Introduction to Algorithms for Wireless Communication

LU Hardware Software Codesign
WS15/16

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The overall goal

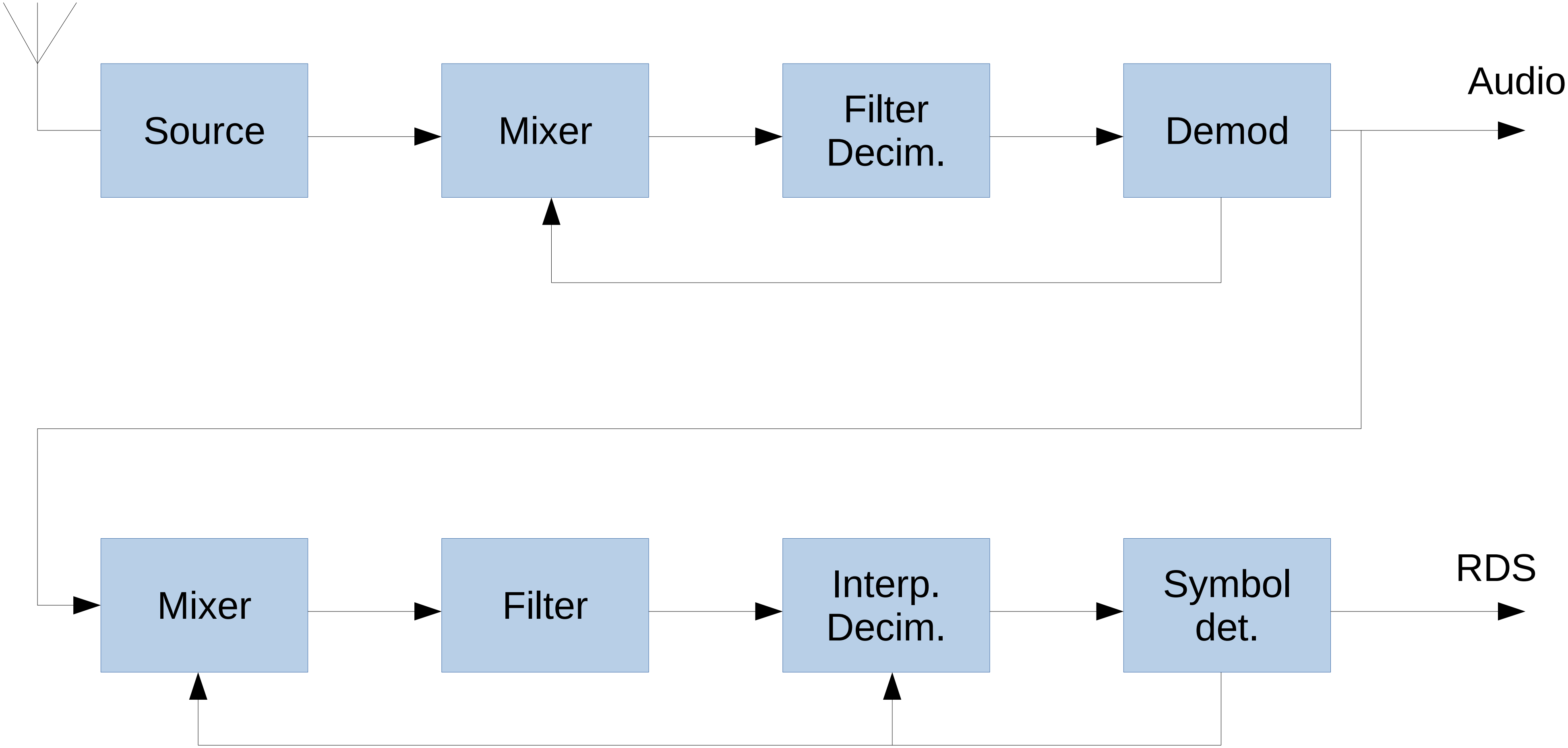
Build an FM receiver:

- Tune to carrier frequency
- Demodulate

Show RDS info

- Tune to subcarrier
- Demodulate
- Decode

System overview



Signal at the input

- IQ samples of baseband signal downmixed from 100.5 MHz
- Sampling rate: 2.5 Msps
- How wide is the spectrum?

  2.5 MHz! (-1.25 MHz to 1.25 MHz)

IQ samples?

- Basically samples as complex numbers
- In-phase (real) and quadrature (imaginary) components
- An RF signal: $A(t) * \cos(2\pi f t + \phi(t))$
  - We need two dimensions for amplitude and phase
- Main advantage:
  - $\sin(x) * \sin(y) = 0.5[\cos(x-y) - \cos(x+y)]$
  - But: $e^x * e^y = e^{x+y}$
- Remember: $e^{i\phi}$ is the unit vector with angle ϕ

IQ sampling

- Generate carrier frequency (100.5 MHz) and a copy of it phase-shifted by $\pi/2$
- Multiply the RF signal by the two, then low-pass-filter and sample separately
- Result is the RF signal frequency-shifted and low-pass-filtered:
  100.5 MHz ± 1.25 MHz
  centered around 0 Hz

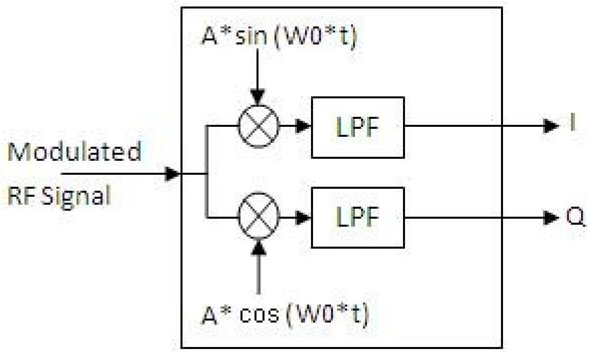
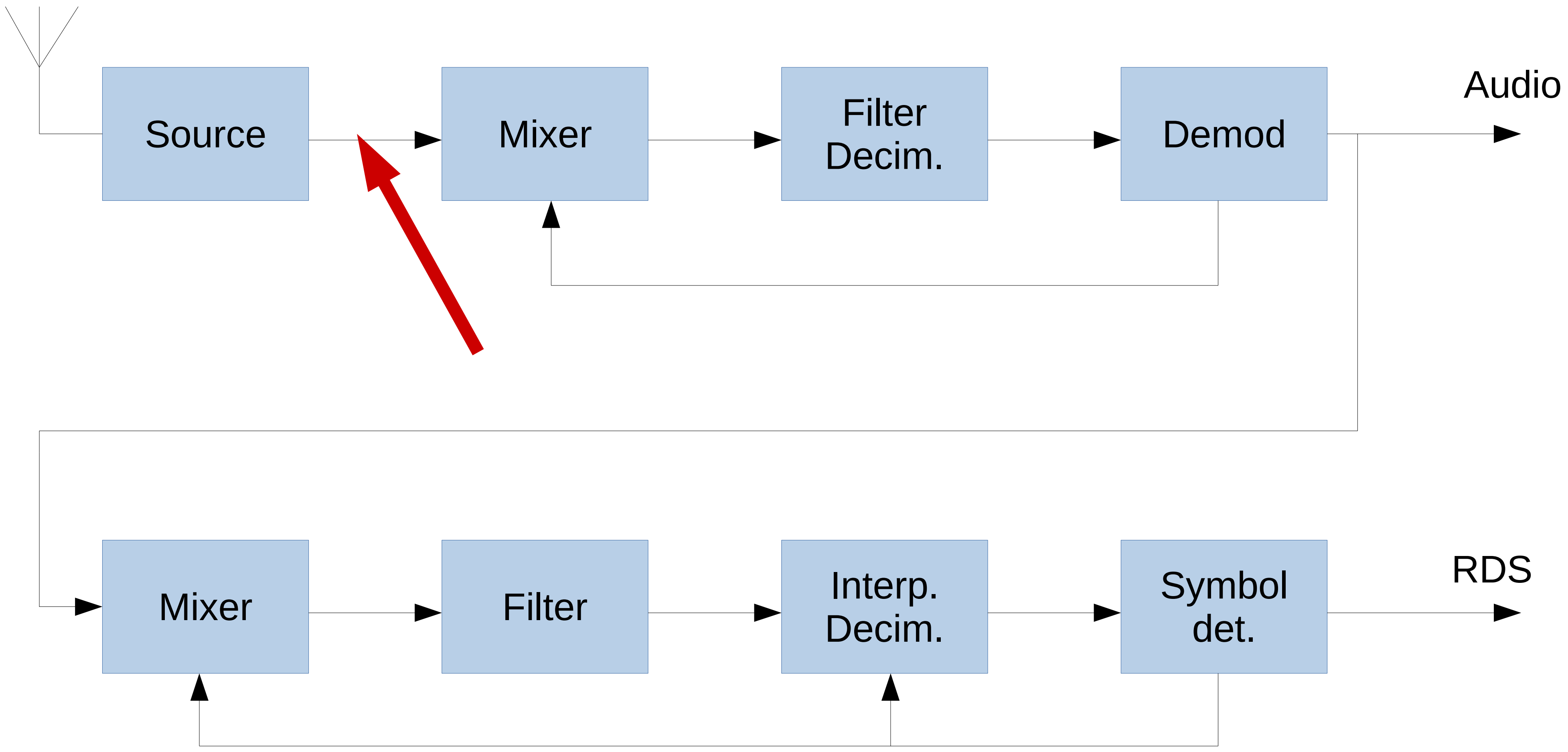


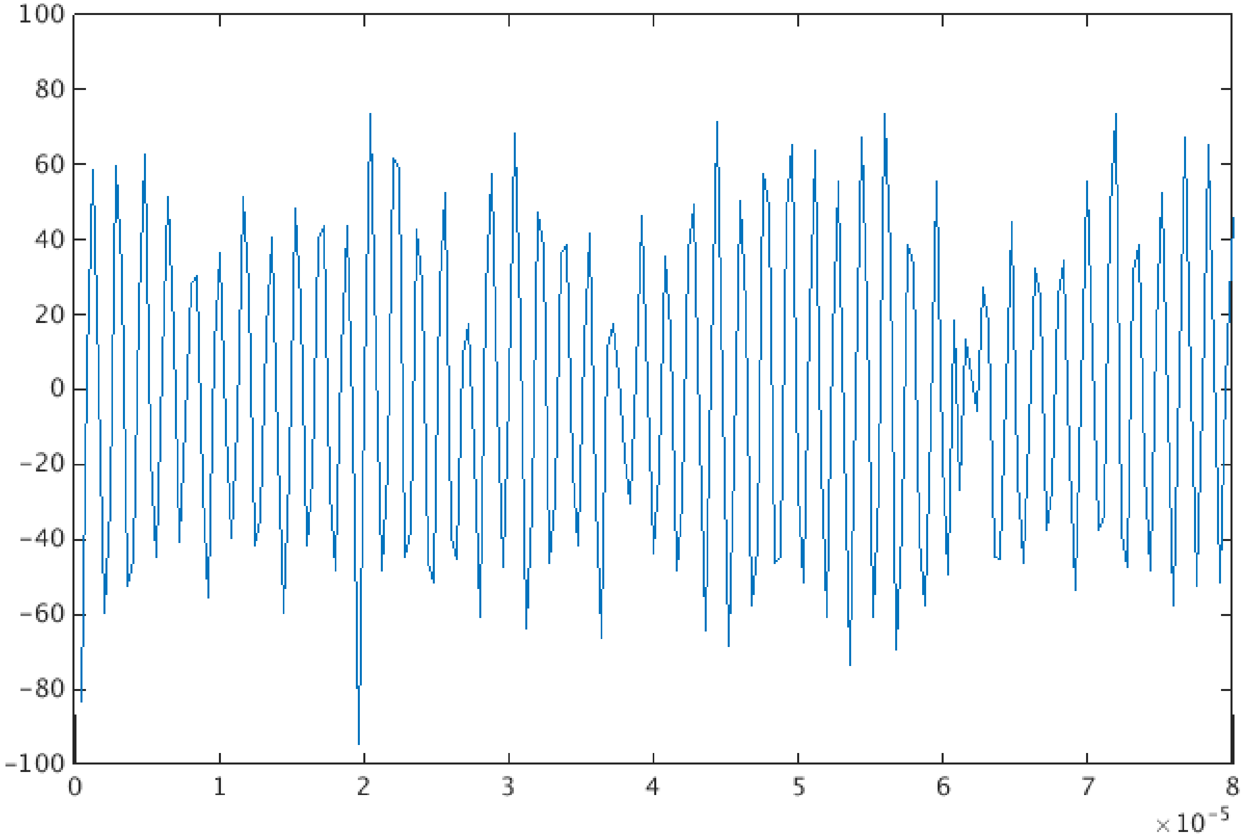
Fig.4 I/Q Demodulator Circuit

[http://www.rfwireless-world.com/Terminology/heterodyne-receiver-versus-homodyne-receiver.html]

Our input signal

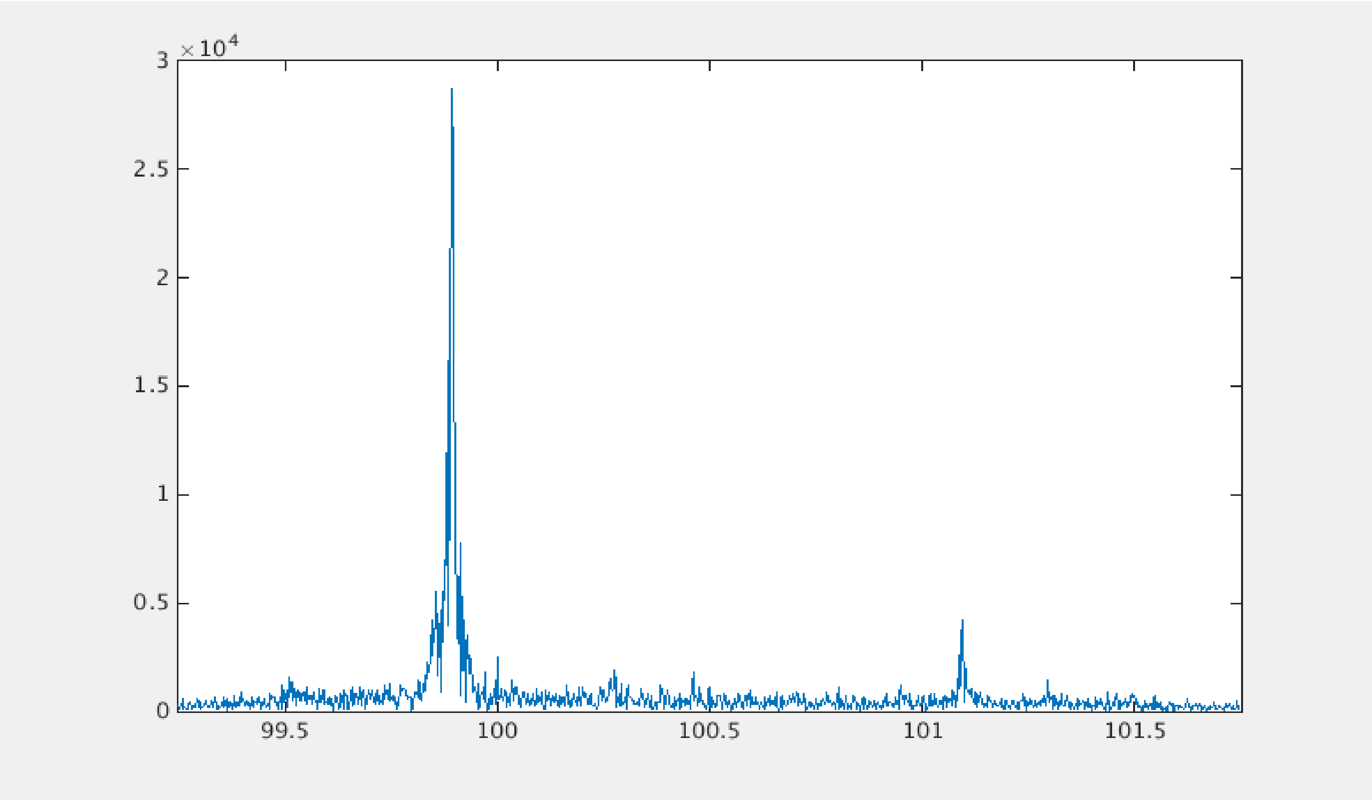


Our input signal



(real part of the signal)

Our input signal



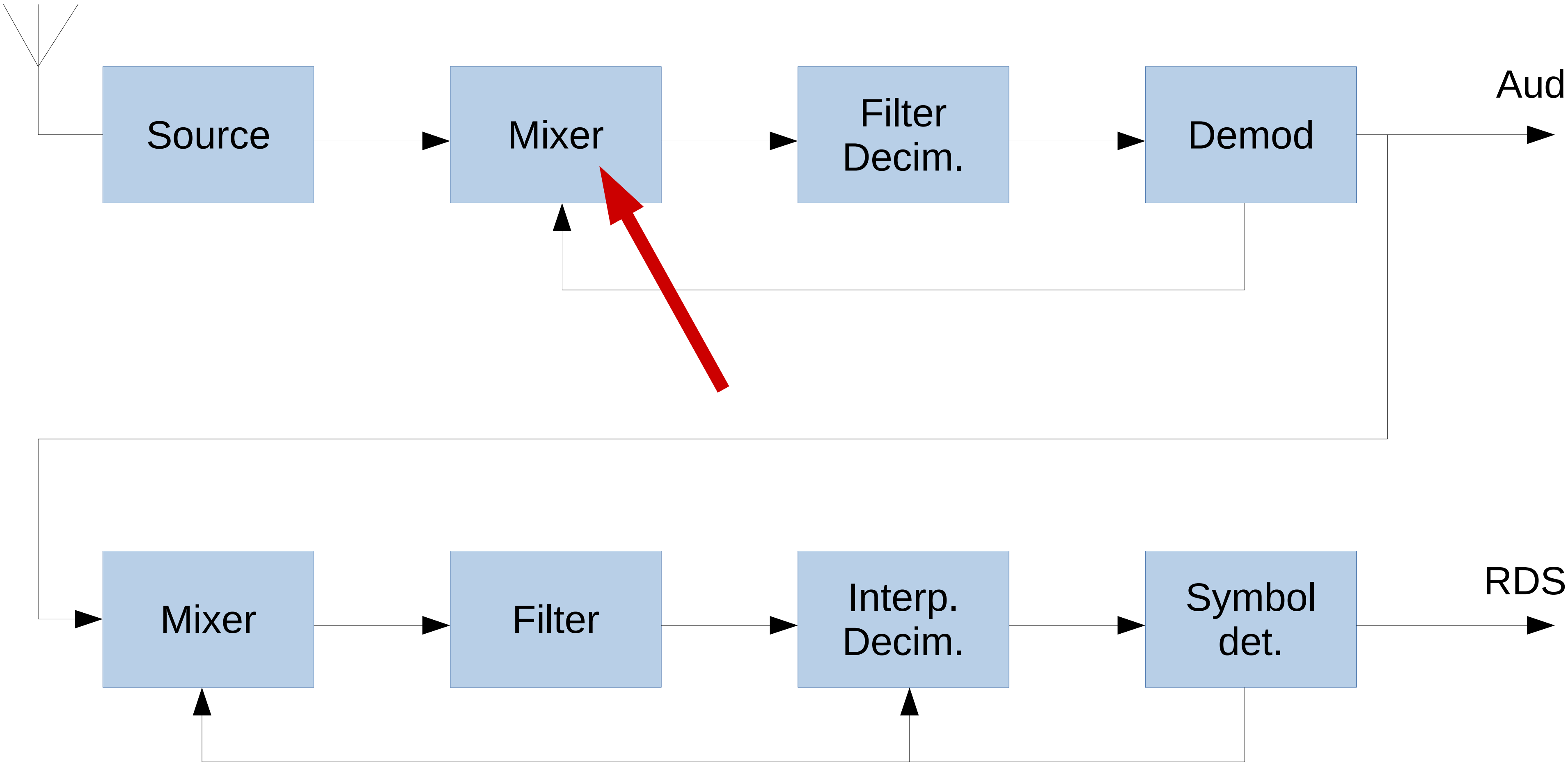
(FFT)

