HW3 (CSCI-C241)

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1. Question One
   (a) This proof is not valid, for the reason that in line six, they are not acknowledging what was assumed in the subproof.
   (b) This proof is valid.
   (c) This proof is not valid, because after they do cases they wrote their direct proof incorrectly.
   (d) This proof is valid.
   (e) This proof is valid
   (f) This proof is not valid, for that reason that when using $\vee - Elimination$, you must have a subproof for each case.

2. Question Two
   (a) Claim: $(P \wedge Q) \to R, P \wedge S, \neg\neg Q \vdash R$
   Goal: R
   Want: $P \wedge Q$ (Application)

   Pf: Assume $(P \wedge Q) \to R, P \wedge S, \neg\neg Q.$

   1. Since we know $\neg\neg Q$, we can conclude Q. (Double Negative)
   2. Since we know $P \wedge S$, we can conclude P. $(\wedge - Elimination)$
   3. From P and Q, we can conclude $P \wedge Q$. $(\wedge - Introduction)$
   4. From $(P \wedge Q) \to R, P$ and Q, we can conclude R. $(\to -Elimination)$

   $\square$

   (b) Claim: $P \to \neg Q, \neg Q \vdash \neg\neg P$
   Goal: $\neg\neg P$

P	Q	$P \to \neg Q$	$\neg Q$	$\neg\neg P$
T	T	F	F	n/a
T	F	T	T	T
F	T	T	F	n/a
F	F	T	T	F

   This is not a valid argument, as shown in the following truth assignment:
   $P = false$
   $Q = false$
   $P \to \neg Q = true$
   $\neg Q = true$
   $\neg\neg P = false$

   (c) Claim: $(A \wedge B) \to C, B, A \wedge \neg D \vdash C \wedge \neg D$
   Goal: $C \wedge \neg D$

Pf: Assume $(A \wedge B) \to C, B, A \wedge \neg D.$

1. From $A \wedge \neg D,$ we can conclude A and $\neg D.$ $(\wedge - Elimination)$
2. From A and $B,$ we can conclude $A \wedge B.$ $(\wedge - Introduction)$
3. From $A \wedge B,$ we can apply $(A \wedge B) \to C$ and conclude $C.$ $(Application)$
4. From A and $\neg D,$ we can conclude $A \wedge \neg D.$ $(\wedge - Introduction)$

$\square$

(d) Claim: $(A \wedge B) \to C, B \vdash (A \wedge \neg D) \to (C \wedge \neg D)$
Goal: $(A \wedge \neg D) \to (C \wedge \neg D)$

Pf: Assume $(A \wedge B) \to C, B.$

1. *Subproof*
2. Assume $A \wedge \neg D.$
3. From $A \wedge \neg D,$ we can conclude A and $\neg D.$ $(\wedge - Elimination)$
3. From A and $B,$ we can conclude $A \wedge B.$ $(\wedge - Introduction)$
4. From $A \wedge B,$ we can apply $(A \wedge B) \to C$ and conclude $C.$ $(Application)$
5. From $A \wedge \neg D$ we can apply $(A \wedge \neg D) \to (C \wedge \neg D)$ and conclude $C \wedge \neg D.$ $(Application)$
6. Under the assumption of $A \wedge \neg D,$ we proved C and $\neg D.$ (Direct Proof)

$\square$

(e) Claim: $\neg P \to (Q \wedge R) \vdash (\neg P \wedge S) \to (R \wedge S)$
Goal: $(\neg P \wedge S) \to (R \wedge S)$

Pf: Assume $\neg P \to (Q \wedge R).$

1. *Subproof*
2. Assume $\neg P \wedge S.$
3. From $\neg P \wedge S,$ we can conclude $\neg P$ and $S.$ $(\wedge - Elimination)$
4. From $\neg P,$ we can apply $\neg P \to (Q \wedge R)$ and conclude $Q \wedge R.$ $(Application)$
5. From $Q \wedge R,$ we can conclude $R.$ $(\wedge - Elimination)$
6. From $\neg P$ and $S,$ we can conclude $\neg P \wedge S.$ $(\wedge - Introduction)$
7. From $Q \wedge R$ we can conclude Q and $R.$ $(\wedge - Elimination)$
8. From R and $S,$ we can conclude $R \wedge S.$ $(\wedge - Introduction)$
9. Under the assumption $\neg P$ and $S,$ we proved $R \wedge S,$ therefore $(\neg P \wedge S) \to (R \wedge S).$ (Direct Proof)

