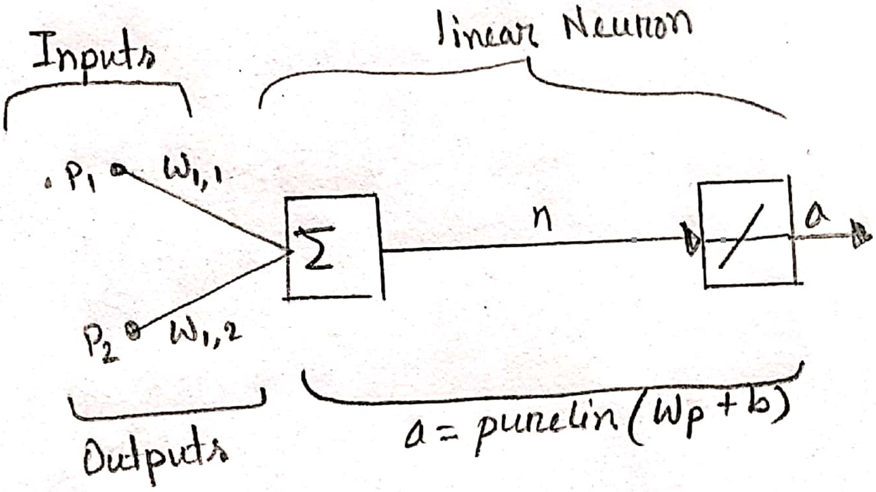
4 Simulation with Concurrent Inputs in a Static Network

In the simplest situation for simulating a network occur when the network to be simulated is static. In this case, you need not to be concerned about whether or not the input vectors occur in a particular time sequence, so ~~you~~ we can treat the inputs as concurrent.



Suppose that the network simulation data set consists of Q=4 concurrent vectors:

$$P_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad P_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \quad P_3 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \quad P_4 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

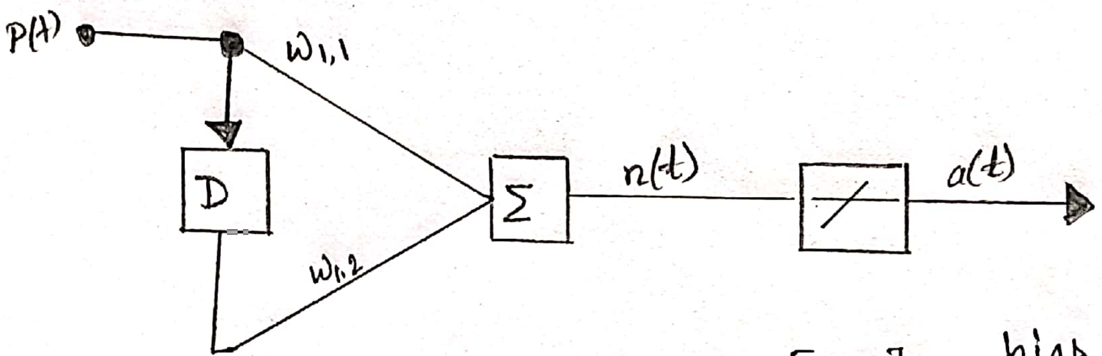
Given $W = [1 \ 2]$ and $b = [0]$

Applying $y = wx + b$ we get

P_1	P_2	P_3	P_4
$A = [1 \ 2]\begin{bmatrix} 1 \\ 2 \end{bmatrix} + 0$	$A = [1 \ 2]\begin{bmatrix} 2 \\ 1 \end{bmatrix} + 0$	$A = [1 \ 2]\begin{bmatrix} 2 \\ 3 \end{bmatrix} + 0$	$A = [1 \ 2]\begin{bmatrix} 3 \\ 1 \end{bmatrix} + 0$
$= 5$	$= 4$	$= 8$	$= 5$

⊞ Simulation with Sequential Inputs in a Dynamic Network

When a network contains delays, the input to the network would normally be a sequence of input vectors that occur in a certain time order. To illustrate this case, the next figure shows a simple network that contains one delay.