First step

- Tune to the frequency we want:
  99.9 MHz (OE 3)
- Multiplication with the negative carrier frequency:

  $A(t)e^{i2\pi fct+i\theta(t)} * e^{i2\pi(-fc)t} = A(t)e^{i\theta(t)}$
- The component name: Mixer

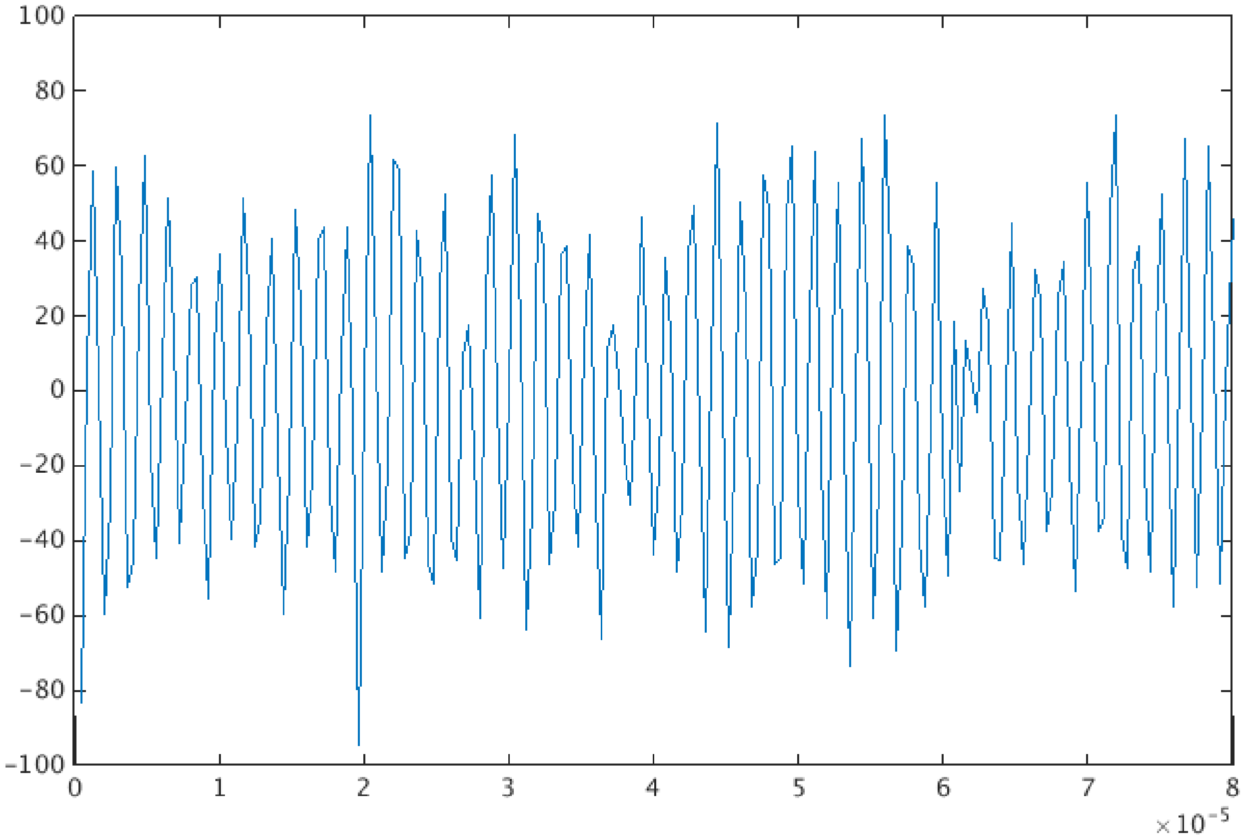
Mixer



Source
Mixer
Filter
Decim.
Demod
Audio
Mixer
Filter
Interp.
Decim.
Symbol
det.
RDS

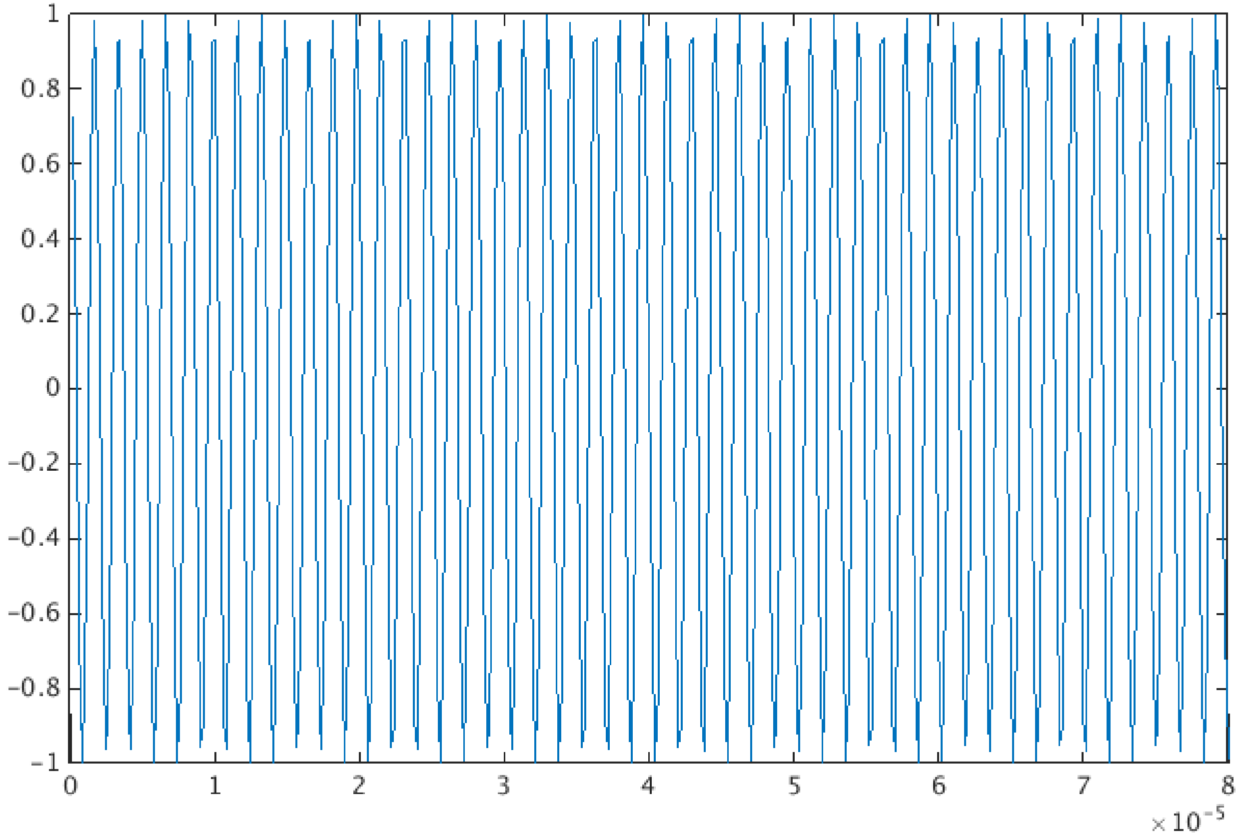
Mixer

Source:



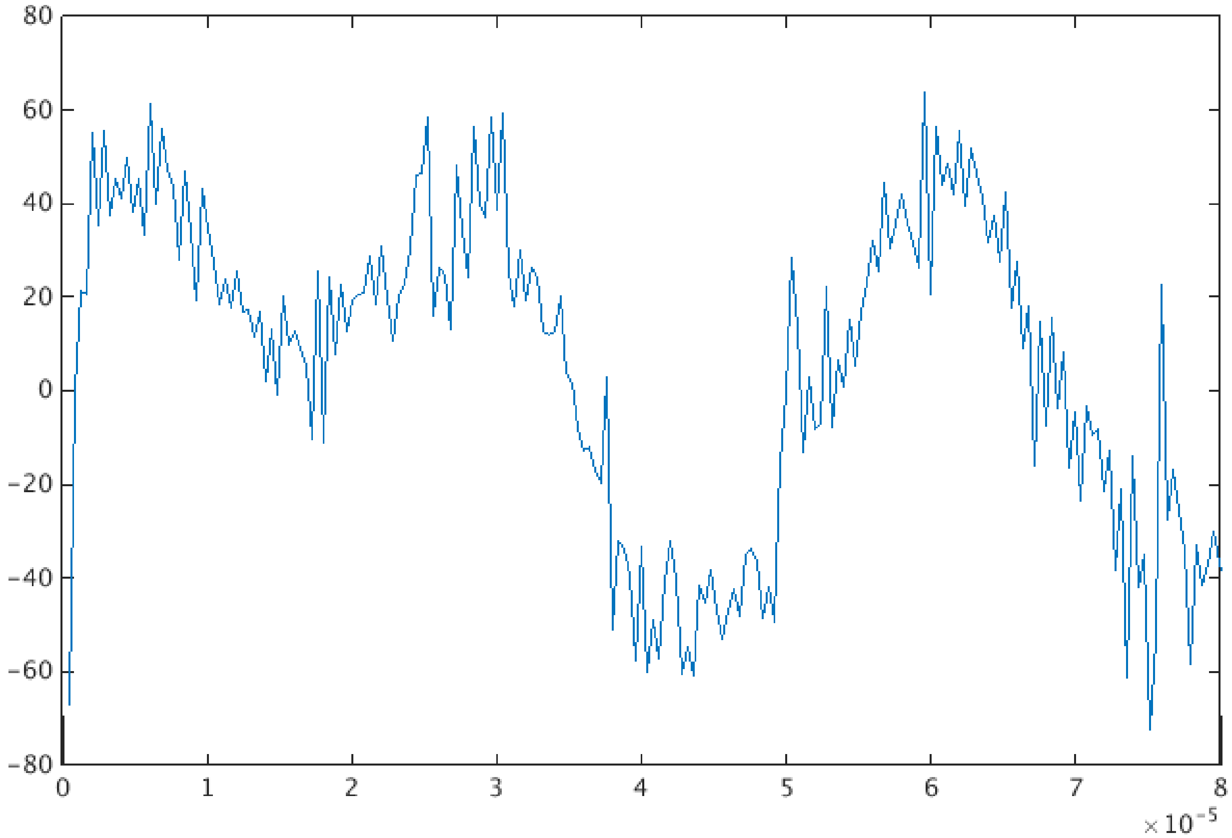
Mixer

Multiplicand (-carrier): (0.6 MHz = -(99.9-100.5))



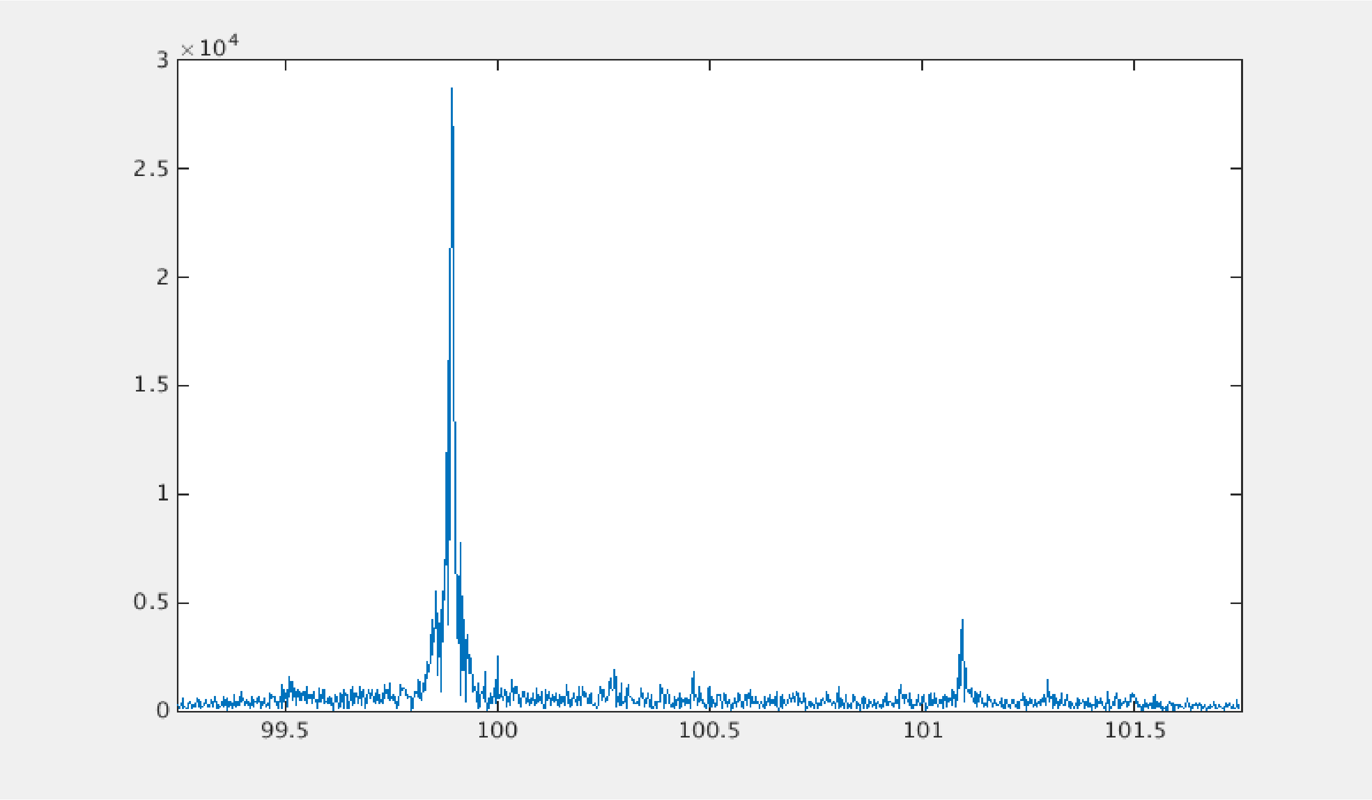
Mixer

Result:



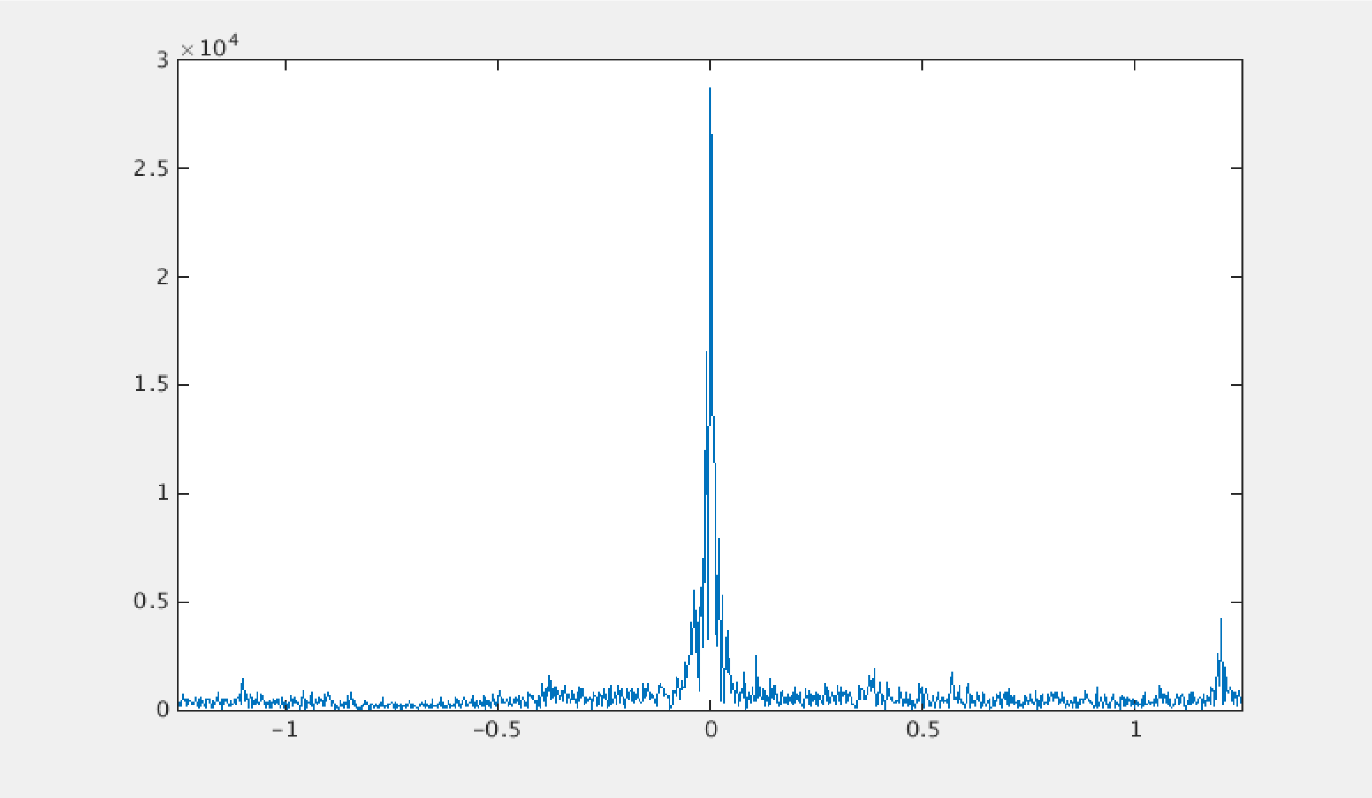
Mixer

FFT before:



Mixer

FFT after:



