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Development of Algorithm for Removing Duplicate Points within a
         specified tolerence in 3D
         After validating the 2D algoritham I improved it to 3D. Here the same logic is applied in 3 dimesions.
         Possible critical test cases considered for which a simpler algorithm
         would have failed
           1. Consider a case where input points are nothining but a single line with distance any two consecutive points is less than the
             tolerence. If I am deleteing the first point instead of second point every time, I will end up with only the exteme last point.
             Which is not a feasible solution. So I am removing the second point while keeping the first point and comparing the rest
             points with the same first point again.
         Problem analysis
         I think the given problem can have multiple solutions depending upon where we start from the first point and if the tolerence is
         not very small. In case of a quadrilateral with side equal to tolerence, it may give two different dialgonal points depending upon
         where we start with.
         Code analysis and testing
         I have tested the code by increaing and reducing
           1. Tolerence
           2. Number of Input points
           3. Bounding box of the input points
         Inference
         The time for the algorithm varies depending upon the number of input points and also the value of tolerence.
         Possible improvement and application
           1. I think the time can further be improved by using range tree in data structures.
           2. We can use this algorithm as an application to mesh cleanup in any FEA or CAD package. Instead of removing the
             duplicate point we can insert the mid-point of those two duplicate points and remove both of them. This is only useful if
             the tolerence is very small compared to point cloud or object dimensions.
 In [1]: # File creation
          # Last updated by Onkar Salunkhe on 25 Oct 2020
          import random
          # Function for creating a random Point with Float values
          def random_floats(min, max, size):
             # min = Minimum value of co-ordinate
             # max = Maximum value of co-ordinate
             # size = 2 for 2D point and 3 for 3D point
             return [random.uniform(min, max) for _ in range(size)]
          # Create a file
          Data= open("Random_Points_3D.txt","w+")
          K = 1000 # Number of Input Points
          # Writing the Points in the file
          with open('Random_Points_3D.txt', 'w+') as filehandle:
              for i in range(K):
                Point = random_floats(0, 1, 3)
                  #filehandle.writelines("%s" % x for x in Point)
                  filehandle.writelines(" ".join(str(x) for x in Point)) filehandle.writelines("\n")
          # Close file
         Data.close
 Out[1]: <function TextIOWrapper.close()>
 In [2]: # Opening an existing file with point cloud
          Data=open("Random_Points_3D.txt", "r")
          # Reading each point
          Point=Data.readlines()
          # Writing each point in the list
         lst3D = []
          for i in Point:
              #print(i)
              m1=i.split()
              arr=[]
              for j in range(3):
                  arr.append(float(m1[j]))
              lst3D.append(arr)
          # Total number of Points in the file
          print(len(lst3D))
         1000
In [11]: # Functions which can also be included in the header file
          # Function to calculate distance between two points in 2D
          def distance2D(Point1, Point2):
           # Point1 = List of co-ordinates of fisrt point in 2D
            # Point2 = List of co-ordinates of second point in 2D
            # return the distance between the points using distance formula
            return abs(math.sqrt((Point1[0]-Point2[0])**2+(Point1[1]-Point2[1])**2))
          # Function to calculate distance between two points in 3D
          def distance3D(Point1, Point2):
           # Point1 = List of co-ordinates of fisrt point in 3D
           # Point2 = List of co-ordinates of second point in 3D
            # return the distance between the points using distance formula
            return abs(math.sqrt((Point1[0]-Point2[0])**2+(Point1[1]-Point2[1])**2+(Point1[2]-Point2[2
          ])**2))
          # Function to calculate distance along x-axis
          def distance_x(Point1, Point2):
           # Point1 = List of co-ordinates of fisrt point in 2D or 3D
           # Point2 = List of co-ordinates of second point in 2D or 3D
            # return the value of absolute difference in x-co-ordinates
            return abs(Point1[0]-Point2[0])
          # Function to calculate distance along y-axis
          def distance_y(Point1, Point2):
           # Point1 = List of co-ordinates of fisrt point in 2D or 3D
           # Point2 = List of co-ordinates of second point in 2D or 3D
            # return the value of absolute difference in y-co-ordinates
            return abs(Point1[1]-Point2[1])
          # Function to calculate distance along z-axis
          def distance_z(Point1, Point2):
           # Point1 = List of co-ordinates of fisrt point in 2D or 3D
            # Point2 = List of co-ordinates of second point in 2D or 3D
            # return the value of absolute difference in z-co-ordinates
            return abs(Point1[2]-Point2[2])
In [12]: # Algorithm for removing the points within tolerence
          import matplotlib as mpl
          from mpl_toolkits.mplot3d import Axes3D
          import matplotlib.pyplot as plt
          import itertools
          import random
          import time
          import math
          # Start the timer
          start=time.time()
          # Sort the list with x-co-ordinates
          updated_list_3D = sorted(lst3D)
          # Number of points in the Point cloud
          print(len(updated_list_3D))
          # Tolerence under which the points to be removed
          tol=0.000001
          # Algorithm
         i=0 # Start with the first element in the updated list as the base point
          while i<len(updated_list_3D)-1:</pre>
           Count=0
            # Creating a slab of x-tolerance and count the number of points in that slab for x-toleren
            while Count<len(updated_list_3D)-i and distance_x(updated_list_3D[i], updated_list_3D[i+Cou</pre>
