

# Technical Questions and Answers in Machine Learning

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June 25, 2019

## 1 Problem 2 \*\*COMPLETED\*\*

Determine the first and second derivative with respect to  $x$  of:  $f(x) = \frac{1}{1+e^{-x}}$

### 1.1 Solution to Problem 2

First Derivative:

$$f(x) = (1 + e^{-x})^{-1}$$

Using Chain Rule

$$f'(x) = -1(1 + e^{-x})^{-2} \times -1e^{-x}$$

$$f'(x) = \frac{e^{-x}}{(1 + e^{-x})^2}$$

Second Derivative:

$$f'(x) = \frac{e^{-x}}{(1 + e^{-x})^2}$$

Using Quotient Rule of Differentiation

$$g(x) = e^{-x} \quad h(x) = (1 + e^{-x})^2$$

$$g'(x) = -e^x \quad h'(x) = -2e^{-x}(1 + e^{-x})$$

$$f''(x) = \frac{h \times g' - g \times h'}{g^2}$$

$$f''(x) = \frac{-e^{-x} \times (1 + e^{-x})^2 + 2e^{-2x} \times (1 + e^{-x})}{e^{-2x}}$$

$$f''(x) = \frac{e^{-2x} - 1}{e^{-x}}$$

## 2 Problem 3 \*\*COMPLETED\*\*

If I break a stick of unit length into three random pieces, what is the expected length of the largest piece? (You may need to state the assumptions that you make.)

### 2.1 Solution to Problem 3

Let the stick be broken at two points  $X$  and  $Y$ .

Therefore, we have two independent random variables  $X, Y$  which are both uniform in  $[0,1]$ .

Let  $A = \min(X, Y)$ ,  $B = \max(X, Y)$  and  $C = \max(A, 1 - B, B - A)$ .

Let  $f_C(a)$  be the probability density function (pdf) of  $C$  and  $F_c(a)$  be the cumulative distribution function (cdf). Then:

$$F_c(a) = P(C \leq a) = P(A \leq a, 1 - B \leq a, B - A \leq a)$$

The cdf for the unit square is then equivalent to:

$$F_c(a) = \begin{cases} (3a - 1)^2 & : \frac{1}{3} \leq a \leq \frac{1}{2} \\ 1 - 3(1 - a)^2 & : \frac{1}{2} \leq a \leq 1 \end{cases}$$

Then the pdf is:

$$f_c(a) = \begin{cases} 6(3a - 1) & : \frac{1}{3} \leq a \leq \frac{1}{2} \\ 6(1 - a) & : \frac{1}{2} \leq a \leq 1 \end{cases}$$

Therefore, the expected length of the largest piece ( $C$ ) is:

$$\int_{\frac{1}{2}}^{\frac{1}{3}} 6a(3a - 1)da + \int_1^{\frac{1}{2}} 6a(1 - a)da = \frac{11}{18}$$

### 3 Problem 8 \*\*COMPLETED\*\*

What are the values of the constants  $a$ ,  $b$  and  $c$  if one writes the following expression in the form:  $a(x - b)^2 + c$

$$3x^2 - 4x + 5 \tag{1}$$

#### 3.1 Solution to Problem 8

$$3(x^2 - \frac{4}{3}x + \frac{5}{3})$$

$$3\left[(x - \frac{2}{3})^2 - \frac{4}{9} + \frac{5}{3}\right]$$

$$3\left[(x - \frac{2}{3})^2 + \frac{11}{9}\right]$$

$$3(x - \frac{2}{3})^2 + \frac{11}{3}$$

$$a = 3; \quad b = \frac{2}{3}; \quad c = \frac{11}{3}$$

### 4 Problem 6 \*\*COMPLETED\*\*

A factory that makes light bulbs contains three machines. The machines manufacture 20%, 30% and 50% of the total production. From their production, 5%, 4%, and 2% respectively are faulty. I choose a collection of light bulbs at random from the output.

#### 4.1 Solution to Problem 6a

If the collection contains two faulty light bulbs, what is the probability that they come from the same machine?

Let  $P_{M_{Af}}$  represent Probability of faulty bulbs produced from the first machine (A).

Similarly for second machine and third machine we have  $P_{M_{Bf}}$  and  $P_{M_{Cf}}$  respectively.

$$P_{M_{Af}} = \frac{5}{20}; \quad P_{M_{Bf}} = \frac{4}{30}; \quad P_{M_{Cf}} = \frac{2}{50}.$$

Let the probability that two faulty light bulbs from a collection come from the same machines be  $P_{M_{2f}}$ .

$$P_{M_{2f}} = \left(\frac{5}{20} \times \frac{5}{20}\right) + \left(\frac{4}{30} \times \frac{4}{30}\right) + \left(\frac{2}{50} \times \frac{2}{50}\right) = 0.00163$$

## 4.2 Solution to Problem 6b

Let the probability that the three faulty light bulbs from a collection come from the different machines be  $P_{M_{3f}}$ .

$$P_{M_{3f}} = \left(\frac{5}{20} \times \frac{4}{30} \times \frac{2}{50}\right) = \frac{1}{750}$$

## 5 Problem 4 **\*\*COMPLETED\*\***

The twenty-first century began on 1 January 2001 (a Monday) and will end on 31 December 2100 (a Friday). What percentage of twenty-first century Wednesdays fall on the last day of a month? (This question requires some coding.)

### 5.1 Solution to Problem 4

The Python source code is shown in Figure 1. Percentage of the 21st century Wednesdays that fall on the last day of the month:

$$\begin{aligned} &= \frac{172}{5218} \times 100\% \\ &3.3\%(1dp) \end{aligned}$$

The screenshot shows a Jupyter Notebook titled 'Untitled13' running on a local host. The code is as follows:

```
In [54]: import calendar                                     #Python Calendar module
all_wednesdays = 0
last_wednesdays = 0
cal= calendar.Calendar()

for year in range(2001, 2101):                               #set range for the 21st century
    for month in range(1,13):
        last_day_month = calendar.monthrange(year,month)[1]  #get the Last day of each month per year

        if (calendar.weekday(year, month, last_day_month)) == 4: #check if the Last day is a wednesday
            last_wednesdays = last_wednesdays + 1           # if the check above is true increment the counter

        for day in cal.itermonthdays(year,month):
            if day > 0:
                if (calendar.weekday(year, month, day)) == 4:  #check for wednesday per month per year
                    all_wednesdays = all_wednesdays + 1

        if calendar.isleap(year):                             #check for Leap year and add one more wednesday to it.
            all_wednesdays = all_wednesdays + 1

print(all_wednesdays,last_wednesdays)

5242 172
```

The output of the code is '5242 172'. The Jupyter interface includes a menu bar (File, Edit, View, Insert, Cell, Kernel, Widgets, Help), a toolbar, and a status bar at the bottom showing the system clock as 11:33 AM on 6/25/2019.

Figure 1: Python Source Code Screenshot