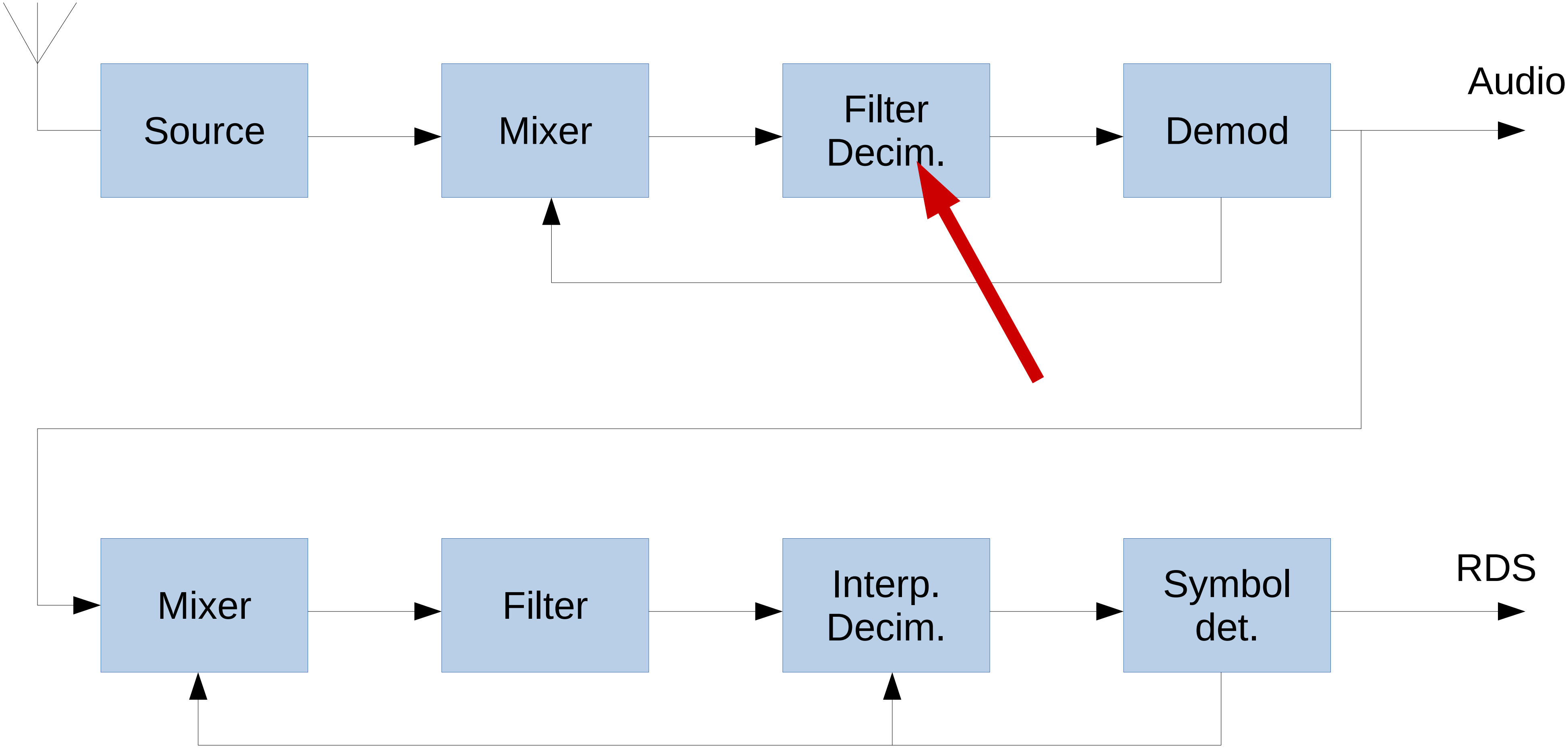
Baseband filter

- Select only relevant signal
- FM bandwidth is max. 200 kHz
  - Low pass filter below 100 kHz
- FIR filter
  - Length is design parameter, affects SNR

Decimation

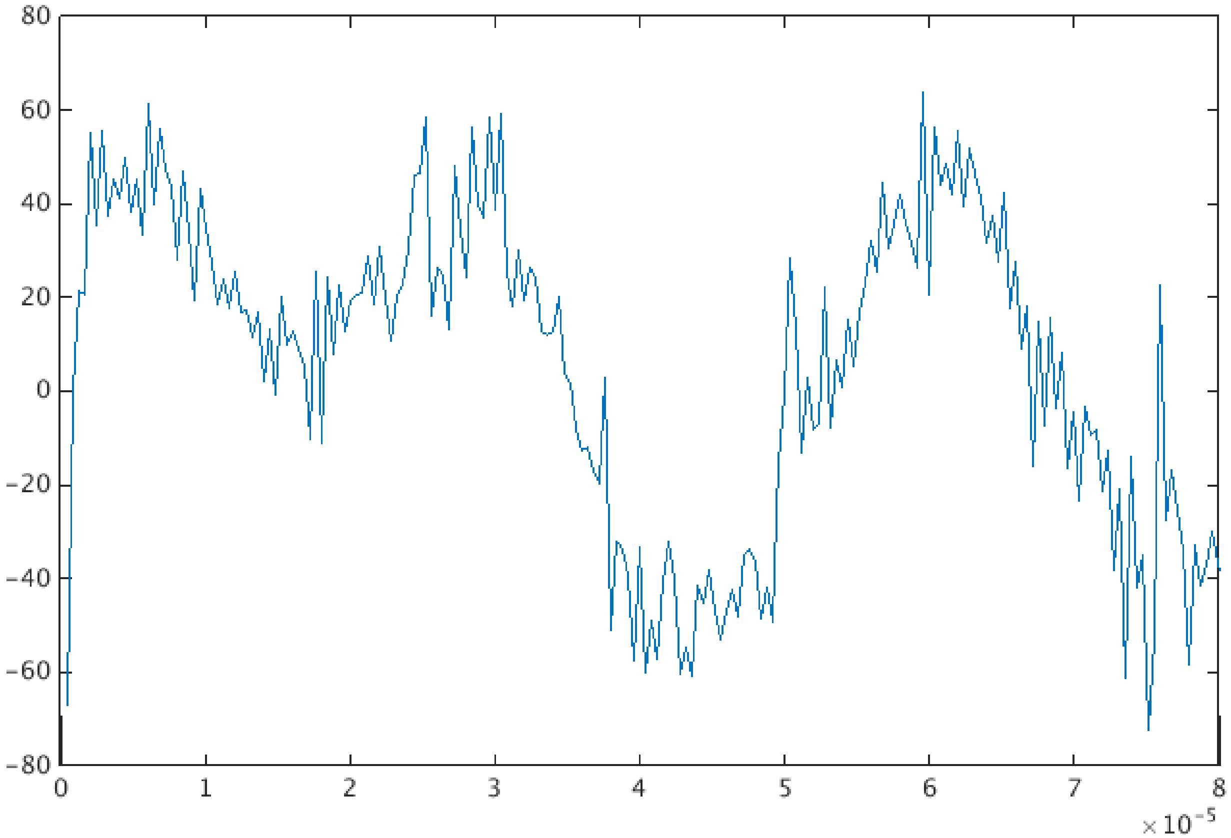
- After filtering, sampling rate unnecessarily high
  - 100 kHz max. frequency, 2.5 Msps
- We can drop every N*th* sample without loss of signal information
  - N is a design parameter, affects algorithmic complexity (demodulation, filters) and computation speed later

Filtering and decimation



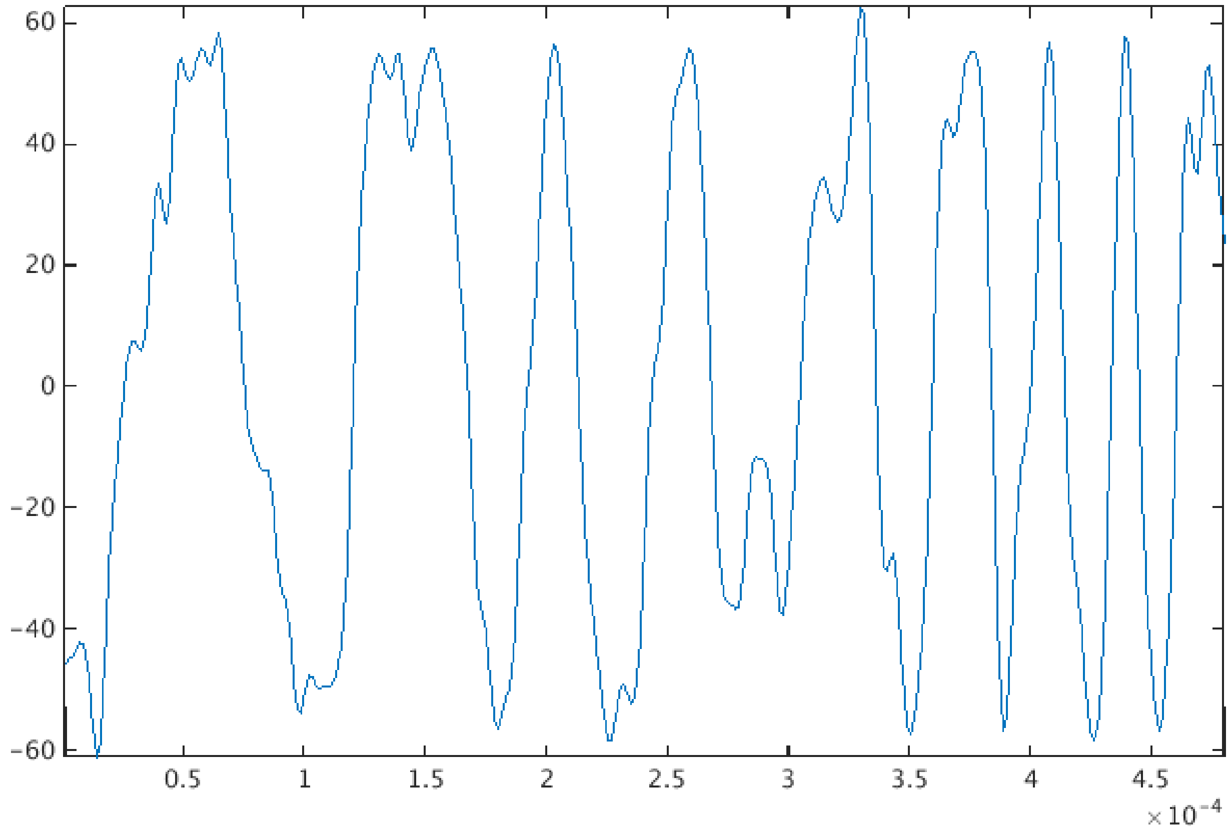
Filtering and decimation

Before:



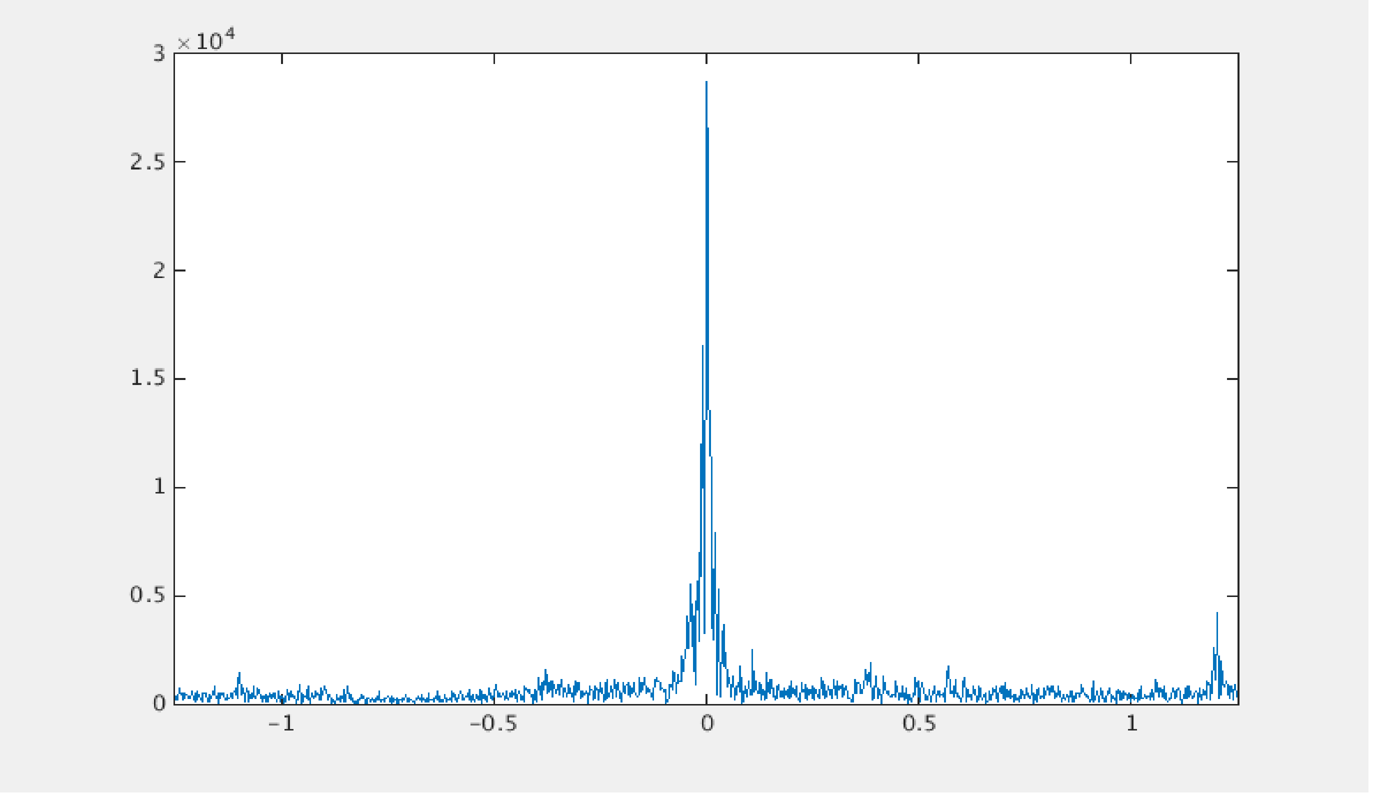
Filtering and decimation

After:



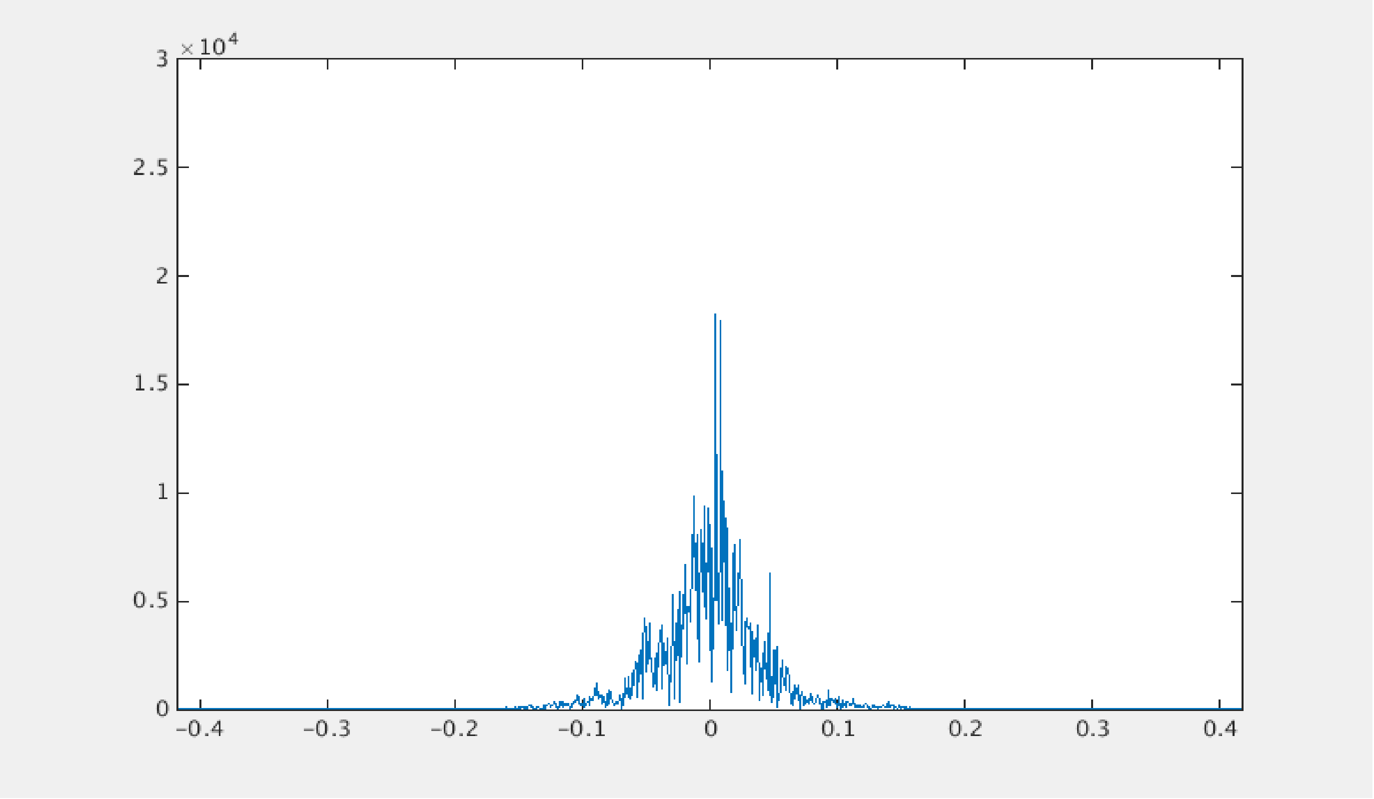
Filtering and decimation

FFT before:



Filtering and decimation

FFT after:

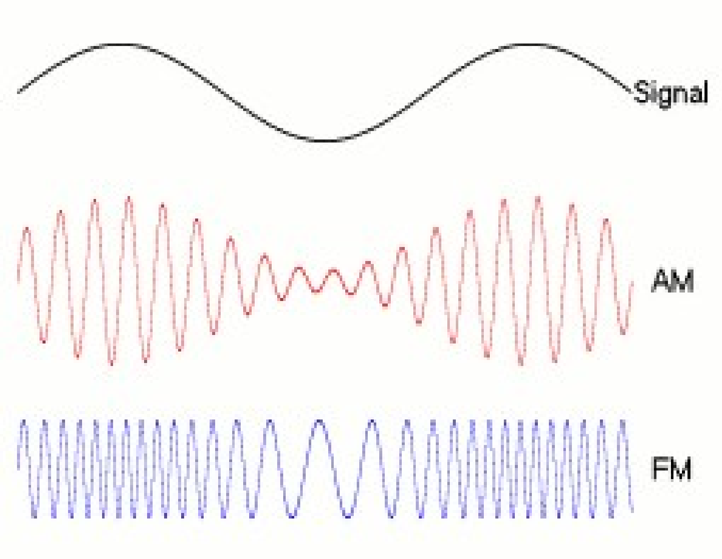


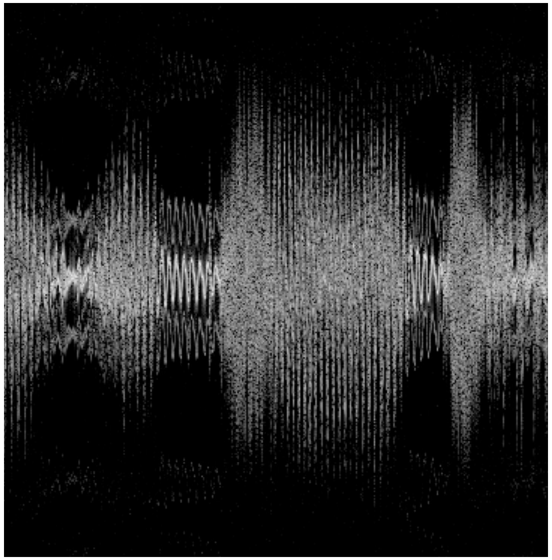
FM Modulation

- The amplitude of the carrier is left unchanged but the frequency is altered:

  $rf(t) = A(t)e^{i2\pi f_c t + i m(t) t}$

  – A(t) constant, frequency deviation m(t) = message





["Amfm3-en-de" by Berserkerus - Own work. Licensed under CC BY-SA 2.5 via Commons - https://commons.wikimedia.org/wiki/File:Amfm3-en-de.gif#/media/File:Amfm3-en-de.gif]

[http://web.stanford.edu/class/ee179/labs/Lab5.html]

FM modulation

- We already removed the carrier:

  $e^{i2\pi f_c t+i m(t) t} * e^{-i2\pi f_c t} = e^{i m(t) t}$

- Frequency ~~deviation~~ = message
- Frequency is phase difference
  - With complex signals phase = angle

FM demodulation #1

- Detect angle difference
- One approximation (has a problem):

    angle(sig(t)) – angle(sig(t-1/fs))

- Simpler:

    angle(sig(t)*conj(sig(t-1/fs)))

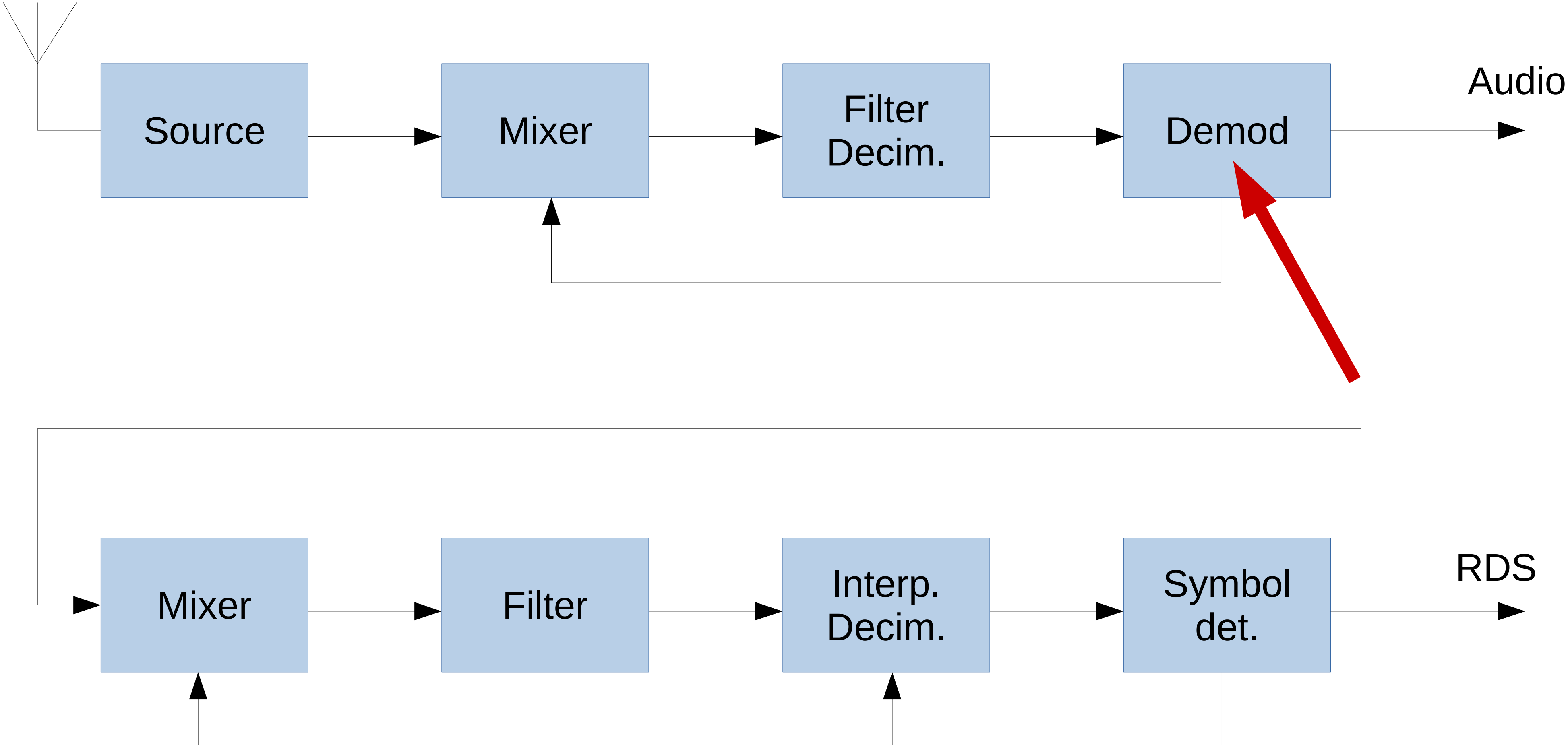
FM demodulation #2

- Differentiate signal and multiply by conjugate of original

  $(d/dt\ e^{im(t)t}) * conj(e^{im(t)t}) =$
  $im(t) * e^{im(t)t} * e^{-im(t)t} =$
  $im(t)$

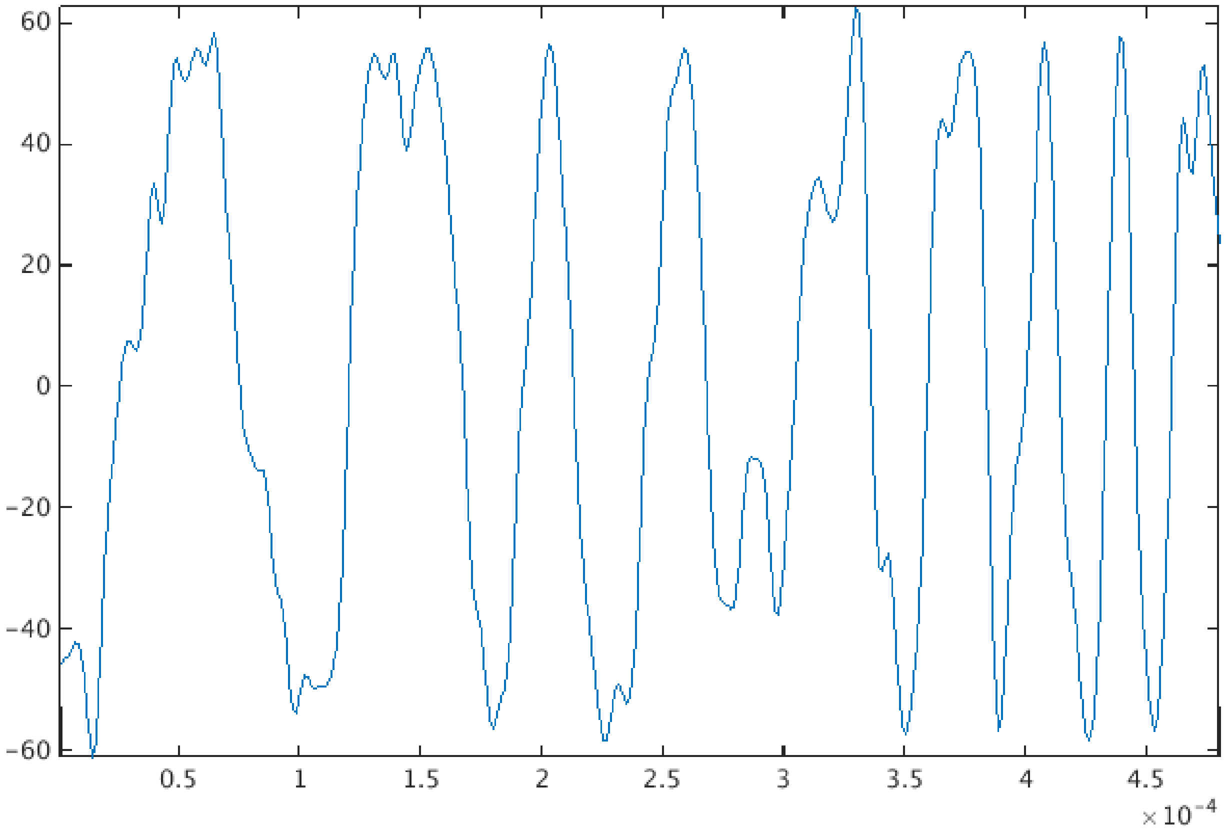
- Differentiation by first order approximation (sig(t)-sig(t-1/fs)) or better with FIR filter
  - See recommended reading http://web.stanford.edu/class/ee179/labs/Lab5.html

Filtering and decimation



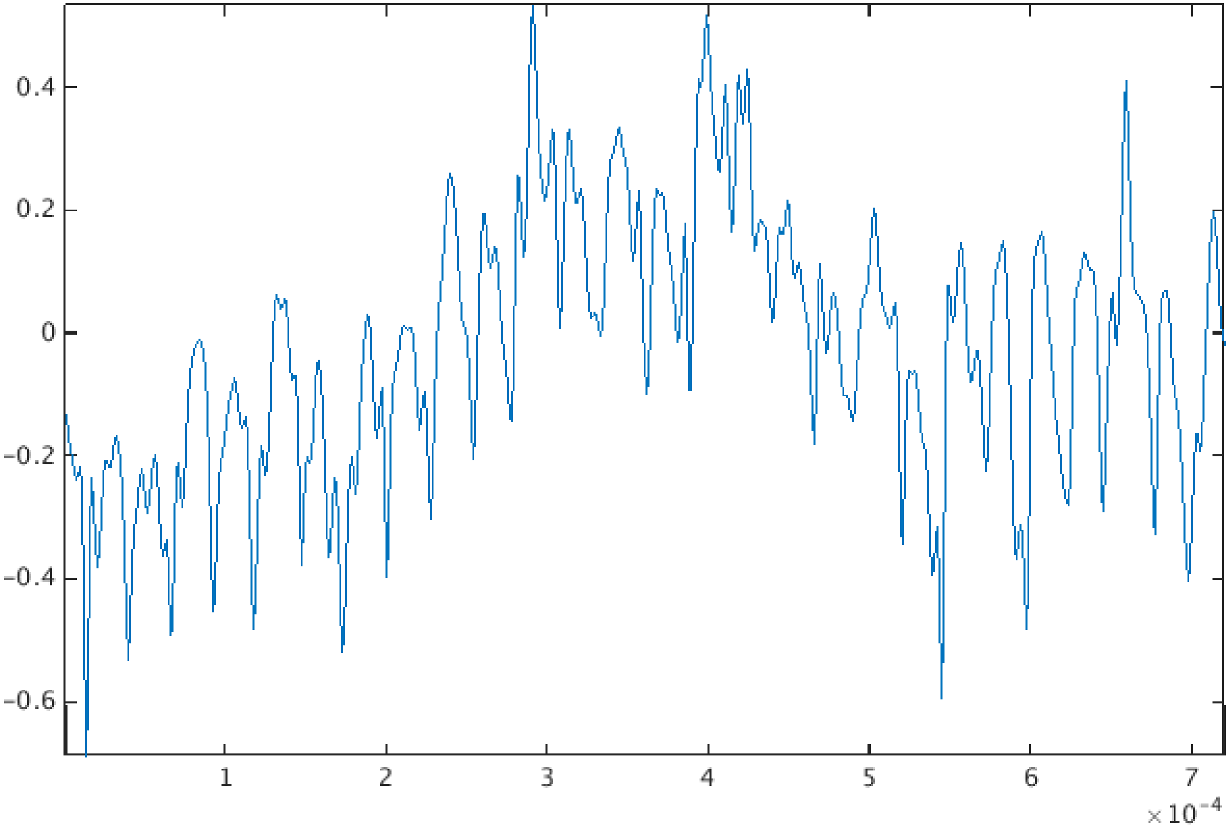
FM demodulation

Before:



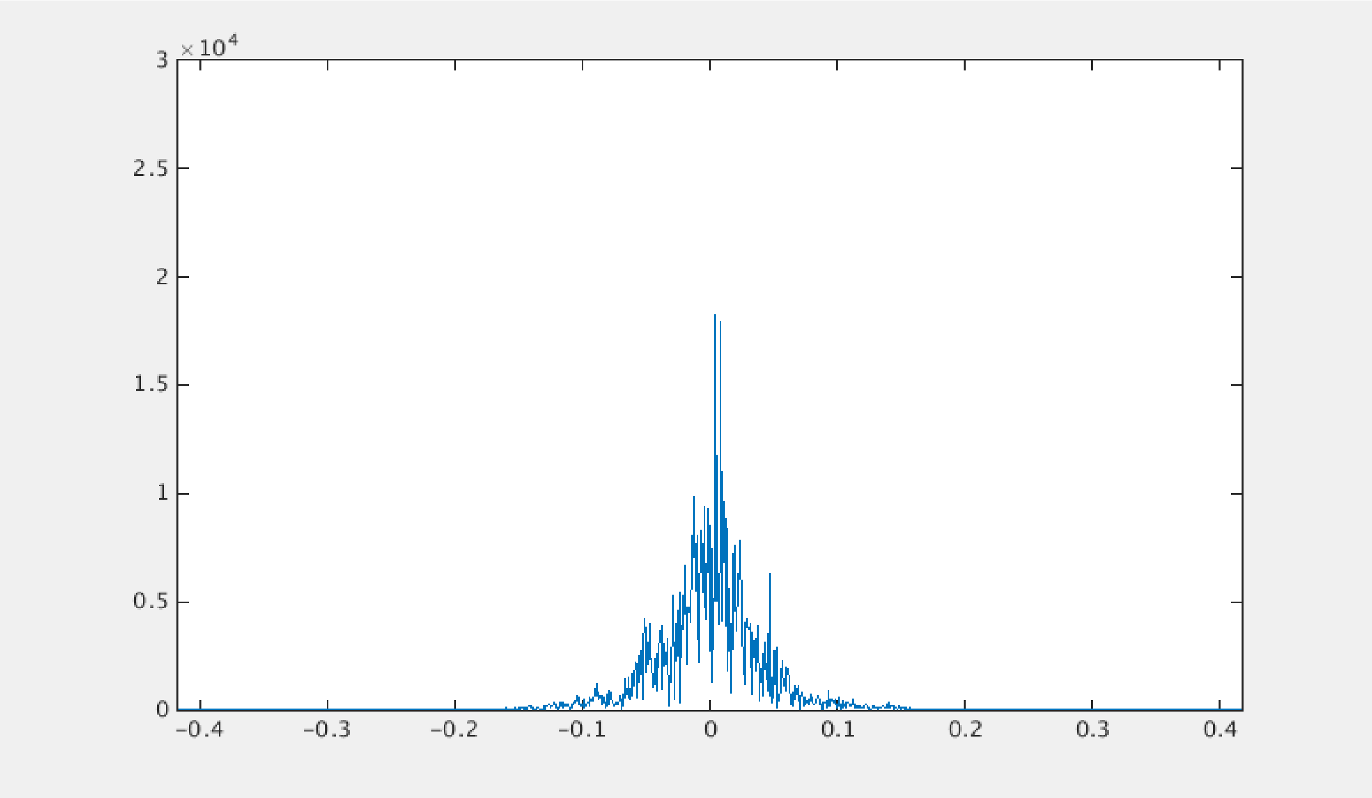
FM demodulation

After (real signal):



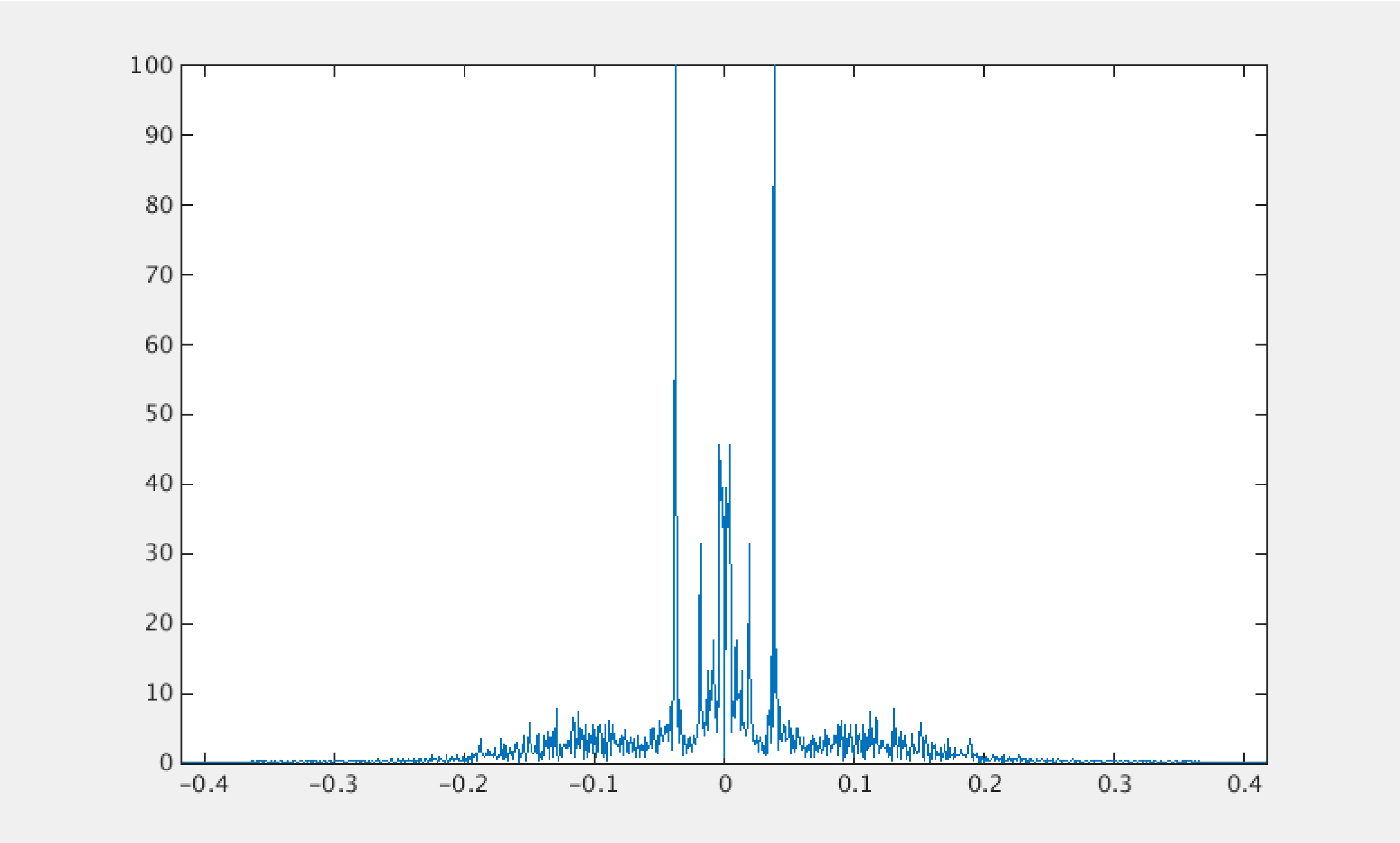
FM demodulation

FFT before:



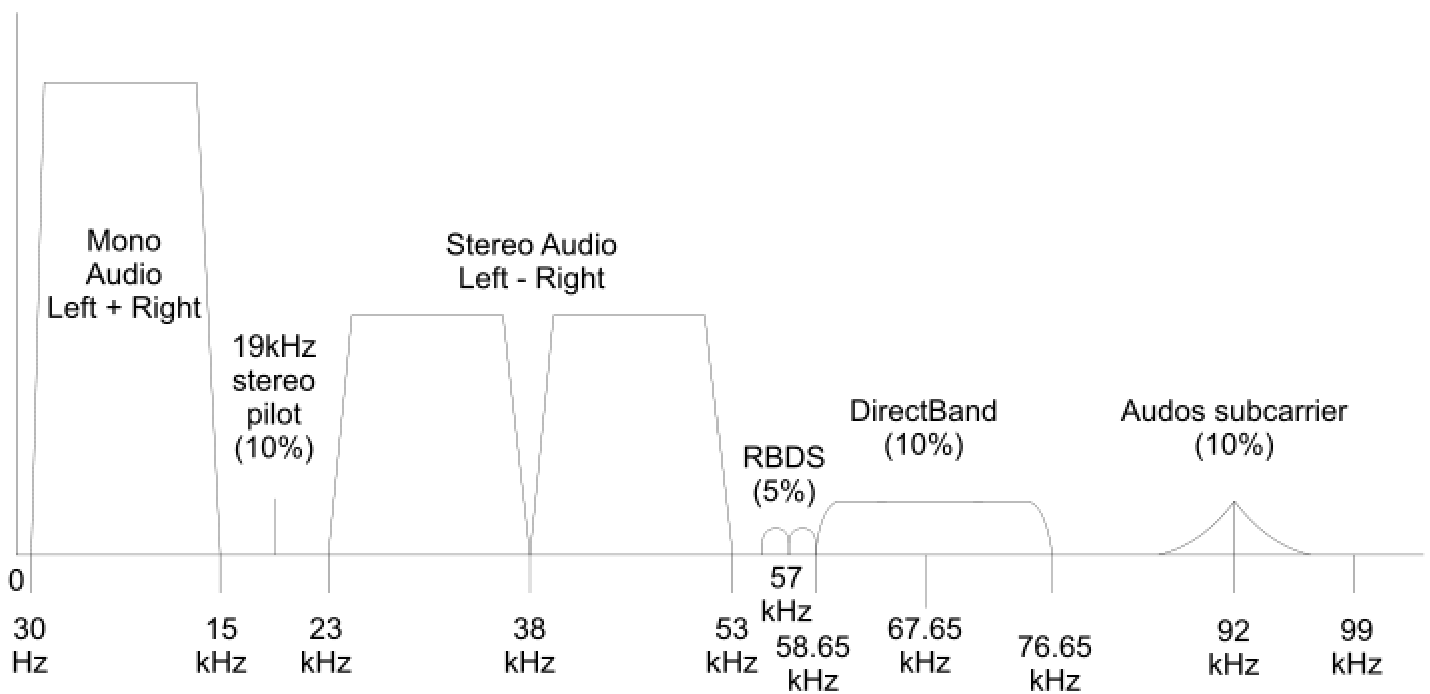
FM demodulation

FFT after:



FM demodulated signal

- Audio output: Low pass below 19 (15) kHz, (resample and) ouput
- RDS: (Again) modulated at 57 ± 2.4 kHz



[Wikipedia]

Carrier frequency error

- When mixing the (estimated) carrier, an error term remains:

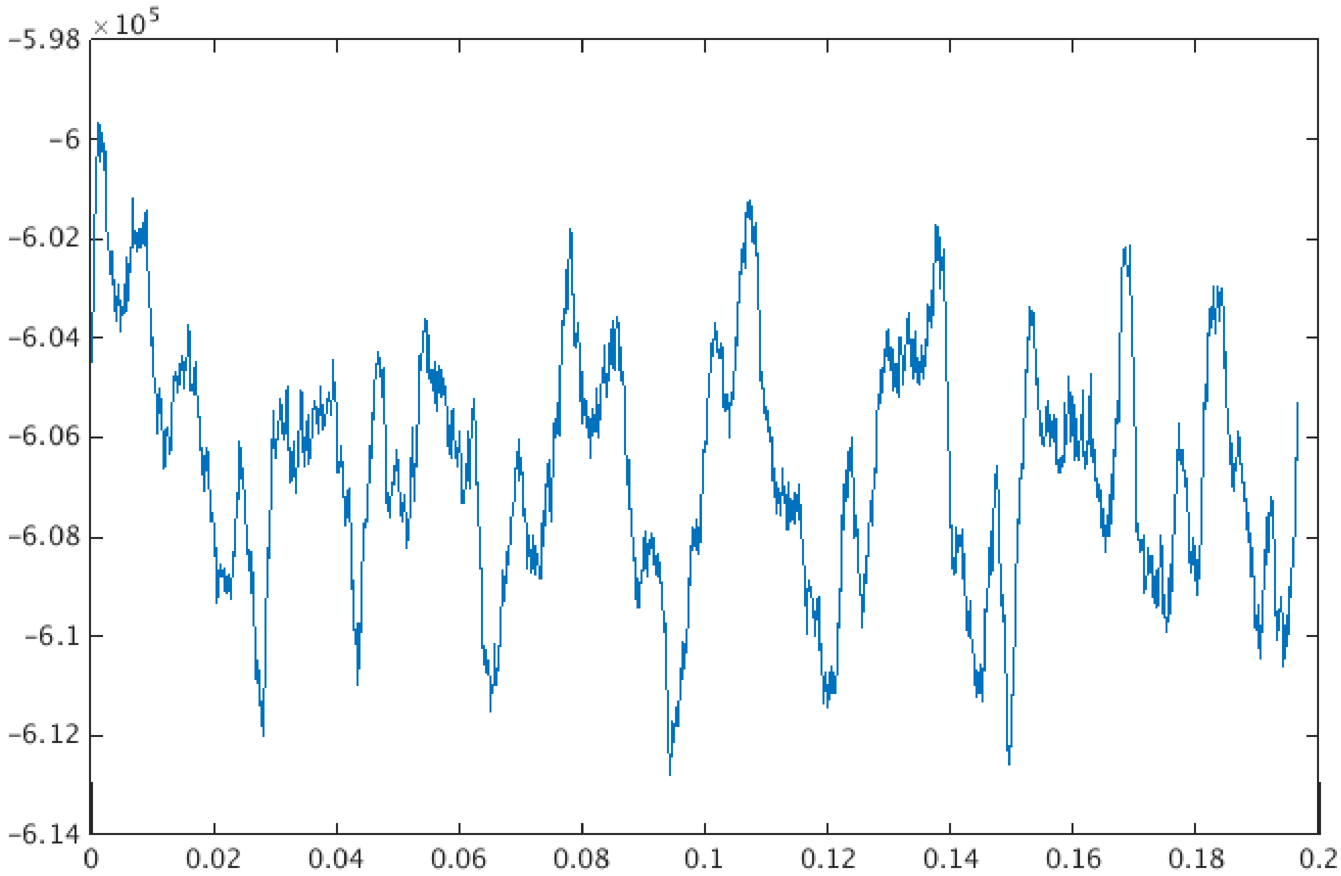
  $e^{i2\pi fc t + i\theta(t)t} * e^{i2\pi(-fc+err)t} = e^{-i\theta(t)t} * e^{i2\pi err t}$

  - A remaining frequency component
- After FM demodulation, this term adds to the message signal
- Assuming it is almost constant, we can correct fc using the low-passed message signal
- Error comes mainly from sampling, not from broadcasting

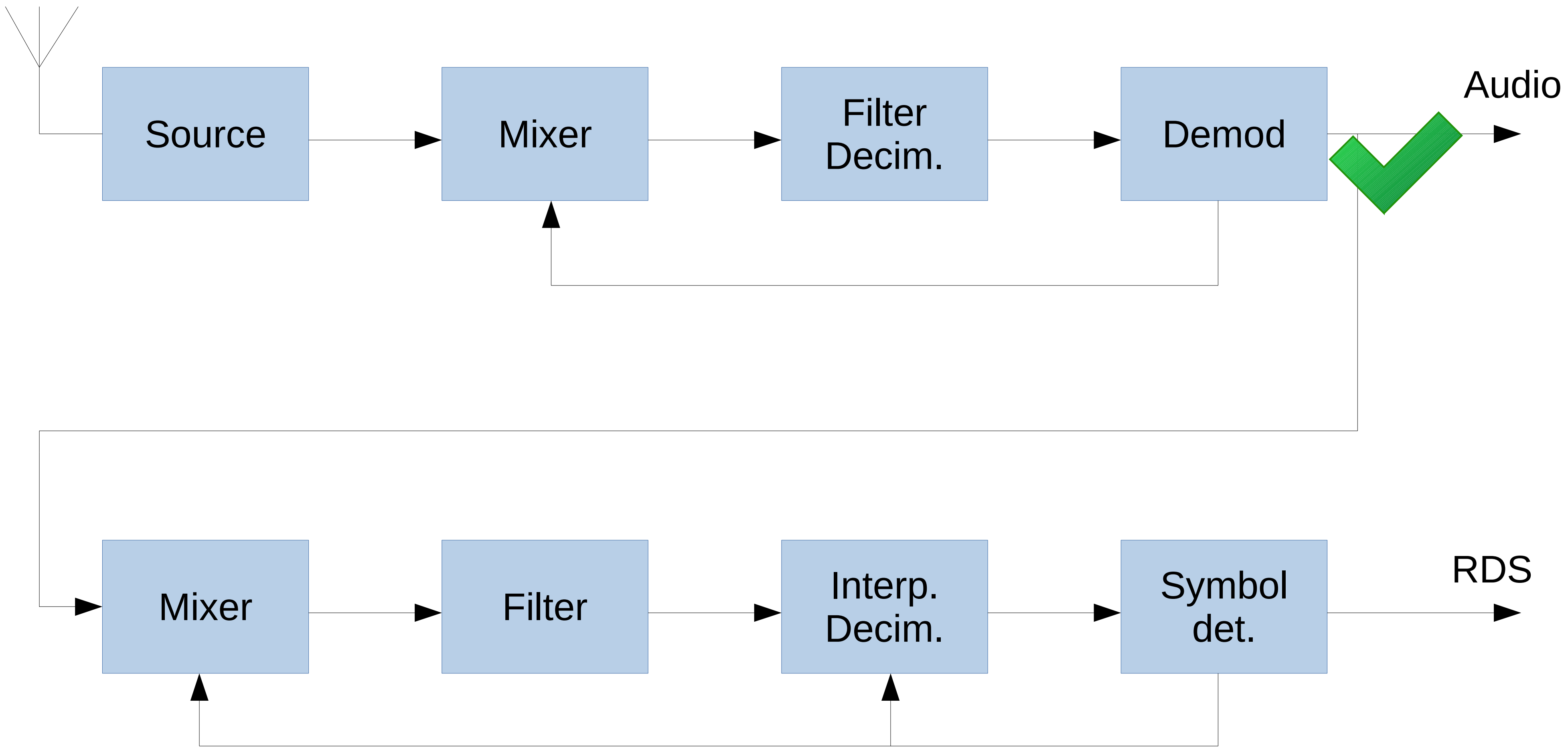
Carrier frequency error

- FM audio starts at 30 Hz – use a low pass filter (well) below that; an IIR filter may be used

fc vs. time



2nd Part


Source
Mixer
Filter
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Demod
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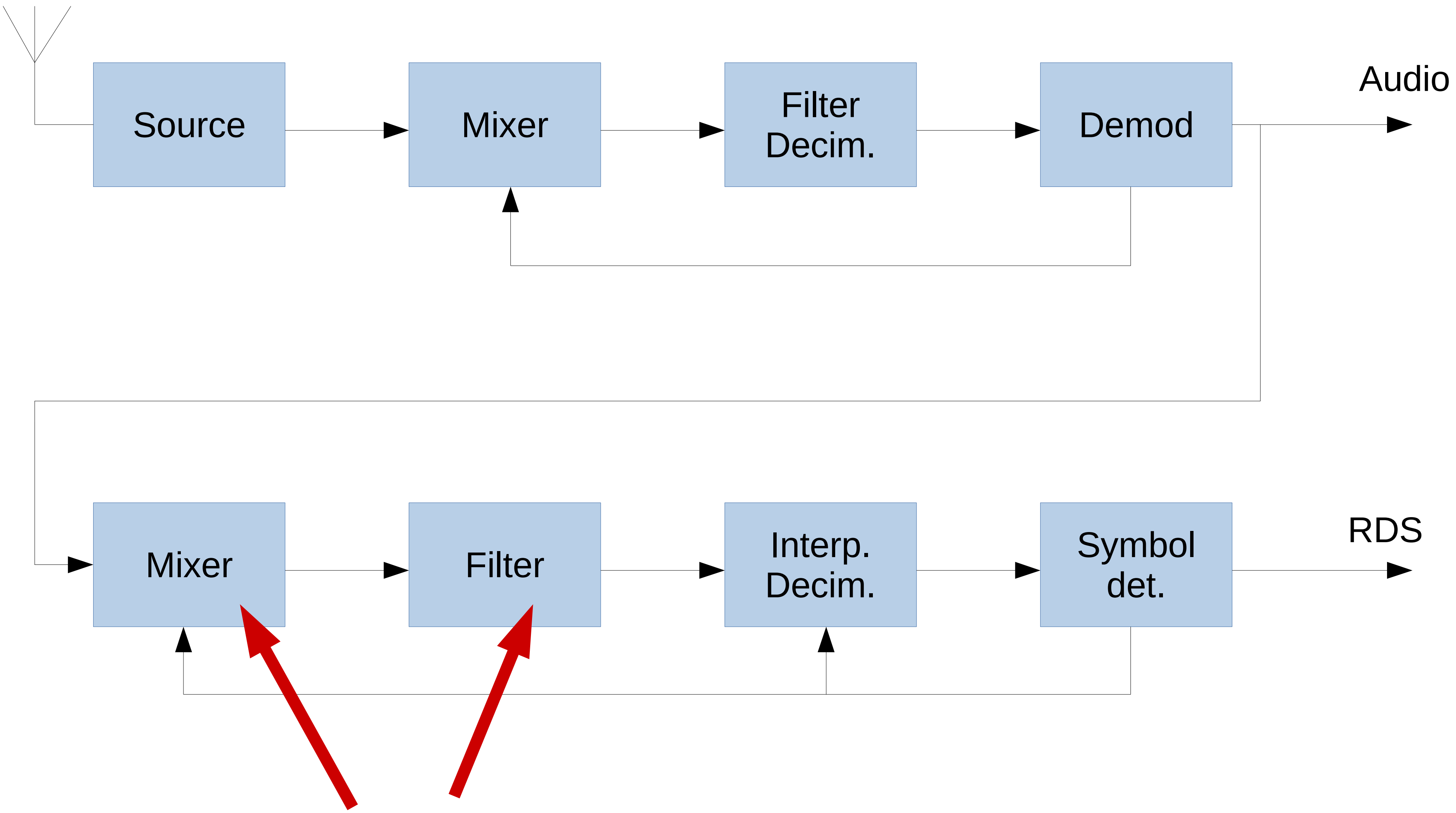
Symbol
det.
RDS

RDS subband

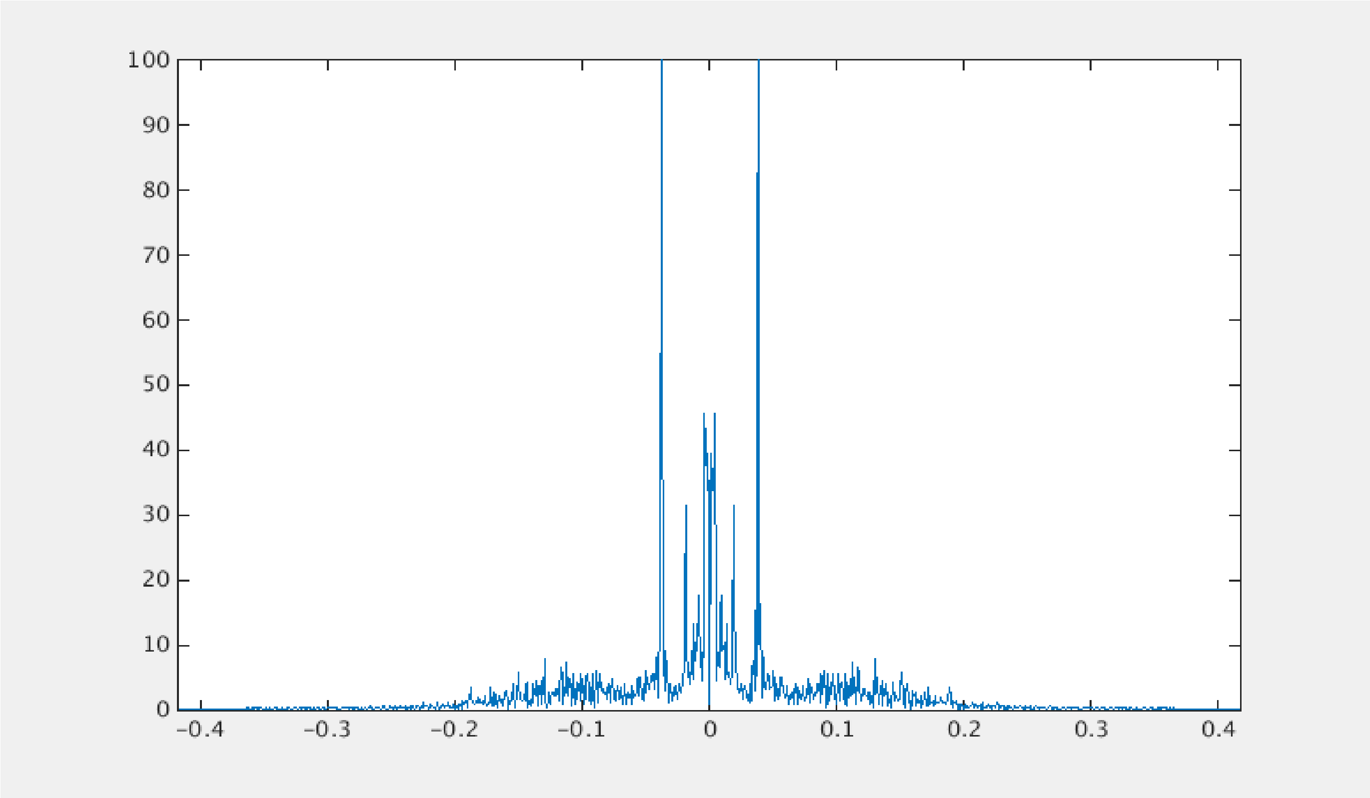
- Subcarrier is 57 kHz → start with mixer
  - We know how to do that already
- Next step is low pass (matched) filtering
  - RDS standard specifies the filter parameters
- The idea behind matched filtering
  - low-pass@TX and low-pass@RX <u>together</u> give a filter beneficial for signal quality