          nt])<tol:
              Count+=1
            # Save the starting point of the x-slab as base point of the slab
            # Maximum range of points in the x-slab of x-tolerence
            n=i+Count+1
            # Traverse upto ponits in the x-slab
            while x<n:
                # Traverse through the slab
                j=x+1
                # Traverse upto the range of x-slab
                while j<n-1:
                  # Check whether the y-co-ordinate is within tolerence or not
                  if distance_y(updated_list_3D[x], updated_list_3D[j])<tol:</pre>
                      # Check the actual distance between the points
                      dist=distance3D(updated_list_3D[x], updated_list_3D[j])
                      # Remove the points under tolerence else check for the next point
                      if dist<tol:</pre>
                           updated_list_3D.pop(j)
                           j -=1
                           n-=1 # Upadating the range of the x-slab
                  j+=1
                x + = 1
            i+=1
          # Stop the timer
          end=time.time()
          # Number of points in the updated list
          print(len(updated_list_3D))
          # Print the total time for the algorithm
          print(end-start)
          # 3D Visualization
          fig = plt.figure()
          ax = fig.gca(projection='3d')
          plt.figure(1)
          for p in lst3D:
              ax.scatter(p[0], p[1], p[2], zdir='z', c='r')
          plt.title('Input')
          fig = plt.figure()
          bx = fig.gca(projection='3d')
          plt.figure(2)
          for p in updated_list_3D:
              bx.scatter(p[0], p[1], p[2], zdir='z', c='r')
          plt.title('Output')
          plt.show()
         1000
         195
         0.05458807945251465
                                                  0.4
                                                  0.2
                                                 0.0
                  0.2 0.4 0.6 0.8 1.0
                                        0.2
                                                  0.8
                                                  0.6
                                                  0.4
                                                  0.2
                  0.2 0.4
                          0.6
                             0.8
                                 1.0 0.0
In [13]: #Brute Force Method with O(n^2)
          #Calculating dist with each point with each
          import matplotlib as mpl
          from mpl_toolkits.mplot3d import Axes3D
          import matplotlib.pyplot as plt
          import itertools
          import random
          import time
          import math
          start=time.time()
          # Sorting the list with x-co-ordinate
          updated_list_ref = sorted(lst3D)
          print(len(updated_list_ref))
          tol=0.000001
         i=0
          while i< len(updated_list_ref)-1:</pre>
            j=i+1
            while j <len(updated_list_ref):</pre>
              if distance_x(updated_list_ref[i], updated_list_ref[j])<tol and distance_y(updated_list_r</pre>
          ef[i],updated_list_ref[j])<tol :</pre>
                if distance_z(updated_list_ref[i], updated_list_ref[j])<tol:</pre>
                  dist = distance3D(updated_list_ref[i], updated_list_ref[j])
                  if dist<tol:</pre>
                    updated_list_ref.pop(j)
                    j-=1
              j+=1
            i+=1
          end=time.time()
          # Number of Points in the updated list
          print(len(updated_list_ref))
          # Print the total time for algorithm
          print(end-start)
          # 3D Visualization
          fig = plt.figure()
          ax = fig.gca(projection='3d')
          plt.figure(1)
          for p in lst3D:
              ax.scatter(p[0], p[1], p[2], zdir='z', c='r')
          plt.title('Input')
          fig = plt.figure()
          bx = fig.gca(projection='3d')
          plt.figure(2)
          for p in updated_list_ref:
              bx.scatter(p[0], p[1], p[2], zdir='z', c='r')
          plt.title('Output')
          plt.show()
         1000
          195
         0.04162168502807617
                                                  0.6
                                                  0.4
                                                  0.2
                  0.2 0.4 0.6 0.8 1.0
                                        0.2
                                                  1.0
                                                  0.4
                                                  0.2
              0.0 0.2 0.4 0.6 0.8 1.0 0.0
         We get the same results for both Brutforce method and Algorithm but with the reduced time.
In [14]: # Create a new file to store the updated points
          Data= open("Updated_Points_3D.txt", "w+")
          # Writing the new Points in the file
          with open('Updated_Points.txt', 'w+') as filehandle:
              for i in range(0,len(updated_list_3D)):
                  filehandle.writelines(" ".join(str(x) for x in updated_list_3D[i]))
                  filehandle.writelines("\n")
          # Close file
         Data.close
Out[14]: <function TextIOWrapper.close()>
          # Tolerence = 0.000001
          import statistics
          nn=[50,100,500,1000,5000,10000] # Number of Points in the Input
          # Time in seconds for each case
          t_{algo} = [0.00044536590576171875, 0.0005443096160888672, 0.0014872550964355469, 0.002753973007202]
```

```
import statistics
nn=[50,100,500,1000,5000,10000] # Number of Points in the Input
# Time in seconds for each case
t_algo=[0.00044536590576171875,0.0005443096160888672,0.0014872550964355469,0.002753973007202
1484,0.01472330093383789,0.03570199012756348]
t_brut=[0.0008940696716308594,0.0030884742736816406,0.059842586517333984,0.21149277687072754
,5.373635292053223,22.34713840484619]
plt.figure(3)
plt.loglog(nn,t_brut)
plt.loglog(nn,t_algo)
plt.title('Log-log plot for comparing Time complexity')
plt.legend(['Brutforce method','Algorithm'])
plt.xlabel('Number of Points')
plt.ylabel('Time')
Out[15]: Text(0, 0.5, 'Time')
Log-log plot for comparing Time complexity
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 10^{4}

Brutforce method

Algorithm

 10^{2}

 10^{3}

Number of Points

10¹

10°

10-1

 10^{-2}

 10^{-3}