

HW8 (CSCI-C241)

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1. Question One

- (a) R_1 is not a function, because $R(1, 2)$ and $R(1, 3)$ but $2 \neq 3$
- (b) R_2 is a total function each member of A is related only to a single member of A
- (c) R_3 is a partial function, because each member of A is related to no more than one member of A , but not every member of A is related to another member.
- (d) P_2 is partial function because if $x, y, z \in \mathbb{R}$, $x \cdot y = 120$, and $x \cdot z = 120$, then $y = z$, but $\neg P_2(0, n)$ where $n \in \mathbb{R}$
- (e) P_3 is a total function because if $x, y, z \in \mathbb{R}^*$, $x \cdot y = 120$, and $x \cdot z = 120$, then $y = z$, and for every $n \in \mathbb{R}^*$, there is some $n_2 \in \mathbb{R}^*$ where $n * n_2 = 120$

2. Question Two

- (a) f_1 is not one-to-one, because $f_1(a) = f_1(d)$ but $a \neq d$
- (b) f_1 is not onto because there is no $x \in B$ where $f_1(x) = a$
- (c) f_2 is one-to-one because there is no $x, y \in B$ where $f_2(x) = f_2(y)$ and $x \neq y$
- (d) f_2 is onto because for every member of the codomain, there is some member of the domain that maps to said member of the codomain
- (e) s_1 is not one-to-one because $s_1(10) = s_1(-10)$ but $10 \neq -10$
- (f) s_2 is one-to-one because if a $x \in [0, \infty)$ and $x^2 - 10 = n$ where $n \in [0, \infty)$, there is no other member of the domain that can be squared and have ten added to equal n
- (g) s_2 is not onto, because there is no $x \in [0, \infty)$ where $s_2(x) = 1$
- (h) c_1 is one-to-one because there is no $x, y \in \mathbb{R}$ where $x \neq y$ and $c_1(x) = c_1(y)$
- (i) c_1 is onto because for every $y \in \mathbb{R}$ there will always be some $x \in \mathbb{R}$ where $y = x^3 - 10$
- (j) c_2 is not onto because there is no $x \in \mathbb{Z}$ where $c_2(x) = 1$
- (k) d is a total function, because for any possible string (including the empty string), there exists a string that contains dashes in between each character
- (l) d is one-to-one, because for any two strings, if they have dashes inserted in between their characters, the resulting strings for each would never be the same
- (m) d is not onto, because there exists no $s \in \text{Str}$ where $d(s) = \text{wall}$
- (n) f is a partial function, because for any $s \in \text{Str}$, there is only one possible first character of that string, but there does not exist $s_2 \in \text{Str}$ such that $f(\text{""}) = s_2$
- (o) f is not one-to-one, because $f(\text{car}) = f(\text{can})$ but $\text{car} \neq \text{can}$

3. Question Three

- (a) $\{(1, a), (2, a), (3, b)\}$
- (b) This is not possible, because in order for a relation to be a function, each member of the domain can't be related to more than a single member of the codomain.
- (c) $\{(a, 2), (b, 3), (c, 1)\}$
- (d) $f(x) = 2x$
- (e) $f(x) = 2x$
- (f) $f(x) = \lfloor \frac{x}{10} \rfloor$
- (g) $f(x) = x + 5$

- (h) $f(s) = s$ is concatenated to itself
- (i) $f(s) = \begin{cases} "" & \text{if } s = "" \\ s \text{ with the last character removed} & \text{otherwise} \end{cases}$
- (j) $f(s) = |s|$
- (k) $|\mathbf{Str}| \geq |\mathbb{N}|$
- (l) $f(s) =$ the sum of the ascii values of each character of s multiplied by the total amount of all ascii values, raised to the position (starting at zero) of said character in s
Note: This is base-256 to base-10, similar to how we convert base-16 to base-10
- (m) $|\mathbf{Str}| = |\mathbb{N}|$

4. Question Four

- (a) k is not one-to-one, because $0, 6 \in \mathbb{R}$ and $k(0) = k(6)$, but $0 \neq 6$
- (b) Claim: k_2 is one-to-one

Proof.

