

# Technical Questions and Answers in Machine Learning

ONI Olalekan Joseph

June 24, 2019

## 1 Problem 2

Determine the first and second derivative with respect to  $x$  of:  $f(x) = \frac{1}{1+e^{-x}}$

### 1.1 Solution to Problem 2

First Derivative:

$$f(x) = (1 + e^{-x})^{-1}$$

Using Chain Rule

$$f'(x) = -1(1 + e^{-x})^{-2} \times -1e^{-x}$$

$$f'(x) = \frac{e^{-x}}{(1 + e^{-x})^2}$$

Second Derivative:

$$f'(x) = \frac{e^{-x}}{(1 + e^{-x})^2}$$

Using Quotient Rule of Differentiation

$$\begin{aligned} g(x) &= e^{-x} & h(x) &= (1 + e^{-x})^2 \\ g'(x) &= -e^x & h'(x) &= -2e^{-x}(1 + e^{-x}) \end{aligned}$$

$$f''(x) = \frac{h \times g' - g \times h'}{g^2}$$

$$f''(x) = \frac{-e^{-x} \times (1 + e^{-x})^2 + 2e^{-2x} \times (1 + e^{-x})}{e^{-2x}}$$

$$f''(x) = \frac{e^{-2x} - 1}{e^{-x}}$$

## 2 Problem 3

If I break a stick of unit length into three random pieces, what is the expected length of the largest piece? (You may need to state the assumptions that you make.)

## 3 Problem 8

What are the values of the constants  $a$ ,  $b$  and  $c$  if one writes the following expression in the form:  $a(x - b)^2 + c$

$$3x^2 - 4x + 5 \tag{1}$$

### 3.1 Solution to Problem 8

$$3(x^2 - \frac{4}{3}x + \frac{5}{3})$$

$$3\left[(x - \frac{2}{3})^2 - \frac{4}{9} + \frac{5}{3}\right]$$

$$3\left[(x - \frac{2}{3})^2 + \frac{11}{9}\right]$$

$$3(x - \frac{2}{3})^2 + \frac{11}{3}$$

$$a = 3; \quad b = \frac{2}{3}; \quad c = \frac{11}{3}$$