HW7 (CSCI-C241)

Lillie Donato

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1. $A \times A = \{(1,1),(1,2),(1,3),(2,1),(2,2),(2,3),(3,1),(3,2),(3,3)\}$
2. Question Two
   (a) True
   (b) False
   (c) True
   (d) True
   (e) False
   (f) False
   (g) False
   (h) True
   (i) True
   (j) False
   (k) $|B \times C| = 21$
   (l) $|R| = 9$
   (m) $\{1,2,3,4,5,6,7\}$
   (n) {"a","b","c"}
   (o) 0
   (p) 21
3. Question Three
   (a) i. Not Reflexive, $\neg R(b,b)$
       ii. Not Anti-Reflexive, $R(d,d)$
       iii. Not Symmetric, $R(a,b)$ but $\neg R(b,a)$
       iv. Anti-Symmetric, the only related pairs where the order does not "matter", are those that pairs of equal items.
       v. Transitive, if one node is related to another, and that node is related to a third, the first node will be related to the third.
   (b) i. Not Reflexive, $\neg S(4,4)$
       ii. Not Anti-Reflexive, $S(1,1)$
       iii. Not Symmetric, $S(2,4)$ but $\neg S(4,2)$
       iv. Not Anti-Symmetric, $S(1,2)$ and $S(2,1)$ but $1 \neq 2$
       v. Not Transitive, $S(1,2)$ and $S(2,4)$ but $\neg S(1,4)$
   (c) i. Not Reflexive, $\neg T(a,a)$
       ii. Not Anti-Reflexive, $T(d,d)$
       iii. Not Symmetric, $T(a,b)$ but $\neg T(b,a)$
       iv. Not Anti-Symmetric, $T(c,d)$ and $T(d,c)$ but $c \neq d$
       v. Not Transitive, $T(b,c)$ and $T(c,d)$ but $\neg T(b,d)$
   (d) i. Not Reflexive, $\neg Q(1,1)$
       ii. Anti-Reflexive, the sum of any integer and itself, is always even.
       iii. Symmetric, order does not affect the sum of two integers.

iv. Not Anti-Symmetric, $Q(1,2)$ and $Q(2,1)$ but $1 \neq 2$

v. Not Transitive, $Q(1,2)$ and $Q(2,3)$ but $\neg Q(1,3)$

(e) i. Not Reflexive, $\neg C(\text{"str"}, \text{"str"})$

ii. Anti-Reflexive, a proper substring of a string can never be related on itself.

iii. Not Symmetric, $C(\text{"car"}, \text{"racecar"})$ but $\neg C(\text{"racecar"}, \text{"car"})$

iv. Anti-Symmetric, this is technically true, as if one string is related to the other, the other cannot be related to the first.

v. Transitive, if a string is a proper substring of another string and that other string is a proper substring of a third string, the first string will be a proper substring of the third string.

(f) i. Not Reflexive, $\neg A(P \wedge \neg P, P \wedge \neg P)$

ii. Anti-Reflexive, because a formula will never be logically equivalent to its negation.

iii. Symmetric, because order does not affect the logical equivalence of two formulas.

iv. Not Anti-Symmetric, $A(P \leftrightarrow Q, P \oplus Q)$ and $A(P \oplus Q, P \leftrightarrow Q)$ but $P \leftrightarrow Q \neq P \oplus Q$

v. Not Transitive, $A(P \leftrightarrow Q, P \oplus Q)$ and $A(P \oplus Q, P \leftrightarrow Q)$ but $\neg A(P \leftrightarrow Q, P \leftrightarrow Q)$

4. Question Four

(a) i. Not Reflexive, $\neg X(\{\}, \{\})$

ii. Not Anti-Reflexive, $X(\{1\}, \{1\})$

(b) i. Symmetric, the order of the sets does not affect the condition.

ii. Not Anti-Symmetric, $X(\{1,2,3\}, \{2,3,4\})$ and $X(\{2,3,4\}, \{1,2,3\})$, but $\{1,2,3\} \neq \{2,3,4\}$

(c) i. Not Transitive, $X(\{1,2\}, \{2,3\})$ and $X(\{2,3\}, \{3,4\})$ but $\neg X(\{1,2\}, \{3,4\})$

5. Question Five

(a) $\mathbb{N}$

(b) $\mathbb{N}$

(c) M is reflexive because a natural number n is a multiple of itself.

(d) Claim: M is anti-symmetric

Proof.

Choose $x, y \in \mathbb{N}$ and Assume $M(x,y)$ and $M(y,x)$ (1)
Since $M(x,y)$ and $M(y,x)$, we know x is a multiple of y and y is a multiple of x (2)
Since x is a multiple of y, there exists some $k \in \mathbb{N}$ such that $x = ky$ (3)
Since y is a multiple of x, there exists some $j \in \mathbb{N}$ such that $y = jx$ (4)
Since $x = ky$ and $y = jx$, we know $y = jky$ (5)
Since $y = jky$, we know $1 = jk$ (6)
Since $jk = 1$, we know $j = 1$ and $k = 1$ (7)
Since $j = 1$ and $k = 1$, we know $x = 1y$ (8)
Since $x = 1y$, weknow $x = y$, therefore M is anti-symmetric (9)

□

(e) Claim: M is transitive

Proof.

