CSCI-C 241 HW #5
Instructors: Wennstrom & LaRacuente Due Tuesday, February 13th, 2023

For questions 1-8, use the following definitions:

$A = \{1, 2, 3, 4, 5\}$
$B = \{0, 2, 4, 6, 8\}$
$C = \{1, 3, 5\}$
$D = \{1, 2, 3\}$
$X = \{x \mid x \in \mathbb{N} \wedge x \leq 5\}$
$Y = \{x \mid x \in \mathbb{N} \wedge x + 2 \leq 5\}$
$W = \{x + 2 \mid x \in \mathbb{N} \wedge x \leq 5\}$
$Q = \{x^3 \mid x \in \mathbb{N}\}$
$P = \{a + b \mid a \in A \wedge b \in B\}$
$H = \left\{\frac{n}{2} \mid n \in \mathbb{N}\right\}$
$S = \big\{\{1\}, \{1, 2\}, \{1, 3\}, \{1, 2, 3\}\big\}$

1. Answer each question. If you think none of the given values are members, say "none".
   (a) Which of the following are members of $A \cap B$?
   1 2 6 9
   (b) Which of the following are members of $A \cup B$?
   1 2 6 9
   (c) Which of the following are members of $A \setminus B$?
   1 2 6 9
   (d) If the universe is $\mathbb{Z}$, which of the following are members of $\overline{A}$?
   1 2 6 9
   (e) Which of the following are members of H?
   $\frac{3}{2}$ $\frac{2}{3}$ 4 2.5 $-\frac{5}{2}$ $\sqrt{2}$
   (f) Which of the following are members of X?
   1 3 4 5 6 9
   (g) Which of the following are members of W?
   1 3 4 5 6 9
   (h) Which of the following are members of Y?
   1 3 4 5 6 9

2. Give an example of each of the following. If you think no such example exists, you must explain why.
   (a) a member of $H \setminus \mathbb{Z}$.
   (b) a member of $\mathbb{Z} \setminus H$.
   (c) a member of $W \cap Y$
   (d) three different members of Q
   (e) a member of S
   (f) a member of $\varnothing$

3. Decide whether the following statements are true or false. No justification is needed.
   (a) $2 = \{2\}$
   (b) $\varnothing = \{\}$

(c) $\varnothing \in A$

(d) $\{2\} \in A$

(e) $\{1\} \in S$

(f) $3 \in S$

(g) $D \in S$

4. Decide whether the following statements are true or false. Give a brief justification (for most of these, one sentence or a single counterexample should be enough) for your answer.

   (a) $11 \in P$

   (b) $14 \in P$

   If you think two sets are *not* equal, you need to give an example of something that is a member of one set, but not the other. If you think they *are* equal, then you need to explain why.

   (c) $D = S$

   (d) $\{3, 5, 1\} = C$

   (e) $\{1, 5, 1, 3, 1, 5, 5, 1, 3\} = C$

   (f) $\varnothing = \{\varnothing\}$

5. For this problem, let the universe be $\mathcal{U} = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Write the following sets in set-list notation:

   (a) $\overline{B}$

   (b) $\overline{C \cup D} \cap \{2, 3, 4\}$

   (c) $\{2, 3, 4\} \setminus (C \cup D)$

   (d) $\overline{\varnothing}$

   (e) $\overline{\mathcal{U}}$

   (f) $(A \setminus B) \cap D$

   (g) $A \cap C$

   (h) $A \cup C$

   (i) $A \setminus C$

   (j) $C \setminus A$

   (k) $B \cap C$

   (l) $B \setminus C$

6. Give a definition of the set B using set-builder notation. There are many possible correct answers here.

7. **Bonus**: Give a second, different definition of B using set-builder notation.

