CSCI-C 241 HW #2
Instructors: Wennstrom & LaRacuente Due Tuesday, August 29th, 2023

1. For each of the following pairs of formulas, decide if they are logically equivalent, and justify your answer with a truth assignment or truth table as appropriate.
   (a) $A \equiv A \vee A$?
   (b) $A \equiv A \oplus A$?
   (c) $A \to B \equiv \neg A \to \neg B$?
   (d) $P \leftrightarrow \neg Q \equiv (P \wedge \neg Q) \vee (\neg P \wedge Q)$?
   (e) $A \vee B \equiv P \vee Q$?
   Hint: How many variables are needed for a truth assignment here?
   (f) $A \vee \neg A \equiv P \to P$?

2. The **converse** of an implication (such as $A \to B$) is the implication you get by switching the positions of the premise (A in our example) and conclusion (B). So the converse of $A \to B$ is $B \to A$.

   The **contrapositive** of an implication is the what you get by switching the premise and conclusion *and* replacing them with their negations. So the contrapositive of $A \to B$ is $\neg B \to \neg A$.

   While the *converse* of an implication is *not* equivalent to the original conditional ($A \to B \not\equiv B \to A$), it turns out that the *contrapositive* of an implication *is* equivalent to the original ($A \to B \equiv \neg B \to \neg A$).

   (a) Prove that an implication is *not* logically equivalent to its converse. In other words, find a counterexample that proves $A \to B$ is not logically equivalent to $B \to A$.
   (b) Prove that an implication *is* logically equivalent to its contrapositive. In other words, show that every truth assignment gives the same outcome for $A \to B$ as it does for $\neg B \to \neg A$.

3. Check whether each argument is valid or not, and justify your answer with a truth assignment or truth table as appropriate.

   (a) $$\begin{array}{l} \neg(A \wedge B) \\ \neg A \\ \hline \neg(B \to A) \end{array}$$

   (b) $$\begin{array}{l} Y \to X \\ X \to Y \\ \hline \neg Y \vee X \end{array}$$

   (c) $$\begin{array}{l} P \to \neg Q \\ \neg Q \\ \hline \neg\neg P \end{array}$$

   (d) $$\begin{array}{l} P \\ Q \\ \hline R \end{array}$$

4. Give an example of each of the following. If you think no such example exists, explain why not.
   (a) A satisfiable formula that is not a tautology.
   (b) A satisfiable formula that is also a tautology.
   (c) A contingency that is also a contradiction.
   (d) A formula that is logically equivalent to $A \leftrightarrow B$.
   (e) A contingency and a tautology that are logically equivalent.
   (f) A consistent set formulas that includes the formula $P \wedge \neg Q$. (There have to be at least two formulas in your set.)
   (g) An inconsistent set of formulas where every formula in the set is satisfiable.
   (h) A consistent set of formulas where one of the formulas is a contradiction.
   (i) A consistent set of formulas that contains both a contingency and a tautology.
   (j) An invalid argument with two premises where the premises and the conclusion are all contingencies. (The contingency requirement is just there to rule out some very silly answers.)
   (k) Two formulas (call them "p" and "q") such that the argument $\begin{array}{l} p \\ q \\ \hline X \wedge Y \end{array}$ is valid.

      Write your answer in the form: "$p =$ ______ and $q =$ ______".
   (l) Two formulas (call them "p" and "q") where the set $\{p, q\}$ is consistent, but the argument $\dfrac{p}{q}$ is invalid.

      Write your answer in the form: "$p =$ ______ and $q =$ ______".

5. Answer each of the following questions. If the answer is yes, you must explain why. If the answer is no, you must give a counterexample.
   (a) Is every tautology also satisfiable?
   (b) Is every satisfiable formula also a tautology?
   (c) Is every contingency required to be satisfiable?

6. Consider the following argument. Without building a truth table, decide whether or not you *think* it is valid. Write down your answer, and explain in your own words why you think the argument is valid or invalid. **Remember, do not build a truth table!** This problem is just to get you thinking. You will not be graded on whether you are correct, but you may lose points if you don't really try to come up with an answer and an explanation, or if we can't understand what you're talking about.

$$\begin{array}{l} A \wedge B \\ A \rightarrow C \\ C \rightarrow (D \wedge E) \\ \hline D \wedge B \end{array}$$

7. **Bonus:** Look at the two formulas below. Do *not* actually build their truth tables (We're not that mean). Which one would have a bigger truth table? Why?
   (a) $(A \wedge B) \vee \big(C \rightarrow (D \wedge E)\big)$
   (b) $\big((A \wedge \neg A) \leftrightarrow (B \wedge \neg B)\big) \rightarrow \big((A \rightarrow A) \vee \neg\neg (A \vee B)\big)$