Math :

exo 1 : $f(x) = \ln(1+x^2)$: d sur $[0; +\infty[$

1) $\lim_{+\infty} f(x) = +\infty$

$$\lim_{+\infty} \frac{f(x)}{x} = \lim_{+\infty} \frac{\ln(1+x^2)}{x} = \lim_{+\infty} \frac{\ln\left(x^2\left(1+\frac{1}{x^2}\right)\right)}{x} = \frac{\ln(x^2)}{x} + \frac{\ln\left(1+\frac{1}{x^2}\right)}{x}$$

$$= \frac{2\ln(x)}{x} + \frac{\ln\left(1+\frac{1}{x^2}\right)}{x} = 0$$

$\Rightarrow$ branche parabolique de direction $O\vec{i}$ au voisinage $+\infty$

2) a) $f'(x) = \left(\ln(1+x^2)\right)' = \frac{(1+x^2)'}{1+x^2} = \frac{2x}{1+x^2}$

b) $f''(x) = \left(\frac{2x}{1+x^2}\right)' \Rightarrow \left\{ \frac{f}{g} = \frac{f' \cdot g - g' f}{g^2} \right.$

$(2x)' = 2 \quad f$

$(1+x^2)' = 2x \quad g$

$$= \frac{2(1+x^2) - 2x \times 2x}{(1+x^2)^2} = \frac{2+2x^2-4x^2}{(1+x^2)^2}$$

$$= \frac{2-2x^2}{(1+x^2)^2} = \boxed{\frac{2(1-x^2)}{(1+x^2)^2}}$$