HW8 (CSCI-C241)

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1. Question One
   (a) R_1 is not a function, because $R(1, 2)$ and $R(1, 3)$ but $2 \neq 3$
   (b) R_2 is a total function each member of A is realated only to a single member of A
   (c) R_3 is a partial function, because each member of A is related to no more than one member of A, but not every member of A is related to another member.
   (d) P_2 is partial function because if $x, y, z \in \mathbb{R}$, $x \cdot y = 120$, and $x \cdot z = 120$, then $y = z$, but $\neg P_2(0, n)$ where $n \in \mathbb{R}$
   (e) P_3 is a total function because if $x, y, z \in \mathbb{R}^*$, $x \cdot y = 120$, and $x \cdot z = 120$, then $y = z$, and for every $n \in \mathbb{R}^*$, there is some $n_2 \in \mathbb{R}^*$ where $n * n_2 = 120$
2. Question Two
   (a) f_1 is not one-to-one, because $f_1(a) = f_1(d)$ but $a \neq d$
   (b) f_1 is not onto because there is no $x \in B$ where $f_1(x) = a$
   (c) f_2 is one-to-one because there is no $x, y \in B$ where $f_2(x) = f_2(y)$ and $x \neq y$
   (d) f_2 is onto because for every member of the codomain, there is some member of the domain that maps to said member of the codomain
   (e) s_1 is not one-to-one because $s_1(10) = s_1(-10)$ but $10 \neq -10$
   (f) s_2 is one-to-one because if a $x \in [0, \infty)$ and $x^2 - 10 = n$ where $n \in [0, \infty)$, there is no other member of the domain that can be squared and have ten added to equal n
   (g) s_2 is not onto, because there is no $x \in [0, \infty)$ where $s_2(x) = 1$
   (h) c_1 is one-to-one because there is no $x, y \in \mathbb{R}$ where $x \neq y$ and $c_1(x) = c_1(y)$
   (i) c_1 is onto because for every $y \in \mathbb{R}$ there will always be some $x \in \mathbb{R}$ where $y = x^3 - 10$
   (j) c_2 is not onto necause there is no $x \in \mathbb{Z}$ where $c_2(x) = 1$
   (k) d is a total function, because for any possible string (including the empty string), there exists a string that contains dashes in between each character
   (l) d is one-to-one, because for any two strings, if they have dashes inserted in between their characters, the resulting strings for each would never be the same
   (m) d is not onto, because there exists no $s \in \text{Str}$ where $d(s) = \text{wall}$
   (n) f is a partial function, because for any $s \in \text{Str}$, there is only one possible first character of that string, but there does not exist $s_2 \in \text{Str}$ such that $f(\text{""}) = s_2$
   (o) f is not one-to-one, because $f(\text{car}) = f(\text{can})$ but $\text{car} \neq \text{can}$
3. Question Three
   (a) $\{(1, a), (2, a), (3, b)\}$
   (b) This is not possible, because in order for a relation to be a function, each member of the domain can't be related to more than a single member of the codomain.
   (c) This is not possible, because a function from A to B that is not one-to-one, would only be a partial function (an example of a partial function from A to B that is one-to-one: $\{(a, 2), (b, 3), (c, 1)\}$)
   (d) $f(x) = 2x$
   (e) $f(x) = 2x$
   (f) $f(x) = \lfloor \frac{x}{10} \rfloor$

(g) $f(x) = x + 5$

(h) $f(s) = s$ is concatenated to itself

(i) $f(s) = \begin{cases} \texttt{""} & \text{if } s = \texttt{""} \\ s \text{ with the last character removed} & \text{otherwise} \end{cases}$

(j) $f(s) = |s|$

(k) $|\texttt{Str}| \geq |\mathbb{N}|$

(l) $f(s) =$ the sum of the ascii values of each character of s multiplied by the total amount of all ascii values, raised to the position (starting at zero) of said character in s
Note: This is base-256 to base-10, similar to how we convert base-16 to base-10

(m) $|\texttt{Str}| = |\mathbb{N}|$

4. Question Four

(a) k is not one-to-one, because $0, 6 \in \mathbb{R}$ and $k(0) = k(6)$, but $0 \neq 6$

(b) Claim: k_2 is one-to-one

Proof.

