parallel CPHD filter

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Outline

CPHD filter

Analytic Implementations of the Cardinalized Probability Hypothesis Density Filter
`Ba-Tuong Vo, Ba-Ngu Vo, and Antonio Cantoni` 2007

introduction

Due to the inherent conbinatorial nature of multitarget densities and the multiple integrations on the multitarget state and observation space,the multitarget Bayes recursion is intractable in most ptactical applications. To alleviate this intractability the PHD recursion was developd as a first moment approximation to the multiarget bayes recursion.

THe PHD recursion propagates cardinality information with only a single parameter,and thus,it effectively approximates the cardinality distribution by a Poisson distribution.

When the number of targets present is high,the PHD filter estimates the cardinality with a correspondingly high variance.

RFSs

suppose at time k there are N(k) targets with states $x_{k,1},....x_{k,N(k)}$ each taing values in a state space $\chi \subseteq R$ Suppose also at time k that M(k) measurements $z_{k,1}.....z_{k,M(k)}$ are received each taking values in an observation space.

$$X_k = \left\{x_{k,1},...,x_{k,N(k)}\right\} \in f(\chi) \tag{1}$$

$$Z_k = \left\{z_{k,1},...,z_{k,M(k)}\right\} \in f(z) \tag{2}$$

Intuitively, an RFS is simply a finite-set-valued random variable which can be completely characterized by a discrete probability distribution and a family of joint probability densities.

RFSs

we consider multitarget dynamics modeled by

$$X_k = \left[\bigcup_{\zeta \in X_{k-1}} S_{k|k-1}(\zeta) \right] \bigcup \Gamma_k \tag{3}$$

Similarly,the multitarget sensor observations are modeled by

$$Z_k = \left[\bigcup_{x \in X_k} \Theta_k(x) \right] \bigcup K_k \tag{4}$$

RFSs

multitarget transition density $f_{k|k-1}(\bullet|\bullet)$ descirbes the time evolution of the multitarget state and encapsulates the underlying models of target motions,birth and deaths.

Similarly, the *likelihood* $g_k(\bullet|\bullet)$ describes the multitarget sensor measurement and encapsulates the underlying models of detections,false alarms,and target generated measurements.

RFSs

The *multitarget Bayes recursion* propagates the multitarget posterior density $\pi_k(\bullet|Z_{1:k})$

$$\pi_{k|k-1}(X_k|Z_{1:k-1}) = \int f_{k|k-1}(X_k|X)\pi_{k-1}(X|Z_{1:k-1})\mu_s(dX) \tag{5}$$

$$\pi_k(X_k|Z_{1:k}) = \frac{g_k(Z_k|X_k)\pi_{k|k-1}(X_k|Z_{1:k-1})}{\int g_k(Z_k|X_k)\pi_{k|k-1}(X_k|Z_{1:k-1})\mu_s(dX)} \tag{6}$$

solution to the CPHD recursion

The CPHD recursion rests on the following assumptions:

- each target evolves and generates measurements independently of one another
- the birth RFS and the surviving RFSs are independent of each other
- the clutter RFS is an i.i.d cluster process and independent of measurement RFSs
- the prior and predicted multitatget RFSs are i.i.d cluster processes.

solution to the CPHD recursion

proposition 1: suppose at time k-1 that the posterior intensity v_{k-1} and posterior cardiniality distribution p_{k-1} are given.

$$p_{k|k-1}(n) = \sum_{j=0}^{n} p_{\Gamma,k}(n-j)\Pi_{k|k-1}\left[v_{k-1}, p_{k-1}\right](j) \tag{7}$$

$$v_{k|k-1}(n) = \int p_{S,k}(\zeta) f_{k|k-1}(x|\zeta) v_{k-1}(\zeta) d\zeta + \gamma_k(x) \tag{8}$$

where $\Pi_{k|k-1}\left[v, p\right](j) = \sum_{\iota=j}^{\infty} C_j^{\iota} \frac{<p_{S,k},v>^{j}<1-p_{S,k},v>^{\iota-j}}{<1,v>^{\iota}} p(\iota)$

solution to the CPHD recursion

proposition 2: update

$$p_k(n) = \frac{\Upsilon_k^0[v_{k|k-1}, Z_k](n) p_{k|k-1}(n)}{< \Upsilon_k^0[v_{k|k-1}, Z_k], p_{k|k-1} >} \tag{9}$$

$$v_k(x) = \frac{< \Upsilon_k^1[v_{k|k-1}, Z_k], p_{k|k-1} >}{< \Upsilon_k^0[v_{k|k-1}, Z_k], p_{k|k-1} >} \times [1 - p_{D,k}(x)] v_{k|k-1}(x) + \sum_{z \in Z_k} \frac{< \Upsilon >}{} \tag{10}$$

