Q.1. Insertion sort

let

A = [10, 7, 3, 8, 1, 9, 0]

step 1 : A [10, 7, 3, 8, 1, 9, 0]

step 2 : [7, 10, 3, 8, 1, 9, 0]

step 3 : [3, 7, 10, 8, 1, 9, 0]

step 4 : [3, 7, 8, 10, 1, 9, 0]

Step 5 : [1, 3, 7, 8, 10, 9, 0]

Step 6 : [1, 3, 7, 8, 9, 10, 0]

$\boxed{\text{Final Array : A [0, 1, 3, 7, 8, 9, 10]}}$

Q. 2. Merge sort algorithm.

1. Divide & conquer Stage:

A = [13, 57, 39, 85, 70, 22, 64, 48]

control flow of prog.

[13, 57, 39, 85, ∝] [70, 22, 64, 48, ∝]

[13, 57, ∝] [39, 85, ∝] [70, 22, ∝] [64, 48, ∝]

[13, ∝] [57, ∝] [39, ∝] [85, ∝] [70, ∝] [22, ∝] [64, ∝] [48, ∝].

2. Combine stage:

we will consider only the significant portion of arry at each step.

Step 1 :

A' = [13, 57,] ~~39, 85, 70, 22, 64, 48~~]

L = [13, ∝] R = [57, ∝]

i j

step 2 : $A' = [13, 57]$

$L = [13, \infty]$ $R = [57, \infty]$ .... as $L[i] < R[j]$
$\quad i \qquad\qquad j$

put $L[i]$,
encreament i.

step 3 : $A' = [13, 57]$

$L = [13, \infty]$, $R = [57, \infty]$
$\qquad i \qquad\qquad j$

step 4 : $A' = [39, 85]$.

$L = [39, \infty]$ $R = [85, \infty]$
$i \qquad\qquad j$

step 5 : $A' = [39, 85]$

$L = [39, \infty]$ $R = [85, \infty]$
$\qquad i \qquad\qquad j$

step 6 : $A' = [39, 85]$

$L = [39, \infty]$ $R = [85, \infty]$
$\qquad i \qquad\qquad\qquad j$

step 7 $A' = [13, 57, 39, 85]$.

$L = [13, 57, \infty]$ $R = [39, 85, \infty]$.

i j

step 8 :

$A' = [13, 57, 39, 85]$

$L = [13, 57, \infty]$ $R = [39, 85, \infty]$

i j

step 9 : $A' = [13, 39, 39, 85]$

$L = [13, 57, \infty]$ $R = [39, 85, \infty]$

i j

step 10 : $A' = [13, 39, 57, 85]$

$L = [13, 57, \infty]$ $R = [39, 85, \infty]$

i j

step 11 : $A' = [13, 39, 57, 85]$

$L = [13, 57, \infty]$ $R = [39, 85, \infty]$

i j

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Step 12 : $A' = [70, 22]$.

$L = [70, \infty]$ $R = [22, \infty]$
i j

Step 13 : $A' = [22, 22]$

$L = [70, \infty]$ $R = [22, \infty]$
i j

Step 14 : $A' = [22, 70]$.

$L = [70, \infty]$ $R = [22, \infty]$
i j

Step 15 : $A' = [64, 48]$

$L = [64, \infty]$ $R = [48, \infty]$
i j

Step 16 $A' = [48, 48]$

$L = [64, \infty]$ $R = [48, \infty]$
i j

step 17 : $A' = [48, 64]$

$L = [64, \infty]$ $R = [48, \infty]$

i j

step 18 : $A' = [22, 70, 48, 64]$

$L = [22, 70, \infty]$ $R = [48, 64, \infty]$

i j

step 19 $A' = [22, 70, 48, 64]$

$L = [22, 70, \infty]$ $R = [48, 64, \infty]$

i j

step 20 $A' = [22, 48, 48, 64]$

$L = [22, 70, \infty]$ $R = [48, 64, \infty]$

i j

step 21 : $A' = [22, 48, 64, 64]$

$L = [22, 70, \infty]$ $R = [48, 64, \infty]$

i j

Step 22 $A' = [22, 48, 64, 70]$

$L = [22, 70, \infty]$ $R = [48, 64, \infty]$

i j

Step 23 : $A' = A = [13, 39, 57, 85, 22, 48, 64, 70]$

$L = [13, 39, 57, 85, \infty]$ $R = [22, 48, 64, 70, \infty]$.

