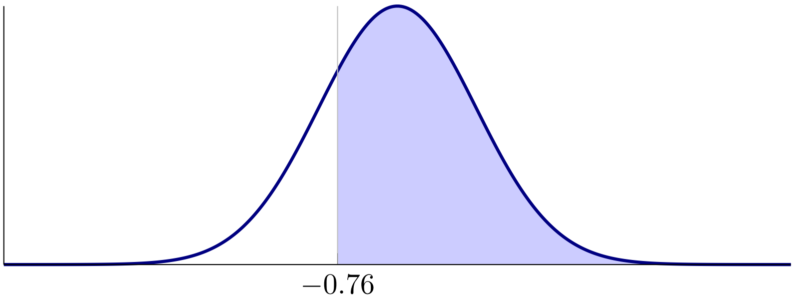
$z_{\text{Jonathan}} = \dfrac{x - \mu}{\sigma} = \dfrac{26 - 20.9}{5.3} = 0.96$

Since $z_{\text{Idonna}} > z_{\text{Jonathan}}(1.32 > 0.96)$, it appears that Idonna did better.

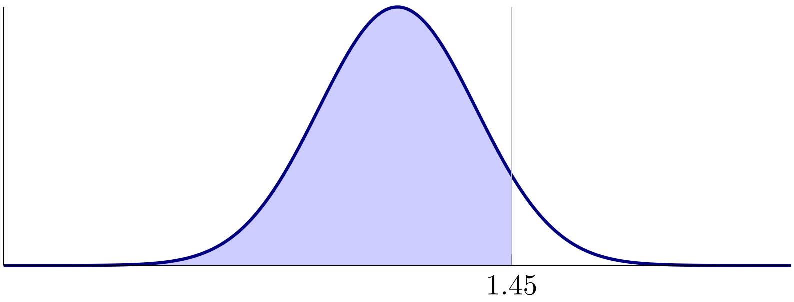
3.10 (a) From the Table A, we get that the value for for $z = -0.76$ is 0.2236. This means that 22.36% area will be covered to the left of the point -0.76.



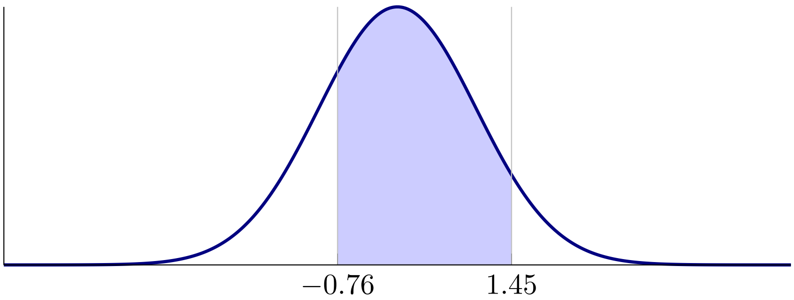
(b) From the Table A, we get that the value for for $z = -0.76$ is 0.2236. This means that 22.36% area will be covered to the left of the point -0.76 leaving $100\% - 22.36\% = 77.64\%$ to the right.



(c) From the Table A, we get that the value for for $z = 1.45$ is 0.9265. This means that 92.65% area will be covered to the left of the point 1.45.



(d) From the Table A, we get that the value for for $z = -0.76$ is 0.2236 and for $z = 1.45$ is 0.9265. Therefore, we shade the region in-between -0.76 and 1.45 and get the total area in percentages equal to $(0.9265 - 0.2236) \times 100\% = 70.30\%$.



3.11 (a) $z = \dfrac{x - \mu}{\sigma} = \dfrac{697 - 852}{82} = -1.89$. From the Table A, we have that the standard normal cumulative proportion is 0.0294 which in percentages, is $0.0294 \times 100\% = 2.94\%$. Therefore, we have that in 2.94% of all years, India will have 697 mm or less monsoon rain.

(b) z for 683 is $\dfrac{683 - 852}{82} = -2.06$.

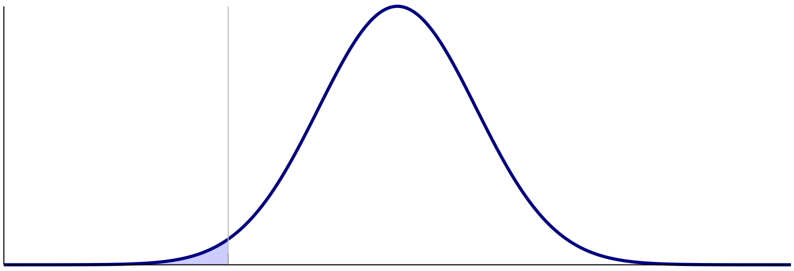
z for 1022 is $\dfrac{1022 - 852}{82} = 2.07$

We now look at the Table A and find that for $z = -2.06$, the standard normal cumulative proportion value is 0.0197 and for $z = 2.07$, 0.9808. Finally, we get that $(0.9808 - 0.0197) \times 100\% = 96.11\%$ of all years, the rainfall is "normall," or between 683mm and 1022mm.

