# HW4 (CSCI-C241)

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1. Question One
   (a) This proof is valid.
   (b) This proof is not valid, as in their proof by contradiction, they are not proving the entire conclusion and only one part of it.
   (c) This proof is valid.
   (d) This proof is valid.
2. Question Two
   (a) Claim: $P \wedge Q \vdash \neg(P \rightarrow \neg Q)$
   Goal: $\neg(P \rightarrow \neg Q)$

   *Pf: Assume $P \wedge Q$.*

   1. From $P \wedge Q,$ we can conclude $P$ and $Q$ $(\wedge - Elimination)$
   2. $Subproof$
   3. Suppose towards a contradiction that $P \rightarrow \neg Q$
   4. From $P,$ we can apply $P \rightarrow \neg Q$ and conclude $\neg Q$ (Application)
   5. Under the assumption of $P \rightarrow \neg Q,$ we proved an impossibility, so therefore $\neg(P \rightarrow \neg Q)$ (Proof By Contradiction)

   $\square$

   (b) Claim: $\vdash \neg((P \wedge Q) \wedge (P \rightarrow \neg Q))$
   Goal: $\neg(X \wedge Z)$

   *Proof.*

   1. $Subproof$
   2. Suppose $(P \wedge Q) \wedge (P \rightarrow \neg Q)$ towards a contradiction
   3. From $(P \wedge Q) \wedge (P \rightarrow \neg Q),$ we can conclude $P \wedge Q$ and $P \rightarrow \neg Q$ $(\wedge - Elimination)$
   4. From $P \wedge Q,$ we can conclude $P$ and $Q$ $(\wedge - Elimination)$
   5. From $P,$ we can apply $P \rightarrow \neg Q$ and conclude $\neg Q$ (Application)
   6. Under the assumption of $(P \wedge Q) \wedge (P \rightarrow \neg Q)$ we proved an impossibility of $Q$ and $\neg Q$ so therefore $\neg((P \wedge Q) \wedge (P \rightarrow \neg Q))$ (Proof By Contradiction)

   $\square$

   (c) With the truth assignment $(A = false,\ B = true)$, the proof is invalid.
   (d) Claim: $(W \wedge X) \rightarrow \neg Y, X \vdash \neg(W \wedge Y)$
   Goal: $\neg(W \wedge Y)$

*Assume* $(W \wedge X) \rightarrow \neg Y, X$.

1. *Subproof*
2. Suppose $W \wedge Y$ towards a contradiction
3. From $W \wedge Y$, we can conclude $W$ and $Y$ ($\wedge - Elimination$)
4. From $W$ and $X$, we can conclude $W \wedge X$ ($\wedge - Introduction$)
5. From $W \wedge X$, we can apply $(W \wedge X) \rightarrow \neg Y$ and conclude $\neg Y$ (Application)
6. Under the assumption of $W \wedge Y$ we proved an impossibility of $Y$ and $\neg Y$ so therefore $\neg(W \wedge Y)$ (Proof By Contradiction)

$\square$

(e) Claim: $(W \wedge X) \rightarrow \neg Y \vdash X \rightarrow \neg(W \wedge Y)$
Goal: $X \rightarrow \neg(W \wedge Y)$

*Assume* $(W \wedge X) \rightarrow \neg Y$.

1. Assume $X$
2. Suppose $W \wedge Y$ towards a contradiction
3. From $W \wedge Y$, we can conclude $W$ and $Y$ ($\wedge - Elimination$)
4. From $W$ and $X$, we can conclude $W \wedge X$ ($\wedge - Introduction$)
5. From $W \wedge X$, we can apply $(W \wedge X) \rightarrow \neg Y$ and conclude $\neg Y$ (Application)
6. Under the assumption of $W \wedge Y$ we proved an impossibility of $Y$ and $\neg Y$ so therefore $\neg(W \wedge Y)$ (Proof By Contradiction)
7. Under the assumption of $X$, we proved $\neg(W \wedge Y)$, so therefore $X \rightarrow \neg(W \wedge Y)$ (Direct Proof)

$\square$

(f) Claim: $U \rightarrow V, \neg V \vdash \neg(U \wedge W)$
Goal: $\neg(U \wedge W)$

*Assume* $U \rightarrow V, \neg V$.