i j

Step 24: $A' = [13, 39, 57, 85, 22, 48, 64, 70]$

$L = [13, 39, 57, 85, \infty]$ $R = [22, 48, 64, 70, \infty]$

i j

Step 25 $A' = [13, 22, 57, 85, 22, 48, 64, 70]$

$L = [13, 39, 57, 85, \infty]$ $R = [22, 48, 64, 70, \infty]$

i j

Step 26 $A = [13, 22, 39, 85, 22, 48, 64, 70]$

$L = [13, 39, 57, 85, \infty]$ $R = [22, 48, 64, 70, \infty]$

i j

step 27 A = [13, 22, 39, 48, 22, 48, 64, 70]

L = [13, 39, 57, 85, ∞] R = [22, 48, 64, 70, ∞]

i j

step 28 A = [13, 22, 39, 48, 57, 48, 64, 70]

L = [13, 39, 57, 85, ∞] R = [22, 48, 64, 70, ∞]

i j

step 29 A = [13, 22, 39, 48, 57, 64, 64, 70]

L = [13, 39, 57, 85, ∞] R = [22, 48, 64, 70, ∞]

i j

step 30 A = [13, 22, 39, 48, 57, 64, 70, 70]

L = [13, 39, 57, 85, ∞] R = [22, 48, 64, 70, ∞]

i j

⇒ **step 31** $\boxed{\text{A = [13, 22, 39, 48, 57, 64, 70, 85] sorted}}$

L = [13, 39, 57, 85, ∞] R = [22, 48, 64, 70, ∞]

i j

Q.3. Let

$$A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} \quad ; \quad B = \begin{bmatrix} 5 & 8 \\ 1 & 3 \end{bmatrix}$$

Let the 10 strassen's matrices be:

$S_1 = B_{12} - B_{22} = 8 - 3 = 05$ $\quad S_6 = B_{11} + B_{22} = 5 + 3 = 8$

$S_2 = A_{11} + A_{12} = 2 + 3 = 5$ $\quad S_7 = A_{12} - A_{22} = 3 - 5 = -2$

$S_3 = A_{21} + A_{22} = 4 + 5 = 9$ $\quad S_8 = B_{21} + B_{22} = 1 + 3 = 4$

$S_4 = B_{21} - B_{11} = 1 - 5 = -4$ $\quad S_9 = A_{11} - A_{21} = 2 - 4 = -2$

$S_5 = A_{11} + A_{22} = 2 + 5 = 7$ $\quad S_{10} = B_{11} + B_{12} = 5 + 8 = 13$

Let

$P_1 = A_{11} \cdot S_1 = 2 \times 5 = 10$ $\quad P_5 = S_5 \cdot S_6 = 7 \times 8 = 56$

$P_2 = S_2 \cdot B_{22} = 5 \times 3 = 15$ $\quad P_6 = S_7 \cdot S_8 = (-2) \times 4 = -8$

$P_3 = S_3 \cdot B_{11} = 9 \times 5 = 45$ $\quad P_7 = S_9 \cdot S_{10} = (-2) \times 13 = -26$

$P_4 = A_{22} S_4 = 5 \times (-4) = -20$

Let $C_{11} = P_5 + P_4 + P_6 - P_2 = 56 - 20 - 8 - 15 =$

$C_{12} = P_1 + P_2 = 10 + 15 =$ $\quad ; C_{21} = P_3 + P_4 = 45 - 20 = 2$

$C_{22} = P_5 + P_1 - P_3 - P_7 = 56 + 10 - 45 + 2 =$

let

$C_{11} = P_5 + P_4 + P_6 - P_2$

$= 56 + (-20) + (-8) - 15 = \boxed{13 = C_{11}}$

$C_{12} = P_1 + P_2$

$= 10 + 15 = \boxed{25 = C_{12}}$

$C_{21} = P_3 + P_4$

$= 45 - 20 = \boxed{25 = C_{21}}$

$C_{22} = P_5 + P_1 - P_3 - P_7$

$= 56 + 10 - 45 - (-26) = \boxed{47 = C_{22}}$

$\therefore \quad C = A \cdot B$

$$\boxed{\therefore \quad C = \begin{bmatrix} 13 & 25 \\ 25 & 47 \end{bmatrix}}$$

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Q.4.

