Module 7 Notes (MATH-211)

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General Notes (and Definitions)

- Working with Integrals
  A function $f(x)$ is **even** if $f(-x) = f(x)$.
  A function $f(x)$ is **odd** if $f(-x) = -f(x)$.
  Let $a \in \mathbb{R}$ such that $a > 0$ and let f be an integrable function on the interval $[-a, a]$.

$$\text{If } f \text{ is even, } \int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$$

$$\text{If } f \text{ is odd, } \int_{-a}^{a} f(x)\,dx = 0$$

  The average value of an integrable function f on the interval $[a, b]$ is

$$\overline{f} = \frac{1}{b-a}\int_{a}^{b} f(x)\,dx$$

  Let f be continuous on the interval $[a, b]$. There exists a point c in (a, b) such that (Mean Value Theorem)

$$f(c) = \overline{f} = \frac{1}{b-a}\int_{a}^{b} f(t)\,dx$$

Examples

1. Use symmetry to evaluate integrals

$$\int_{-200}^{200} 2x^5\,dx = 0$$

$$\begin{aligned}
\int_{-2}^{2} \left(x^2 + x^3\right)\,dx &= \int_{-2}^{2} x^2\,dx + \int_{-2}^{2} x^3\,dx && (1)\\
&= 2\int_{0}^{2} x^2\,dx + 0 && (2)\\
&= 2\frac{x^3}{3} && (3)\\
&= \frac{16}{3} && (4)
\end{aligned}$$

2. A derivative calculation

$$s(t) = -16t^2 + 64t$$

$$t = 4$$

$$[0, 4]$$

$$
\begin{aligned}
v(t) &= s'(t) && (1)\\
\overline{v} &= \frac{1}{4}\int_0^4 v(t)\,dx && (2)\\
&= \frac{1}{4}\int_0^4 s'(t)\,dx && (3)\\
&= \frac{1}{4}s(t) && (4)\\
&= \frac{1}{4}\left(s(4)-s(0)\right) && (5)\\
&= 0 && (6)
\end{aligned}
$$

3. Applying MVT for integrals

$$f(x) = e^x$$

$$[0,2]$$

$$
\begin{aligned}
\overline{f} &= \frac{1}{2}\left(\int_0^2 e^x\,dx\right) && (1)\\
&= \frac{e^x}{2} && (2)\\
&= \frac{e^2}{2}-\frac{e^0}{2} && (3)\\
&= \frac{e^2-1}{2} && (4)\\
e^x &= \frac{e^2-1}{2} && (5)\\
\ln e^x &= \ln\frac{e^2-1}{2} && (6)
\end{aligned}
$$

Related Exercises

1. Example

A copy of my notes (in LaTeX) are available on my GitHub