HW7 (Corrections) (CSCI-C241)

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22 March 2024

- Question 2
    - An empty relation
    - $B \times C$ (The Cartesian Product)

- Question 5
    - Part (d)
    Claim: M is anti-symmetric

    Proof.

Choose $x, y \in \mathbb{N}$ and Assume $M(x,y)$ and $M(y,x)$	(1)
Since $M(x,y)$ and $M(y,x)$, we know x is a multiple of y and y is a multiple of x	(2)
Since x is a multiple of y, there exists some $k \in \mathbb{N}$ such that $x = ky$	(3)
Since y is a multiple of x, there exists some $j \in \mathbb{N}$ such that $y = jx$	(4)
Since $x = ky$ and $y = jx$, we know $y = jky$	(5)
Since $y = jky$, we know $jk = 1$ or $y = 0$	(6)
Case 1: $jk = 1$	(7)
Since $jk = 1$ and $j, k \in \mathbb{N}$, we know $j = 1$ and $k = 1$	(8)
Since $j = 1$ and $k = 1$, we know $x = 1y$	(9)
Since $x = 1y$, we know $x = y$	(10)
Case 2: $y = 0$	(11)
Since $y = 0$ and $x = ky$, we know $x = 0k$	(12)
Since $x = 0k$, we know $x = 0$	(13)
Since $y = 0$ and $x = 0$, we know $x = y$	(14)
In either case, we proved $x = y$, so we know $x = y$ in general	(15)
Under the assumption of $M(x,y)$ and $M(y,x)$, we proved $x = y$, so M is anti-symmetric	(16)

    $\square$

- Question 7
    - Part (c)
    Claim: I has property F

    Proof.

Choose $x, y, z \in \mathbb{R}$ and Assume $I(x,y)$ and $I(x,z)$	(1)
Since $I(x,y)$ and $I(x,z)$, we know $x \cdot y = 1$ and $x \cdot z = 1$	(2)
Since $x \cdot y = 1$ and $x \cdot z = 1$, we know $x \cdot y = x \cdot z$	(3)
Since $x \cdot y = x \cdot z$, we know $y = z$	(4)
Under the assumption of $I(x,y)$ and $I(x,z)$, we proved $y = z$, therefore I has property F	(5)

    $\square$