Lecture No.- 06

ARJUNA

NEET FASTRACK 2024

Physics [redacted]

Motion in Straight Line

[redacted]

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TODAYS TARGETS

① Angle Made by Axis.

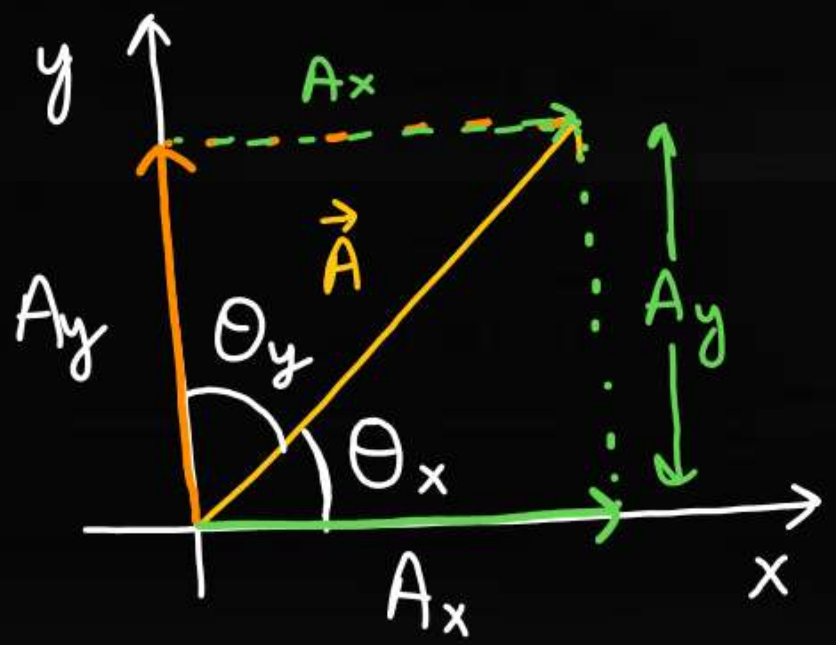
② Addition & Subtraction of Vector in Unit Vector Form.

③ Magnitude of Vector.

④ Dot-Product

⑤ Cross-Product.

Angle Made With Axis :



$$A = A_x\hat{i} + A_y\hat{j}$$

$$\boxed{\theta_x = \tan^{-1}\left(\frac{A_y}{A_x}\right)}$$

$$\tan(\theta_x) = \frac{A_y}{A_x}$$

$$\boxed{\theta_y = \tan^{-1}\left(\frac{A_x}{A_y}\right)}$$

$$\tan\theta_y = \frac{A_x}{A_y}$$

θ_x = Angle with x-axis.

θ_y = Angle with y-axis.

Inverse trigonometric Function

$\rightarrow$ $\tan\left(\frac{\pi}{6}\right) = \frac{1}{\sqrt{3}}$

$$\boxed{\frac{\pi}{6} = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)}$$

$\tan\theta = \frac{2}{5}$

$$\boxed{\theta = \tan^{-1}\left(\frac{2}{5}\right)}$$

$\sin\theta = \frac{2}{3}$

$$\boxed{\theta = \sin^{-1}\left(\frac{2}{3}\right)}$$

$$\tan(\theta) = \sqrt{3}$$

$$\theta = \tan^{-1}(\sqrt{3})$$

$$\boxed{\theta = \frac{\pi}{6} = 60^\circ}$$

$$\sin\theta = \frac{1}{2}$$

$$\theta = \sin^{-1}\left(\frac{1}{2}\right)$$

$$\boxed{\theta = \frac{\pi}{6} = 30^\circ}$$

$$\cos\theta = \frac{1}{\sqrt{2}}$$

$$\theta = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

$$\boxed{\theta = \frac{\pi}{4} = 45^\circ}$$

$$\tan\theta = \frac{1}{\sqrt{2}}$$

$$\boxed{\theta = \tan^{-1}\left(\frac{1}{\sqrt{2}}\right)}$$

Q: → Find Angle with x-axis & y-Axis with following Vectors.

