HW2 (CSCI-C241)

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1. Question One

   (a) The pair ($A \equiv A \vee A$) is logically equivalent as shown in the truth table below:

A	$A \vee A$
true	true
false	false

   (b) The pair ($A \equiv A \oplus A$) is not logically equivalent as shown in the truth assignment below:

   A = true

   $A \equiv A \oplus A$

   $true \equiv true \oplus true$

   $true \equiv false$

   (c) The pair ($A \to B \equiv \neg A \to \neg B$) is not logically equivalent as shown in the truth assignment below:

   A = true

   B = false

   $A \to B \equiv \neg A \to \neg B$

   $true \to false \equiv false \to true$

   $false \equiv true$

   (d) The pair ($P \leftrightarrow \neg Q \equiv (P \wedge \neg Q) \vee (\neg P \wedge Q)$) is logically equivalent as shown in the truth table below:

P	Q	$P \leftrightarrow \neg Q$	$(P \wedge \neg Q) \vee (\neg P \wedge Q)$
true	true	false	false
true	false	true	true
false	true	true	true
false	false	false	false

   (e) The pair ($A \vee B \equiv P \vee Q$) is not logically equivalent as shown in the truth assignment below:

   A = true

   B = true

P = false
Q = false
$A \lor B \equiv P \lor Q$
$true \lor true \equiv false \lor false$
$true \equiv false$

(f) The pair ($A \lor \neg A \equiv P \to P$) is logically equivalent as shown in the truth table below:

A	P	$A \lor \neg A$	$P \to P$
true	true	true	true
true	false	true	true
false	true	true	true
false	false	true	true

2. Question Two

(a) The argument is not logically equivalent as shown in the truth assignment below:
A = true
B = false
$A \to B = true \to false =$ false
$B \to A = false \to true =$ true
$A \to B \equiv B \to A = false \equiv true =$ false

(b) The argument is logically equivalent as shows in the truth table below:

A	B	$A \to B$	$\neg B \to \neg A$
true	true	true	true
true	false	false	false
false	true	true	true
false	false	true	true

3. Question Three

(a) This argument is not valid because in the truth assignment below, the premise is true but the conclusion is not:
A = false
B = false
Premise $= \neg(A \land B) \land \neg A = \neg(false \land false) \land \neg false = true \land true$ = true
Conclusion $= \neg(B \to A) = \neg(false \to false) = \neg true =$ false

(b) This argument is valid because in the truth table below, each premise that is true, has a corresponding conclusion that is also true:

X	Y	$Y \to X$	$X \to Y$	$\neg Y \land X$
true	true	true	true	true
true	false	true	false	true
false	true	false	true	false
false	false	true	true	true

(c) This argument is not valid because in the truth assignment below, the premise is true but the conclusion is not:

P = false

Q = false

Premise = $(P \to \neg Q) \land (\neg Q) = (false \to true) \land (\neg false) = true \land true$ = true

Conclusion = $\neg\neg P = \neg\neg false$ = false

(d) This argument is not valid because in the truth assignment below, the premise is true but the conclusion is not:

P = true

Q = true

R = false

Premise = $P \land Q$ = true

Conclusion = R = false

4. Question Four

   (a) $A \oplus B$

   (b) $A \lor \neg A$

   (c) This is not possible, because for a formula to be a contingency it must be both satisfiable and not satisfiable, where a contradiction must be only not satisfiable.

   (d) $(A \land B) \lor (\neg A \land \neg B)$

   (e) This is not possible, because for a formula to be a tautology it must be satisfiable for every assignment, where a contingency must be both satisfiable and not satisfiable.

   (f) $\{\ P \land \neg Q,\ P,\ \neg Q,\ \neg(\neg P \lor Q)\ \}$

   (g) $\{\ P \lor \neg Q,\ \neg P \land Q\ \}$

   (h) This is not possible, because for a formula to be consistent there must be one assignment where all of the formula's in the set are satisfiable at once, but for a contradiction there must be no satisfiable assignments.

   (i) $\{\ (A \lor B) \lor (\neg A \lor \neg B),\ A \oplus B\ \}$

   (j) $$\begin{array}{c} P \\ Q \\ \hline \neg P \land \neg Q \end{array}$$

   (k) $p = X \lor Y$ and $q = \neg(X \oplus Y)$

   (l) $p = P \lor Q$ and $q = \neg P \oplus \neg Q$

5. Question Five

(a) Yes, this is because for a formula to be a tautology means every assignment must be satisfiable, therefore meaning that the formula is satisfiable.

(b) This is not true, because in order for a formula to be a tautology, each and every assignment must be satisfiable, where for a formula to be satisfiable it only requires one assignment.

(c) Yes, this is the case because in order for a formula to be a contingency, it must be not only satisfiable but also not satisfiable for a different assignment.

6. Question Six - all letters true due to that being the only way one can be true

   (a) The formula is valid, because in order for the premise to evaluate to true, both A and B must be true, in turn meaning C also must be true. When looking at the last statement in the premise, if C is true so must D & E. With that being said, if all the letters are true so would the conclusion.

7. Question Seven

   (a) $(A \wedge B) \vee (C \rightarrow (D \wedge E))$ would have a larger truth table due to the truth table for this formula having a greater number of different atomic propositions. Furthermore, the equation to calculate the amount of rows in a truth table, is 2^n where n is the number of different atomic propositions.