$\square$

(f) Claim: $X \wedge (X \to (Z \wedge Y)) \vdash X \wedge Y$
Goal: $X \wedge Y$

Pf: Assume $X \wedge (X \to (Z \wedge Y)).$

1. From $X \wedge (X \to (Z \wedge Y))$ we can conclude X and $X \to (Z \wedge Y)$
2. From X we can apply $X \to (Z \wedge Y)$ and conclude $Z \wedge Y.$ $(Application)$
3. From $Z \wedge Y$ we can conclude $Y.$ $(\wedge - Elimination)$
4. From X and Y we can conclude $X \wedge Y.$ $(\wedge - Introduction)$

$\square$

(g) Claim: $\vdash (X \wedge (X \to (Z \wedge Y))) \to (X \wedge Y)$
Goal: $(X \wedge (X \to (Z \wedge Y))) \to (X \wedge Y)$

Proof.

1. *Subproof*
2. Assume $X \wedge (X \rightarrow (Z \wedge Y))$
3. From $X \wedge (X \rightarrow (Z \wedge Y))$ we can conclude X and $X \rightarrow (Z \wedge Y)$
4. From X we can apply $X \rightarrow (Z \wedge Y)$ and conclude $Z \wedge Y$. $(Application)$
5. From $Z \wedge Y$ we can conclude Y. $(\wedge - Elimination)$
6. From X and Y we can conclude $X \wedge Y$. $(\wedge - Introduction)$
7. Under the assumption $X \wedge (X \rightarrow (Z \wedge Y))$, we proved $(X \wedge Y)$, therefore $(X \wedge (X \rightarrow (Z \wedge Y))) \rightarrow (X \wedge Y)$ (Direct Proof)

□

(h) Claim: $P \vee (Q \wedge \neg\neg R), P \rightarrow R \vdash R$
Goal: R

Pf: Assume $P \vee (Q \wedge \neg\neg R), P \rightarrow R.$

1. Case 1: P
2. From P, we can apply $P \rightarrow R$, and conclude R. $(Application)$
3. Case 2: $Q \wedge \neg\neg R$
4. From $Q \wedge \neg\neg R$, we can conclude $\neg\neg R$. $(\wedge - Elimination)$
5. From $\neg\neg R$, we can conclude R. (Double Negation)
6. In either case of $P \vee (Q \wedge \neg\neg R)$, we proved R, so R is true in general. (Proof By Cases)

□

(i) Claim: $C \rightarrow (D \vee E), D \rightarrow E, \neg\neg C \vdash E \wedge C$
Goal: $E \wedge C$

Pf: Assume $C \rightarrow (D \vee E), D \rightarrow E, \neg\neg C.$

1. From $\neg\neg C$, we can conclude C. (Double Negation)
2. From C, we can apply $C \rightarrow (D \vee E)$, and conclude $D \vee E$. $(Application)$
3. Case 1: D
4. From D, we can apply $D \rightarrow E$, and conclude E. $(Application)$
5. Case 2: E
6. We know E.
7. In either case of $D \vee E$, we proved E, so E is true in general. (Proof By Cases)
8. From E and C, we can conclude $E \wedge C$. $(\wedge - Introduction)$

□

(j) Claim: $X \rightarrow Y \vdash (X \vee Z) \rightarrow Y$
Goal: $(X \vee Z) \rightarrow Y$

X	Y	Z	$X \rightarrow Y$	$(X \vee Z) \rightarrow Y$
T	T	T	T	true
T	T	F	T	true
T	F	T	F	n/a
T	F	F	F	n/a
F	T	T	T	true
F	T	F	T	true
F	F	T	T	false
F	F	F	T	true

This is not a valid argument, as shown in the following truth assignment:
$X = false$
$Y = false$

$Z = true$
$X \to Y = true$
$(X \vee Z) \to Y = false$

(k) Claim: $J \to K \vdash (J \vee \neg\neg K) \to K$
Goal: $(J \vee \neg\neg K) \to K$

Pf: Assume $J \to K$.

1.	*Subproof*	
2.	Assume $J \vee \neg\neg K$.	
3.	Case 1: J	
4.	From J, we can apply $J \to K$, and conclude K.	(*Application*)
5.	Case 2: $\neg\neg K$	
6.	From $\neg\neg K$, we can conclude K.	(Double Negation)
7.	In either case of $J \vee \neg\neg K$, we proved K, so K is true in general.	(Proof By Cases)
8.	Under the assumption $J \vee \neg\neg K$, we proved K, so therefore $(J \vee \neg\neg K) \to K$.	(Direct Proof)

$\square$