Math 317 - Chapter 14 Homework 1

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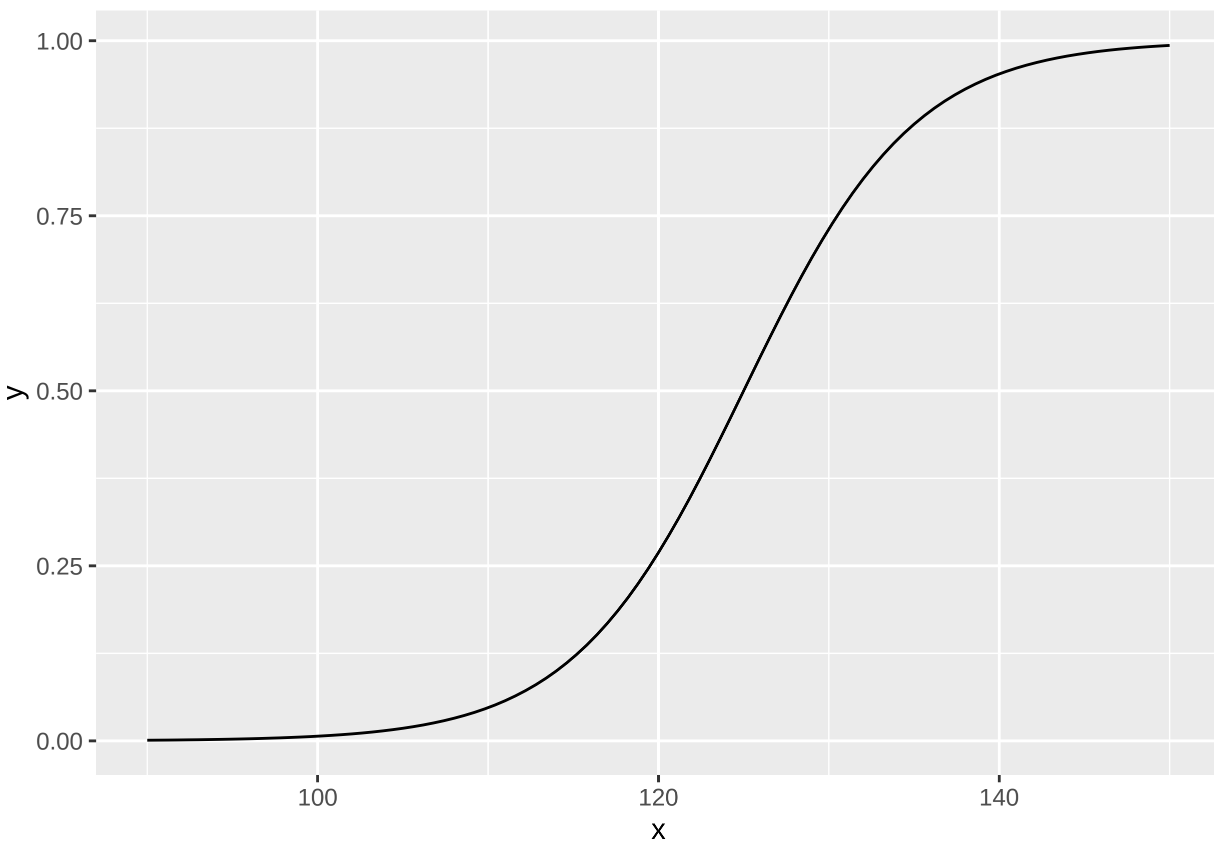
Plot the logistic mean function, $E\{Y\} = exp(\beta_0 + \beta_1 X_1)/(1 + exp(\beta_0 + \beta_1 X_1))$, when $\beta_0 = -25$ and $\beta_1 = 0.2$.

```r
library(ggplot2)

X = seq (90, 150)

sigmoid = function(x) {
  exp(-25 + 0.2 * x) / (1 + exp(-25 + 0.2 * x))
}

ggplot(data.frame(x=X), aes(x=x)) +
      stat_function(fun=sigmoid, geom="line") +
      xlab("x") + ylab("y")
```



What is the predicted proabability for X=115?

```r
print(sigmoid(115))
```

```
## [1] 0.1192029
```

For what value of X is the mean response equal to 0.5? (Solve for X algebraically, then calculate in R)

```r
# The formula is $X = (ln(P / (1 - P)) - B_0) / B_1$
print((log(0.5 / (1 - 0.5)) - (-25)) / 0.2)
```

```
## [1] 125
```

Find the odds when $X = 130$, when $X = 131$, and the odds ratio for X=131 vs. X=130. Verify that this odds ratio is equal to $exp(\beta_1)$.

```r
# The odds formula is $p / (1 - p)$
odds = function(x) {
  x / (1 - x)
}

odds130 = odds(sigmoid(130))
odds131 = odds(sigmoid(131))

oddsRatio = odds131 / odds130

print(oddsRatio)
```

```
## [1] 1.221403
```

```r
print(exp(0.2))
```

```
## [1] 1.221403
```

```r
# Odds ratio and beta_1 are equal (at least within 12 decimal points)
print(round(oddsRatio, 12) == round(exp(0.2), 12))
```

```
## [1] TRUE
```