Assign the weight matrix to be $W = [1, 2]$ bias $b = 0$

Suppose that, the input sequence is:

$P_1 = [1]$, $P_2 = [2]$, $P_3 = [3]$, $P_4 = [4]$

Time span,

$T_1 = [0]$, $T_2 = [1]$ $T_3 = [2]$ $T_4 = [3]$

$\therefore \quad P_1' = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad P_2' = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \quad P_3' = \begin{bmatrix} 3 \\ 2 \end{bmatrix} \quad P_4' = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$

$a_1 = W * P_1' + b = [1, 2] \begin{bmatrix} 1 \\ 0 \end{bmatrix} = 1$

$a_2 = W * P_2' + 0 = [1, 2] \begin{bmatrix} 2 \\ 1 \end{bmatrix} = 4$

$a_3 = W * P_3' + 0 = [1, 2] \begin{bmatrix} 3 \\ 2 \end{bmatrix} = 7$

$a_4 = W * P_4' + 0 = [1, 2] \begin{bmatrix} 4 \\ 3 \end{bmatrix} = 10$

⊞ Simulation with Concurrent Inputs in a Dynamic Network

Input array, $P = \{ [1 \ 4] \ [2 \ 3] \ [3 \ 2] \ [4 \ 1] \}$

Let, t = time span

$P_1(1) = [1]$ $\quad P_1(2) = [2]$ $\quad P_1(3) = [3]$ $\quad P_1(4) = 4$

$P_2(1) = [4]$ $\quad P_2(2) = [3]$ $\quad P_2(3) = [2]$ $\quad P_2(4) = 1$

$t_1(1) = [0]$ $\quad t_1(2) = [1]$ $\quad t_1(3) = [2]$ $\quad t_1(4) = 3$

$t_2(1) = [0]$ $\quad t_2(2) = [4]$ $\quad t_2(3) = [3]$ $\quad t_2(4) = 2$

Weight $W = [1 \ 2]$

$\cancel{\therefore P_n'(m) = [P_n}$

$$\therefore P_n'(m) = \begin{bmatrix} P_n(m) \\ t_n(m) \end{bmatrix}$$

$$P_1'(1) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad \therefore A_1'(1) = [1 \ 2] * \begin{bmatrix} 1 \\ 0 \end{bmatrix} = 1$$

$$P_2'(1) = \begin{bmatrix} 4 \\ 0 \end{bmatrix} \qquad \therefore A_2'(1) = [1 \ 2] * \begin{bmatrix} 4 \\ 0 \end{bmatrix} = 4$$

$$\therefore A(1) = [1 \ 4]$$

$$P_1'(2) = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \qquad \therefore A_1'(2) = [1 \ 2] * \begin{bmatrix} 2 \\ 1 \end{bmatrix} = 4$$

$$P_2'(2) = \begin{bmatrix} 3 \\ 4 \end{bmatrix} \qquad \therefore A_2(2) = [1 \ 2] * \begin{bmatrix} 3 \\ 4 \end{bmatrix} = 11$$

$$\therefore A(2) = [4 \ 11]$$

$P_1'(3) = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ $\therefore A_1'(3) = \begin{bmatrix} 1 & 2 \end{bmatrix} * \begin{bmatrix} 3 \\ 2 \end{bmatrix} = 7$

$P_2'(3) = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ $\therefore A_2'(3) = \begin{bmatrix} 1 & 2 \end{bmatrix} * \begin{bmatrix} 2 \\ 3 \end{bmatrix} = 8$

$\therefore A(3) = \begin{bmatrix} 7 & 8 \end{bmatrix}$

$P_1'(4) = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$ $\therefore A_1'(4) = \begin{bmatrix} 1 & 2 \end{bmatrix} * \begin{bmatrix} 4 \\ 3 \end{bmatrix} = 10$

$P_2'(4) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ $\therefore A_2'(4) = \begin{bmatrix} 1 & 2 \end{bmatrix} * \begin{bmatrix} 1 \\ 2 \end{bmatrix} = 5$

$\therefore A(4) = \begin{bmatrix} 10 & 5 \end{bmatrix}$

The resulting network output would be

$A = \{ \begin{bmatrix} 1 & 4 \end{bmatrix} \begin{bmatrix} 4 & 1 \end{bmatrix} \begin{bmatrix} 7 & 8 \end{bmatrix} \begin{bmatrix} 10 & 5 \end{bmatrix} \}$

Incremental Training of Static Network

Suppose we want to train the network to create the linear function

$t = 2P_1 + P_2$

$P_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $P_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, $P_3 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$, $P_4 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

