2017 Asia and Pacific Mathematical Contest in Modeling

Problem B: Spray Trajectory Planning Issues

Glaze spraying process sprays glaze into the mist under compressed air with the glaze spray gun or glaze spray machine, so that the glaze adheres onto the clay body. It is a process easily realizing automation in the ceramic production process. As the uneven glaze will crack in the firing process, causing workpiece scrapping, so the thickness of the sprayed glaze in the spraying process is required to be uniform as far as possible.

In actual air spraying, compressed air is usually arranged on both sides of the spray gun mouth, the mist cone is squashed into an elliptical cone, the region on the plane covered by the spray cone formed by the paint mist is an ellipse, with semi-major axis of *a* and semi-minor axis of *b*, as shown in Fig. 1.

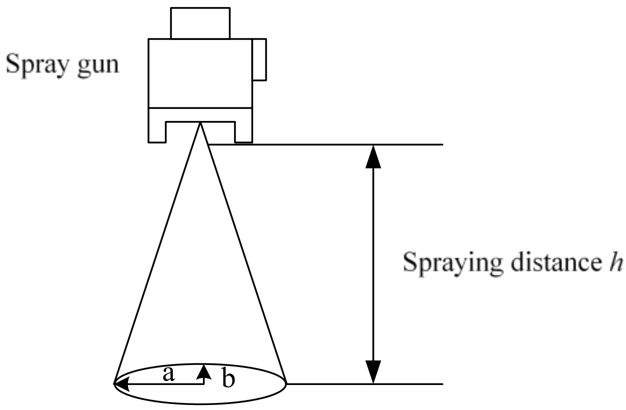

Fig. 1 Schematic diagram of spraying

It meets elliptic double β distribution model in the elliptic distribution region:

$$z(x,y) = z_{\max}\left(1-\frac{x^2}{a^2}\right)^{\beta_1-1}\left[1-\frac{y^2}{b^2\left(1-\frac{x^2}{a^2}\right)}\right]^{\beta_2-1}$$

Where: a- semi-major axis of spray ellipse (mm); b- semi-minor axis of spray ellipse (mm); $z_{\max}$- maximum thickness of paint film; β_1 – the index of β distribution in the section in the x direction; β_2 – the index of β distribution in the section in the y direction.

Some studies have shown that atomization pressure P_1, diaphragm pump pressure P_2 and spray distance h are the main factors affecting the above parameters, and they have the relationship as follows:

$$\begin{bmatrix} 129.8665 & -55.2435 & 1.7436 & -297.3908 \\ 52.5130 & -5.7480 & 0.7394 & -128.6368 \\ 59.7245 & 393.9655 & -0.1244 & 150.0184 \\ -7.0125 & 34.5045 & 0.0284 & -9.5229 \\ -4.6130 & 18.3620 & 0.0113 & -0.3924 \end{bmatrix} \times \begin{bmatrix} P_1 \\ P_2 \\ h \\ 1 \end{bmatrix} = \begin{bmatrix} a \\ b \\ z_{\max} \\ \beta_1 \\ \beta_2 \end{bmatrix}$$

The above model is a model of the spray gun spraying at a single point. In practice, however, the spray gun needs to move along the planned path so that the surface of the workpiece to be sprayed is uniformly covered with glaze, as shown in Fig. 2.

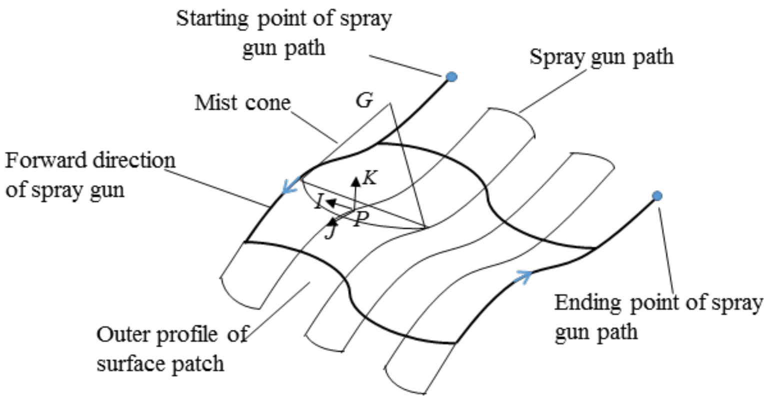

Fig. 2 Working schematic of spray gun

As the thickness in the region of the mist cone is large in the middle and thin on both sides during single-point spraying, so in order to ensure spray surface to be uniform, the mist cone will overlap in the adjacent paths in Fig. 3.