Closed-Form Solution to the CPHD Recursion

Each target follows a linear Gaussian dynamical model

$$f_{k|k-1}(x|\xi) = N(x; F_{k-1}\zeta, Q_{k-1}) \tag{11}$$

$$g_k(z|x) = N(z; H_k x, R_k) \tag{12}$$

where $N(; m, P)$ denotes a Gaussian density with mean m and covariance P,F_{k-1} is the state transition matrix,

The intensity of the birth RFS is a Gaussian mixture of the form:

$$\gamma_k(x) = \sum_{i=1}^{J_{\gamma,k}} \omega_{\gamma,k}^{(i)} N(x; m, P) \tag{13}$$

where ω ,m,P are the weights,means,and covariances of the mixture birth intensity.

Closed-Form Solution to the CPHD Recursion

Suppose at time k-1 that the posterior intensity v_{k-1} and posterior cardinality distribution p_{k-1} are given,and that v_{k-1} is a Gaussian mixture of the form

$$v_{k-1} = \sum_{i=1}^{j_{k-1}} \omega_{k-1}^{(i)} N(x; m_{k-1}^{(i), P_{k-1}^{(i)}}) \tag{14}$$

the CPHD prediction simplifies to

$$p_{k|k-1}(n) = \sum_{j=0}^{n} p_{\Gamma,k}(n-j) \sum_{\iota=j}^{\infty} C_j^{\iota} p_{S,k}^{j} (1-p_{s,k})^{\iota-j} \tag{15}$$

$$v_{k|k-1}(x) = v_{S,k|k-1}(x) + \gamma_k(x) \tag{16}$$

Closed-Form Solution to the CPHD Recursion

where

(17)

code

$x_k = [p_{x,k}, p_{y,k}, \overline{p}_{x,k}, \overline{p}_{y,k}]$

$$F_k = \begin{bmatrix} I_2 & \Delta I_2 \\ 0 & I_2 \end{bmatrix} Q_k = \sigma_v^2 \begin{bmatrix} \frac{\Delta^4}{4} I_2 & \frac{\Delta^3}{2} I_2 \\ \frac{\Delta^3}{2} I_z & \Delta^2 I_2 \end{bmatrix}$$

$p_{S,k} = 0.99$

$H_k = [I_2 0_2]$ $R_k = \sigma_\varepsilon^2 I_2$

Bullet Points and Numbered Lists

- Lorem ipsum dolor sit amet, consectetur adipiscing elit
- Aliquam blandit faucibus nisi, sit amet dapibus enim tempus eu

1. Nulla commodo, erat quis gravida posuere, elit lacus lobortis est, quis porttitor odio mauris at libero
2. Nam cursus est eget velit posuere pellentesque
3. Vestibulum faucibus velit a augue condimentum quis convallis nulla gravida

Verbatim

How to include a theorem in this presentation:

```
\mybox{0.8\textwidth}{
\begin{theorem}[Murphy (1949)]
Anything that can go wrong, will go wrong.
\end{theorem}
}
```

Displaying Information

Table

Treatments	Response 1	Response 2
Treatment 1	0.0003262	0.562
Treatment 2	0.0015681	0.910
Treatment 3	0.0009271	0.296

Table 1: Table caption

Theorem

The most common definition of Murphy's Law is as follows.

Theorem (Murphy (1949))
Anything that can go wrong, will go wrong.

Proof. A special case of this theorem is proven in the textbook. □

Remark
This is a remark.

Algorithm
This is an algorithm.

Citations

An example of the `\cite` command to cite within the presentation:

This statement requires citation [1].

References

[1] J. M. Smith and A. B. Jones. *Book Title*. Publisher, 7th edition, 2012.

Questions?