Topology

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Assignment №3

Section 18

2. Suppose that $f : X \to Y$ is continuous. If x is a limit point of of the subset A of X, is it necessarily true that $f(x)$ is a limit point of $f(A)$?

   It is not. Consider the constant continuous function $f : \mathbb{R} \to \mathbb{R} : x \mapsto 0$. Then 0 is the limit point of A, however, $f(0) = 0$ is not a limit point of $f(A) = \{0\}$ since there is no neighborhood of 0 that intersects $\{0\}$ at point other than 0.

5. Show that the subspace (a, b) of $\mathbb{R}$ is homeomorphic with $(0, 1)$ and the subspace $[a, b]$ of $\mathbb{R}$ is homeomorphic with $[0, 1]$.

   Recall that a homeomorphism is a bijective and continuous function whose inverse is also continuous. Therefore, all we have to do here is to find bijective and continuous function(s) which would map (a, b) to $(0, 1)$ in the first case and $[a, b]$ to $[0, 1]$ in the second case.

   Let's first show that the subspace (a, b) of $\mathbb{R}$ is homeomorphic with $(0, 1)$.
   Consider the function $f : (a, b) \to (0, 1) : x \mapsto \dfrac{x - a}{b - a}$. Then notice that it is both injective and surjective hence is a bijection. Besides, it is also a continuous function (it can be verified using epsilon-delta definition of continuity). The inverse of f is a function $f^{-1} : (0, 1) \to x \mapsto (a, b) : (b-a)x+a$ which is obviously bijective and also continuous (once again, can be verified using epsilon-delta definition of continuity). Finally, we have that the subspace (a, b) of $\mathbb{R}$ is homeomorphic with $(0, 1)$. $\square$

   Now, let's show that the subspace $[a, b]$ of $\mathbb{R}$ is homeomorphic with $[0, 1]$. Let's take the exact same function f but let's reconstruct it in the way that it maps

$[a, b]$ to $[0, 1]$. We have, $f : [a, b] \to [0, 1] : x \mapsto \dfrac{x - a}{b - a}$. Once again, this is a continuous bijective function whose inverse is also continuous and therefore the subspace $[a, b]$ of $\mathbb{R}$ is homeomorphic with $[0, 1]$. $\square$

Section 19

3. Prove theorem 19.3.

<u>Theorem 19.3</u>

"Let A_α be a subspace of X_α, for each $\alpha \in J$. Then $\prod A_\alpha$ is a subspace of $\prod X_\alpha$ if both products are given the box topology, or if both products are given the product topology."

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