8. Calculate the following. If the cardinality is infinite, just say "infinite".
   (a) $|B|$
   (b) $|S|$
   (c) $|X|$
   (d) $|\{x \mid x \in \mathbb{N} \wedge x \leq 1000\}|$
   (e) $|\varnothing|$
   (f) $|A \setminus B|$
   (g) $|Q|$
   (h) $|\mathbb{Z}|$

For problems 9-12, use the following definitions:

$A = \{\texttt{'a'}, \texttt{'b'}, \texttt{'c'}, \texttt{'d'}, \texttt{'e'}\}$

$C = \{\texttt{'b'}, \texttt{'c'}, \texttt{'d'}\}$

$V = \{\texttt{'a'}, \texttt{'e'}, \texttt{'i'}, \texttt{'o'}, \texttt{'u'}\}$

$X = \big\{\{\texttt{'a'}, \texttt{'b'}\}, \{\texttt{'a'}, \texttt{'c'}\}, \{\texttt{'b'}, \texttt{'c'}\}\big\}$

Str = the set of all strings

$S_4 = \{s \mid s \in \mathsf{Str} \wedge |s| = 4\}$

$S_{\text{even}} = \{s \mid s \in \mathsf{Str} \wedge |s| \text{ is even}\}$

$Y = \big\{\varnothing, \{\texttt{'a'}\}, \{\texttt{'a'}, \texttt{'b'}\}, \{\texttt{'a'}, \texttt{'b'}, \texttt{'c'}\}\big\}$

The vertical line notation $|\cdot|$ means different things in different contexts. If x is a *real number*, then $|x|$ means the *absolute value* of x. If A is a *set*, then $|A|$ means the *cardinality* of A. If s is a *string*, then $|s|$ means the *length* of s.

Also, remember that $\varnothing$ is the empty set, and ε is the empty string.

9. Give an example of each of the following. If you think no such example exists, you must explain why.
   (a) a proper subset of A that is not empty
   (b) a proper subset of S_4 that has at least three members
   (c) a proper subset of Y that has at least two members
   (d) a proper subset of S_{even} with infinitely many members (You must write your answer in set-builder notation.)
   (e) a superset of C
   (f) a member of $\mathcal{P}(V)$
   (g) a member of $\mathcal{P}(S_{\text{even}})$
   (h) a member of $\mathcal{P}(X)$

10. Decide whether the following statements are true or false. No justification is needed.
   (a) $C \subseteq A$
   (b) $\{\texttt{'c'}, \texttt{'b'}\} \subseteq C$
   (c) $\texttt{'d'} \subseteq A$

(d) $\{\texttt{'d'}\} \subseteq A$

(e) $\texttt{'b'} \in Y$

(f) $\{\texttt{'a'}, \texttt{'b'}\} \in X$

(g) $\{\texttt{'b'}\} \in X$

(h) $\{\texttt{'a'}, \texttt{'b'}\} \subseteq X$

(i) $\{\texttt{'a'}\} \subseteq Y$

(j) $\big\{\{\texttt{'a'}\}, \{\texttt{'a'}, \texttt{'b'}\}\big\} \in Y$

(k) $\big\{\{\texttt{'a'}\}\big\} \in Y$

(l) $\big\{\{\texttt{'a'}\}, \{\texttt{'a'}, \texttt{'b'}\}\big\} \subseteq Y$

(m) $\big\{\{\texttt{'a'}\}\big\} \subseteq Y$

(n) $\big\{\{\texttt{'a'}\}\big\} \subseteq X$

(o) $\varnothing \in X$

(p) $\varnothing \in Y$

(q) $\varnothing \subseteq X$

(r) $\varnothing \subseteq Y$

11. Decide whether the following statements are true or false. Give a brief justification (for most of these, one sentence should be enough) for your answer.

    Remember that if you think one set is *not* a subset of another, you need to give an example of something that is a member of the supposed subset, but not a member of the supposed superset. If you think one set *is* a subset of another, you don't need to write a proof, just give an explanation.

    (a) $V \subseteq A$

    (b) $S_4 \subseteq S_{\text{even}}$

    (c) $S_{\text{even}} \subseteq S_4$

    (d) $\texttt{'b'} \in \mathcal{P}(A)$

    (e) $\{\texttt{'a'}, \texttt{'b'}\} \in \mathcal{P}(A)$

    (f) $\{\texttt{'a'}\} \in \mathcal{P}(A)$

    (g) $X \subseteq Y$

12. Calculate the following. If the cardinality is infinite, just say "infinite".

    (a) $|\mathcal{P}(C)|$

    (b) $|\mathcal{P}(V)|$