Filtering and decimation



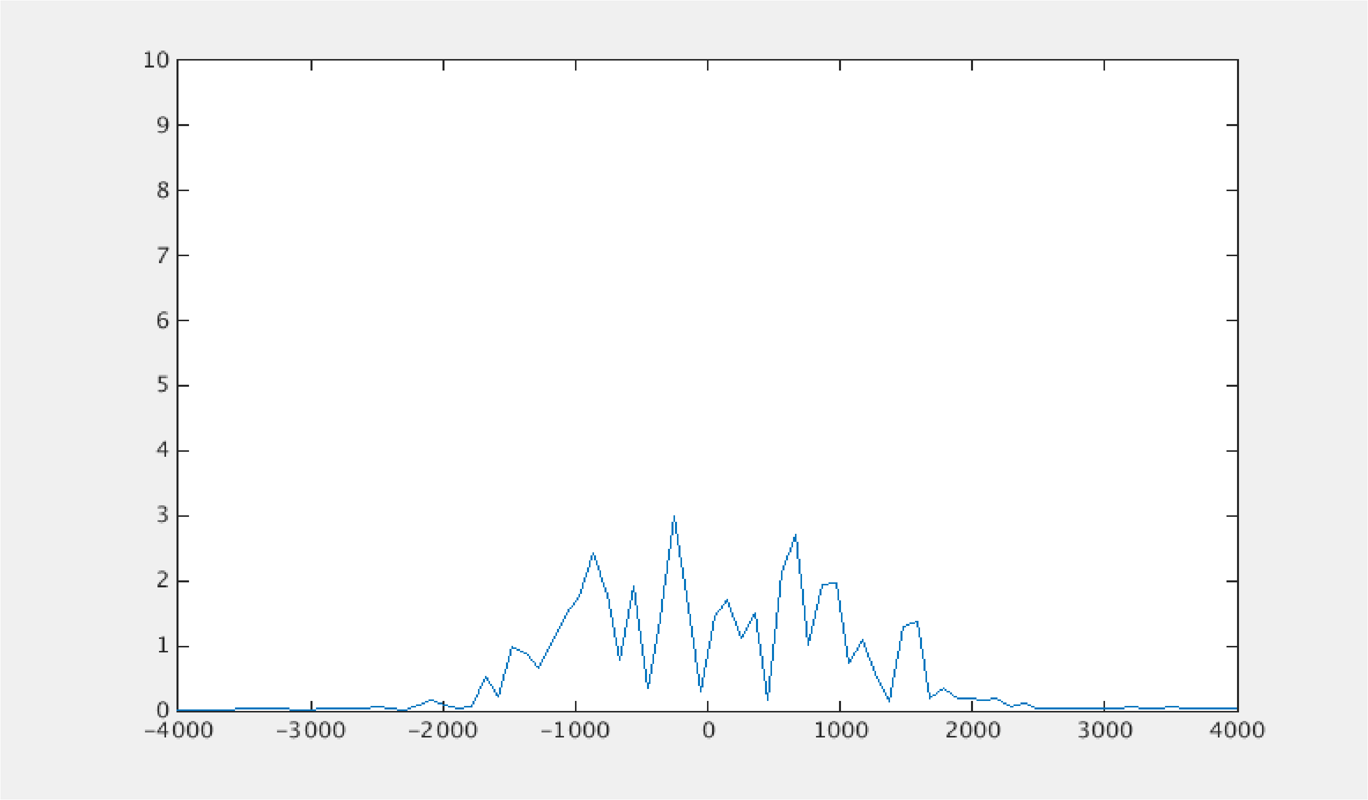
RDS mixing and filtering

FFT input:



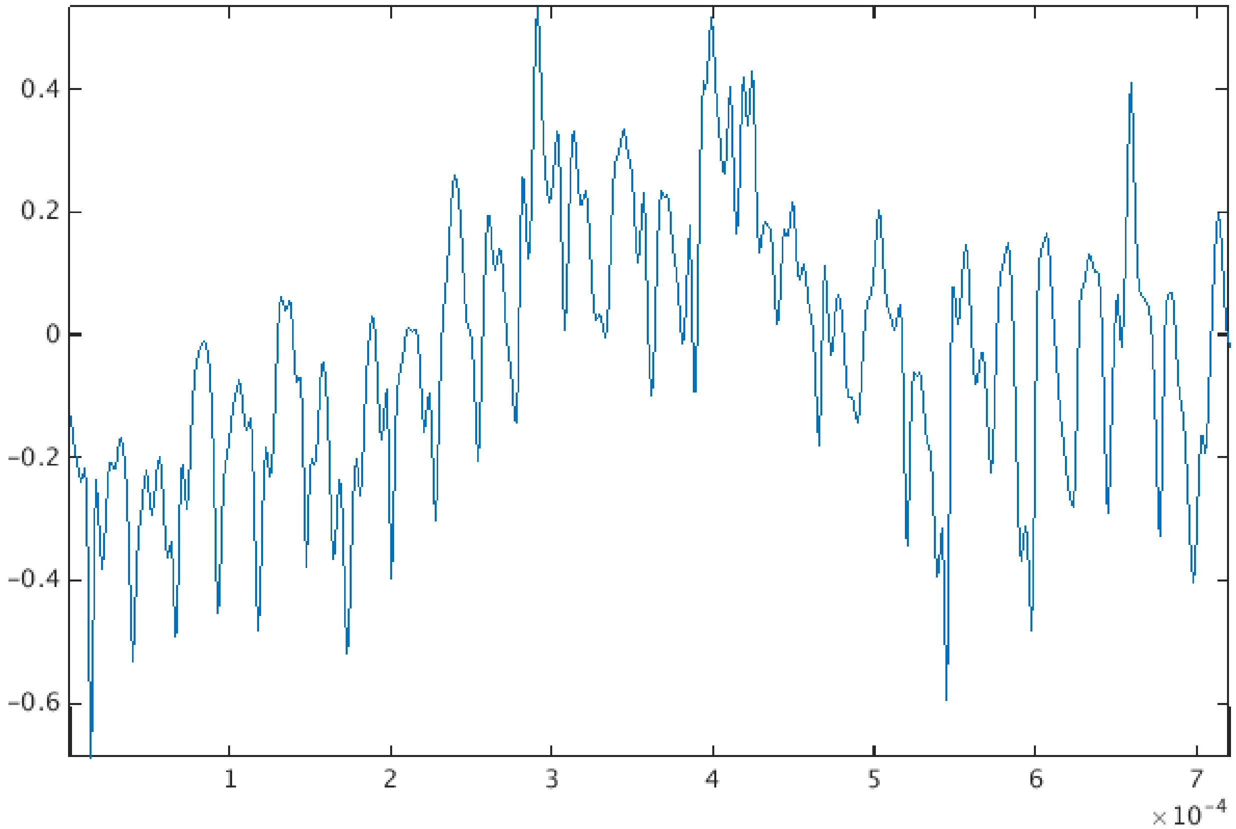
RDS mixing and filtering

FFT after mixing and filtering:



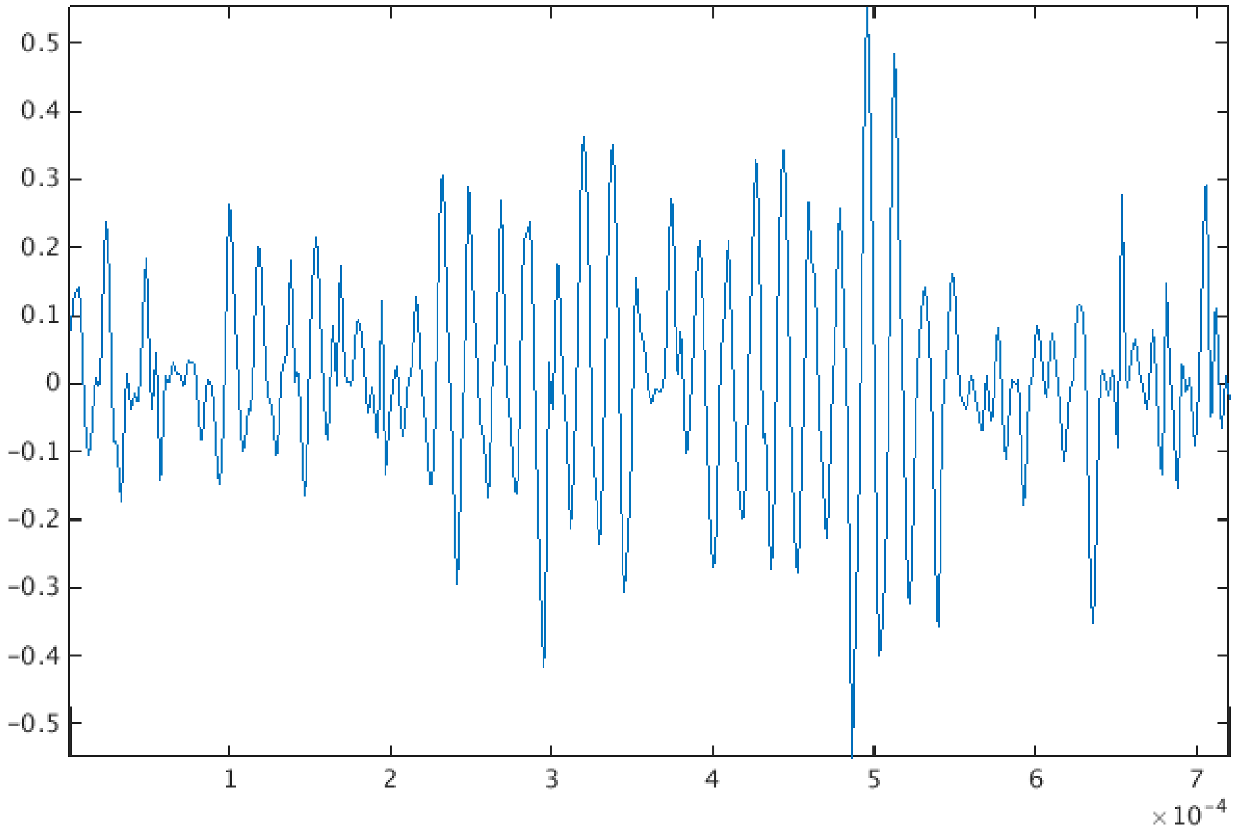
RDS mixing and filtering

Signal at input:



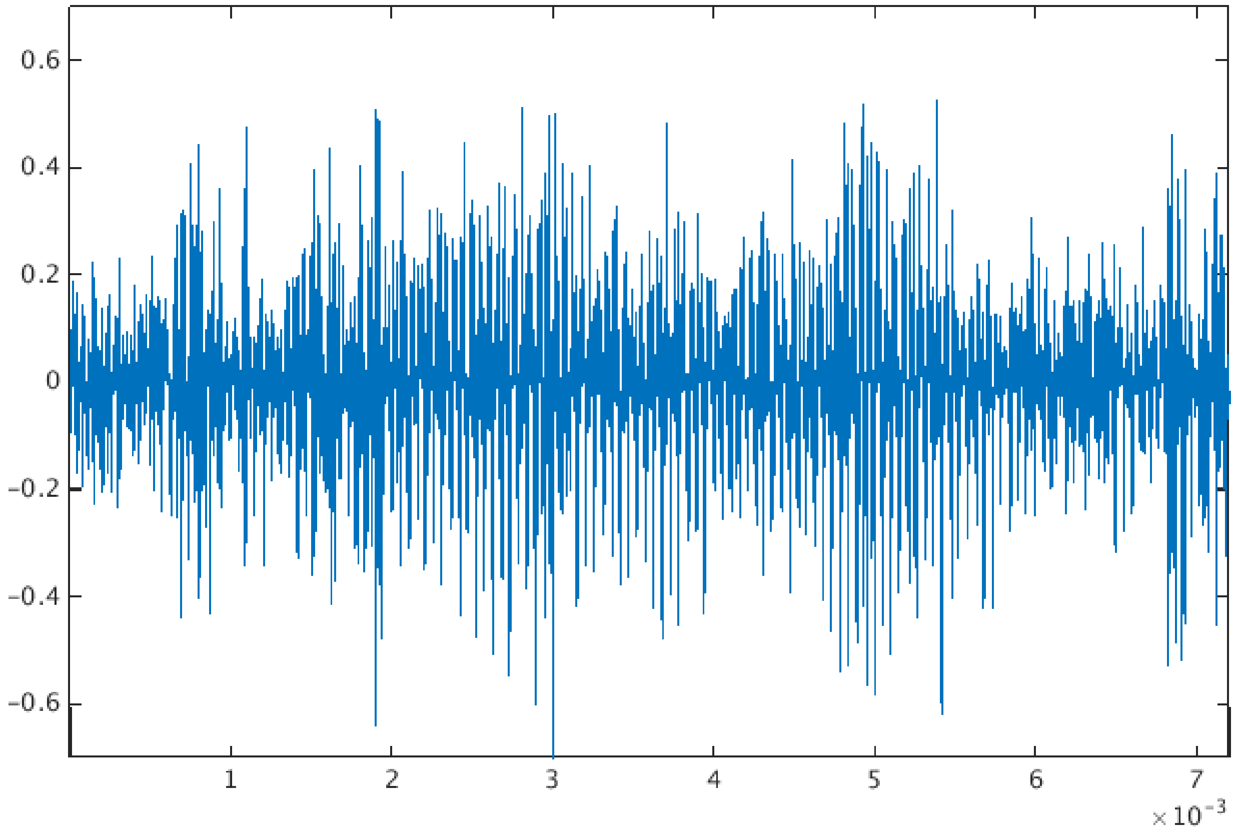
RDS mixing and filtering

Signal after mixing:



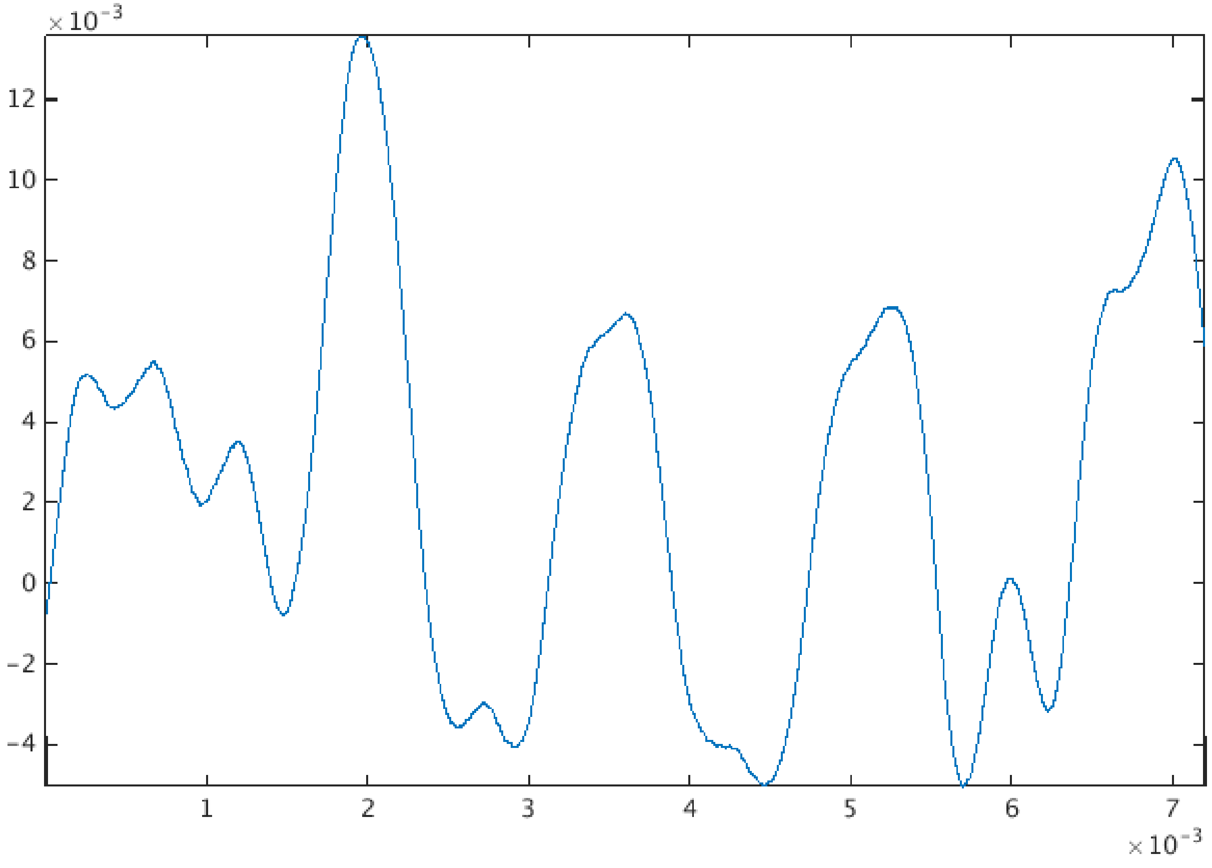
RDS mixing and filtering

Signal after mixing (zoom out):



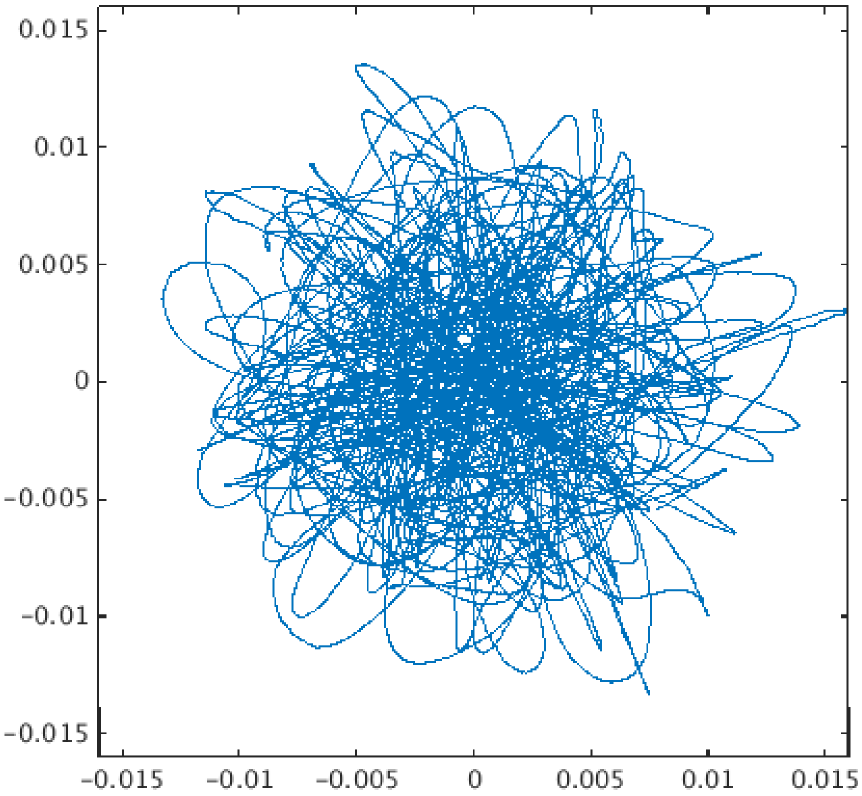
RDS mixing and filtering

Signal after mixing and filtering (real part):



