Face recognition using eigenfaces Algorithm:

1. Given a set of m images of shape(A×A) (lets us suppose a square image), we convert each of the m images into vector of size $A^2 \times 1$.
2. We do Principal component analysis on the resultant vector of shape(m × A^2). The way it is done is explained below:
   a. Calculate average of all the face vectors.
   b. Subtract the average from each vector to form a resultant face vector F.
   c. We calculate the covariance matrix by multiplying F with F^T. F has dimensions A^2 * M, thus F^T has dimensions M * A^2. When we multiplied this gives us matrix of A^2 * A^2, which gives us A^2 eigenvectors of A^2 size which is not computationally efficient to calculate. So we calculate our covariance matrix by multiplying F^T and F. This gives us M * M matrix which has M eigenvectors of size M.
   d. We then calculate eigenvalues and eigenvectors of the above covariance matrix.
   e. The principal components are these eigenvectors in decreasing order of the eigenvalues. The physical meaning of the principal component vectors is that they are the key directions that we can construct the columns of a matrix. The greater the eigenvalue, the more useful the principal component vector.
3. For any given face picture, we can project its mean-subtracted version onto the eigenface picture using a vector dot-product. For the K eigenfaces, we can find K dot-product for any given face picture. We can present the result as weights of this face picture with respect to the eigenfaces.
4. We then calculate the weight vector for each input picture.
5. We compare the weight vector of the picture in the query to that of each existing picture and find the one with the smallest euclidean distance as the best match.

Results obtained:
We used AT&T Database of Faces for this face recognition. This database of faces consists of 10 pictures each of 40 different faces in different shapes and lightning conditions. We divided the data into a training set and testing set. Testing set consisted of a total of 49 images, among which 39 were image 1 of each first 39 people and remaining 10 were of the last person's.
In total we had 351 training images and 49 testing images. Among 49, 39 were faces of people from training class and 10 were of a person which was not in training class.

Accuracy:
- Accuracy of match for all the out of sample images which belong to existing class in training data is **82.05 %**
- Accuracy of match for all the out of sample images which belong to existing class in training data is 100 % which implies that the algorithm correctly detects that the image of the person does not exist.

References:

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