

Explanation of the Practical Steps with Code:

This practical involves calculating Fibonacci numbers both using a **recursive** approach and an **iterative** approach, and then analyzing their **time** and **space complexity**.

1. **Fibonacci Series Overview:**

The Fibonacci series is an integer sequence where each number is the sum of the two preceding ones, usually starting with 0 and 1. The first few Fibonacci numbers are:

...

0, 1, 1, 2, 3, 5, 8, 13, 21, ...

...

In mathematical terms, the Fibonacci sequence is defined as:

- $F(0) = 0$

- $F(1) = 1$

- $F(n) = F(n-1) + F(n-2)$, for $n > 1$

2. **Recursion vs. Iteration:**

- **Recursive Approach**: The function `fibonacciRecursive(int n)` calls itself to calculate the Fibonacci number for smaller values of `n` until it reaches the base case (`n = 0` or `n = 1`).

- **Iterative Approach**: The function `fibonacciIterative(int n)` calculates the Fibonacci number by iterating from 2 up to `n`, storing the last two numbers in the sequence and adding them to get the next number.

3. **Code Walkthrough:**

Recursive Fibonacci Function (`fibonacciRecursive`):

```
```cpp
```

```
int fibonacciRecursive(int n) {
```

```

 if (n <= 1) return n; // Base case: if n is 0 or 1, return n.

 return fibonacciRecursive(n - 1) + fibonacciRecursive(n - 2); // Recursive case: F(n) = F(n-1) + F(n-2)
}
...

```

- For any `n`, if `n` is 0 or 1, the function simply returns `n`.
- Otherwise, it calls itself twice (once for `n-1` and once for `n-2`) and adds the results to calculate `F(n)`.

#### \*\*Iterative Fibonacci Function (`fibonacciIterative`):\*\*

```

```cpp
int fibonacciIterative(int n) {
    if (n <= 1) return n; // Base case: if n is 0 or 1, return n.

    int prev1 = 0, prev2 = 1, result;
    for (int i = 2; i <= n; i++) {
        result = prev1 + prev2; // Calculate the next Fibonacci number.
        prev1 = prev2;         // Update prev1 to be the previous Fibonacci number.
        prev2 = result;         // Update prev2 to be the current Fibonacci number.
    }
    return result; // Return the nth Fibonacci number.
}
...

```

- The iterative approach starts with `prev1 = 0` and `prev2 = 1`, then iteratively calculates the next Fibonacci number by summing the two previous values until the `n`th number is reached.

Main Function (`main`):

```

```cpp
int main() {
 int n;

 cout << "Enter the position of the Fibonacci number: ";

```

```

cin >> n;

// Time measurement for recursive approach
auto start_recursive = high_resolution_clock::now();
int result_recursive = fibonacciRecursive(n);
auto end_recursive = high_resolution_clock::now();
auto duration_recursive = duration_cast<nanoseconds>(end_recursive - start_recursive);

cout << "Recursive Fibonacci of " << n << " is " << result_recursive << endl;
cout << "Time taken by recursive approach: " << duration_recursive.count() << " nanoseconds" <<
endl;
cout << "Space complexity of recursive approach: O(n) (due to recursion stack)" << endl;

// Time measurement for iterative approach
auto start_iterative = high_resolution_clock::now();
int result_iterative = fibonacciIterative(n);
auto end_iterative = high_resolution_clock::now();
auto duration_iterative = duration_cast<nanoseconds>(end_iterative - start_iterative);

cout << "Iterative Fibonacci of " << n << " is " << result_iterative << endl;
cout << "Time taken by iterative approach: " << duration_iterative.count() << " nanoseconds" << endl;
cout << "Space complexity of iterative approach: O(1) (constant space)" << endl;

return 0;
}
...

```

- The user is prompted to enter a number `n` for which the Fibonacci number is to be calculated.
- Time is measured for both recursive and iterative approaches using the `chrono` library.
- Results and their respective time and space complexities are displayed.

#### ### 4. **Time and Space Complexity Analysis:**

##### #### **Recursive Approach:**

###### - **Time Complexity:**

The time complexity of the recursive Fibonacci function is  $O(2^n)$ . This is because each call to `fibonacciRecursive(n)` results in two more calls, leading to an exponential growth in the number of calls.

###### - **Space Complexity:**

The space complexity is  $O(n)$ . This is because of the recursion stack. At most, there will be `n` function calls on the stack.

##### #### **Iterative Approach:**

###### - **Time Complexity:**

The time complexity of the iterative approach is  $O(n)$ . This is because the loop runs `n-1` times to calculate the Fibonacci number.

###### - **Space Complexity:**

The space complexity is  $O(1)$  because the iterative approach only requires a constant amount of space (for `prev1`, `prev2`, and `result`), regardless of the value of `n`.

#### ### 5. **Conclusion:**

- The Fibonacci series can be computed both recursively and iteratively.
- The recursive approach is less efficient due to its exponential time complexity and higher space complexity due to the recursion stack.
- The iterative approach is more efficient with linear time complexity and constant space complexity.

#### ### 6. **Review Questions and Answers:**

1. **What is the Fibonacci series?**

The Fibonacci series is a sequence of numbers where each number is the sum of the two preceding ones, starting from 0 and 1.

2. **What is recursion?**

Recursion is a programming technique where a function calls itself in order to solve smaller instances of the same problem.

3. **Analyze time and space complexity of the Fibonacci series?**

- Recursive Approach:

- Time complexity:  $O(2^n)$  (exponential)

- Space complexity:  $O(n)$  (due to the recursion stack)

- Iterative Approach:

- Time complexity:  $O(n)$  (linear)

- Space complexity:  $O(1)$  (constant)