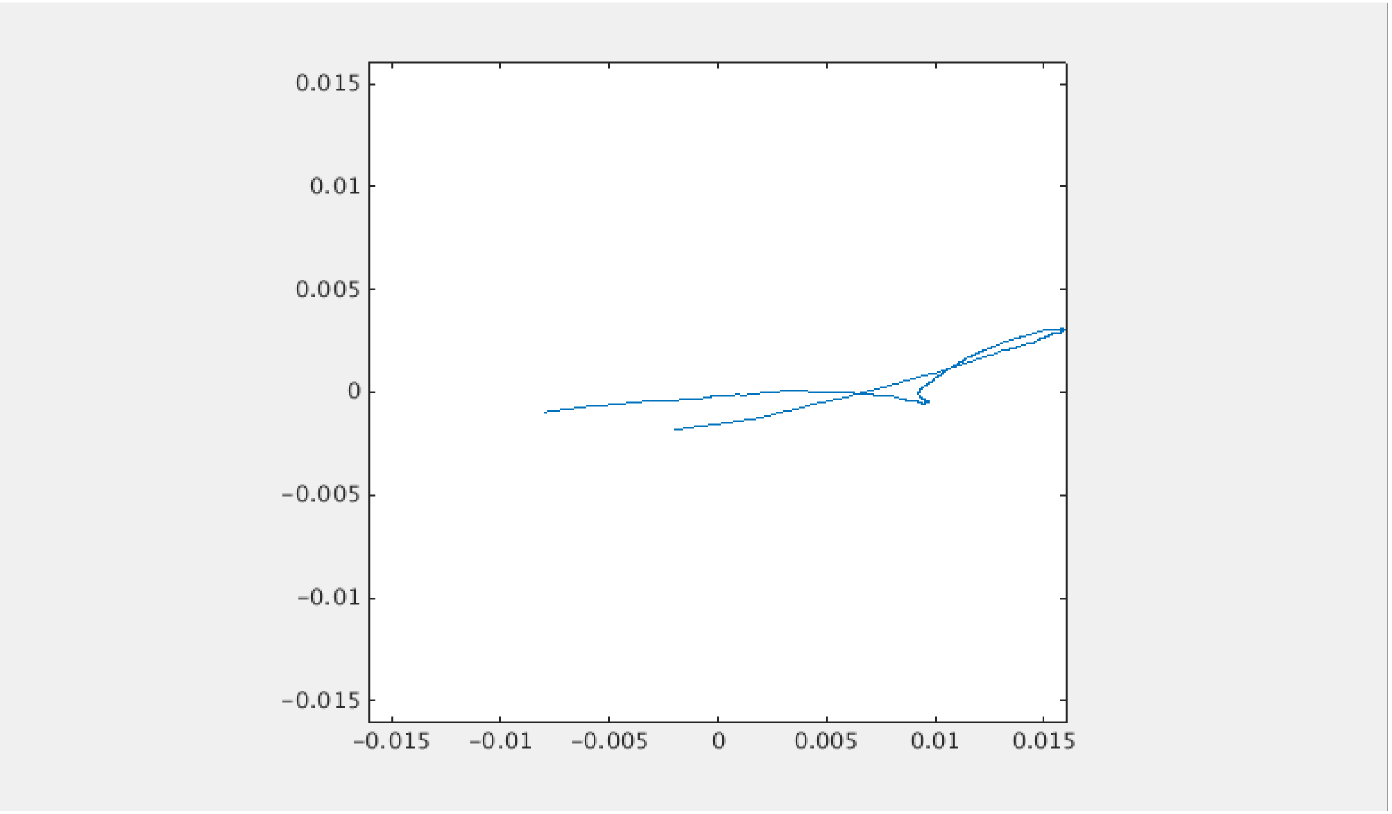
RDS baseband signal

Plotted in complex plane:



RDS baseband signal

Plotted in complex plane:



RDS baseband signal

- RDS binary data is modulated as:
  - AM: $\{-1, 1\} * e^{i 2\pi f_c t}$ → after mixing: $\{-1, 1\}$
  - "FM" (2PSK): $e^{i 2\pi f_c t - i\{0, \pi\}}$ → after mixing: $\{-1, 1\}$
- In the complex plane, the signal should stay on the real axis
- Rotation caused by carrier estimation error, we will compensate for that

Interpolation and decimation

- RDS symbol rate is 1187.5 Hz * 2 (bi-phase)
  - Lower than and not a multiple of the sampling rate
- Decimation (skipping samples) and interpolation (reconstructing signal inbetween samples) will return symbols with correct rate and sampling points
  - Need to know (symbol) clock phase and frequency
    - → clock recovery
  - Interpolation with first order estimate vs. filter

Symbol detection

- When the signal stays on the real axis and the symbol timing is correct, detection is easy:

  out = 1 when real(symbol) > 0   else out = 0

- Assuming the detection was correct, the angle of the (complex) symbol is a phase error feedback source

Carrier estimation correction

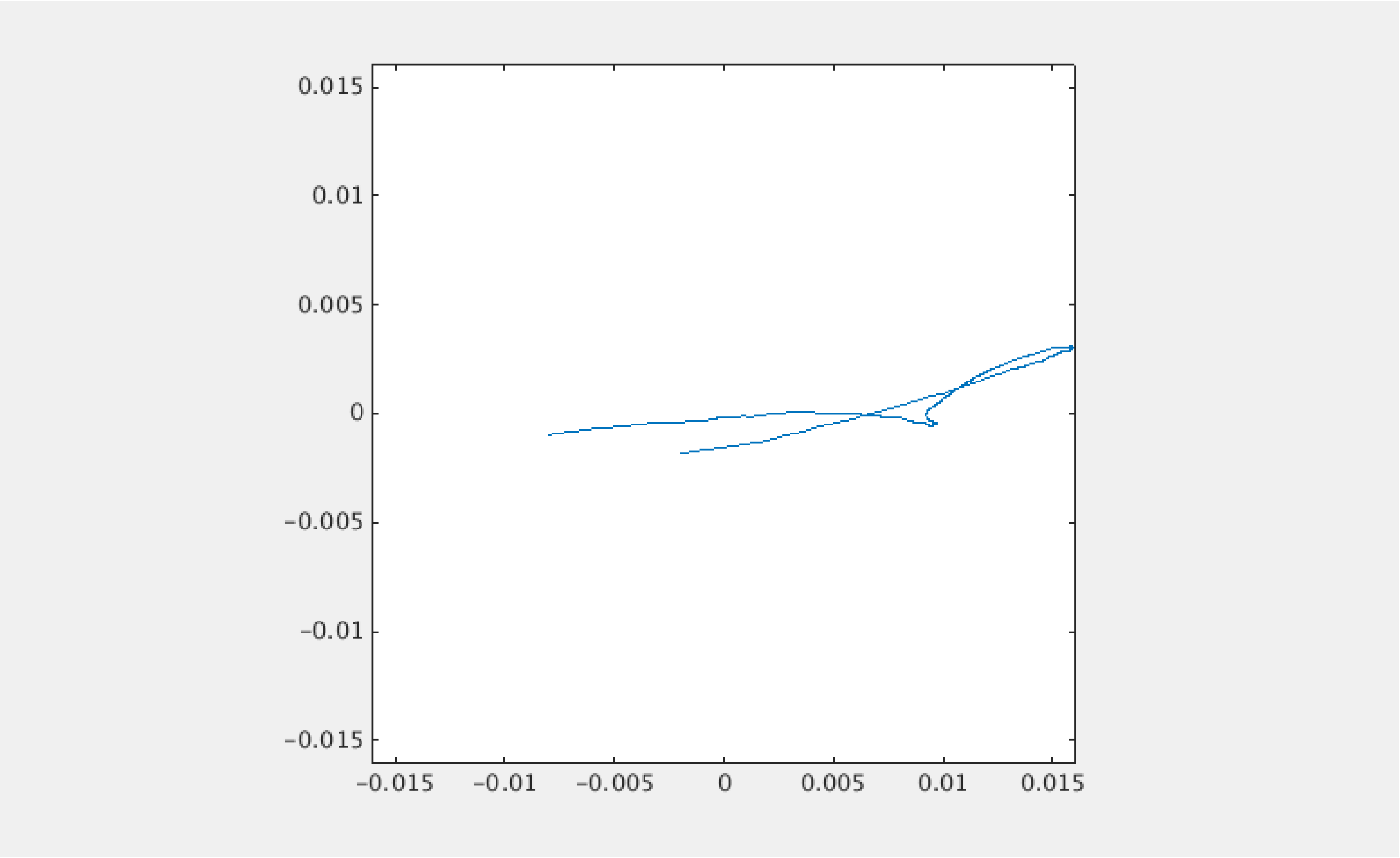
- The phase error from a detected symbol can be used to correct the phase of further samples
  - Low-pass filtering (IIR)
- A constant (very low frequency) term in the phase error means frequency error
  - Use for carrier frequency correction
  - Low-pass filtering (IIR)

Clock recovery

- With a good carrier phase and frequency estimation, the signal has (real) zero crossings in-phase with the bit-clock
  - Detect zero crossing
  - Adjust current clock phase
  - Low-frequency term of phase error = frequency error → adjust
- Bi-phase coding (1→10; 0→01)
  - Zero crossing in every bit

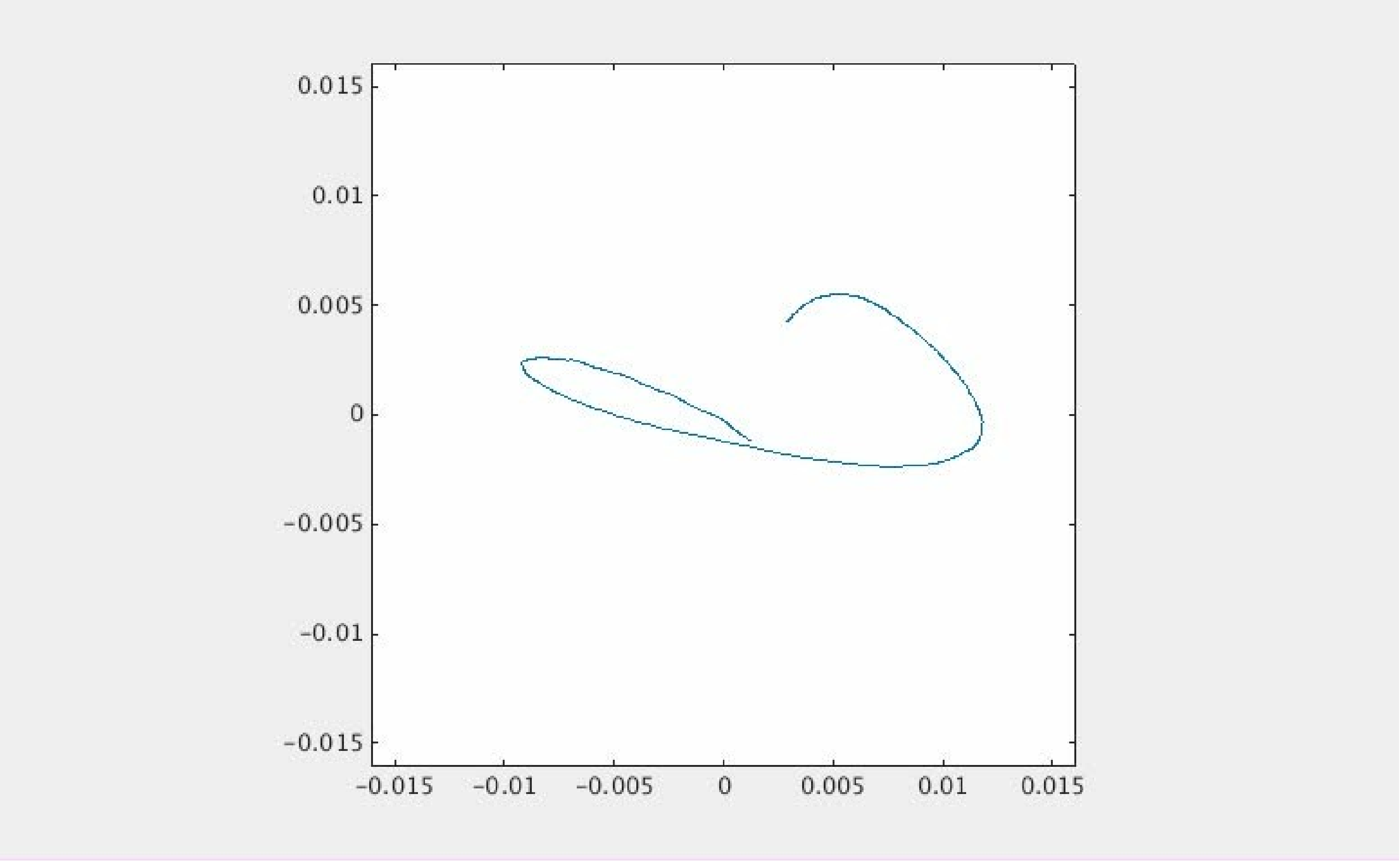
RDS baseband signal

Without f / θ correction:



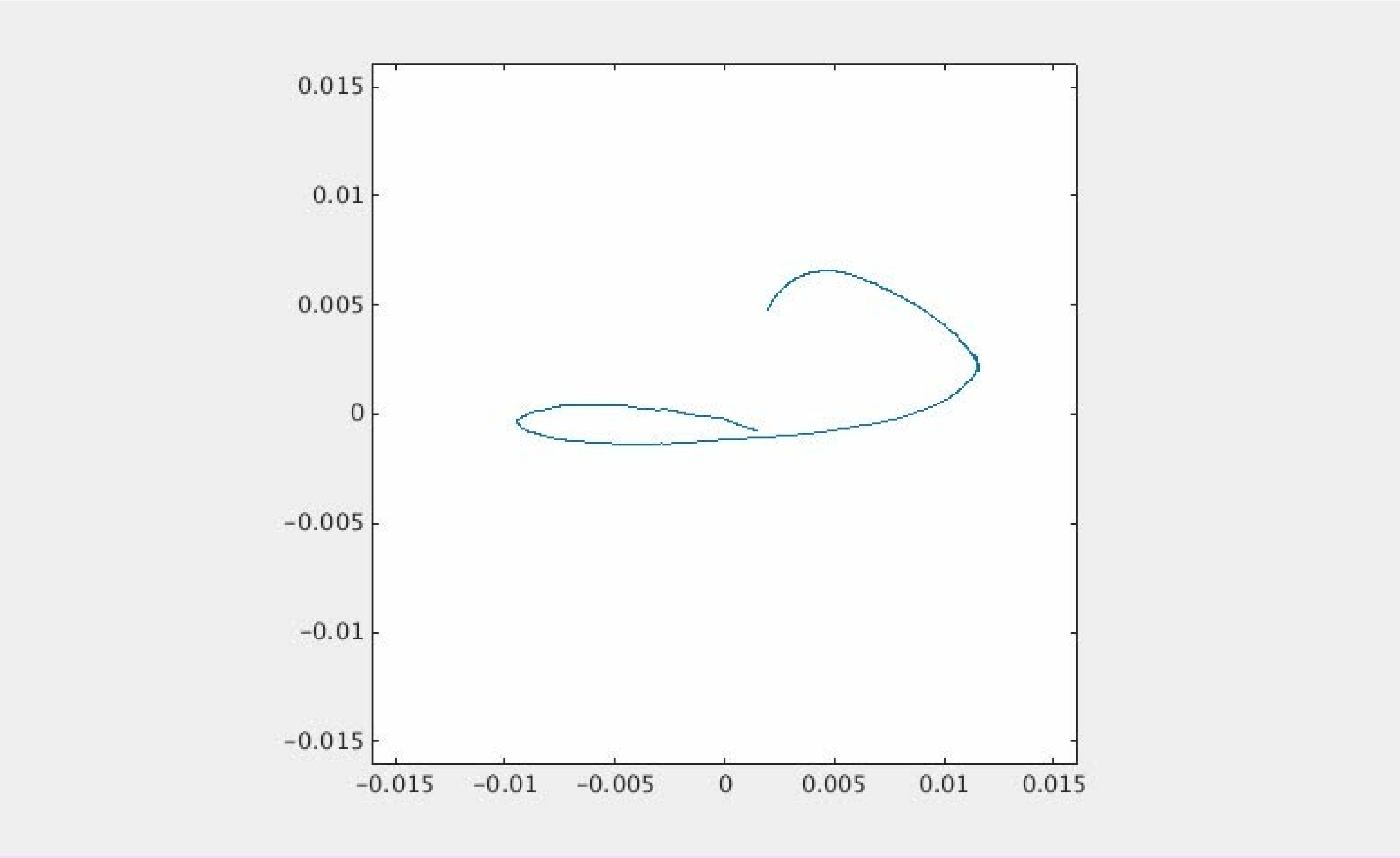
RDS baseband signal

With f correction:



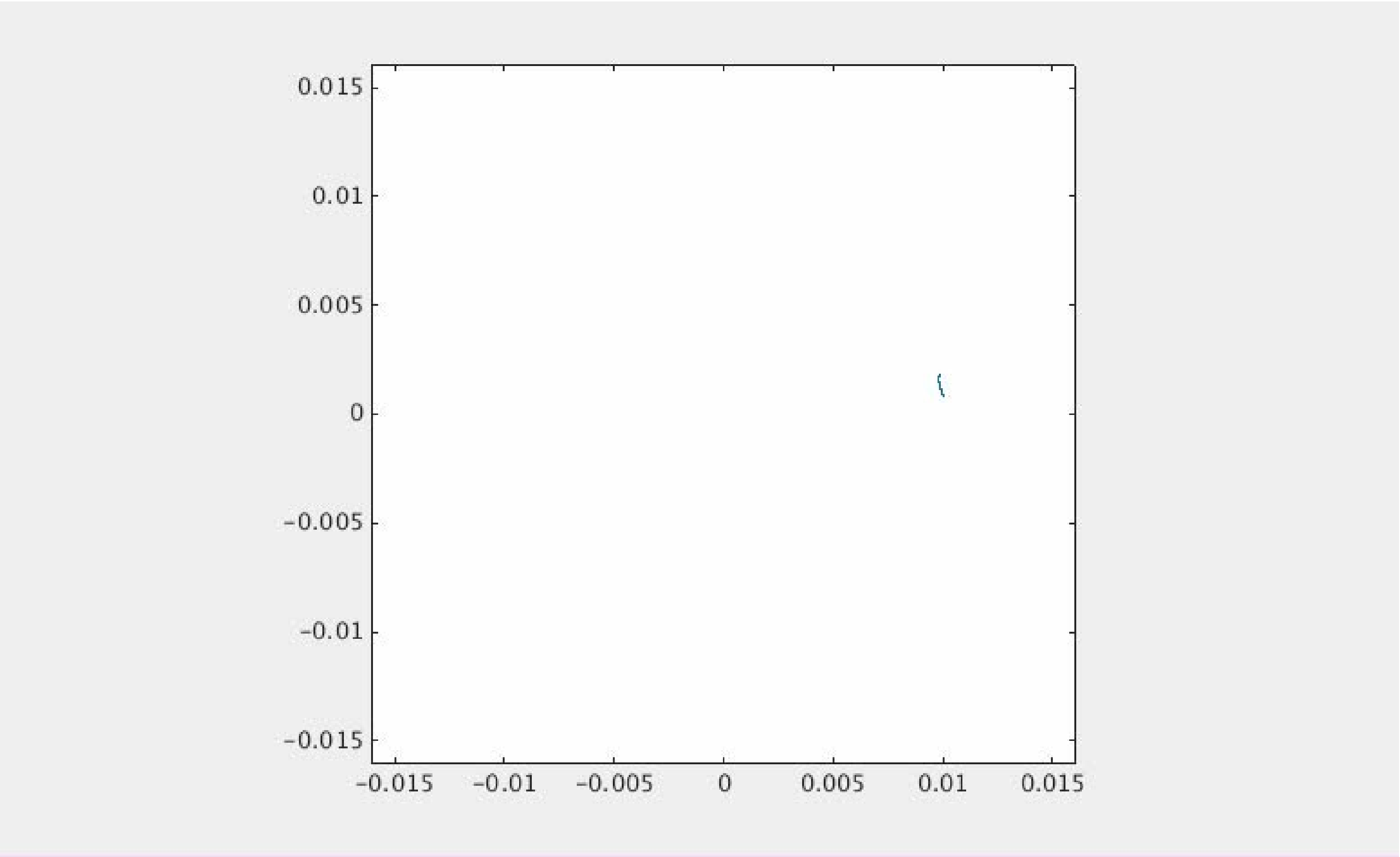
RDS baseband signal

With f and θ correction:



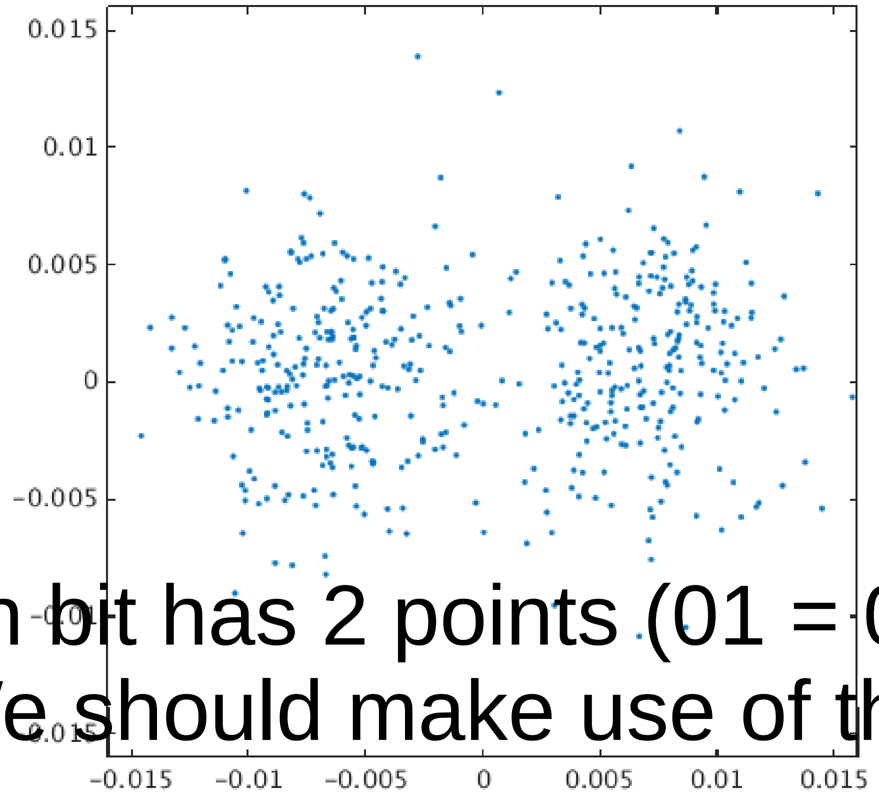
RDS baseband signal

After clock recovery, interpolation and decimation:



RDS baseband signal

Constellation diagram (IQ plot):



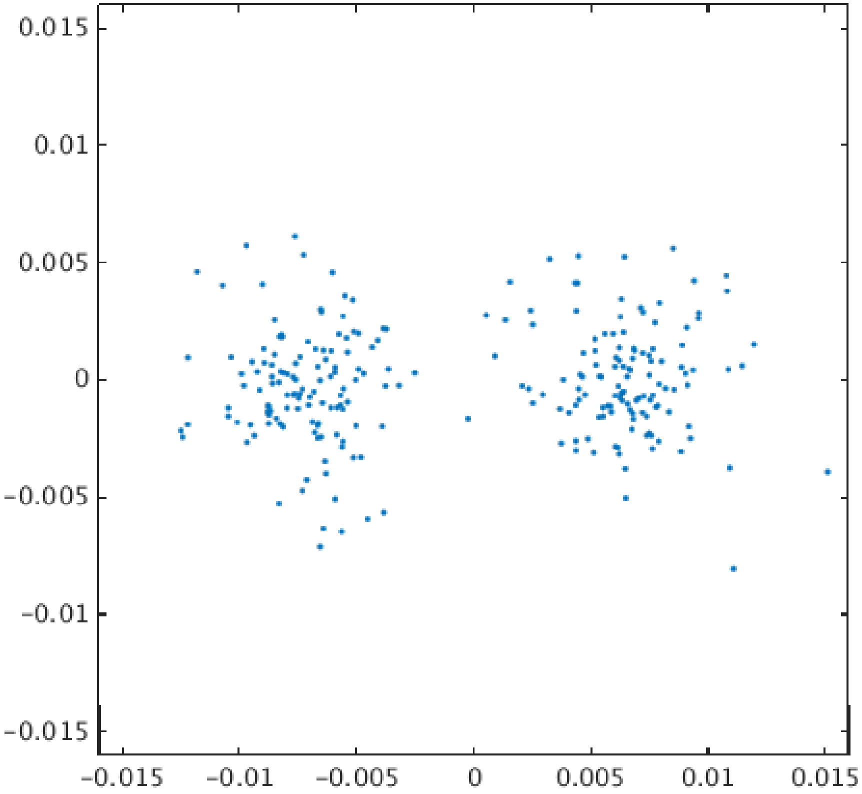
But each bit has 2 points (01 = 0; 10 = 1)

We should make use of this!

RDS baseband signal

Constellation diagram (IQ plot),

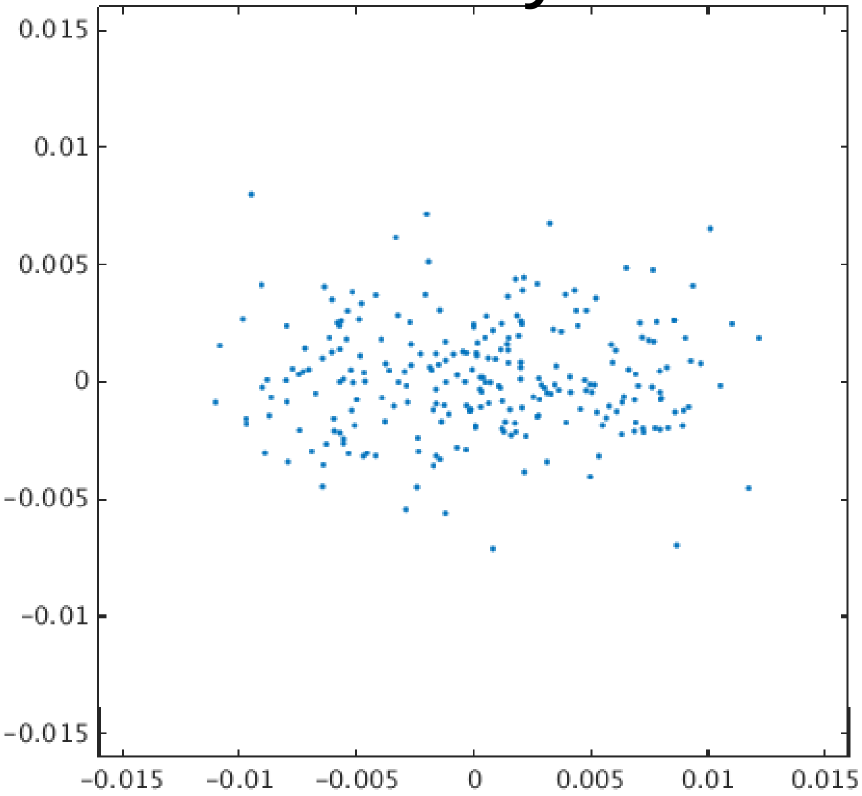
bi-phase decoded:



RDS baseband signal

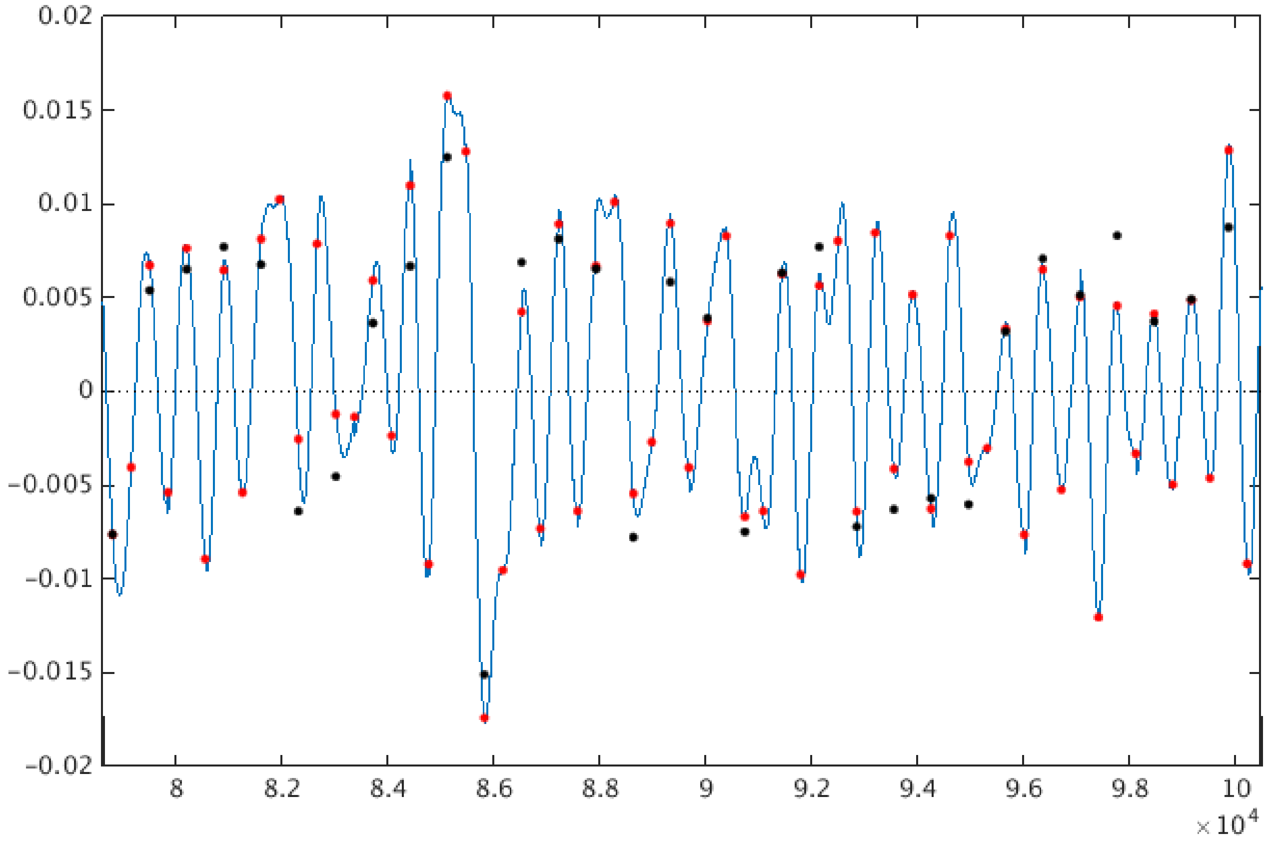
Constellation diagram (IQ plot),

bi-phase decoded – badly:



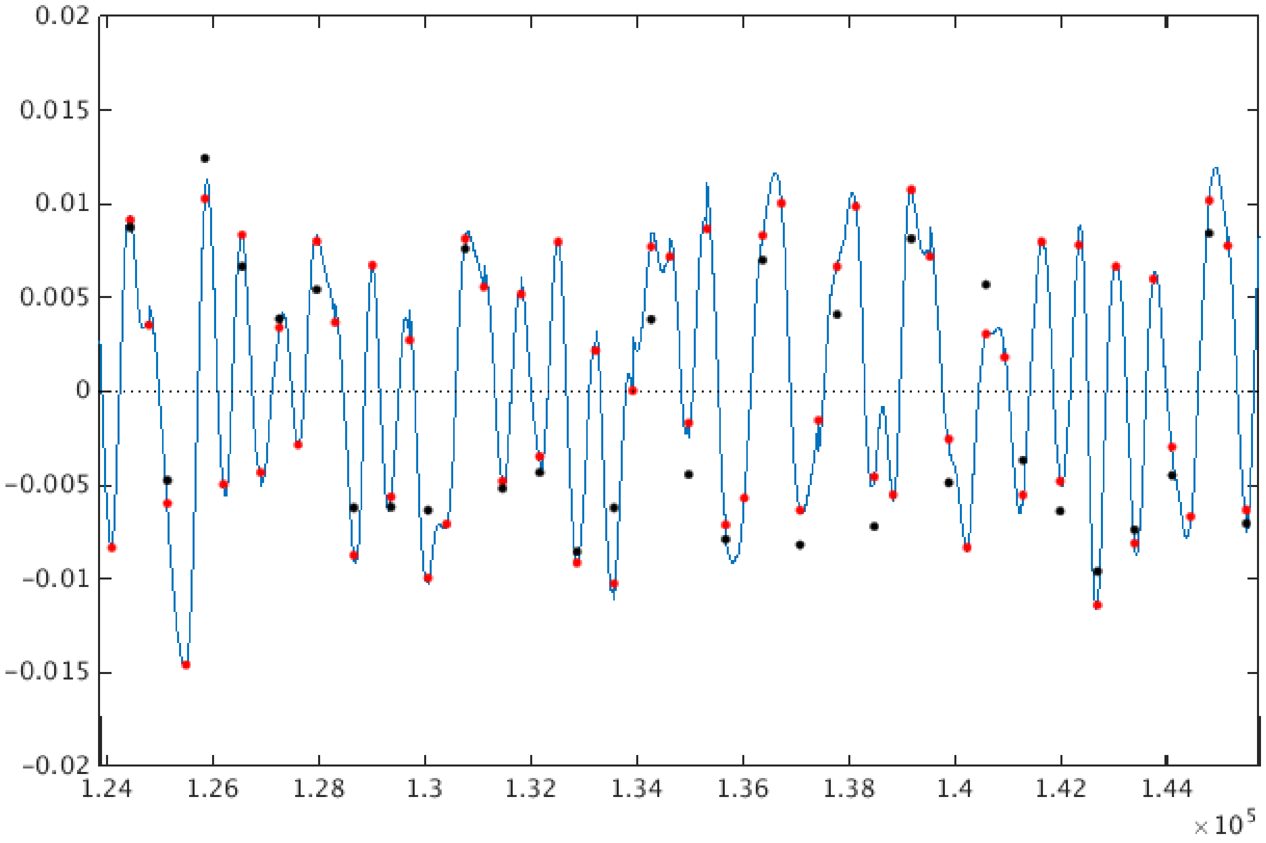
RDS baseband signal

Real part of the signal



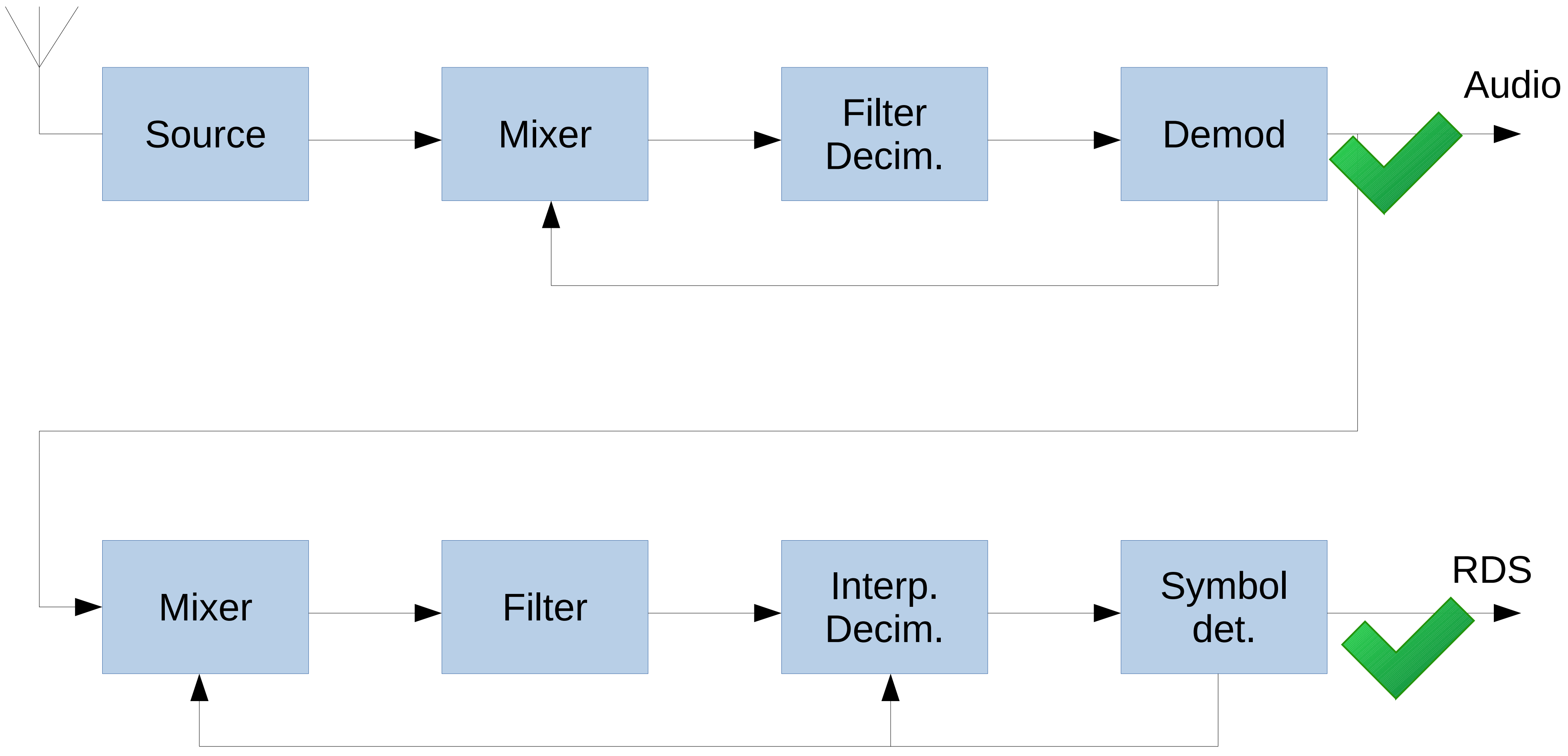
RDS baseband signal

Real part of the signal



Black dots are resulting 1s and 0s

All done!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Wien, 16. 10. 2015