The targets would be

$t_1 = [4]$, $t_2 = [5]$ $t_3 = [7]$, $t_4 = [7]$

$\therefore$ learning rate $= 0.1$

weight $w_0 = [0 \quad 0]$

bias $b_0 = 0$

$a_1 = [0 \quad 0] \begin{bmatrix} 1 \\ 2 \end{bmatrix} + 0 = 0$

$e_1 = 4 - 0 = 4$

$\Delta w = 0.1 * 4 * \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0.4 \\ 0.8 \end{bmatrix}$

$w_1 = \Delta w^T + w_0 = [0.4 \quad 0.8] + [0 \quad 0] = [0.4 \quad 0.8]$

$b_1 = 0.1 * e_1 + b_0 = 0.1 * 4 + 0$

$= 0.4$

$a_2 = [0.4 \quad 0.8] * \begin{bmatrix} 2 \\ 1 \end{bmatrix} + 0.4 = 2$

$e_2 = 5 - 2 = 3$

$\Delta w = 0.1 * 3 * \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.6 \\ 0.3 \end{bmatrix}^T = [0.6 \quad 0.3]$

$w_2 = [0.4 \quad 0.8] + [0.6 \quad 0.3] = [1 \quad 1.1]$

$b_2 = 0.1 * 3 + 0.4 = 0.7$

$$a_3 = \begin{bmatrix} 1 & 1.1 \end{bmatrix} * \begin{bmatrix} 2 \\ 3 \end{bmatrix} + 0.7$$

$$= 6$$

$$e_3 = 7 - 6 = 1$$

$$\Delta w = 0.1 \times 1 \times \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 0.2 \\ 0.3 \end{bmatrix}^T = \begin{bmatrix} 0.2 & 0.3 \end{bmatrix}$$

$$w_3 = \begin{bmatrix} 1 & 1.1 \end{bmatrix} + \begin{bmatrix} 0.2 & 0.3 \end{bmatrix} = \begin{bmatrix} 1.2 & 1.4 \end{bmatrix}$$

$$b_3 = 0.1 * 1 + 0.7 = 0.8$$

$$a_4 = \begin{bmatrix} 1.2 & 1.4 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} + 0.8$$

$$= 5 + 0.8$$

$$= 5.8$$

$$e_4 = 1.2$$

$$a = [0] \quad [2] \quad [6] \quad [5.8]$$

$$e = [4] \quad [3] \quad [1] \quad [1.2]$$

Incremental Training with Dynamic Networks

To train the network incrementally, present the inputs and targets as elements of cell arrays, Here are the initial input Pi and the inputs P and targets T as elements of cell arrays.

$P_i = \{1\}$
$P = \{2 \; 3 \; 4\}$
$T = \{3 \; 5 \; 7\}$

Initial weight $w_0 = [0 \; 0]$
learning rate $= 0.1$

the input matrix will be

$$P_n = \begin{bmatrix} P_n \\ P_{n-1} \end{bmatrix}$$

$$P_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \qquad P_2 = \begin{bmatrix} 3 \\ 2 \end{bmatrix} \qquad P_3 = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

$$\therefore a_1 = [0 \; 0] \begin{bmatrix} 2 \\ 1 \end{bmatrix} = 0$$

$$e_1 = (3-0) = 3$$

$$\Delta w = 0.1 * 3 * \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.6 \\ 0.3 \end{bmatrix}^T = [0.6 \;\; 0.3]$$

$$w_1 = [0.6 \;\; 0.3] + [0 \;\; 0] = [0.6 \;\; 0.3]$$

$$a_2 = [0.6 \;\; 0.3] * \begin{bmatrix} 3 \\ 2 \end{bmatrix} = 1.8 + 0.6 = 2.4$$

$$e_2 = 5 - 2.4 = 2.6$$

$$\Delta w = 0.1 * 2.6 * \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 0.78 \\ 0.52 \end{bmatrix}^T = [0.78 \;\; 0.52]$$

$$w_2 = [0.78 \;\; 0.52] + [0.6 \;\; 0.3] = [1.38 \;\; 0.82]$$

$$a_3 = \begin{bmatrix} 1.38 & 0.82 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \end{bmatrix} = 7.98$$