$$
\begin{aligned}
&\text{Choose } x_1, x_2 \in (3, \infty) \text{ and Assume } k_2(x_1) = k_2(x_2) && (1)\\
&\quad \text{Since } k_2(x_1) = k_2(x_2), \text{ we know } (x_1 - 3)^2 = (x_2 - 3)^2 && (2)\\
&\quad \text{Suppose towards a contradiction } x_1 \neq x_2 && (3)\\
&\qquad \text{Since } (x_1 - 3)^2 = (x_2 - 3)^2, \text{ we know } \pm(x_1 - 3) = \pm(x_2 - 3) && (4)\\
&\qquad \text{Case 1: } (x_1 - 3) = (x_2 - 3) && (5)\\
&\qquad\quad \text{Since } (x_1 - 3) = (x_2 - 3), \text{ we know } x_1 = x_2 && (6)\\
&\qquad \text{In the case of } (x_1 - 3) = (x_2 - 3), \text{ we proved } x_1 = x_2, && (7)\\
&\qquad \text{which contradicts our assumption}\\
&\qquad \text{Case 2: } (x_1 - 3) = -(x_2 - 3) && (8)\\
&\qquad\quad \text{Since } (x_1 - 3) = -(x_2 - 3), \text{ we know } x_1 - 3 = -x_2 + 3 && (9)\\
&\qquad\quad \text{Since } x_1 - 3 = -x_2 + 3, \text{ we know } x_1 - 6 = -x_2 && (10)\\
&\qquad\quad \text{Since } x_1 - 6 = -x_2 \text{ and } x_2 > 3, \text{ we know } x_1 < 3 && (11)\\
&\qquad \text{In the case of } (x_1 - 3) = -(x_2 - 3), \text{ we proved an impossibility of } x_1 < 3, && (12)\\
&\qquad \text{which contradicts our domain}\\
&\qquad \text{Case 3: } -(x_1 - 3) = (x_2 - 3) && (13)\\
&\qquad\quad \text{Since } -(x_1 - 3) = (x_2 - 3), \text{ we know } -x_1 + 3 = x_2 - 3 && (14)\\
&\qquad\quad \text{Since } -x_1 + 3 = x_2 - 3, \text{ we know } -x_1 = x_2 - 6 && (15)\\
&\qquad\quad \text{Since } -x_1 = x_2 - 6 \text{ and } x_1 > 3, \text{ we know } x_2 < 3 && (16)\\
&\qquad \text{In the case of } -(x_1 - 3) = (x_2 - 3), \text{ we proved an impossibility of } x_2 < 3, && (17)\\
&\qquad \text{which contradicts our domain}\\
&\qquad \text{Case 4: } -(x_1 - 3) = -(x_2 - 3) && (18)\\
&\qquad\quad \text{Since } -(x_1 - 3) = -(x_2 - 3), \text{ we know } -x_1 + 3 = -x_2 + 3 && (19)\\
&\qquad\quad \text{Since } -x_1 + 3 = -x_2 + 3, \text{ we know } -x_1 = -x_2 && (20)\\
&\qquad\quad \text{Since } -x_1 = -x_2, \text{ we know } x_1 = x_2 && (21)\\
&\qquad \text{In the case of } -(x_1 - 3) = -(x_2 - 3), \text{ we proved } x_1 = x_2, && (22)\\
&\qquad \text{which contradicts our assumption}\\
&\qquad \text{In any case of } (x_1 - 3)^2 = (x_2 - 3)^2, \text{ we proved an impossibility of } x_1 \neq x_2 && (23)\\
&\quad \text{Under the assumption of } x_1 \neq x_2, \text{ we proved an impossibility, so } x_1 = x_2 && (24)\\
&\text{Under the assumption of } k_2(x_1) = k_2(x_2), \text{ we proved } x_1 = x_2, \text{ so } k_2 \text{ is one-to-one} && (25)
\end{aligned}
$$

□

5. Question Five

(a) h is not one-to-one, because `"X-Y-Z"`, `"X--Y--Z"` $\in$ `Str` and $h($`"X-Y-Z"`$) = h($`"X--Y--Z"`$)$, but `"X-Y-Z"` $\neq$ `"X--Y--Z"`

(b) h is not onto, because `"-"` $\in$ `Str` but there does not exist a $s \in$ `Str` where $h(s) =$ `"-"`

6. Question Six

   (a) a is not one-to-one, because `"76"`, `"67"` $\in$ `Str` and $a($`"76"`$) = a($`"67"`$)$, but `"76"` $\neq$ `"67"`

   (b) Claim: a is onto

   Proof.

   Choose $n \in \mathbb{N}$ (1)

   Let s be an empty string (2)

   As long as $n > 9$, concatenate `"9"` to the end of s and decrease n by 9 (3)

   n is concatenated to the end of s (4)

   For every $x \in \mathbb{N}$, there is some string containing digits that add up to x (5)

   □

7. Question Seven

   (a) Claim: $|A| \neq |B|$

   Proof.

   Choose the sets A, B (1)

   Let F be a function from $A \to B$ and G be a function from $B \to A$ (2)

   □