Choose $x, y, z \in \mathbb{N}$ and Assume $M(x,y)$ and $M(y,z)$ (1)
Since $M(x,y)$ and $M(y,z)$, we know x is a multiple of y and y is a multiple of z (2)
Since x is a multiple of y, there exists some $k \in \mathbb{N}$ such that $x = ky$ (3)
Since y is a multiple of z, there exists some $j \in \mathbb{N}$ such that $y = jz$ (4)
Since $x = ky$ and $y = jz$, we know $x = kjz$ (5)
Let $n = kj$, such that $n \in \mathbb{N}$ (6)
Since $x = kjz$, we know $x = nz$ (7)
Since $x = nz$, we know x is a multiple of z, so $M(x,z)$, therefore M is transitive (8)

□

6. Question Six
   (a) True
   (b) False
   (c) True
   (d) True
   (e) True
   (f) True
   (g) Claim: E_5 is reflexive

   Proof.

   $$\text{Choose } x \in \mathbb{Z} \quad (1)$$
   $$\text{Since } x - x = 0 \text{ and } 0 \text{ is a multiple of any number, } 0 \text{ is a multiple of } 5 \quad (2)$$
   $$\text{Since } 0 \text{ is a multiple of } 5\text{, we know } E_5(x, x)\text{, therefore } E_5 \text{ is reflexive} \quad (3)$$

   □

   (h) Claim: E_5 is symmetric

   Proof.

   $$\text{Choose } x, y \in \mathbb{Z} \text{ and Assume } E_5(x, y) \quad (1)$$
   $$\text{Since } E_5(x, y)\text{, we know } x - y \text{ is a multiple of } 5 \quad (2)$$
   $$\text{Let } z = y - x\text{, such that } z \in \mathbb{Z} \quad (3)$$
   $$\text{Since } y - x = z\text{, we know } x - y = -z \quad (4)$$
   $$\text{Since } -z \text{ is a multiple of } 5\text{, we know } z \text{ must be a multiple of } 5 \quad (5)$$

   □

   (i) Claim: E_5 is transitive

   Proof.

   $$\text{Choose } x, y, z \in \mathbb{Z} \text{ and Assume } E_5(x, y) \text{ and } E_5(y, z) \quad (1)$$
   $$\text{Since } E_5(x, y) \text{ and } E_5(y, z)\text{, we know } x - y \text{ is a multiple of } 5 \text{ and } y - z \text{ is a multiple of } 5 \quad (2)$$
   $$\text{Let } a = x - y \text{ and } b = y - z\text{, such that } a, b \in \mathbb{Z} \quad (3)$$
   $$\text{Since } a \text{ is a multiple of } 5 \text{ and } b \text{ is a multiple of } 5\text{, we know } a + b \text{ must be a multiple of } 5 \quad (4)$$
   $$\text{Let } n = a + b\text{, such that } n \in \mathbb{Z} \text{ and } n \text{ is divisible by } 5 \text{ because } a + b \text{ is a multiple of } 5 \quad (5)$$
   $$\text{Since } n = a + b\text{, we know } n = (x - y) + (y - z) \quad (6)$$
   $$\text{Since } n = (x - y) + (y - z)\text{, we know } n = x - y + y - z \quad (7)$$
   $$\text{Since } n = x - y + y - z\text{, we know } n = x - z \quad (8)$$
   $$\text{Since } n = x - z\text{, we know } x - z \text{ is divisible by } 5 \quad (9)$$
   $$\text{Since } x - z \text{ is divisible by } 5\text{, we know } E_5(x, z)\text{, therefore } E_5 \text{ is transitive} \quad (10)$$

   □

7. Question Seven
   (a) Q does not have property F, because $Q(a, 1)$ and $Q(a, 2)$ but $1 \neq 2$
   (b) L has property F, because a string s will always have the same length.
   (c) Claim: I has property F

   Proof.

   $$\text{Choose } x, y, z \in \mathbb{R} \text{ and Assume } I(x, y) \text{ and } I(x, z) \quad (1)$$
   $$\text{Since } I(x, y) \text{ and } I(x, z)\text{, we know } x \cdot y = x \cdot z \quad (2)$$
   $$\text{Since } x \cdot y = x \cdot z\text{, we know } y = z\text{, therefore } I \text{ has property } F \quad (3)$$

   □

8. Question Eight

   (a) $\{(a,a),(a,b),(b,a),(b,c),(c,d),(d,a)\}$ on A

   (b)

$$\begin{tikzcd}
& a \arrow[dl] \arrow[dr] & \\
b \arrow[rr, leftrightarrow] \arrow[dr] & & c \arrow[dl] \\
& d &
\end{tikzcd}$$

   (c) $\{(s,t) \mid s \text{ shares a character with } t\}$

   (d) $\{(s,t) \mid |s| > |t|\}$

   (e) $\{(x,y) \mid x + y = 10\}$

   (f) $\{(p,q) \mid p \equiv q\}$

   (g) $\{(p,q) \mid p \text{ has more implication statements than } q\}$

   (h) $\{(p,q) \mid \text{the name of } p \text{ is a substring of the name of } q \text{ but nobody else has the exact same name as } p \text{ or } q\}$