*

	Γ	Δ	Θ	Λ	Ξ	Π	Σ	Υ	Φ	Ψ	Ω
`\mit`	$\mathit{\Gamma}$	$\mathit{\Delta}$	$\mathit{\Theta}$	$\mathit{\Lambda}$	$\mathit{\Xi}$	$\mathit{\Pi}$	$\mathit{\Sigma}$	$\mathit{\Upsilon}$	$\mathit{\Phi}$	$\mathit{\Psi}$	$\mathit{\Omega}$

alpha	A	α	beta	B	β
gamma	Γ	γ	delta	Δ	δ
epsilon	E	ϵ, ε	zeta	Z	ζ
eta	H	η	theta	Θ	θ, ϑ
iota	I	ι	kappa	K	κ
lambda	Λ	λ	mu	M	μ
nu	N	ν	omicron	O	o
xi	Ξ	ξ	pi	Π	π, ϖ
rho	P	ρ, ϱ	sigma	Σ	σ, ς
tau	T	τ	upsilon	Υ	υ
phi	Φ	ϕ, φ	chi	X	χ
psi	Ψ	ψ	omega	Ω	ω

*: wangjunjie2013@gmail.com

1 A filter for distinguishable and independent populations

The level of information maintained by the filter on any particular target depends on its status:

- *distinguishable*: individuals are those for which specific information is available, usually through the sensor observation process(past or present).
- *indistinguishable*: individuals, on the other hand, are those that are known only as members of a larger population whose individuals share a common description.

1.1 Bayesian Estimation with Stochastic Populcations

In the context of the DISP filter, we shall consider the following assumptions.

Modelling assumptions. *Individuals in the population $\mathcal{X}$ of interest:*

- (M1) behave independently;
- (M2) enter the scene at most once during the scenario
- (M3) all have state ψ before $t = 0$

Following these assumptions, the population $\mathcal{X}$ shall be decomposed at any time $t \geq 0$

$$\mathcal{X} = \mathcal{X}_t^{\psi} \cup \mathcal{X}_t^{\bullet} \cup \mathcal{X}_t^{*}$$

Modelling assumptions. *The observation process at any time $t \geq 0$ is such that*

- (M4) an individual produces at most one observation(if not, it is miss-detected);
- (M5) an observation originates from at most individual(if not, it is a false alarm);
- (M6) individuals outside the scene produce no observations;
- (M7) observations are produced independently;
- (M8) observations are distinct
- (M9) the number of observations is finite

Modelling assumptions. *First detection*

- (M10) an individual is detected upon entering the scene

Modelling assumptions. *Prior information*

- (M11) global information on the population $\mathcal{X}_t^{\bullet}$ is available, but no specific information is available on any of its individuals

Modelling assumptions. *At any time $t \geq 0$, the knowledge of the operator about:*

- (M12) the evolution of the individuals in $\mathcal{X}$ since time $t-1$ is described by a Markov kernel $m_{t-1,t}$
- (M13) the observation process is described by a likelihood $g_t(z, \cdot)$ and a probability of false alarm $p_{fa,t}$

1.2 DISP filter: Target Representation

The DISP filter maintains a representation of the appearing individuals $\mathcal{X}_t^{\bullet}$ as a stochastic population of indistinguishable targets, composed of:

- a cardinility distribution $\hat{c}_t^a$, describing the number of appearing targets;
- a single probability distribution $\hat{p}_t^a$ collectively describing the initial state of any appearing targets.

A distinguishable target which entered the scene at some birth time $0 \neq t_{\bullet} \neq t$ is characterised by the following 3-tuple or track

$$(t_{\bullet}, y, p_t^y)$$

where $t_{\bullet}$ is its time of birth, y its observation path, and p_t^y a probability distribution describing its current state. The observation path y is s sequence of observations

$$y = (\phi, ..., \phi, z_{td}, z_{t_{d+1}}, ..., z_t)$$

The current set of all possible observation paths is denoted by Y_t. An *hypothesis* h is defined as a given subset of observation paths in Y_t that represents a realisation of the stochastic population,i.e. a possible representation of the estimated population, described by the product measure

$$p_t^h = \bigotimes_{y \in h} p_t^y$$

The DISP filter maintains various representations of rhe estimated populcation through a weighted set of hypotheses H_t; whose weights are given by a distribution c_t on H_t satisfying

$$\sum_{h \in H_t} c_t(h) = 1$$

For any hypothesis $h \in H_t$ the scalar $c_t(h)$ assesses its credibility,i.e. the likelihood that the tracks in h represent the individuals from the estimation population. It is called the probability of existence of hypotheis h.