1. *Subproof*
2. Suppose $U \wedge W$ towards a contradiction
3. From $U \wedge W$, we can conclude $U$ and $W$ ($\wedge - Elimination$)
4. From $U$, we can apply $U \rightarrow V$ and conclude $V$ (Application)
5. Under the assumption of $U \wedge W$ we proved an impossibility of $V$ and $\neg V$ so therefore $\neg(U \wedge W)$ (Proof By Contradiction)

$\square$

(g) With the truth assignment ($U = false$, $V = false, W = true$), the proof is invalid.

(h) Claim: $U \rightarrow V, W \rightarrow V, \neg V \vdash \neg U \wedge \neg W$
Goal: $\neg U \wedge \neg W$

*Assume* $U \to V, W \to V, \neg V$.

1. *Subproof*
2. Suppose $U$ towards a contradiction
3. From $U,$ we can apply $U \to V$ and conclude $V$ (Application)
4. Under the assumption of $U$ we proved an impossibility of $V$ and $\neg V$ so therefore $\neg U$ (Proof By Contradiction)
5. *Subproof*
6. Suppose $W$ towards a contradiction
7. From $W,$ we can apply $W \to V$ and conclude $V$ (Application)
8. Under the assumption of $U$ we proved an impossibility of $V$ and $\neg V$ so therefore $\neg W$ (Proof By Contradiction)
9. From $\neg U$ and $\neg W,$ we can conclude $\neg U \wedge \neg W$ ($\wedge - Introduction$)

□

3. Question Three
   (a) This would be true because the main connective in this statement holds the same meaning as the word "and".
   (b) This would not be the case, as there are no assignments that satisfy both $A$ and $\neg A$, but there are assignments that satisfy $B \leftrightarrow (X \oplus \neg Q)$.
   (c) This argument would be valid for the reason that there are no assignments where the premises are satisfied and the conclusion is not.
   (d) Yes, it would be possible but only if the premises were contradictions as well.
   (e) Yes, this would be possible but only if that other formula was also a contradiction.
4. Question Four
   Claim: $\vdash P \vee \neg P$
   Goal: $P \vee \neg P$

*Proof.*

1. Suppose $\neg(P \vee \neg P)$ towards a contradiction
2. Suppose $P$ towards a contradiction
3. From $P,$ we can conclude $P \vee \neg P$ (Weakening)
4. Under the assumption of $P$ we proved an impossibility of $(P \vee \neg P) \wedge \neg(P \vee \neg P)$ so therefore $\neg P$ (Proof By Contradiction)
5. From $P,$ we can conclude $P \vee \neg P$ (Weakening)
6. Under the assumption of $\neg(P \vee \neg P)$ we proved and impossibility of $(P \vee \neg P) \wedge \neg(P \vee \neg P),$ so therefore $P \vee \neg P$ (Proof By Contradiction)

□

5. Question Five
   (a) Claim: $(A \to B) \to C \equiv (\neg A \to C) \wedge (B \to C)$

*Pf: Assume* $(A \to B) \to C \equiv (\neg A \to C) \wedge (B \to C)$.

| | | | |
|---|---|---|---|
| 1. | $(A \to B) \to C$ | $\equiv \neg(\neg A \vee B) \vee C$ | (Material Implication) |
| 2. | | $\equiv (\neg\neg A \wedge \neg B) \vee C$ | (De Morgan's) |
| 3. | | $\equiv (A \wedge \neg B) \vee C$ | (Double Negation) |
| 4. | $(\neg A \to C) \wedge (B \to C)$ | $\equiv (\neg\neg A \vee C) \wedge (\neg B \vee C)$ | (Material Implication) |
| 5. | | $\equiv (A \vee C) \wedge (\neg B \vee C)$ | (Double Negation) |
| 6. | | $\equiv (A \wedge \neg B) \vee C$ | (Distributive) |

□

(b) With the truth assignment ($A = true$, $B = false$, $C = false$), the two formulas are not equivlaent.

