

# HW3 (Corrections) (CSCI-C241)

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- Question One

- 1c

- This proof is valid

- 1e

- This proof is not valid, as they are doing proof by cases, with cases that are part of an implication premise (meaning it's something that they do not have).

- Question Two

- 2d

- Claim:  $(A \wedge B) \rightarrow C, B \vdash (A \wedge \neg D) \rightarrow (C \wedge \neg D)$

- Goal:  $(A \wedge \neg D) \rightarrow (C \wedge \neg D)$

- Pf:* Assume  $(A \wedge B) \rightarrow C, B$ .

- 1. *Subproof*

- 2. Assume  $A \wedge \neg D$ .

- 3. From  $A \wedge \neg D$ , we can conclude  $A$  and  $\neg D$ . ( $\wedge$  – *Elimination*)

- 3. From  $A$  and  $B$ , we can conclude  $A \wedge B$ . ( $\wedge$  – *Introduction*)

- 4. From  $A \wedge B$ , we can apply  $(A \wedge B) \rightarrow C$  and conclude  $C$ . (*Application*)

- 5. From  $C$  and  $\neg D$ , we can conclude  $C \wedge \neg D$ . ( $\wedge$  – *Introduction*)

- 6. Under the assumption of  $A \wedge \neg D$ , we proved  $C$  and  $\neg D$ . (Direct Proof)

□

- 2g

- Claim:  $\vdash (X \wedge (X \rightarrow (Z \wedge Y))) \rightarrow (X \wedge Y)$

- Goal:  $(X \wedge (X \rightarrow (Z \wedge Y))) \rightarrow (X \wedge Y)$

- Proof.*

- 1. *Subproof*

- 2. Assume  $X \wedge (X \rightarrow (Z \wedge Y))$

- 3. From  $X \wedge (X \rightarrow (Z \wedge Y))$  we can conclude  $X$  and  $X \rightarrow (Z \wedge Y)$  ( $\wedge$  – *Elimination*)

- 4. From  $X$  we can apply  $X \rightarrow (Z \wedge Y)$  and conclude  $Z \wedge Y$ . (*Application*)

- 5. From  $Z \wedge Y$  we can conclude  $Y$ . ( $\wedge$  – *Elimination*)

- 6. From  $X$  and  $Y$  we can conclude  $X \wedge Y$ . ( $\wedge$  – *Introduction*)

- 7. Under the assumption  $X \wedge (X \rightarrow (Z \wedge Y))$ , we proved  $(X \wedge Y)$ ,  
therefore  $(X \wedge (X \rightarrow (Z \wedge Y))) \rightarrow (X \wedge Y)$  (Direct Proof)

□