1.3 Prediction

$t = 0$, the set of hypotheses H_{-1} is reduced to the singletion

$$H_{-1} = \left\{ \emptyset^d \right\}$$

and

$$c_{-1}(\emptyset^d) = 1$$

1.3.1 Track prediction

The information gathered so far by the operator on any target $y \in Y_{t-1}$, described by some distribution p^y_{t-1} on the former state space $\bar{X}_{t-1}$, is then transferred to the current state space $\bar{X}_t$ through the Markov kernel $m_{t-1,t}$

1.3.2 Hypothesis prediction

Since the observation set Z_t is not available yet, neither the observation paths in Y_{t-1} nor the composition of the hypotheses in H_{t-1} are modified by the prediction step.

1.4 DISP filter: Data Update

Input

2 Novel Multi-Object Filtering Approach for Space Situational Awareness

Popular track-based solutions include the MHT and JPDA filters and follow an intuitive construction in which sequences of observations that may represent the data originating from a single specific object are maintained as tracks. They do not maintain a probabilisitc description of the dynamical evolution of the population of objects and rely on herustics and expert knowledge in order to determine, for example, at which point a stream of observations is assumed to be sufficient evidence for the creation of new track, or at which point a track is considered lost.

2.1 Multi-Object Estimation with DISP filter

3 Multi-target filtering with linearised complexity

4 Multi-object filtering with stochastic populations

5 Hypothesised filter for independent stochastic populations

J. Houssineau, P. Del Moral, and D. E. Clark. General multi-object filtering and association measure. In Computational Advances in Multi-Sensor Adaptive Processing (CAMSAP), 5th IEEE International Workshop on, 2013.

The HISP filter allows to characterise separately the individuals of a population while preserving a sufficiently general modelling of the population dynamics.

$\mathcal{P}(E)$ stands for the set of probability measures on a given measurable space $(E, \mathcal{E})$, and $u(f) = \int u(dx)f(x)$ for any $u \in \mathcal{P}(E)$ and any bounded measurable function f on $(E, \mathcal{E})$.

Let $\mathfrak{X}$ be the population of interest. At time t, individuals in $\mathfrak{X}$ are described in the extended space $\bar{X}_t = \{\psi\} \cup X_t$, where X_t is a complete separable metric space, ψ represents the individuals with no image in X_t. At time t, the observation process represents individuals of $\mathfrak{X}$ in the extended observation space $\bar{Z}_t = \{\phi\} \cup Z_t$.

Let t_+ denote the time at which an individual in $\mathfrak{X}$ appeared in the state space.

As the estimation within the HISP filter is concerned with individuals only, the space $\bar{X}_t$ is further augmented by the point ψ to account for the unvertainity of the presence of an individual in $\bar{X}_t$ and we denote

$$X_t^+ = \{\varphi, \psi\} \cup X_t = \{\varphi\} \cup \bar{X}_t \tag{1}$$

For any $p_t^x \in \mathcal{P}(X_t^+)$, the scalar $p_t^x(\bar{X}_t)$ is the probability for x to be an individual of $\mathfrak{X}$. The set of all potential individuals at time t, before the update, is denoted X_t. The set of updated potential individuals represented by $x = (T, y)$ with $T \in [0, t]$ and $y \in Y_t$.

5.1 The HISP filter

5.1.1 Initialisation

At time $t = 0$, no observation has been made available yet so that no individual can be distinguished and the set of individual stochastic representations X_0 is such that $X_0 = x_0$, with $x_0 = (\{0\}, ())$. The associated law $p_0^{x_0}$ is denoted p_0^b as individuals with representation x_0 are thought as being newborn individuals at time 0.

5.1.2 Time update

Given the independence of the individuals in the population $\mathfrak{X}$, the law $\hat{p}_t^x$ of an individual with representation $x \in \hat{X}_{t-1}$ can be predicted straightforwardly by using the Chapman-Kolmogorov equation with a Markov kernel $M_{t|t-1}$ from X_{t-1}^+ to X_t^+.