$$e_3 = 7 - 7.98 = -0.98$$

$$a = [0] \quad [2.4] \quad [7.98]$$

$$e = [3] \quad [2.6] \quad [-0.98]$$

Batch Training with Static Network

for batch training

$$P_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad P_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \quad P_3 = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \quad P_4 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$T = [4 \quad 5 \quad 7 \quad 7]$$

$$W = [0 \quad 0]$$

$$b = 0$$

Learning rate $= 0.01$

Calculation for first epoch:

$$a_1 = [0 \quad 0] * \begin{bmatrix} 1 \\ 2 \end{bmatrix} + 0 = 0$$

$$e_1 = \cancel{0.01 *} \; 4 - 0 = 4$$

$$\Delta W_1 = 0.01 * 4 * \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0.04 \\ 0.08 \end{bmatrix}^T = [0.04 \quad 0.08]$$

$$\Delta b_1 = 0.01 * 4 = 0.04$$

$$a_2 = [0 \quad 0] * \begin{bmatrix} 2 \\ 1 \end{bmatrix} + 0 = 0$$

$$e_2 = 5 - 0 = 5$$

$$\Delta W_1 = 0.01 * 5 * \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.10 \\ 0.05 \end{bmatrix}^T = [0.10 \quad 0.05]$$

$$\Delta b_1 = 0.01 * 5 = 0.05$$

$$a_3 = [0 \quad 0] * \begin{bmatrix} 2 \\ 3 \end{bmatrix} + 0 = 0$$

$$e_3 = 7 - 0 = 7$$

$$\Delta W_1 = 0.01 * 7 * \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 0.14 \\ 0.21 \end{bmatrix}^T = [0.14 \quad 0.21]$$

$$\Delta b_1 = 0.01 * 7 = 0.07$$

$$a_4 = [0 \quad 0] * \begin{bmatrix} 3 \\ 1 \end{bmatrix} + 0 = 0$$

$$e_4 = 7 - 0 = 7$$

$$\Delta w_4 = 0.01 * 7 * \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.21 \\ 0.07 \end{bmatrix}^T = [0.21 \quad 0.07]$$

$$\Delta b_4 = 0.01 \times 07 = 0.07$$

$$w = \Delta w_1 + \Delta w_2 + \Delta w_3 + \Delta w_4$$

$$= [0.04 \quad 0.08] + [0.10 \quad 0.05] + [0.14 \quad 0.21] + [0.21 \quad 0.07]$$

$$= [0.49 \quad 0.41]$$

$$b = \Delta b_1 + \Delta b_2 + \Delta b_3 + \Delta b_4$$

$$= 0.04 + 0.05 + 0.07 + 0.07$$

$$= 0.230$$

III Batch Training With Dynamic Network

Initial $P_i = [1]$

Input vector $P = [2 \quad 3 \quad 4]$

bias $= 0.02$

Weight $= [0 \quad 0]$

Target $T = [3 \quad 5 \quad 6]$

Calculation of first epoch:

$$P_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\therefore a_1 = [0 \quad 0] * \begin{bmatrix} 2 \\ 1 \end{bmatrix} + 0 = 0$$

$$e_1 = 3$$

$$\Delta w_1 = 0.02 * 3 * \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.12 \\ 0.06 \end{bmatrix}^T = [0.12 \quad 0.06]$$

$$P_2 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$$\therefore a_2 = [0 \quad 0] * \begin{bmatrix} 3 \\ 2 \end{bmatrix} + 0 = 0$$

$$e_2 = 5$$

$$\Delta w_2 = \left(0.02 * 5 * \begin{bmatrix} 3 \\ 2 \end{bmatrix}\right)^T \cancel{= \begin{bmatrix} .02 \\ .08 \end{bmatrix}^T = [0.12 \quad 0.08]}$$

$$= [0.3 \quad 0.2]$$

$$P_3 = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

$$\therefore a_3 = [0 \quad 0] * \begin{bmatrix} 4 \\ 3 \end{bmatrix} + 0 = 0$$

$$e_3 = 6$$

$$\Delta w_3 = 0.02 * 6 * \begin{bmatrix} 4 \\ 3 \end{bmatrix} = \begin{bmatrix} 0.48 \\ 0.36 \end{bmatrix}^T = [0.48 \quad 0.36]$$

$$\therefore W = \Delta w_1 + \Delta w_2 + \Delta w_3 = [0.90 \quad 0.62]$$