Course Syllabus

Course Title (in English)	Numerical Linear Algebra
Course Title (in Russian)	Вычислительная линейная алгебра
Lead Instructor(s)	Oseledets, Ivan

Is this syllabus complete, or do you plan to edit it again before sending it to the Education Office?	The syllabus is a work in progress (draft)
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1. Annotation

Course Description

Numerical linear algebra forms the basis for all modern computational mathematics. It is not possible to develop new large scale algorithms and even use existing ones without knowing it.

In this course I will show, how numerical linear algebra methods and algorithms are used to solve practical problems. Matrix decompositions play the key role in numerical linear algebra. We will study different matrix decompositions in details: what are they, how to compute them efficiently and robustly, and most importantly, how they are applied to the solution of linear systems, eigenvalue problems and data analysis applications. For large-scale problems iterative methods will be described. I will try to highlight recent developments when it is relevant to the current lecture.

This course should serve as a basis for other IT Skoltech courses. It will also serve as a first-time place where programming environment and infrastructure is introduced in a consistent manner.

Course Prerequisites / Recommendations	Basic Calculus, basic linear algebra knowledge. The assignments will involve computer programming. The main language will be Python, and the assignments will be distributed in the form of IPython notebooks. We will spend several lectures on the technical details on the Python ecosystem (syntax, plotting, basic libraries, sharing code).

2. Structure and Content

Course Academic Level	Master-level
Number of ECTS credits	6

Topic	Summary of Topic	Lectures (# of hours)	Seminars (# of hours)	Labs (# of hours)
Examples of applications	Examples of applications of numerical linear algebra. This includes applications in data analysis (such as PCA and ICA or recommender systems) and numerical PDEs.	1		
Basic course concepts	Recall of basis concepts that are the core of numerical linear algebra: vector and matrix norms, dot product, numerical complexity of algorithms, representation of numeric values in computer, numerical stability. Main NLA software: BLAS, LAPACK, MKL.	2	4	
Singular value decomposition (SVD) and related topics	SVD, proof of existence of SVD, Eckart-Young theorem and key properties. Topics related to SVD such as rank and skeleton decomposition. SVD applications: principal component analysis, recommender systems, word2vec, integral equations.	1	4	
Matrix decompositions	LU decomposition and its connection with Gaussian elimination, Cholesky decomposition. QR decomposition, Givens rotations and Housholder reflections. Schur and eigenvalue decompositions. Applications of matrix decompositions for solving medium scale linear systems and eigenvalue problems.	1	4	
Least squares	Least squares problem. Moore-Penrose pseudoinverse. Ill-conditioned systems. Regularization techniques.	1	2	
Iterative methods for large-scale sparse eigenvalue problems	Sparse matrices and basic sparse formats. Power method and Page Rank application. Krylov subspace. Arnoldi and Lancos methods. Jacobi-Davidson method.	1	4	
Iterative methods for large-scale linear systems	Richardson and Chebyshev iterative methods. Krylov-type methods: conjugate gradient (CG), minres, GMRES, bicgstab. Basics of their convergence theory.	1	4	
Structured matrices	Fast Fourier Transform (FFT). Toeplitz and circulant matrices. Discrete convolution theorem. Application of convolution in signal and image processing.	1	4	
Matrix functions	General definition of matrix functions. Matrix exponential and its applications. Matrix exponential via Schur-Parlett algorithm, Pade approximation, rational Krylov methods.	1	2	
Advanced topics	Wavelets and compressed sensing.	1		

3. Assignments

4. Grading

Type of Assessment	Graded

Grade Structure

Activity Type	Activity weight, %
Midterm Exam	20
Homework Assignments	40
Final Exam	20
Final Project	20

Grading Scale

A:	86
B:	76
C:	66
D:	56
E:	46
F:	0
Attendance Requirements	Optional

5. Basic Information

Course Stream	Science, Technology and Engineering (STE)
Course Term (in context of Academic Year)	Term 2

Students of Which Programs do You Recommend to Consider this Course as an Elective?

Masters Programs	PhD Programs
Data Science	Computational and Data Science and Engineering

Course Tags	Math Programming Engineering

6. Textbooks and Internet Resources

Required Textbooks	ISBN-10 or ISBN-13
Demmel, James W. Applied numerical linear algebra. Siam, 1997.	978-0898713893
Tyrtyshnikov, Eugene E. A brief introduction to numerical analysis. Springer Science & Business Media, 2012.	9781461264132

7. Facilities

8. Learning Outcomes

Knowledge
Know applications of numerical linear algebra tools in data analysis and numerical PDEs
Find which linear algebra tools are appropriate for the particular application

Skill
Solve medium-scale numerical linear algebra problems (solve linear systems, compute eigenvalues and eigenvectors) using matrix factorizations
Solve large scale numerical linear algebra problems using iterative methods

Experience
Use scientific tools in Python (including numpy, scipy, pandas, scikit-learn, matplotlib)

9. Assessment Criteria

Input or Upload Example(s) of Assigment 1:

Select Assignment 1 Type Problem Set

Input or Upload Example(s) of Assigment 2:

Select Assignment 2 Type Problem Set

Input or Upload Example(s) of Assigment 3:

Input or Upload Example(s) of Assigment 4:

Input or Upload Example(s) of Assigment 5:

10. Additional Notes