For any $x \in X_{t-1}$ and any $x' \in X_t$,

$$\begin{aligned}
&M_{t|t-1}(dx'|x) = p_{S,t} m_{t|t-1}(dx'|x) \\
&M_{t|t-1}(\varphi|x) = 1 - p_{S,t} \\
&M_{t|t-1}(\psi|\psi) = 1 \\
&M_{t|t-1}(\phi|\phi) = 1
\end{aligned} \tag{2}$$

The birth at time t is modelled by a unique individual stochastic representation $x = (\{t\}, ())$ with p_t^b with cardinality distribution c_t^b. We assume that $p_t^b(\varphi) = 0$ as newborn individuals exist almost surely.

5.2 Observation update

At time t, a set of observation in Z_t is received. We denote π the partition of Z_t corresponding to the sensor resolution cells. Each resolution cell is represented by a point z_w in Z_t, which nay be the centre of the cell, and the set Z_t^+ is defined as $\{z_w, s.t. w \in \pi\}$.

For any $z \in \bar{Z}_t$ and any $x \in X_t^+$, we denote the prior probability of association $p_t^{x,z}$ expressed as

$$p \tag{3}$$

6 Tracking with MIMO sonar systems: applications to harbour surveillance

The filter follows the usual multi-target tracking assumptions,i.e

- each target's dynamics and observation follow a hidden Markov model, the observation depends only on the current state
- targets are independent from each other and generate at most one observation per time step following a Bernoulli process
- the clutter is independent from the targets
- targets appear anywhere in the field of view and their disapperarance follows another Bernoulli process

The HISP filter can be seen as propagating a collection of hypotheses

$$\left\{w_t^i, p_t^i\right\}_{i \in \mathbb{I}_t} \tag{4}$$

consisting of single-object probability laws p_t^i on the state space X to which is associated a probability of existence w_t^i,i.e., a probability for a given law to represent a true object. The index set $\mathbb{I}_t$ can be given an explicit expression based on the time of creation and of the observation history of a given hypothesis.

The time prediction of the HISP filter can be expressed simply as

$$\begin{aligned} p_t^i(dx) &\stackrel{f}{=} \int M_t(y, dx) p_{t-1}^i dy \\ w_t^i &= \hat{w}_{t-1}^i \end{aligned} \tag{5}$$

For any given $i \in \mathbb{I}_t$ and $z \in Z_t$, the update of the hypothesis with index i by the observation takes the form

$$\begin{aligned} \hat{p}_t^j(dx) &\stackrel{f}{=} \frac{l_z(x) p_t^i(dx)}{\int l_z(y) p_t^i(dy)} \\ \hat{w}_t^i &= \frac{w_{ex}(i,z) w_t^{iz}}{\sum_{z \in Z_t} w_{ex}(i,z) w_t^{iz}} \end{aligned} \tag{6}$$

where j is an index in the observation-updated set $\hat{\mathbb{I}}_t$, where $w_t^{i,z} = \int l_z(x) p_t^i(dx)$ is the compatibility between the prior law ith index i and the observation. $w_{ex}(i,z)$ is a scalar in the interval $[0,1]$ describing the compatibility between the hypotheses indexed by $\mathbb{I}_t \setminus \{i\}$ and $Z_t \setminus \{z\}$

7 A SEQUENTIAL MONTE CARLO APPROXIMATION OF THE HISP FILTER

It is assumed that each observation does not correspond to more than one individual, so that, if two individuals have their projection on Z_t in the same resolution cell, then only one of them can be detected at the same time.

7.1 Population modelling

7.1.1 State

We start with individuals that have already been detected once and can therefore be distinguished by their observation history, or *observation path*. At time $t \in \mathbb{T}$, the set all possible observation paths can be indexed by the set $\bar{Y} = \bar{Z}_0 \times ... \times \bar{Z}_t$. An interval of existence $T \in \mathbb{T}$ of the form $[t^{'}, t]$ is conveniently added to the characterisation of individuals.

We are now in position to build a full index set in which each individual in the extended population,i.e., the one containing the objective population and the spurious-observation generators, is given a unique index. Before the observation update at time t, this index set is defined as

$$\mathbb{I}_t = \mathbb{I}_t^m \cup \left\{ i_t^a, i_t^b \right\} \tag{7}$$

where, denoting $[\cdot, t]$ the abstrct time interval ending at time t, $\mathbb{I}_t^m = \{(\sharp, [\cdot, t], y) : y \in Y_{t-1}\}$ corresponds to the detected individuals, where $i_t^a = (\sharp, t, \phi_t)$ describes newborn individuals, where the spurious-observation generators index is $i_t^b = (b, \emptyset, \phi_t)$.