**CSCI-C 241 HW #3**
**Instructors: Wennstrom & LaRacuente** Due Tuesday, January 30th, 2023

1. Here are some attempts at proofs inspired by answers that students have given in previous semesters. For each proof, I would like you to explain which steps (if any) are logically incorrect and why. Note that I am not asking you to to explain how they "should" have written the proof.

   I've included some proofs where every step is completely correct, even though the proofs include weird strategies, false starts, or other oddities. These proofs may not be *good* proofs, and you might know of a *better* strategy, but that doesn't make the proofs *wrong.* As long as every step correctly applies a valid inference rule to the available formulas, then you should state that the proof is correct.

   I've included line numbers just to make it easier to talk about the different steps.

   To give you an idea of what I'm looking for, here's an example of a proof and the kind of answer I want from you:

   *Proof.*
   1. Assume $(P \land Q) \to R$ and $Q \land P$.
   2. Since $(P \land Q) \to R$, we can conclude $P \to R$. ($\land$-Elim.)
   3. From $Q \land P$, we know $P$. ($\land$-Elim.)
   4. Because we have $P \to R$ and $P$, we know $R$. (Appl.) □

   **Answer:** Line 2 is wrong because $\land$ isn't the main connective of $(P \land Q) \to R$ and so you can't use $\land$-Elim. on it.

   Finally, note that there are no mistakes in format, phrasing, citations, or other aspects of presentation, so don't worry about that sort of thing. Just pay attention to what formulas and/or subproofs are being used, what rule is being used, what formula is being concluded, and whether that rule can be used on those formulas/subproofs to deduce that formula.

   (a) *Proof.*
   1. Assume $A \to \neg\neg C$ and $B$.
   2.     Assume $A$.
   3.     From $A$ and $B$, we get $A \land B$. ($\land$-Intro.)
   4.     Because $A$ and $A \to \neg\neg C$, we can conclude $\neg\neg C$ (Appl.)
   5.     Since $\neg\neg C$, we know $C$. (Dbl. Neg.)
   6. Since we have $A \land B$ and $C$, we know $(A \land B) \to C$ (Dir. Pf.) □

   (b) *Proof.*
   1. Assume $(A \lor B) \to (C \land D)$.
   2.     Assume $B$.
   3.     From $B$, we get $A \lor B$. ($\lor$-Intro.)
   4.     $(A \lor B) \to (C \land D)$ and $A \lor B$, together imply $C \land D$. ($\to$-Elim.)
   5.     Because $C \land D$, $C$. ($\land$-Elim.)
   6. Assuming $B$, we proved $C$, and hence $B \to C$. ($\to$-Intro.) □

(c) *Proof.*

1. Assume $J \vee K$ and $K \to L$.
2. Assume $J \to L$.
3. First, consider the case where $J$ is true.
4. From $J \to L$ and $J$, we get $L$. (Appl)
5. Next, consider the case where $K$ is true.
6. From $K \to L$ and $K$, we get $L$. (Appl)
7. In either case of $J \vee K$, we were able to prove $L$, so $L$ is true in general. (cases)
8. Assuming $J \to L$, we proved $L$, and hence $(J \to L) \to L$. (dir. pf) $\square$

(d) *Proof.*

1. Assume $J \vee K$ and $K \to L$.
2. Case 1: $J$.
3. Assume $J \to L$.
4. From $J \to L$ and $J$, we get $L$. (Appl)
5. The assumption $J \to L$ lead to $L$, and so $(J \to L) \to L$. (dir. pf)
6. Case 2: $K$.
7. Assume $J \to L$.
8. From $K \to L$ and $K$, we get $L$. (Appl)
9. The assumption $J \to L$ lead to $L$, and so $(J \to L) \to L$. (dir. pf)
10. In either case of $J \vee K$, we were able to prove $(J \to L) \to L$, so $(J \to L) \to L$ is true in general. (cases) $\square$

(e) *Proof.*

1. Assume $(A \vee \neg B) \to C$ and $A$.
2. Case 1: Assume $A$.
3. From $A$, we can derive $A \vee \neg B$. (Weak.)
4. From $A \vee \neg B$ and $(A \vee \neg B) \to C$, we get $C$. (Appl.)
5. Case 2: Assume $\neg B$.
6. From $\neg B$, we can derive $A \vee \neg B$. (Weak.)
7. From $A \vee \neg B$ and $(A \vee \neg B) \to C$, we get $C$. (Appl.)
8. In either case $A$ or $\neg B$, we get $C$. (Cases)
9. From $C$ and $A$, we can conclude $A \wedge C$. ($\wedge$-Intro.) $\square$

(f) *Proof.*

1. Assume $(F \to G) \vee \neg\neg G$ and $F$.
2. Since $(F \to G) \vee \neg\neg G$ is true, so is $\neg\neg G$. ($\vee$-Elim.)
3. From $\neg\neg G$, we can derive $G$. (Dbl. Neg.)
4. Because we have $F$ and $G$, we can conclude $F \wedge G$. ($\wedge$-Intro.) $\square$

2. Some of the following claims are true and some are false. If the claim is true, prove it by giving a semi-formal Natural Deduction proof. If the claim is false, prove this by giving a truth assignment.

   **Hint**: Start by trying to write a proof. If you get stuck, then switch to trying to find a proof assignment to disprove the claim.

Remember that when writing semi-formal proofs for this class, you must follow the guidelines we discussed in the lectures. You can also find those guidelines in the lecture notes, so please use those as a reference. If your proofs do not follow those guidelines **they will receive 0 points**, and you will be asked to resubmit the assignment.

(a) $(P \land Q) \to R, P \land S, \neg\neg Q \vdash R$

(b) $P \to \neg Q, \neg Q \vdash \neg\neg P$

(c) $(A \land B) \to C, B, A \land \neg D \vdash C \land \neg D$

(d) $(A \land B) \to C, B \vdash (A \land \neg D) \to (C \land \neg D)$

(e) $\neg P \to (Q \land R) \vdash (\neg P \land S) \to (R \land S)$

(f) $X \land \big(X \to (Z \land Y)\big) \vdash X \land Y$

(g) $\vdash \Big(X \land \big(X \to (Z \land Y)\big)\Big) \to \Big(X \land Y\Big)$

**Hint:** A claim with no premises requires a proof with no initial assumptions. Typically, the first line of such a proof will be to start a *subproof*.

(h) $P \lor (Q \land \neg\neg R), P \to R \vdash R$

(i) $C \to (D \lor E), D \to E, \neg\neg C \vdash E \land C$

(j) $X \to Y \vdash (X \lor Z) \to Y$

(k) $J \to K \vdash (J \lor \neg\neg K) \to K$