Technical Questions and Answers in Machine Learning

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1 Problem 2

Determine the first and second derivative with respect to x of: $f(x) = \frac{1}{1+e^{-x}}$

1.1 Solution to Problem 2

First Derivative:

$$f(x) = (1 + e^{-}x)^{-1}$$

Using Chain Rule

$$f'(x) = -1(1 + e^{-x})^{-2} \times -1e^{-x}$$

$$f'(x) = \frac{e^{-x}}{(1 + e^{-x})^2}$$

Second Derivative:

$$f'(x) = \frac{e^{-x}}{(1 + e^{-x})^2}$$

Using Quotient Rule of Differentiation

$g(x) = e^{-x} \qquad h(x) = (1 + e^{-x})^2$

$g'(x) = -e^{x} \qquad h'(x) = -2e^{-x}(1 + e^{-x})$

$$f''(x) = \frac{h \times g' - g \times h'}{g^2}$$

$$f''(x) = \frac{-e^{-x} \times (1+e^{-x})^2 \ \ + \ \ 2e^{-2x} \times (1+e^{-x})}{e^{-2x}}$$

$$f''(x) = \frac{e^{-2x} - 1}{e^{-x}}$$

2 Problem 3

If I break a stick of unit length into three random pieces, what is the expected length of the largest piece? (You may need to state the assumptions that you make.)

2.1 Solution to Problem 3

Let the stick be broken at two points X and Y.
Therefore, we have two independent random variables X, Y which are both uniform in [0,1].
Let $A = min(X, Y)$, $B = max(X, Y)$ and $C = max(A, 1-B, B-A)$.

Let $f_C(a)$ be the probability density function (pdf) of C and $F_c(a)$ be the cummulative distribution function (cdf). Then:

$$F_c(a) = P(C \leq a) = P(A \leq a, 1-B \leq a, B-A \leq a)$$

The cdf for the unit square is then equivalent to:

$$F_c(a) = \begin{cases} (3a-1)^2 & : \frac{1}{3} \leq a \leq \frac{1}{2} \\ \\ 1-3(1-a)^2 & : \frac{1}{2} \leq a \leq 1 \end{cases}$$

Then the pdf is:

$$f_c(a) = \begin{cases} 6(3a-1) & : \frac{1}{3} \leq a \leq \frac{1}{2} \\ \\ 6(1-a) & : \frac{1}{2} \leq a \leq 1 \end{cases}$$

Therefore, the expected length of the largest piece (C) is:

$$\int_{\frac{1}{2}}^{\frac{1}{3}} 6a(3a-1)da + \int_{1}^{\frac{1}{2}} 6a(1-a)da = \frac{11}{18}$$

3 Problem 8

What are the values of the constants a, b and c if one writes the following expression in the form: $a(x-b)^2+c$

$$3x^2 - 4x + 5 \tag{1}$$

3.1 Solution to Problem 8

$$3(x^2 - \frac{4}{3}x + \frac{5}{3})$$

$$3\Big[(x - \frac{2}{3})^2 - \frac{4}{9} + \frac{5}{3}\Big]$$

$$3\Big[(x - \frac{2}{3})^2 + \frac{11}{9}\Big]$$

$$3\big(x - \frac{2}{3}\big)^2 + \frac{11}{3}$$

$a = 3$; $b = \frac{2}{3}$; $c = \frac{11}{3}$

4 Problem 6

A factory that makes light bulbs contains three machines. The machines manufacture 20%, 30% and 50% of the total production. From their production, 5%, 4%, and 2% respectively are faulty. I choose a collection of light bulbs at random from the output.

4.1 Solution to Problem 6a

If the collection contains two faulty light bulbs, what is the probability that they come from the same machine?

Let $P_{M_{Af}}$ represent Probability of faulty bulbs produced from the first machine (A).

Similarly for second machine and third machine we have $P_{M_{Bf}}$ and $P_{M_{Cf}}$ respectively.

$P_{M_{Af}} = \frac{5}{20}$; $P_{M_{Bf}} = \frac{4}{30}$; $P_{M_{Cf}} = \frac{2}{50}$.

Let the probability that two faulty light bulbs from a collection come from the same machines be $P_{M_{2f}}$.

$$P_{M_{2f}} = \left(\frac{5}{20} \times \frac{5}{20}\right) + \left(\frac{4}{30} \times \frac{4}{30}\right) + \left(\frac{2}{50} \times \frac{2}{50}\right) = 0.00163$$

4.2 Solution to Problem 6b

Let the probability that the three faulty light bulbs from a collection come from the different machines be $P_{M_{3f}}$.

$$P_{M_{3f}} = \left(\frac{5}{20} \times \frac{4}{30} \times \frac{2}{50}\right) = \frac{1}{750}$$