

1. Which of the following are propositions?
 - (a) Take out the papers and the trash.
 - (b) You will take out the papers and the trash.
 - (c) If someone wins the lottery, then they must have purchased a ticket.
 - (d) Who wrote the Book of Love?
 - (e) Never drink water.
 - (f) Every human on the planet.
 - (g) Every human on the planet who is less than six feet tall.
 - (h) Every human on the planet is less than six feet tall.
 - (i) Every sentence is a proposition.
 - (j) The presence of water on Mars.
 - (k) Water is present on Mars.
 - (l) Is Water present on Mars?
 - (m) $6 \cdot 9 = 42$
 - (n) $6 \cdot 9 - 42$
 - (o) $6 \cdot 9 \geq 42$
 - (p) $6 \cdot 9 \neq 42$
2. Translate the following English sentences into propositional formulas. Remember to define what your propositional variables mean!

For example, if the sentence was “The subject passed test 1 and test 2.”, a good answer might look like this:

P : The subject passed test 1.
 Q : The subject passed test 2.
 $P \wedge Q$

Note that your propositional variables must be defined as complete propositions, so you should *not* just write:

P : test 1
 Q : test 2
 $P \wedge Q$

Sometimes (like in this case), it is necessary to slightly paraphrase so that your definitions are complete sentences.

Feel free to use abbreviations (e.g., “ P : The subj. passed test 1.”) or to omit unimportant words (e.g., “ Q : Subj. passed test 2.”) if you’re feeling lazy.

Words like “not”, “no”, or “doesn’t” should not appear in the definitions of your propositional variables; you should capture the negation in your translation using \neg .

- (a) I had good intentions and the highest hopes.
- (b) The card must either have the same color as the previous card or the same number as the previous card.
- (c) I will eat my tie if the Cubs win the World Series.
- (d) It smelled funny, but he ate it anyway.
- (e) All prizes will be awarded provided that enough eligible entries are received.
- (f) Participants were not timed on this task; however, most finished in less than 8 minutes.
- (g) The image is recovered if and only if the receiver has an authenticated secret key.
- (h) The printer can hold a black ink cartridge or a color cartridge, but not both.
- (i) Changing batteries is a tedious task if the nodes are deployed in remote and/or hostile environments.

Hint: You'll need three separate atomic propositions to fully translate this sentence.

- (j) The people will give up their arms only when the tyrant resigns and we get our money back.

Hint: You'll need three separate atomic propositions to fully translate this sentence.

- (k) The existence of a Lyapunov function is a necessary and sufficient condition for the stability of the system.

Hint: For this proposition (and the following three propositions), you'll need to do some paraphrasing to ensure that your atomic propositions are complete sentences.

- (l) Passing the Turing Test is a sufficient condition for a being's intelligence.
- (m) The absence of antibodies in the subject's body implies that they are not susceptible to infection.
- (n) Disciplinary knowledge and organizational skills are necessary conditions for effective teaching. **Hint:** You should have three separate atomic propositions here.

- (o) **Bonus:** Your meal comes with one of the following: rice, mashed potatoes, or fried plantains.

Hint: This one is trickier than you might think. I recommend testing your answer out by building a truth table.

- (p) **Bonus:** Ready or not, here I come.

Use the following definitions for your atomic variables:

R: I am ready.

C: Here I come.

- (q) **Bonus:** One false move and the whole thing explodes.

Use the following definitions for your atomic variables:

M: Someone makes a false move.

X: The whole thing explodes.

3. Let I , M , and E have the following meanings:

I : The command returns an integer.

M : The command displays a message.

E : The command throws an exception.

Express each of the following propositions as a natural-sounding English sentence.

(a) $I \vee M$

(b) $M \leftrightarrow I$

(c) $\neg E \rightarrow I$

(d) $\neg E \wedge \neg M$

(e) $E \vee (I \wedge M)$

4. When you're trying to explain why a suggested translation is not accurate, the best way to do this is to point out a situation where the translated formula is true and the original sentence is false, or a situation where the translated formula is false and the original sentence is true.

For example, consider the following attempted translation of the sentence "The movie was long, but I enjoyed it."

- L : "The movie was long."
- E : "I enjoyed the movie."

Translation: $L \vee E$

Here's a good explanation of why this is not an accurate translation:

If the movie was actually a short movie and you enjoyed the movie, then the original sentence would be false, but the translation would be true.

- (a) Consider the sentence: "I want ice cream because it is hot," and the following proposed translation:

- I : "I want ice cream."
- H : "It is hot."

Translation: $H \rightarrow I$

Explain in your own words why this is not an accurate translation of this sentence.

- (b) Using the same original sentence and variable definitions, consider the translation: $I \wedge H$.

Explain in your own words why this is not an acceptable translation.

- (c) In your own words, do you think there can be *any* acceptable translation of this sentence into propositional logic? Why or why not? You won't be graded on whether your answer is right or wrong, but on how you justify your answer.

5. Answer the following questions. Justify your answers with a truth assignment (or two) or an entire table as appropriate.

Note 1: In this class, when your justification requires a single truth assignment, **you must explicitly give the truth assignment**. Don't just circle a row in the table. We want to make sure you know how to pull the appropriate assignment from the table.

Note 2: If an answer can be justified with a single truth assignment, you do not need to provide an entire truth table. You can provide a table if that's part of the work you used to find the assignment, but once you find the assignment, you do not need to finish the table.

- (a) Is $A \rightarrow \neg B$ satisfiable?
- (b) Is $(A \wedge \neg A) \vee (B \wedge \neg B)$ satisfiable?
- (c) Is $X \rightarrow \neg X$ a contradiction?
- (d) Is $\neg A \rightarrow \neg(A \vee B)$ a tautology?
- (e) Is $\neg A \rightarrow \neg(A \vee B)$ a contingency?
- (f) Is $((A \rightarrow B) \wedge (C \vee \neg B)) \rightarrow (A \rightarrow C)$ a tautology?
- (g) Is $((A \rightarrow B) \wedge (C \vee \neg B)) \rightarrow (A \rightarrow C)$ satisfiable?
- (h) Is $((A \rightarrow B) \wedge (C \vee \neg B)) \rightarrow (A \rightarrow C)$ a contingency?
- (i) Is $(A \rightarrow B) \rightarrow (\neg A \rightarrow \neg B)$ a contradiction?
- (j) Is $(A \rightarrow B) \rightarrow (\neg A \rightarrow \neg B)$ a tautology?
- (k) Is $\neg A \vee \left((D \vee \neg D) \rightarrow ((B \wedge \neg B) \leftrightarrow (C \rightarrow C)) \right)$ satisfiable?

Hint: No, we don't expect you to create an entire 32-row table.

6. Check whether each set of formulas is consistent or not, and justify your answer with a truth assignment or truth table as appropriate.

- (a) $\{A \wedge \neg B, A \vee B\}$
- (b) $\{P \rightarrow Q, P, \neg Q\}$
- (c) $\{H \rightarrow E, \neg G \leftrightarrow H, G\}$