

HW7 (Corrections) (CSCI-C241)

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- Question 2

- An empty relation
- $B \times C$ (The Cartesian Product)

- Question 5

- Part (d)
Claim: M is anti-symmetric

Proof.

- Choose $x, y \in \mathbb{N}$ and Assume $M(x, y)$ and $M(y, x)$ (1)
- Since $M(x, y)$ and $M(y, x)$, we know x is a multiple of y and y is a multiple of x (2)
- Since x is a multiple of y , there exists some $k \in \mathbb{N}$ such that $x = ky$ (3)
- Since y is a multiple of x , there exists some $j \in \mathbb{N}$ such that $y = jx$ (4)
- Since $x = ky$ and $y = jx$, we know $y = jky$ (5)
- Since $y = jky$, we know $jk = 1$ or $y = 0$ (6)
- Case 1: $jk = 1$ (7)
- Since $jk = 1$ and $j, k \in \mathbb{N}$, we know $j = 1$ and $k = 1$ (8)
- Since $j = 1$ and $k = 1$, we know $x = 1y$ (9)
- Since $x = 1y$, we know $x = y$ (10)
- Case 2: $y = 0$ (11)
- Since $y = 0$ and $x = ky$, we know $x = 0k$ (12)
- Since $x = 0k$, we know $x = 0$ (13)
- Since $y = 0$ and $x = 0$, we know $x = y$ (14)
- In either case, we proved $x = y$, so we know $x = y$ in general (15)
- Under the assumption of $M(x, y)$ and $M(y, x)$, we proved $x = y$, so M is anti-symmetric (16)

□

- Question 7

- Part (c)
Claim: I has property F

Proof.

- Choose $x, y, z \in \mathbb{R}$ and Assume $I(x, y)$ and $I(x, z)$ (1)
- Since $I(x, y)$ and $I(x, z)$, we know $x \cdot y = 1$ and $x \cdot z = 1$ (2)
- Since $x \cdot y = 1$ and $x \cdot z = 1$, we know $x \cdot y = x \cdot z$ (3)
- Since $x \cdot y = x \cdot z$, we know $y = z$ (4)
- Under the assumption of $I(x, y)$ and $I(x, z)$, we proved $y = z$, therefore I has property F (5)

□