S.No	Vector	Angle with X-axis	Angle with y-axis
1	$\vec{V} = 2\hat{i} + 3\hat{j}$	$\theta_x = \tan^{-1}\left(\frac{3}{2}\right)$	$\theta_y = \tan^{-1}\left(\frac{2}{3}\right)$
2	$\vec{B} = 3\hat{i} + 4\hat{j}$	$\theta_x = \tan^{-1}\left(\frac{4}{3}\right)$ $\theta_x = 53^\circ$	$\theta_y = \tan^{-1}\left(\frac{3}{4}\right)$ $\theta_y = 37^\circ$
3	$\vec{a} = 2\hat{i} - 5\hat{j}$	$\theta_x = \tan^{-1}\left(\frac{-5}{2}\right)$	$\theta_y = \tan^{-1}\left(\frac{2}{-5}\right)$
4	$\vec{S} = 10\hat{i} - 5\hat{j}$	$\theta_x = \tan^{-1}\left(\frac{-5}{10}\right)$	$\theta_y = \tan^{-1}\left(\frac{10}{-5}\right)$

$\tan(37^\circ) = \frac{3}{4}$

$\tan(53^\circ) = \frac{4}{3}$

Addition of Vectors in Unit Vector form :

$$\vec{A} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{B} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

$$\vec{C} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$$

$$\boxed{\vec{A} + \vec{B} + \vec{C} = (a_1 + b_1 + c_1)\hat{i} + (a_2 + b_2 + c_2)\hat{j} + (a_3 + b_3 + c_3)\hat{k}}$$

Subtraction of Vectors :

$$\vec{A} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{B} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

$$\boxed{\vec{A} - \vec{B} = (a_1 - b_1)\hat{i} + (a_2 - b_2)\hat{j} + (a_3 - b_3)\hat{k}}$$

$$\boxed{\vec{B} - \vec{A} = (b_1 - a_1)\hat{i} + (b_2 - a_2)\hat{j} + (b_3 - a_3)\hat{k}}$$

Q: →

$$\vec{A} = 2\hat{i} - 3\hat{j} + 4\hat{k}$$

$$\vec{B} = 4\hat{i} - 2\hat{j} + 3\hat{k}$$

(iii) $\vec{B} - \vec{A} = (4-2)\hat{i} + (-2-(-3))\hat{j} + (3-4)\hat{k}$

$$\boxed{\vec{B} - \vec{A} = 2\hat{i} + \hat{j} - \hat{k}}$$

$$\vec{B} - \vec{A} = 2\hat{i} + 1\hat{j} - 1\hat{k}$$

find (i) $\vec{A} + \vec{B} = (2+4)\hat{i} + (-3+(-2))\hat{j} + (4+3)\hat{k}$

$$\boxed{\vec{A} + \vec{B} = 6\hat{i} - 5\hat{j} + 7\hat{k}}$$

(ii) $\vec{A} - \vec{B} = (2-4)\hat{i} + (-3-(-2))\hat{j} + (4-3)\hat{k}$

$$\boxed{\vec{A} - \vec{B} = -2\hat{i} - 1\hat{j} + 1\hat{k}}$$

$$\boxed{\vec{A} - \vec{B} = -2\hat{i} - \hat{j} + \hat{k}}$$

Q:→ Tree Force acting on the body, Find Value of a, b, c such that.

Body is in equillibrium / Body Do-not move $(\vec{F}_{net} = 0)$

$\vec{F}_1 = \underline{5}\hat{i} - 3\hat{j} - 4\hat{k}$, $\vec{F}_2 = \underline{3}\hat{i} + 6\hat{j} + 8\hat{k}$, $\vec{F}_3 = \underline{a}\hat{i} + b\hat{j} + c\hat{k}$

Solution $\quad F_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$

$$\vec{F}_{net} = \underline{(8+a)}\hat{i} + (3+b)\hat{j} + (\underline{4+c})\hat{k} = \underline{0}\hat{i} + 0\hat{j} + 0\hat{k}$$

① $8+a=0$ ② $3+b=0$ ③ $4+c=0$

$\boxed{a=-8}$ $\quad \boxed{b=-3}$ $\quad \boxed{c=-4}$

Q:→ Two forces $\vec{F}_1 = 2\hat{i} - 5\hat{j} + 3\hat{k}$ & $\vec{F}_2 = \hat{i} + 3\hat{j} + 2\hat{k}$ are acting on a body. Find value of third force such that. Body Remains in Equillibrium / Rest.

$$\vec{F}_{net} = 0$$

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \underline{\vec{F}_3} = 0$$

$$\vec{F}_3 = -\vec{F}_1 - \vec{F}_2$$

$$\vec{F}_3 = -(\vec{F}_1 + \vec{F}_2)$$

$$\vec{F}_1 + \vec{F}_2 = (2+1)\hat{i} + (-5+3)\hat{j} + (3+2)\hat{k}$$

$$\boxed{\vec{F}_1 + \vec{F}_2 = 3\hat{i} - 2\hat{j} + 5\hat{k}}$$

$$\vec{F}_3 = -(\vec{F}_1 + \vec{F}_2) = -(3\hat{i} - 2\hat{j} + 5\hat{k})$$

$$\boxed{\vec{F}_3 = -3\hat{i} + 2\hat{j} - 5\hat{k}}$$

Q:→ Two Forces Acting on the body are given as

$$\vec{F}_1 = 4\hat{i} + 2\hat{j} - 6\hat{k}$$

$$\vec{F}_2 = -2\hat{i} + \hat{j} + 4\hat{k}$$

Find Minimum Value of Third Force such that Particle Moves along X-axis.