Choose $x_1, x_2 \in (3, \infty)$ and Assume $k_2(x_1) = k_2(x_2)$ (1)

Since $k_2(x_1) = k_2(x_2)$, we know $(x_1 - 3)^2 = (x_2 - 3)^2$ (2)

Suppose towards a contradiction $x_1 \neq x_2$ (3)

Since $(x_1 - 3)^2 = (x_2 - 3)^2$, we know $\pm(x_1 - 3) = \pm(x_2 - 3)$ (4)

Case 1: $(x_1 - 3) = (x_2 - 3)$ (5)

Since $(x_1 - 3) = (x_2 - 3)$, we know $x_1 = x_2$ (6)

In the case of $(x_1 - 3) = (x_2 - 3)$, we proved $x_1 = x_2$, (7)

which contradicts our assumption

Case 2: $(x_1 - 3) = -(x_2 - 3)$ (8)

Since $(x_1 - 3) = -(x_2 - 3)$, we know $x_1 - 3 = -x_2 + 3$ (9)

Since $x_1 - 3 = -x_2 + 3$, we know $x_1 - 6 = -x_2$ (10)

Since $x_1 - 6 = -x_2$ and $x_2 > 3$, we know $x_1 < 3$ (11)

In the case of $(x_1 - 3) = -(x_2 - 3)$, we proved an impossibility of $x_1 < 3$, (12)

which contradicts our domain

Case 3: $-(x_1 - 3) = (x_2 - 3)$ (13)

Since $-(x_1 - 3) = (x_2 - 3)$, we know $-x_1 + 3 = x_2 - 3$ (14)

Since $-x_1 + 3 = x_2 - 3$, we know $-x_1 = x_2 - 6$ (15)

Since $-x_1 = x_2 - 6$ and $x_1 > 3$, we know $x_2 < 3$ (16)

In the case of $-(x_1 - 3) = (x_2 - 3)$, we proved an impossibility of $x_2 < 3$, (17)

which contradicts our domain

Case 4: $-(x_1 - 3) = -(x_2 - 3)$ (18)

Since $-(x_1 - 3) = -(x_2 - 3)$, we know $-x_1 + 3 = -x_2 + 3$ (19)

Since $-x_1 + 3 = -x_2 + 3$, we know $-x_1 = -x_2$ (20)

Since $-x_1 = -x_2$, we know $x_1 = x_2$ (21)

In the case of $-(x_1 - 3) = -(x_2 - 3)$, we proved $x_1 = x_2$, (22)

which contradicts our assumption

In any case of $(x_1 - 3)^2 = (x_2 - 3)^2$, we proved an impossibility of $x_1 \neq x_2$ (23)

Under the assumption of $x_1 \neq x_2$, we proved an impossibility, so $x_1 = x_2$ (24)

Under the assumption of $k_2(x_1) = k_2(x_2)$, we proved $x_1 = x_2$, so k_2 is one-to-one (25)

□

5. Question Five

- (a) h is not one-to-one, because $"X-Y-Z", "X--Y--Z" \in \mathbf{Str}$ and $h("X-Y-Z") = h("X--Y--Z")$, but $"X-Y-Z" \neq "X--Y--Z"$

(b) h is not onto, because $"-" \in \mathbf{Str}$ but there does not exist a $s \in \mathbf{Str}$ where $h(s) = "-"$

6. Question Six

(a) a is not one-to-one, because $"76", "67" \in \mathbf{Str}$ and $a("76") = a("67")$, but $"76" \neq "67"$

(b) Claim: a is onto

Proof.

Choose $n \in \mathbb{N}$ (1)

Let s be an empty string (2)

As long as $n > 9$, concatenate $"9"$ to the end of s and decrease n by 9 (3)

n is concatenated to the end of s (4)

For every $x \in \mathbb{N}$, there is some string containing digits that add up to x (5)

□

7. Question Seven

(a) Claim: $|A| \neq |B|$

Proof.

Choose the sets A, B (1)

Let F be a function from $A \rightarrow B$ and G be a function from $B \rightarrow A$ (2)

□