13	9	5	7	3	1	10	6	12	2

Let
$i = -1$, $p = 0$, $j = 0$, $r = 9$, $x = A[9] = 2$.

i p j x r

13	9	5	7	3	1	10	6	12	2

i j

13	9	5	7	3	1	10	6	12	2

i j

13	9	5	7	3	1	10	6	12	2

i j

13	9	5	7	3	1	10	6	12	2

i j

13	9	5	7	3	1	10	6	12	2

i p j x r

13	9	5	7	3	1	10	6	12	2

p i j x r

1	9	5	7	3	13	10	6	12	2

i j

1	9	5	7	3	13	10	6	12	2

i j

1	9	5	7	3	13	10	6	12	2

A[i] < A[r] ← A[j] > A[r]. → unsorted A[r].

1	9	5	7	3	13	10	6	12	2

i

1	2	5	7	3	13	10	6	12	9

As there's only one ~~term~~ element on left side, we do not need to sort it.

1	2

i p j							x.r
5	7	3	13	10	6	12	9

i	j						
5	7	3	13	10	6	12	9

		j					
5	7	3	13	10	6	12	9

		i	j				r
5	7	3	13	10	6	12	9

				j			
5	7	3	13	10	6	12	9

		i			j		
5	7	3	13	10	6	12	9

			i			j	
5	7	3	6	10	13	12	9

			i				
5	7	3	6	10	13	12	9

1	2	5	7	3	6	9	13	12	10

sorted
elements in position

Consider two sub arrays that are not sorted

1	2	5	7	3	6	9	13	12	10
1	2	5	7	3	6	9	13	12	10
1	2	5	7	3	6	9	13	12	10
1	2	5	3	7	6	9	10	13	12
1	2	5	3	6	7				

elements in position & sorted.

Consider the two sub arrays that are not sorted

5	3	13	12
5	3	13	12
3	5	12	13

elements sorted & in position

∴ Final Array:

1	2	3	5	6	7	9	10	12	13

Q.6. Let x be the number on the face of the die.

it is given that

$$P(X=x) \propto \frac{1}{x}.$$

$$\therefore P(X=x) = k.\frac{1}{x}.$$

Also.

$$P(X>3) = P(X=\{4,5,6\}).$$

$$\therefore P(X>3) = P(4)+P(5)+P(6)$$

$$= k.\frac{1}{4} + k.\frac{1}{5} + k.\frac{1}{6}.$$

$$= k.\left(\frac{1}{4}+\frac{1}{5}+\frac{1}{6}\right)$$

$$= k\left(\frac{15}{60}+\frac{12}{60}+\frac{10}{60}\right).$$

$$= k.\left(\frac{37}{60}\right). \quad \text{— (1)}.$$

we also know that $\Sigma P(X=x)=1$

$$\therefore \sum_{x=0}^{1} P(X=x)=1$$

$$\therefore K\cdot\left(\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}\right)+\frac{1}{5}+\frac{1}{6}\Big)=1$$

$$\therefore K\left(\frac{60}{60}+\frac{30}{60}+\frac{20}{60}+\frac{15}{60}+\frac{12}{60}+\frac{10}{60}\right)=1$$

$$\therefore \boxed{K=\frac{60}{147}} \quad -②$$

putting eq^n ② in ①.

$$\therefore \frac{60}{147}\times\frac{37}{60}.$$

$$\therefore \boxed{P(X>3)=0.25170068}$$

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Q. 5. Let A be the sorted arry such that

$A = [a_1, a_2, a_3 \dots a_n]$.

$size[A] = n$ & $a_1 \le a_2 \le a_3 \dots \le a_n$.

In order to get the algorithmic complexity $O(\log n)$, i will use binary search.

Algorithm:

for every index i in array:
middle = $\lfloor (size/2) \rfloor$

if size of array = 1
check if A[1] = 1
return 1.

else, if size > 1
if A[middle] > index:
search ~~left~~ subarray [a_1 to $a_{\cancel{middle}\ midle-1}$]

if A[middle] < index:
search subarray [a_{mid+1} to a_n]

if A[middle] = index:

if ~~Middle~~ Middle = ~~key~~ index
return index.

☆ Psuedocode:

```
for i = 1 to n
    left = 1
    right = 1
    mid = ⌊(left + right) / 2⌋
    while (left <= right)
        if (A[middle] < ~~key in~~ i
            left = mid + 1

        elseif (A[mid] = i AND mid = i)
            found = i

        else
            right = midd - 1

        mid = ⌊(left + right) / 2⌋

return found.
```