Module 5 Notes (MATH-211)

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General Notes (and Definitions)

- Maxima and Minima
  Absolute Maximum: Assume a function f is defined on a set D, and $x = c$ is a point in D. Then, $y = f(c)$ is an **absolute maximum value** of f on D if $f(c) \geq f(x)$ for every x in D. Changing the set on which f is defined may change the absolute maximum value.
  Absolute Minimum: Assume a function f is defined on a set D, and $x = c$ is a point in D. Then, $y = f(c)$ is an **absolute minimum value** of f on D if $f(c) \leq f(x)$ for every x in D. Changing the set on which f is defined may change the absolute minimum value.
  Extreme Value Theorem: A function that is continuous on a closed interval is guarenteed to have both an absolute maximum value and an absolute minmum value.
  A discontinuous function, or a function defined on an interval that is not closed, may still have absolute extrema.
  Local Maximum and Minimum Values: Assume $x = c$ is an interior point (not an endpoint) of some interval I in the domain of f. Then, $y = f(c)$ is a **local maximum value** of f if $f(c) \geq f(x)$ for every x in I, and $y = f(c)$ is a **local minimum value** of f if $f(c) \leq f(x)$ for every x in I.
  Critical Points: An interior point $x = c$ of the domain of f is called a **critical point** of f if either $f'(c) = 0$ or $f'(c)$ does not exist.
  Local Extreme Value Theorem: If a function f has a local maximum or a local minimum at a point $x = c$, then either $f'(c) = 0$ or $f'(c)$ does not exist.
  If f has a local extreme, it must occur at a critical point.
  Not every critical point is the location of a local extreme value.
  For a continuous function f on a closed interval $[a, b]$, absolute extremes are guaranteed to exist, and they must occur either at the endpoints of interval or at critical points of f within the interval.

Examples

1. Locate absolute maxima and minima from a graph
   Absolute Maximum: $f(c)$ and occurs at $x = c$
   Absolute Minimum: None, as the $f(b)$ does not exist

2. Locate local maxima and minima from a graph
   Absolute Min at $(a, f(a))$
   Absolute Max at $(p, f(p))$
   Local Max at $(p, f(p))$
   Local Max at $(r, f(r))$
   Local Min at $(q, f(q))$
   Local Min at $(s, f(s))$

3. Find critical points of a function

$$f(t) = t^2 - 2\ln\left(t^2 + 1\right)$$

$$f'(t) = \frac{2t\,(t+1)\,(t-1)}{t^2 + 1}$$

   Critical Point at $x = -1$
   Critical Point at $x = 0$
   Critical Point at $x = 1$

4. Find absolute extremes of a continuous function on a closed interval

$$f(x) = \frac{x}{(x^2+9)^5}$$

$$f'(x) = \frac{-9x^2+9}{(x^2+9)^6}$$

$$[-2, 2]$$

$$f(-2) \approx -0.000005$$

$$f(2) \approx 0.000005$$

$$f(-1) = -0.00001$$

$$f(1) = 0.00001$$

   Absolute Min at $(-1, f(-1))$
   Absolute Max at $(1, f(1))$

5. Application of finding absolute extreme values

$$P(x) = 2x + \frac{128}{x}$$

$$P'(x) = 2 + \frac{-128}{x^2}$$

$$(0, \infty)$$

$$f(8) = 18$$

   Absolute min at $(8, 32)$ or a perimeter of 32 units

Related Exercises

1. (Section 4.1, Exercise 11)
   Absolute Min at $x = c_2$
   Absolute Max at $x = b$

2. (Section 4.1, Exercise 14)
   Absolute Min at $x = c$
   Absolute Max at $x = b$

3. (Section 4.1, Exercise 15)
   Absolute Max at $x = b$
   Absolute Min at $x = a$
   Local Max at $x = p$
   Local Max at $x = r$
   Local Min at $x = q$
   Local Min at $x = s$

4. (Section 4.1, Exercise 18) Absolute Max at $x = p$
   Absolute Min at $x = u$
   Local Max at $x = p$
   Local Max at $x = r$
   Local Max at $x = t$
   Local Min at $x = q$
   Local Min at $x = s$
   Local Min at $x = u$

5. (Section 4.1, Exercise 35)

$$f(x) = \frac{1}{x} + \ln x$$

$$f'(x) = \frac{x-1}{x^2}$$

   Critical Points at $x = 1$

6. (Section 4.1, Exercise 36)

$$f(t) = t^2 - 2\ln\left(t^2+1\right)$$

$$f'(t) = \frac{2t\,(t+1)\,(t-1)}{t^2+1}$$

Critical Points at $t=-1$, $t=0$ and $t=1$

7. (Section 4.1, Exercise 46)

$$f(x) = x^4 - 4x^3 + 4x^2$$

$$f'(x) = 4x^3 - 12x^2 + 8x$$

$$[-1, 3]$$

$$f(-1) = 9$$

$$f(0) = 0$$

$$f(1) = 1$$

$$f(2) = 0$$

$$f(3) = 9$$

Absolute Max at $(-1,9)$ and $(3,9)$
Absolute Min at $(0,0)$ and $(2,0)$

8. (Section 4.1, Exercise 52)

$$f(x) = 3x^{\frac{2}{3}}$$

$$f'(x) = \frac{2}{x^{\frac{1}{3}}}$$

$$[0, 27]$$

$$f(0) = 0$$

$$f(27) = 27$$

Absolute Min at $(0,0)$
Absolute Min at $(27,27)$

9. (Section 4.1, Exercise 73)

$$s(t) = -16t^2 + 64t + 192$$

$$s'(t) = -32t + 64$$

$$0 \le t \le 6$$

$$s(0) = 192$$

$$s(2) = 256$$

$$s(6) = 0$$

The stone will reach its maximum height at 2 seconds

A copy of my notes (in LaTeX) are available on my GitHub