$$\boxed{\vec{F}_3 = a\hat{i} + b\hat{j} + c\hat{k}}$$

$$\boxed{\vec{F}_3 = -3\hat{j} + 2\hat{k}}$$

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = x\hat{i} + 0\hat{j} + 0\hat{k}$$

$$= (4 + (-2) + a)\hat{i} + (2 + 1 + b)\hat{j} + (-6 + 4 + c)\hat{k}$$

$$\vec{F}_{net} = (2+a)\hat{i} + (3+b)\hat{j} + (-2+c)\hat{k} = x\hat{i} + 0\hat{j} + 0\hat{k}$$

$3 + b = 0$

$\boxed{b = -3}$

$-2 + c = 0$

$\boxed{c = +2}$

$2 + a = x$

For Minimum Value of F_3

$\boxed{a = 0}$

* Magnitude of Vector in Unit Vector form :

$$\vec{A} = a\hat{i} + b\hat{j} + c\hat{k}$$

$$\boxed{|\vec{A}| = \sqrt{a^2 + b^2 + c^2}}$$

Q:→ Find Magnitude of Following Vectors :

① $\vec{A} = 2\hat{i} - \hat{j} + 3\hat{k}$

$|\vec{A}| = \sqrt{2^2 + (-1)^2 + (3)^2}$

$|\vec{A}| = \sqrt{4+1+9} = \sqrt{14}$

② $\vec{v} = 2\hat{i} + 3\hat{j}$

$|\vec{v}| = \sqrt{(2)^2 + (3)^2}$

$= \sqrt{4+9} = \sqrt{13}$

③ $\vec{B} = 2\hat{i} + 3\hat{j} - 4\hat{k}$

$|\vec{B}| = \sqrt{2^2 + 3^2 + (-4)^2}$

$|\vec{B}| = \sqrt{4+9+16} = \sqrt{29}$

④ $\vec{a} = \hat{i} + \hat{j}$

$|\vec{a}| = \sqrt{(1)^2 + (1)^2} = \sqrt{2}$.

Unit Vector:

$$\boxed{\hat{A} = \frac{\vec{A}}{|\vec{A}|}}$$

e.g $\vec{A} = a\hat{i} + b\hat{j} + c\hat{k}$

$$\hat{A} = \frac{\vec{A}}{|\vec{A}|} = \frac{a\hat{i} + b\hat{j} + c\hat{k}}{\sqrt{a^2 + b^2 + c^2}}$$

(1) $\vec{A} = 2\hat{i} + 3\hat{j} - 5\hat{k}$

$$\hat{A} = \frac{\vec{A}}{|\vec{A}|} = \frac{2\hat{i} + 3\hat{j} - 5\hat{k}}{\sqrt{(2)^2 + (3)^2 + (-5)^2}}$$

$$\hat{A} = \frac{2\hat{i} + 3\hat{j} - 5\hat{k}}{\sqrt{4 + 9 + 25}}$$

$$\boxed{\hat{A} = \frac{2\hat{i} + 3\hat{j} - 5\hat{k}}{\sqrt{38}}}$$

Q:→ $\vec{v} = 3\hat{i} - 4\hat{j}$

Find Direction of $\vec{v}$.

Solution

$$\hat{v} = \frac{\vec{v}}{|\vec{v}|} = \frac{3\hat{i} - 4\hat{j}}{\sqrt{(3)^2 + (-4)^2}}$$

$$\hat{v} = \frac{3\hat{i} - 4\hat{j}}{\sqrt{25}} = \frac{3\hat{i} - 4\hat{j}}{5}$$

Q:→ $\vec{a} = \hat{i} + \hat{j}$, Find $\hat{a}$.

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{\hat{i} + \hat{j}}{\sqrt{(1)^2 + (1)^2}}$$

$$\boxed{\hat{a} = \frac{\hat{i} + \hat{j}}{\sqrt{2}}}$$

Q: →

$\vec{c} = \hat{i} - 2\hat{j}$, find $\hat{c} = ?$

$$\hat{c} = \frac{\vec{c}}{|\vec{c}|} = \frac{\hat{i} - 2\hat{j}}{\sqrt{(1)^2 + (-2)^2}}$$

$$\boxed{\hat{c} = \frac{\hat{i} - 2\hat{j}}{\sqrt{5}}}$$


THANK You