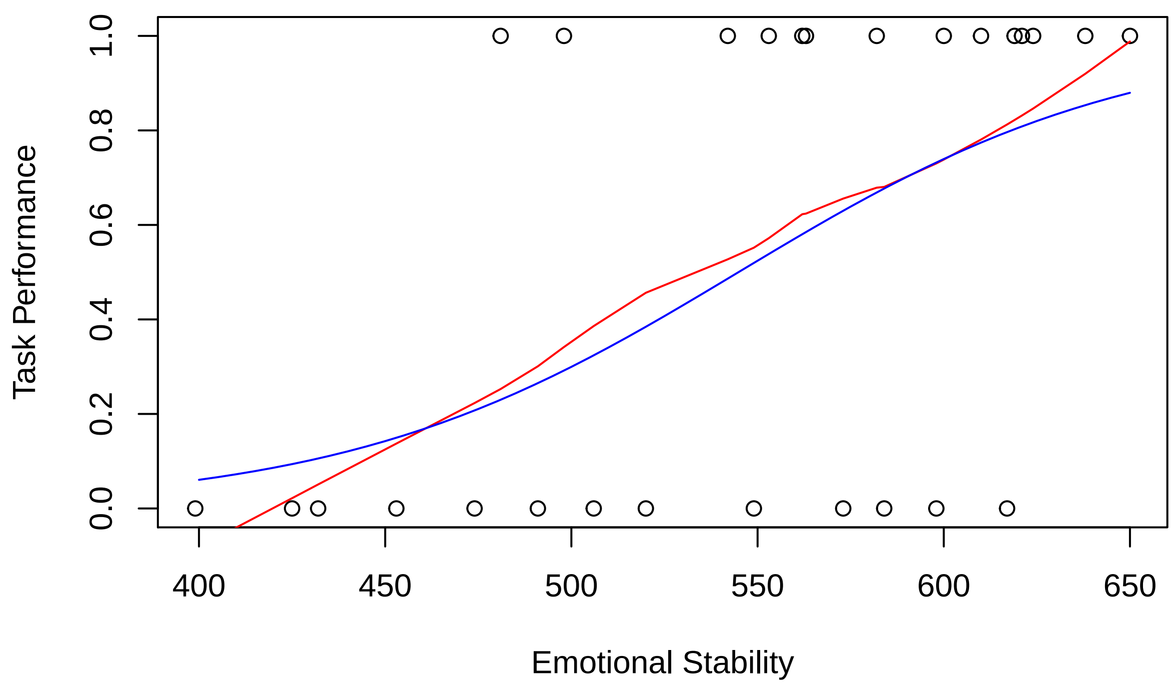
A psychologist conducted a study to examine the nature of the relation, if any, between an employee's emotional stability (X) and the employee's ability to perform in a task group (YJ. Emotional stability was measured by a written test for which the higher the score, the greater is the emotional stability. Ability to perform in a task group (Y = 1 if able, Y = 0 if unable) was evaluated by the supervisor.

```r
taskperf = c(0, 0, 0, 1, 1, 0, 0, 1, 1,
             1, 1, 1, 1, 0, 1, 0, 1, 0,
             1, 0, 0, 0, 1, 0, 1, 0, 1)

emostab = c(474, 432, 453, 481, 619, 584, 399, 582, 638,
            624, 542, 650, 553, 425, 563, 549, 498, 520,
            610, 598, 491, 617, 621, 573, 562, 506, 600)

plot (emostab, taskperf, xlab="Emotional Stability", ylab="Task Performance")
lines (lowess (taskperf ~ emostab), col='red')
logistic.fit = glm (taskperf ~ emostab, family=binomial)
summary (logistic.fit)
```

```
##
## Call:
## glm(formula = taskperf ~ emostab, family = binomial)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -1.7845  -0.8350   0.5065   0.8371   1.7145
##
## Coefficients:
```

```
##                 Estimate Std. Error z value Pr(>|z|)
## (Intercept) -10.308925   4.376997  -2.355   0.0185 *
## emostab       0.018920   0.007877   2.402   0.0163 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 37.393  on 26  degrees of freedom
## Residual deviance: 29.242  on 25  degrees of freedom
## AIC: 33.242
##
## Number of Fisher Scoring iterations: 4
```

```r
emostab.seq = seq(400, 650, by=5)
X <- cbind(1, emostab.seq)
betahat = coefficients(logistic.fit)
Xb <- X %*% betahat
prob <- exp(Xb)/(1+exp(Xb))

lines(emostab.seq, prob, col='blue')
```



Obtain $exp(\beta_1)$ and interpret that number.

```r
# Obtaining exp(beta_1) value
print(exp(0.018920))
```

```
## [1] 1.0191
```

$exp(b_1) = 1.0191$. This means that the odds of employee's estimated task performance increases by 1.91% (1.91 percent) with each additional unit increase in employee's emotional stability.