

Technical Questions and Answers in Machine Learning

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June 25, 2019

1 Problem 2 **COMPLETED**

Determine the first and second derivative with respect to x of: $f(x) = \frac{1}{1+e^{-x}}$

1.1 Solution to Problem 2

First Derivative:

$$f(x) = (1 + e^{-x})^{-1}$$

Using Chain Rule

$$f'(x) = -1(1 + e^{-x})^{-2} \times -1e^{-x}$$

$$f'(x) = \frac{e^{-x}}{(1 + e^{-x})^2}$$

Second Derivative:

$$f'(x) = \frac{e^{-x}}{(1 + e^{-x})^2}$$

Using Quotient Rule of Differentiation

$$g(x) = e^{-x} \quad h(x) = (1 + e^{-x})^2$$

$$g'(x) = -e^x \quad h'(x) = -2e^{-x}(1 + e^{-x})$$

$$f''(x) = \frac{h \times g' - g \times h'}{g^2}$$

$$f''(x) = \frac{-e^{-x} \times (1 + e^{-x})^2 + 2e^{-2x} \times (1 + e^{-x})}{e^{-2x}}$$

$$f''(x) = \frac{e^{-2x} - 1}{e^{-x}}$$

2 Problem 3 **COMPLETED**

If I break a stick of unit length into three random pieces, what is the expected length of the largest piece? (You may need to state the assumptions that you make.)

2.1 Solution to Problem 3

Let the stick be broken at two points X and Y .

Therefore, we have two independent random variables X, Y which are both uniform in $[0,1]$.

Let $A = \min(X, Y)$, $B = \max(X, Y)$ and $C = \max(A, 1 - B, B - A)$.

Let $f_C(a)$ be the probability density function (pdf) of C and $F_c(a)$ be the cumulative distribution function (cdf). Then:

$$F_c(a) = P(C \leq a) = P(A \leq a, 1 - B \leq a, B - A \leq a)$$

The cdf for the unit square is then equivalent to:

$$F_c(a) = \begin{cases} (3a - 1)^2 & : \frac{1}{3} \leq a \leq \frac{1}{2} \\ 1 - 3(1 - a)^2 & : \frac{1}{2} \leq a \leq 1 \end{cases}$$

Then the pdf is:

$$f_c(a) = \begin{cases} 6(3a - 1) & : \frac{1}{3} \leq a \leq \frac{1}{2} \\ 6(1 - a) & : \frac{1}{2} \leq a \leq 1 \end{cases}$$

Therefore, the expected length of the largest piece (C) is:

$$\int_{\frac{1}{2}}^{\frac{1}{3}} 6a(3a - 1)da + \int_1^{\frac{1}{2}} 6a(1 - a)da = \frac{11}{18}$$

3 Problem 8 **COMPLETED**

What are the values of the constants a , b and c if one writes the following expression in the form: $a(x - b)^2 + c$

$$3x^2 - 4x + 5 \quad (1)$$

3.1 Solution to Problem 8

$$3(x^2 - \frac{4}{3}x + \frac{5}{3})$$

$$3\left[(x - \frac{2}{3})^2 - \frac{4}{9} + \frac{5}{3}\right]$$

$$3\left[(x - \frac{2}{3})^2 + \frac{11}{9}\right]$$

$$3(x - \frac{2}{3})^2 + \frac{11}{3}$$

$$a = 3; \quad b = \frac{2}{3}; \quad c = \frac{11}{3}$$

4 Problem 6 **COMPLETED**

A factory that makes light bulbs contains three machines. The machines manufacture 20%, 30% and 50% of the total production. From their production, 5%, 4%, and 2% respectively are faulty. I choose a collection of light bulbs at random from the output.

4.1 Solution to Problem 6a

If the collection contains two faulty light bulbs, what is the probability that they come from the same machine?

Let $P_{M_{Af}}$ represent Probability of faulty bulbs produced from the first machine (A).

Similarly for second machine and third machine we have $P_{M_{Bf}}$ and $P_{M_{Cf}}$ respectively.

$$P_{M_{Af}} = \frac{5}{20}; \quad P_{M_{Bf}} = \frac{4}{30}; \quad P_{M_{Cf}} = \frac{2}{50}.$$

Let the probability that two faulty light bulbs from a collection come from the same machines be $P_{M_{2f}}$.

$$P_{M_{2f}} = \left(\frac{5}{20} \times \frac{5}{20}\right) + \left(\frac{4}{30} \times \frac{4}{30}\right) + \left(\frac{2}{50} \times \frac{2}{50}\right) = 0.00163$$

4.2 Solution to Problem 6b

Let the probability that the three faulty light bulbs from a collection come from the different machines be $P_{M_{3f}}$.

$$P_{M_{3f}} = \left(\frac{5}{20} \times \frac{4}{30} \times \frac{2}{50}\right) = \frac{1}{750}$$

5 Problem 4 ****COMPLETED****

The twenty-first century began on 1 January 2001 (a Monday) and will end on 31 December 2100 (a Friday). What percentage of twenty-first century Wednesdays fall on the last day of a month? (This question requires some coding.)

5.1 Solution to Problem 4

The Python code snippet is shown in Figure 1. Percentage of the 21st century Wednesdays that fall on the last day of the month:

$$\begin{aligned} &= \frac{172}{5242} \times 100\% \\ &3.3\%(1dp) \end{aligned}$$

The screenshot shows a Jupyter Notebook titled 'Untitled13' running on a local host. The code is in a cell labeled 'In [54]:'. It imports the 'calendar' module and initializes 'all_wednesdays' and 'last_wednesdays' to 0. It then iterates over years from 2001 to 2101 and months from 1 to 13. For each month, it finds the last day of the month and checks if it is a Wednesday. If so, it increments 'last_wednesdays'. It also iterates over the days of the month to check for Wednesdays and increments 'all_wednesdays'. Finally, it prints the total count of Wednesdays and the last Wednesday of the century.

```
In [54]: import calendar                                     #Python Calendar module
all_wednesdays = 0
last_wednesdays = 0
cal= calendar.Calendar()

for year in range(2001, 2101):                                #set range for the 21st century
    for month in range(1,13):
        last_day_month = calendar.monthrange(year,month)[1]    #get the Last day of each month per year

        if (calendar.weekday(year, month, last_day_month)) == 4: #check if the Last day is a wednesday
            last_wednesdays = last_wednesdays + 1             # if the check above is true increment the counter

        for day in cal.itermonthdays(year,month):
            if day > 0:
                if (calendar.weekday(year, month, day)) == 4:    #check for wednesday per month per year
                    all_wednesdays = all_wednesdays + 1

        if calendar.isleap(year):                                #check for Leap year and add one more wednesday to it.
            all_wednesdays = all_wednesdays + 1

print(all_wednesdays,last_wednesdays)

5242 172
```

Below the code cell, there is an input prompt 'In []:' followed by an empty text box.

Figure 1: Python Source Code Screenshot