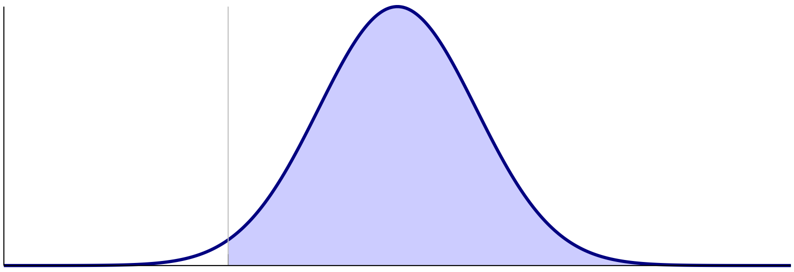
3.14 (a) Since the distribution is normal, its mean and median are equal. Therefore, $M = \overline{x} = 25.3$. To calculate the IQR, we need Q_1 and Q_3. To calculate Q_1, we look at the Table A to find the value closest to 0.25. It appears to be 0.2514 which corresponds to $z = -0.67$. For Q_3, we find the value closest to 0.75 which turns out to be $z = 0.67$ (it makes sense since these two points must match if we fold the graph across the $y-$axis). Finally, we have that $Q_1 = -0.67\sigma = -0.67 \times 6.5 = -4.355$, $Q_2 = 0.67\sigma = 0.67 \times 6.5 = 4.355$, and $\text{IQR} = 0.67\sigma - (-0.67\sigma) = 1.34\sigma = 1.34 \times 6.5 = 8.71$

(b) We will first need to find a values close to 0.10. It appears to be 0.1003 for $z = -1.28$. Since the normal distribution is <u>perfectly</u> symmetric, value close to 0.90 will be $z = 1.28$ (one can verify this by looking up the value in the Table A). Therefore, we have that the interval is $(-1.28, 1.28)$.

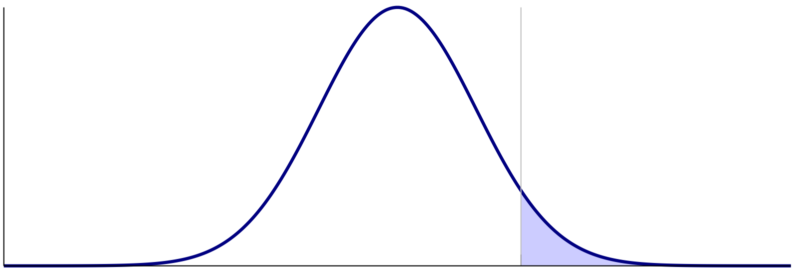
3.28 (a) From the Table A, we get that the value for for $z = -2.15$ is 0.0158. This means that 1.58% area will be covered to the left of the point -2.15.



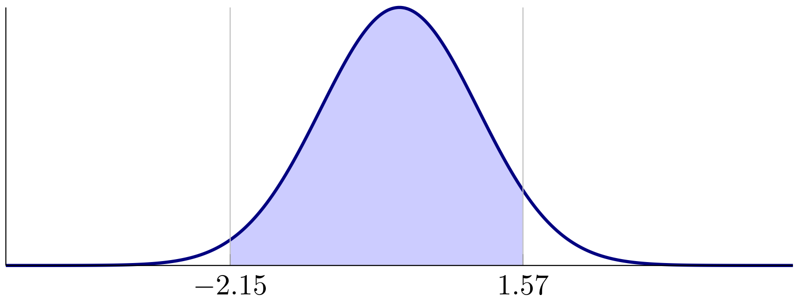
(b) From the Table A, we get that the value for for $z = -2.15$ is 0.0158. This means that 1.58% area will be covered to the left of the point -2.15 leaving $100\% - 1.58\% = 98.42\%$ to the right.



(c) From the Table A, we get that the value for for $z = 1.57$ is 0.9418. This means that 94.18% area will be covered to the left of the point 1.45 leaving $100 - 94.18 = 5.82\%$ to the left.



(d) From the Table A, we get that the value for for $z = -2.15$ is 0.0158 and for $z = 1.57$ is 0.9418. Therefore, we shade the region in-between -2.15 and 1.57 and get the total area in percentages equal to $(0.9418 - 0.0158) \times 100\% = 92.6\%$.



3.30 (a) First, let's find the z-value. We have, $z = \dfrac{0.6 - 0.8}{0.078} \approx -2.56$. From the Table A, we get that this value corresponding to $z = -2.56$ is 0.0052. Then since this value shows us proportion to the left of the normal distribution, we got that 0.0052 is the proportion of flies that have thorax length less than 0.6mm.

(b) First, let's find the z-value. We have, $z = \dfrac{0.9 - 0.8}{0.078} \approx 1.28$. From the Table A, we get that this value corresponding to $z = 1.28$ is 0.8997. Then since this value shows us proportion to the left of the normal distribution, we got that $1 - 0.8997 = 0.1003$ is the proportion of flies that have thorax length greater than 0.9mm.

(c) The z-value for 0.6 is 2.56 and for 0.9 is 1.28. From the Table A, we get that the proportion for $z = 0.6$ is 0.0052 and for $z = 0.9$ is 0.8997. The proportion in-between is $0.8997 - 0.0052 = 0.8945$.