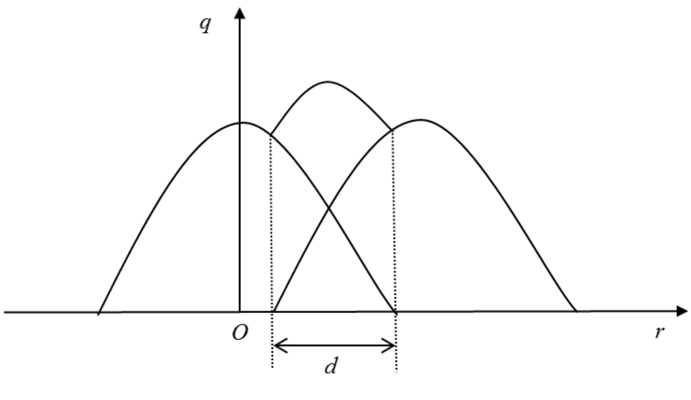

Fig. 3 Schematic of overlapping

We try to explore four issues as follows based on the background above:

1. According to the above materials, if the spraying direction of the spray gun always keeps unchanged (as shown in Fig. 4), please calculate the cumulative situation of spraying in the plane and find out the suitable overlap interval of the spray gun trajectory (P_1 and P_2 takes 0.2Mpa, and h takes 225mm).

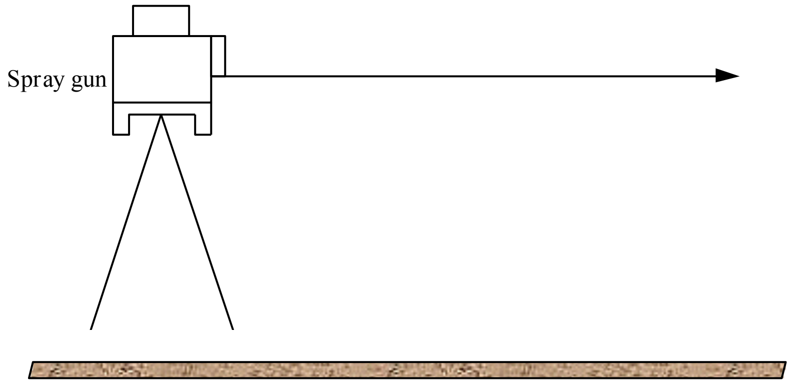

Fig. 4 Spraying direction of spray gun (always the same)

2. For curved surface $z = -x^2 + x - xy(-10 \le x \le 10, -10 \le y \le 10)$, determine whether the spraying interval calculated in issue 1 is applicable. If not, please plan the spray gun trajectory again, and calculate the overlap interval, so that the glaze thickness difference is less than 10% (intervals for different trajectories can be different, P_1 and P_2 takes 0.2Mpa, h can be selected according to actual needs).

3. If the spraying direction of the spray gun is always the normal direction of the spraying point of the mist cone center (as shown in Fig. 5) during spraying, and other conditions remain unchanged, please recalculate the result of issue 2.

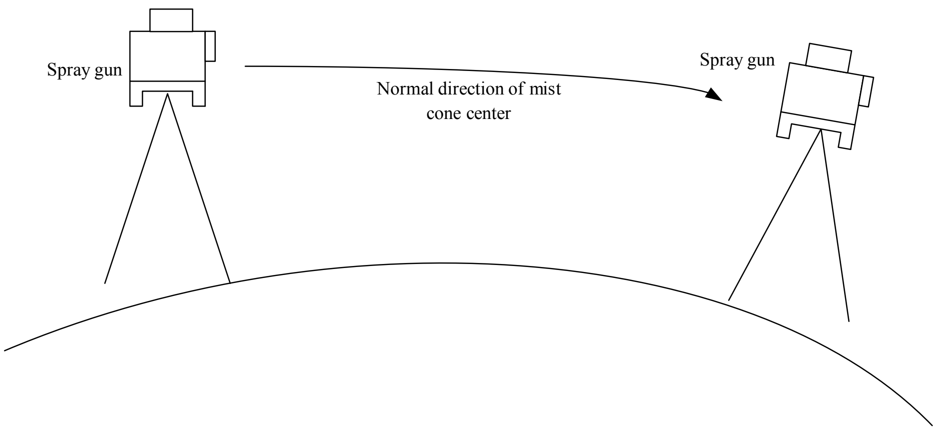

Fig. 5 Spraying direction of spray gun (normal direction of mist cone center)

4. Whether the result of issue 3 is applicable for any curved surface $z = f(x, y)$? Is there a general solution to the spray path planning?