Technical Questions and Answers in Machine Learning

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June 24, 2019

1 Problem 2

Determine the first and second derivative with respect to x of: $f(x) = \frac{1}{1+e^{-x}}$

1.1 Solution to Problem 2

First Derivative:

$$f(x) = (1 + e^{-}x)^{-1}$$

Using Chain Rule

$$f'(x) = -1(1 + e^{-x})^{-2} \times -1e^{-x}$$

$$f'(x) = \frac{e^{-x}}{(1 + e^{-x})^2}$$

Second Derivative:

$$f'(x) = \frac{e^{-x}}{(1 + e^{-x})^2}$$

Using Quotient Rule of Differentiation

$g(x) = e^{-x} \qquad h(x) = (1 + e^{-x})^2$

$g'(x) = -e^{x} \qquad h'(x) = -2e^{-x}(1 + e^{-x})$

$$f''(x) = \frac{h \times g' - g \times h'}{g^2}$$

$$f''(x) = \frac{-e^{-x} \times (1+e^{-x})^2 \ \ + \ \ 2e^{-2x} \times (1+e^{-x})}{e^{-2x}}$$

$$f''(x) = \frac{e^{-2x} - 1}{e^{-x}}$$

2 Problem 3

If I break a stick of unit length into three random pieces, what is the expected length of the largest piece? (You may need to state the assumptions that you make.)

2.1 Solution to Problem 3

Let the stick be broken at two points X and Y.
Therefore, we have two independent random variables X, Y which are both uniform in [0,1].
Let $A = min(X, Y)$, $B = max(X, Y)$ and $C = max(A, 1 - B, B - A)$.

Let $f_C(a)$ be the probability density function (pdf) of C and $F_c(a)$ be the cummulative distribution function (cdf). Then:

$$F_c(a) = P(C \leq a) = P(A \leq a, 1 - B \leq a, B - A \leq a)$$

The cdf for the unit square is then equivalent to:

$$F_c(a) = \begin{cases} (3a-1)^2 & : \frac{1}{3} \leq a \leq \frac{1}{2} \\ 1 - 3(1-a)^2 & : \frac{1}{2} \leq a \leq 1 \end{cases}$$

Then the pdf is:

$$f_c(a) = \begin{cases} 6(3a-1) & : \frac{1}{3} \leq a \leq \frac{1}{2} \\ 6(1-a) & : \frac{1}{2} \leq a \leq 1 \end{cases}$$

Therefore, the expected length of the largest piece (C) is:

$$\int_{\frac{1}{2}}^{\frac{1}{3}} 6a(3a-1)da + \int_{1}^{\frac{1}{2}} 6a(1-a)da = \frac{11}{18}$$

3 Problem 8

What are the values of the constants a, b and c if one writes the following expression in the form: $a(x-b)^2+c$

$$3x^2 - 4x + 5 \tag{1}$$

3.1 Solution to Problem 8

$$3(x^2 - \frac{4}{3}x + \frac{5}{3})$$

$$3\Big[(x-\frac{2}{3})^2 - \frac{4}{9} + \frac{5}{3}\Big]$$

$$3\Big[(x-\frac{2}{3})^2 + \frac{11}{9}\Big]$$

$$3\big(x-\frac{2}{3}\big)^2 + \frac{11}{3}$$

$a=3;\;\; b=\frac{2}{3};\;\; c=\frac{11}{3}$