(c) With the truth assignment ($W = true$, $X = false$, $Y = true$, $Z = false$), the two formulas are not equivlaent.

(d) Claim: $\neg((W \wedge \neg X) \to (\neg Y \vee Z)) \equiv (Y \wedge \neg Z) \wedge (W \wedge \neg X)$

*Pf: Assume* $\neg((W \wedge \neg X) \to (\neg Y \vee Z)) \equiv (Y \wedge \neg Z) \wedge (W \wedge \neg X)$.

| | | | |
|---|---|---|---|
| 1. | $\neg((W \wedge \neg X) \to (\neg Y \vee Z))$ | $\equiv \neg(\neg(W \wedge \neg X) \vee (\neg Y \vee Z))$ | (Material Implication) |
| 2. | | $\equiv \neg((\neg W \vee \neg\neg X) \vee (\neg Y \vee Z))$ | (De Morgan's) |
| 3. | | $\equiv (\neg(\neg W \vee \neg\neg X) \wedge \neg(\neg Y \vee Z))$ | (De Morgan's) |
| 4. | | $\equiv (\neg(\neg W \vee X) \wedge \neg(\neg Y \vee Z))$ | (Double Negation) |
| 5. | | $\equiv (\neg\neg W \wedge \neg X) \wedge (\neg\neg Y \wedge \neg Z)$ | (De Morgan's) |
| 6. | | $\equiv (W \wedge \neg X) \wedge (Y \wedge \neg Z)$ | (Double Negation) |
| 7. | | $\equiv (Y \wedge \neg Z) \wedge (W \wedge \neg X)$ | (Commutative) |

$\square$

(e) Claim: $P \wedge (\neg Q \to R) \equiv (P \to \neg R) \to (P \wedge Q)$

*Pf: Assume* $P \wedge (\neg Q \to R) \equiv (P \to \neg R) \to (P \wedge Q)$.

| | | | |
|---|---|---|---|
| 1. | $P \wedge (\neg Q \to R)$ | $\equiv P \wedge (\neg\neg Q \vee R)$ | (Material Implication) |
| 2. | | $\equiv P \wedge (Q \vee R)$ | (Double Negation) |
| 3. | $(P \to \neg R) \to (P \wedge Q)$ | $\equiv \neg(\neg P \vee \neg R) \vee (P \wedge Q)$ | (Material Implication) |
| 4. | | $\equiv (\neg\neg P \wedge \neg\neg R) \vee (P \wedge Q)$ | (De Morgan's) |
| 5. | | $\equiv (P \wedge R) \vee (P \wedge Q)$ | (Double Negation) |
| 6. | | $\equiv P \wedge (Q \vee R)$ | (Distributive) |

$\square$

(f) Claim: $(M \to N) \wedge (\neg M \to N) \equiv N$

*Pf: Assume* $(M \to N) \wedge (\neg M \to N) \equiv N$.

| | | | |
|---|---|---|---|
| 1. | $(M \to N) \wedge (\neg M \to N)$ | $\equiv (\neg M \vee N) \wedge (\neg\neg M \vee N)$ | (Material Implication) |
| 2. | | $\equiv N \vee (\neg M \wedge \neg\neg M)$ | (Distributive) |
| 3. | | $\equiv N \vee (\neg M \wedge M)$ | (Double Negation) |
| 4. | | $\equiv N \vee \bot$ | (Contradiction) |
| 5. | | $\equiv N$ | ($\vee - Identity$) |

$\square$