Technical Questions and Answers in Machine Learning

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1   Problem 2

Determine the first and second derivative with respect to x of: $f(x) = \frac{1}{1+e^{-x}}$

1.1   Solution to Problem 2

First Derivative:
$f(x) = (1 + e^-x)^{-1}$
Using Chain Rule
$f'(x) = -1(1 + e^{-x})^{-2} \times -1e^{-x}$
$f'(x) = \frac{e^{-x}}{(1+e^{-x})^2}$

Second Derivative:
$f'(x) = \frac{e^{-x}}{(1+e^{-x})^2}$
Using Quotient Rule of Differentiation
$g(x) = e^{-x} \qquad h(x) = (1 + e^{-x})^2$
$g'(x) = -e^x \qquad h'(x) = -2e^{-x}(1 + e^{-x})$
$f''(x) = \frac{h \times g' - g \times h'}{g^2}$
$f''(x) = \frac{-e^{-x} \times (1+e^{-x})^2 \ + \ 2e^{-2x} \times (1+e^{-x})}{e^{-2x}}$
$f''(x) = \frac{e^{-2x}-1}{e^{-x}}$

2   Problem 3

If I break a stick of unit length into three random pieces, what is the expected length of the largest piece? (You may need to state the assumptions that you make.)

3 Problem 8

What are the values of the constants a, b and c if one writes the following expression in the form: $a(x - b)^2 + c$

$$3x^2 - 4x + 5 \tag{1}$$

3.1 Solution to Problem 8

$$3(x^2 - \frac{4}{3}x + \frac{5}{3})$$

$$3\left[(x - \frac{2}{3})^2 - \frac{4}{9} + \frac{5}{3}\right]$$

$$3\left[(x - \frac{2}{3})^2 + \frac{11}{9}\right]$$

$$3\left(x - \frac{2}{3}\right)^2 + \frac{11}{3}$$

$a = 3; \;\; b = \frac{2}{3}; \;\; c = \frac{11}{3}$