CSCI-C 241 HW #6
Instructors: Wennstrom & LaRacuente Due Tuesday, February 27th, 2024

1. Give an informal proof of each of the following claims.
   (a) For all real numbers x and y, if $x + 5 < y$, then $2x < 2y$.
   (b) $\left\{x \mid \frac{x+1}{2} \geq 4\right\} \subseteq \{x \mid 2x - 2 > 10\}$
   (For this problem the universe is $\mathbb{R}$, the set of all real numbers.)
   (c) For all sets A, B, C, and D, if $B \cup C \subseteq D$, then $A \cap B \subseteq (A \cup C) \cap D$.
   (d) For all sets X, Y, and Z: if $(X \cap Y) \subseteq \overline{Z}$, then $X \subseteq \overline{Y \cap Z}$.
   You must use (in the appropriate place) proof by contradiction to solve this problem¿
   (e) For all sets H, J, K, L, and M, if $J \subseteq L \setminus M$ and $H \subseteq \overline{M}$, then $H \cup (J \cap K) \subseteq (H \cup K) \setminus M$.
   You must use (in the appropriate place) proof by cases for this problem.

2. Prove that the following claim is *false*.
   Claim: For all sets A, B, and C: if $A \subseteq C$, then $A \cup B \subseteq B \cap C$.
   Hint: I used the word "prove" here to mean that you have to give all the details. That does not mean that you should be writing a *direct* proof where you start with making some generic assumptions and then use them to prove a conclusion. Remember that you already know how to prove a universal claim is false.

3. Provide proofs for the following claims:
   (a) For any integer n, define $S_n = \{nx \mid x \in \mathbb{Z}\}$ (same as in this week's lectures). For any integer n, if a and b are members of S_n, then $5a - b$ is also a member of S_n.
   (b) The set of rational numbers is closed under addition.
   Hint: This is just another way of saying that the sum of any two rational numbers must also be rational.
   (c) Define $T = \{x + y\sqrt{2} \mid x \in \mathbb{Q} \wedge y \in \mathbb{Q}\}$ and $S = \{st \mid s \in T \wedge t \in T\}$. Claim: $S = T$.
   Hint: You'll need two separate proofs (one to prove every member of S is a member of T, and one to prove every member of T is a member of S.
   (d) The sum of an irrational number and a rational number is irrational. (You can assume that all numbers here are real numbers.)
   (e) For any integer n, define $S_n = \{nx \mid x \in \mathbb{Z}\}$ as before. For any integers a, b, and n, if $ab \notin S_n$, then neither a nor b is a member of S_n.
   Hint: Don't try to prove the two conclusions using the same proof by contradiction subproof. You'll want two separate (but very similar) subproofs for this.